

Progress and realization platforms of dynamic topological photonics

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TOPICAL REVIEW — States and new effects in nonequilibrium

Progress and realization platforms of dynamic topological photonics

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Dynamic topological photonics is a novel research field, combining the time-domain optics and topological physics. In this review, the recent progress and realization platforms of dynamic topological photonics have been well introduced. The definition, measurement methods and the evolution process of the dynamic topological photonics are demonstrated to better understand the physical diagram. This review is meant to bring the readers a different perspective on topological photonics, grasp the advanced progress of dynamic topology, and inspire ideas about future prospects.

Keywords: dynamic topological photonics, optical waveguide array, topological optical lattice, ultrafast spectroscopy

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1. Introduction

Time-domain dynamic topological photonics (DTP) is a novel field that studies the topological properties and phase transition behavior of light in time domain. It combines the theory and experimental methods of topological physics and photonics to explore and utilize the topological characteristics of light to develop topological photonic devices and new optical functions. Researches in this field are of great significance for understanding the propagation and control of light in complex structures, as well as the evolution process of topological edge states.

The research of time-domain DTP includes many perspectives. First, it focuses on the propagation behavior of light in topological photonic crystals, topological waveguides and topological optical cavities.[1–5] These structures have special geometric and topological properties, which can guide light to propagate in a specific path, and there is a special topological protected mode in the edge state. By studying the propagation behavior of light in these structures, the characteristics of topological edge states and the topological protection properties of light can be further revealed. Secondly, the researches on time-domain DTP involve the topological phase transition and topological evolution of optics. Topological phase transition refers to the phenomenon of changes in the topological properties of materials or structures when parameters change.[6–8] In topological photonics, the topological phase transition of light can be realized by adjusting the parameters such as the frequency, phase and coupling strength of light. This phase transition process can be observed through optical and spectral measurements, and in experiments, the topological properties of light can be controlled and adjusted by dynamically adjusting the above parameters in time domain. In addition, the time-domain DTP can also include the nonlinear control of light. The optical nonlinear effects can be used to regulate the propagation and interaction behavior of light, thereby achieving dynamic regulation of topological properties.[9–13] By introducing nonlinear materials or composite structures, nonlinear regulation of light can be achieved, which is advantageous to observe dynamic topological phenomena and nonlinear topological phase transitions of light. Moreover, the experimental measurement methods of time-domain DTP rely on advanced optical technology and devices. For example, ultrafast laser technology can provide a high-resolution measurement method for observing the dynamic behavior of light in time domain.[14–16] Devices such as optical waveguides, microcavities, and optical fibers can be used to control and regulate light propagation, which can lead to deep understanding of DTP by combining with simulation and theoretical analysis.

Time-domain DTP not only promotes the development of photonics, but also has important application potential in quantum information processing, optical sensing and photonic devices. By deeply studying the dynamic topological behavior of light, researchers can better understand and use the special properties of light in time domain, and provide novel ideas and methods for the development and application of photonics

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technology.

In this review, we focus on the recent progress and realization platforms of DTP. The remainder of the paper is organized as follows. In Section 2, the topological photonics, dynamic photonics and DTP are well introduced. Section 3 describes the measurement methods of dynamic processes in photonics, as well as in topological photonics. Meanwhile, the evolution process, synthetic dimension and artificial gauge field of DTP are illustrated. In addition to that, Section 4 gives an overall discussion on realization platforms of DTP. At last, conclusions and outlook are given in Section 5.

2. Dynamic topological photonics

Dynamic topological photonics (DTP) is a novel research field, which combines the time-domain optics and topological physics. It mainly explores the topological characteristics and phase transitions of light in the time domain, and studies how these characteristics affect applications such as optical transmission, optical control, and information processing. The DTP can provide a novel way to design and manipulate photonic devices, which brings great potential for the development of optical communication, quantum computing, photonic chips and other fields. In this section, the topological photonics, dynamic photonics and the significance of DTP will be introduced.

2.1. Topological photonics

Topology, as a branch of mathematics, is used to study the geometric kinds of different objects. If some continuous operations are applied to an object, including bending and stretching, but not splitting or connecting, all the geometric shapes of the object can be considered as topologically equivalent. It was initially introduced into the condensed-matter physics to describe the topological phases as well as the topological phase transitions. Integer quantum Hall effect (IQHE), quantum spin Hall effect (QSHE), anomalous quantum Hall effect (AQHE) and other effects[17–19] can all be well described by topological theory. Topological insulators have excellent properties of bulk insulation, surface conductivity and resistance to scattering in electronic systems.[20] In 2008, Haldane and Raghu first proposed the possibility[21] of analogy for implementing quantum Hall effects in photonic configurations. Subsequently, topological photonics has been developed rapidly, providing a new stage for the exploration of photonics.

Different types of photonic topological insulators have different implementation principles and methods. The topological photonic insulator has essential differences and many different phenomena in one-dimensional (1D), two-dimensional (2D), three-dimensional (3D) and even higher-dimensional systems. According to the photonics system, the research of topological photonics can be divided into several directions. The research teams of Segev, Szameit, Chen, Song and Rechtsman aim to study the topological phenomena in 1D and 2D optical waveguide arrays. The research teams of Lu, Soljačić, Chen, Dong, Ma, Hu, Zhang and Yong study the 2D and high-dimensional topological photonic crystals in optical and microwave bands. Zhang's group, as a representative, studies the optical topological properties of metasurface materials.

In general, there are three typical methods to realize topological phase in optical wavelength. Firstly, the internal degree of freedom of photons can be treated as the pseudo-spin, which is similar to the spins in the quantum spin Hall effect. This model does not need to break the time reversal symmetry, but each pseudo spin can be limited in the artificial magnetic field. For example, Hafezi *et al.* put forward a theoretical assumption in 2011,[22] and realized experimentally at the wavelength of 1550 nm by using silicon materials in 2013.[23] Moreover, in 2015, Hu's group regarded the angular momentum of the electric field in vertical component as pseudo-spin by shrinking or expanding the honeycomb lattice along the center, and also showed a new optical topological phase.[24] In addition, by using the linear combination of transverse electric (TE) and transverse magnetic (TM) modes, the topological pseudo-spin systems can be constructed as well.[25,26] The second method is to apply time modulation to the optical configurations by using Floquet topological insulator. This system can be equivalent to an effective time-independent Hamiltonian, breaking the time reversal symmetry.[27–29] For example, this Floquet system can be realized by using the optical waveguide arrays, as shown in Fig. 1(a).[30] The last method is to use time-varying modulation to realize topological pumping phenomenon.[31] The research shows that the topological pumping of low dimensions corresponds to the quantum Hall effect of higher dimensions, so that the topological phases of higher dimensions can be studied in low-dimensional systems.[32]

2.2. Dynamic photonics

Dynamic photonics describes the evolution of a photonic system, and the concept of the dynamic photonics is based on the non-equilibrium state in photonic systems, which is opposed to the steady state or equilibrium state in photonic systems. Therefore, the equilibrium state and the non-equilibrium state in photonics are introduced first. The concept of the equilibrium state was originated in the thermodynamics. In classical thermodynamics, the concept of the equilibrium state is defined based on the fact that the macroscopic state parameters in a thermodynamic system are constant in time. In other words, if a thermodynamics system is in a thermodynamic equilibrium state, the thermodynamic macroscopic state parameters such as pressure and temperature will not change. In fact,

for any isolated thermodynamic system and given any initial state of the system, if the initial state is not the intrinsic equilibrium state of the system, the isolated system will spontaneously evolve to the equilibrium state after a sufficiently long time T, and the time T of this evolution is the relaxation time of the system. When the system has evolved to an equilibrium state, it cannot leave the equilibrium state spontaneously. Therefore, for any given non-equilibrium initial state, during the process of relaxation, the system will inevitably undergo the non-equilibrium evolution, and a unique physical mechanism will be reflected in the time scale. Such novel physical mechanism has broad research prospects.

The non-equilibrium states in photonics can be analogous to the non-equilibrium states in thermodynamics. The general equilibrium photonics research is mainly focused on the steady state of an optical system. For example, there is a non-equilibrium optical system which has the characteristics of time evolution, and a certain physical quantity in the system is studied by researchers. The initial state of the system is a non-equilibrium state at time $t = 0$, as shown in Fig. 1(b). Before t_1, the observed physical quantity changes and the physical system is undergoing evolution. And from t_1 to t_2, the observed physical quantity does not change in this period of time and the system is in a stable state, as shown in the region of t_1–t_2 in Fig. 1(b). In the region of t_1–t_2, the system is in an equilibrium state. However, if the system is disturbed at t_2, the system will be out of balance and return to a steady state at t_3. The observed physical quantity changes and the physical system is undergoing evolution again before t_3. So, t_1 and $t_3 - t_2$ are the relaxation times in the evolution of the two-stage process of the system, respectively. This phenomenon can be further illustrated by an example, as shown in Fig. 1(c), which shows the establishment process of normal dispersive solitons,[33,34] which is a typical non-equilibrium evolution. During the evolution of solitons, the solitons go through a period of drastic changes and then tend to a steady equilibrium state, which is the evolution process from the non-equilibrium initial state to the equilibrium steady state. In fact, the researches on DP are quite broad, including the dynamic evolution process of optical system[35,36] and physical system in time domain, such as synthetic dimension[37–39] and gauge field.[40] As for the photonic devices, DP is also widely studied in waveguide systems,[41–43] resonant cavity systems,[44–46] photonic crystal systems,[47–49] exciton systems.[50–52]

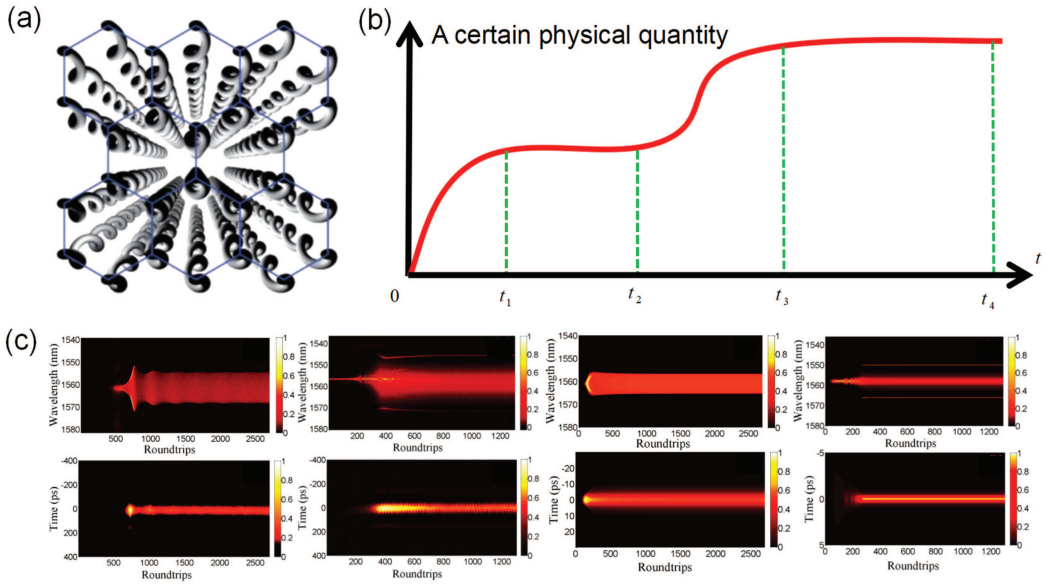


Fig. 1. (a) Floquet system by using the optical waveguide arrays.[30] (b) The non-equilibrium process and the equilibrium process in the evolution of a physical system. (c) The experimental (the first and second columns) and simulated (the third and fourth columns) results of the establishment process of normal dispersive solitons.[33,34] Reprinted with permission from Ref. [30]. Copyright 2013 Springer Nature.

2.3. Dynamic topological photonics and its significance

The non-equilibrium optical state can be related to topological photonics when the researches focus on the time-domain dynamic process. In traditional optics, the spatial distribution and propagation mode of light are usually paid attention to, while the temporal behavior of light is often overlooked. However, the emergence of time-domain DTP has changed this situation, and time has been brought into the scope of optical research. The core is the special dynamic behavior of light in the time domain, and by exploring these time-domain characteristics, researchers can find novel topological structures and topological properties of optics in the time domain, which can achieve a high degree of optimization and precise manipulation of optical transmission and control from a new perspective.

It is of great significance to study time-domain DTP. First of all, it provides novel methods to design and construct photonic devices. By using topological phase transition and

topologically protected modes, researchers can design devices with excellent performance, such as efficient optical transmission channels, optical switches and couplers. These devices have important application prospects in fields such as optical communication and photonic integrated circuits. On the other hand, time-domain DTP studies the topologically protection characteristics of light in time domain. Topological protected states can maintain the stability of transmission in the presence of disturbances, which provides a solution for high-density data transmission in fiber optic communication and photonic integrated circuits on the basis of time-domain properties, improving the reliability and stability of communication. In addition, time-domain DTP brings new possibilities to the field of optical information processing. By controlling the temporal evolution of light, operations such as delay, modulation, and encoding of optical signals can be achieved, thereby improving the speed and efficiency of information processing. In addition, time-domain DTP can also be used to realize topological quantum computing and quantum information transmission, bringing new breakthroughs in the field of quantum communication and quantum computing. Moreover, time-domain DTP provides a platform for studying topological physics. Through the temporal evolution and topological phase transition of light, researchers can simulate and observe various phenomena and effects in topological physics. This provides a controllable and observable system for in-depth understanding and research of bulk states, topological edge states and topological property.

Time-domain DTP, as a new research field, has brought new perspectives and solutions for optical device design, optical transmission, information processing and topological physics. Its development will promote the progress of optical technology, bringing more efficient, stable, and powerful functions to application fields such as optical communication, quantum computing, and photonic integrated circuits. Following sections will describe the recent progress and applications of DTP.

3. Mechanism of dynamic topological photonics

Detecting transient photonic dynamic processes involves observing and measuring transient events to study their dynamic changes. In this section, we will introduce some common methods in photonics to detect the transient dynamic process, including the combination of femtosecond laser and ultrafast spectroscopy, Zewail four-dimensional (4D) electron microscope and polarization dynamics measurement.

3.1. Measurement methods on dynamic process in photonics

Femtosecond laser technology is a technology with extremely short laser pulse time, and the pulse width is usually a few to hundreds of femtoseconds. Femtosecond laser has the characteristics of peak power, high single pulse energy, and high frequency, which can provide sufficient energy in a very short time to trigger the occurrence of transient events. Ultrafast spectroscopy technology utilizes the characteristics of femtosecond lasers to observe transient dynamic processes by time-resolved measurements of optical signals.[53–56]

The femtosecond laser system is first needed in the experiment, generally composed of femtosecond laser, optical system and detector. Femtosecond lasers can generate peak power and high repetition rate femtosecond pulses, usually achieved using Q-switched or mode-locked techniques. The optical system is used to focus the laser beam on the sample and collect the optical signals generated on the sample. Detectors are used to measure the intensity and time information of the optical signals. Then, the femtosecond laser is tuned to focus on the surface or interior of samples, by using appropriate optical elements. The peak power of femtosecond laser can excite electrons, phonons or other excited states in the sample in a very short time. The transient process that occurs in the sample will be fed back through the generated optical signal. The next step is to collect the optical signal generated by the sample. After the sample is excited by femtosecond laser, there will be transitions or dynamic processes between various excited states. These processes may be accompanied by phenomena such as light emission, absorption, and scattering, resulting in specific optical signals. The collected optical signal can be transmitted to the detector through free space or fiber optic transmission. Moreover, the time-resolved measurements of the optical signal are performed using ultrafast spectroscopy technology. Ultrafast spectroscopy technology is based on the high repetition rate and ultra-short pulse width of femtosecond lasers, and measures the time delay of optical signals by interfering or correlating with reference light pulses. This technology can achieve sub femtosecond or femtosecond level time-resolved measurements, visualizing the temporal characteristics of transient dynamic processes. Finally, by analyzing information such as time delay and intensity of the optical signals, dynamic changes that occur during transient processes can be restored. Among the femtosecond laser systems, the two most noted methods are transient absorption spectroscopy[57–60] and photoemission electron microscopy (PEEM) instruments,[61–65] and there are many scientific researches by using these two systems. For example, the ultrafast plasmonic dynamic process was measured by PEEM instrument.[66–69]

Zewail 4D electron microscopy technology is also a high-resolution time-resolved electron microscopy technology proposed by Egyptian scientist Ahmed Zewail.[70,71] The goal of this technology is to realize real-time observation of dynamic processes in materials and biological systems at the nanoscale. Traditional electron microscopes (such as transmission elec-

tron microscopy) are mainly used for static observation of the morphology and structure of samples, but cannot capture the dynamic changes in the material. The Zewail 4D electron microscope technology combines femtosecond laser pulses with electron beams, enabling the electron beam to have time resolution, therefore leading to the high-resolution real-time observation of dynamic processes in the sample.

In addition to the above methods, there are also polarization dynamics measurements,[72] time-resolved fluorescence spectroscopy,[73,74] and time-resolved photoacoustic techniques[75,76] to explore the optical dynamics process of the sample.

3.2. Measurement methods on dynamic process in topological photonics

One of the key aspects of DTP is to realize and control the dynamic process of light on photonic configurations, in which the topological properties of the system can be controlled and manipulated in real time. Here, we would like to introduce some typical methods for the realization of dynamic process in topological photonics.

As mentioned in Subsection 2.1, the Floquet systems are generally utilized to study the dynamic process of topological photonics. As Fig. 1(a) shows, this system can be regarded with two spatial dimensions and one temporal dimension. When light propagates in it, its evolution equation mathematically takes the same form as the 2D time-dependent Schrödinger equation. Therefore, the two transverse dimensions perpendicular to the propagation direction can be considered as spatial dimensions, while the propagation direction can be considered as a temporal dimension. If the waveguide is modified into a spiral shape, it becomes a Floquet system, i.e., a periodically driven system. This can effectively break the time-reversal symmetry and generate band structures with non-trivial topological properties. Moreover, time varying modulation to realize topological pumping phenomenon is also a kind of dynamically regulating topological states. Kraus *et al.* achieved the topological pumping phenomenon for the first time in a 1D optical waveguide array system.[32] They used laser direct writing waveguide technology to precisely control the coupling of waveguide array, so as to build a topological pumping model similar to quasicrystal modulation, successfully observed the topological pumping phenomenon of 1D system, and successfully corresponded to the 2D quantum Hall effect. Here, it can be found that the waveguide array systems are good regulatory platforms for dynamics. By introducing non-trivial topological structures into waveguides or resonant cavities, dynamic manipulation of topological photonic states can be achieved. Adjusting the shape, size, and material parameters of the waveguide or resonant cavity can change the topological properties of the system and regulate the evolution of edge states. Therefore, by measuring the spectral and transmission characteristics of waveguides or resonant cavities in real time, the dynamic process of topological photonic systems can be studied. More details can refer to Subsection 4.1.

In addition to that, optical regulation is an important method for achieving dynamic processes. For example, by adjusting parameters such as intensity, phase, and polarization of the light field, dynamic regulation of topological photon systems can be achieved. This can be realized by using devices such as optical components, electro-optic modulators, and phase modulators. There have been many programmable coupled resonant ring waveguide arrays with integrated Mach–Zehnder interferometer (MZI) to complete the dynamic regulation, which are significant for on-chip lasers,[77,78] quantum computing,[79] and so on.

Furthermore, the method introduced in Subsection 3.1 – ultrafast laser technology – can also be used in measuring DTP, which provides valuable insights into the dynamic behavior of topological photonic systems. Ultrafast laser technology has been used to study the transient response of topological structures, implementing research on direct or indirect topological edge-state evolution. By carefully designing experimental samples and measuring the system's response, the time-domain dynamics of photonic topological edge states can be well understood.

In experiments, nonlinear regulation can usually be used to achieve dynamic regulation of topological properties and phase transitions, thereby exhibiting rich topological phenomena and dynamic behaviors. Nonlinear regulation of time-domain DTP utilizes nonlinear effects in materials or systems to alter topological structures and interactions, achieving the coupling relationship changes. This method has been widely used in the fields of photonics, electronics and phonology. In photonics, nonlinear optical effects such as self-phase modulation can be used to realize nonlinear control. By introducing nonlinear materials or structures into topological photonic crystals, the propagation behavior and mode coupling of light can be changed, leading to the topological properties changes of the system. For example, by adjusting the intensity, phase, or nonlinear coefficient of light, the regulation of topological energy bands and the generation of dynamic topological edge states can be realized. The detailed descriptions of measurement methods on DTP can refer to Section 4, especially for the micro/nano integrated platforms.

3.3. Evolution process of dynamic topological photonics

In the previous article, the time-domain process and the measurement methods of photonics and topological photonics were discussed in detail. Based on the above measuring methods and experimental instruments, researchers can build the

combinational platforms of DTP and carry out the research. In the rest of this section, the evolution behavior in topological-photonic systems based on time-domain processes will be introduced. Moreover, the specific physical systems of DTP are demonstrated in order to help establish concrete physical configurations.

The change in evolutionary process is one of the important research directions in DTP. The research focuses on the changes of the system in the time domain, so as to find the novel physical mechanisms and effects in the time evolution system. This evolution method includes not only the direct observation of the evolution process of the system on the time scale, but also the observation of the change of the system with time on the space scale. The current research direction mainly includes the non-equilibrium phase transition, the photonic time crystal, and the Thouless pump. Non-equilibrium phase transitions can be observed in atomic and molecular systems. For example, the Rb atoms can be used to simulate a non-equilibrium phase transition in the spin 1D Dicke model.[80] The experimental setup is shown in Fig. 2(a). The Rb atoms in a high finesse cavity are driven from the side by two linearly polarized lasers with the electric fields along the axis of the cavity. The external pumping of the laser results in the system not forming an isolated system, so a non-equilibrium state is introduced in the system. This working experiment found that the spin 1D Dicke model has a rich phase diagram of non-equilibrium phases and phase transitions. The phase transitions studied in the non-equilibrium systems are not limited to non-equilibrium phases, the topological phase transitions can also be studied in the non-equilibrium systems. Existing work has shown that topological edge states of non-equilibrium polaritons can be simulated in the optical lattices,[81] and the digital quantum simulations of Floquet topological phases can be performed using solid-state quantum simulators.[82] Figure 2(b) shows the work of simulating topological phase transitions in a honeycomb lattice array where there are two boundary states, which are the topological currents at the inner and outer edges, and the pump frequency can be tuned to switch between currents.[81] Therefore, the dynamic photonics can provide the theoretical basis to the multiple phase change system.

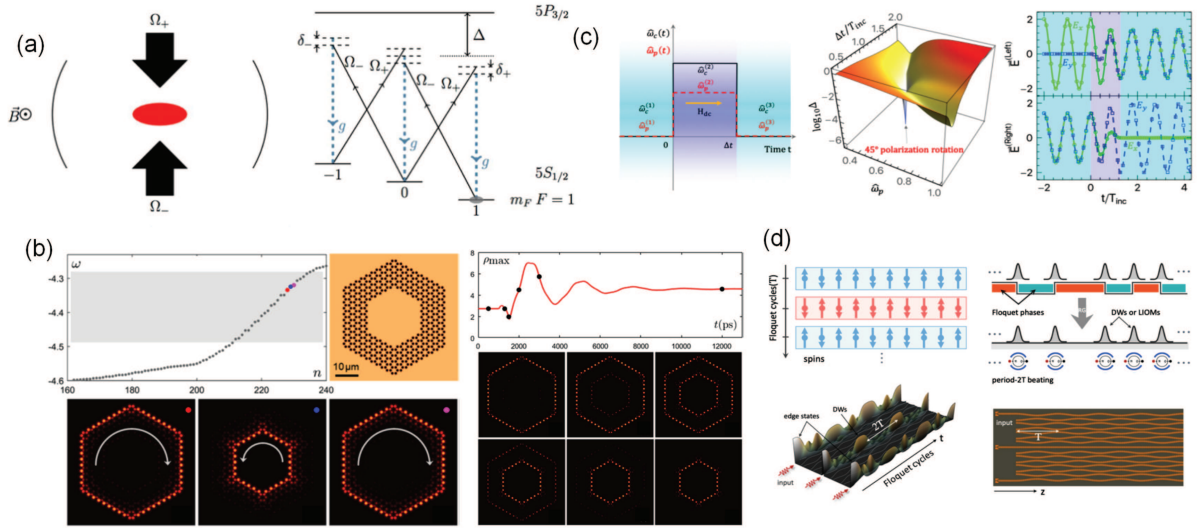


Fig. 2. (a) The schematic diagram that the Rb atoms can be used to simulate a non-equilibrium phase transition in the spin 1D Dicke model.[80] (b) The simulating of the topological phase transition in a honeycomb lattice array. The pump frequency can be tuned to switch between these two kinds of topological currents.[81] (c) The study of time-domain nonreciprocity based on photonic time crystals.[85] (d) The topological properties and topological phases of the system are studied based on Floquet time crystals.[86] Reprinted with permission from Ref. [85]. Copyright 2022 American Physical Society. References [80,81,86] are open access articles distributed under the terms of the Creative Commons CC BY license.

The photonic time crystal is also an excellent platform to study the time domain properties, because the photonic time crystal (PTC) is a kind of material whose dielectric constant is periodically modulated in time. The photonic time crystal itself has the characteristics of evolution in time. The periodic modulation of the refractive index causes these time refraction and time reflection to interfere, thereby creating band gaps in momentum, and new physical mechanisms come along.[83,84] Therefore, in recent years, the time modulation work based on photonic time crystals is rich, such as the study of time-domain non-reciprocity based on photonic time crystals. In 2022, a time-domain non-reciprocal transmission method is proposed.[85] As shown in Fig. 2(c), the polarized light propagating from left to right in the x direction is transformed into linear polarized light with 45° $E_x = E_y$ through the time plate, while the $E_x = E_y$ 45° linear polarized light propagating from right to left is transformed into linear polarized light in the y direction through the same time plate. In addition, the photonic time crystals can also be used as a research platform for studying time domain topology in topological photonics. For

example, period drive can be introduced into SSH chain, and the topological properties and topological phases of the system can be studied based on the Floquet time crystals, as shown in Fig. 2(d), which illustrates the period Floquet time-crystalline phases in topologically protected single-particle pictures.[86] As a self-carrying time dimension platform, the photonic time crystals have unique advantages in time domain research.

In the process of studying the time evolution of the system, in addition to the above mentioned content, the non-Abelian Thouless pumping based on the waveguide system is established,[87] and the non-Abelian braiding is realized through the evolution of light in the waveguide.[88] The evolution based on the waveguide systems will be discussed separately in Section 4.

3.4. Synthetic dimension and gauge field of dynamic topological photonics

In this section, the specific physical systems of DTP are demonstrated here, including the configurations on artificial synthetic dimensions and gauge field. The concept of "synthetic dimensions" has recently emerged as a powerful way to model the novel phenomena of topological phase of the matter, which are now of great interest in many areas of physics. The main idea of synthetic dimensions is to couple together some appropriate degrees of freedom to simulate the motion of particles along the extra spatial dimensions. This method provides a way to design the lattice Hamiltonian and provides the possibility to realize the high-dimensional topological models in low-dimensional platforms.[89] The concept of synthetic dimensions can be seen through a simple example, as shown in Fig. 3(a).[90] Each resonator supports a set of modes whose frequencies are equally spaced in frequency Ω, forming a frequency comb. It is also assumed that only modes with the same frequency between the nearest neighbor resonators are coupled. In addition, we assume that each resonator is modulated at frequency Ω, which leads to a coupling between modes in the same resonator with frequencies separated by Ω, thus coupling together the new degrees of freedom. We can prove that the Hamiltonian of the resonator system is equivalent to the tightly binding Hamiltonian in two dimensions under the action of an effective magnetic field outside the plane. The two dimensions of the resonator system correspond to one-dimensional space and frequency axis, respectively. In this structure, the frequency axis becomes an additional synthetic dimension. The idea of synthesizing dimensions provides us with an important method to study high-dimensional systems by using low-dimensional systems, and it also provides us with the possibility to manipulate the light fields flexibly and explore novel physical mechanisms and effects.

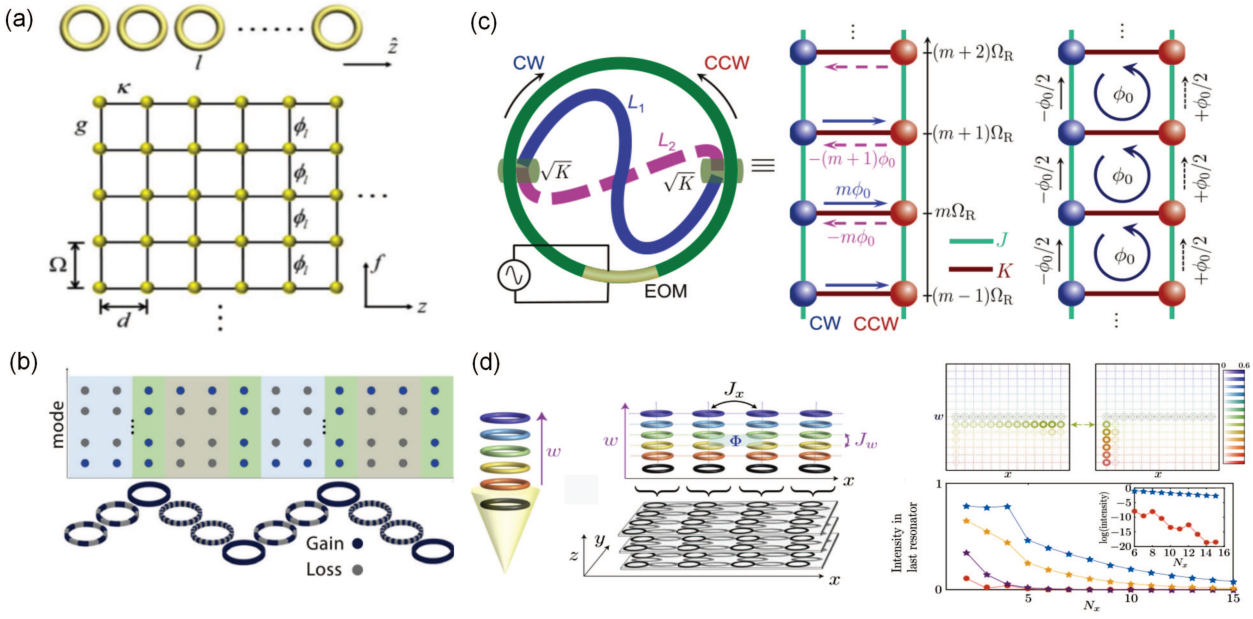


Fig. 3. (a) The diagram of the synthesized dimension. Resonance rings are used to build a new dimension.[90] (b) A robust laser with topology protection based on synthetic dimensions.[91] (c) A modulated ring resonator with CW–CCW mode-coupling and its corresponding lattice in synthetic dimensions.[92] (d) The artificial gauge field is realized by external time-dependent modulation based on the synthesized dimension.[93] Reprinted with permission from Ref. [90]. Copyright 2016 Optical Society of America. Reprinted with permission from Ref. [91]. Copyright 2021 Optical Society of America. Reprinted with permission from Ref. [92]. Copyright 2019 AAAS. Reprinted with permission from Ref. [93]. Copyright 2016 American Physical Society.

Based on the idea of synthetic dimensions, the topological photonics is with a new sight. The concept of synthetic dimensions plays an important role in the topological photonics. For example, a robust laser that can achieve topological protection with the method of synthetic dimensions is shown in Fig. 3(b).[91] The synthetic dimension can also be combined with time modulation as a tool for studying DTP. We can directly add a time modulation factor to the synthesis dimen-

sion, and then study the physical properties of the system in the time domain. Figure 3(c) shows a system consisting of a time-modulated ring resonator with spatial coupling between clockwise and anticlockwise modes, such that the two modes form a pseudo-spin degree of freedom.[92] Figure 3(c) also shows the corresponding lattice in the synthetic dimension of the system. This work takes advantage of the time modulation property of the ring resonator. The external time modulation driver can also be applied to the study of DTP based on synthetic dimensions. Figure 3(d) shows a study that realizes an artificial gauge field via external time-dependent modulation with different modes of silicon ring resonators as additional dimensions of photons. The study shows that the 2D quantum Hall effect and its associated topological edge states can be achieved using 1D multimode resonator chains, which shows a potential to be used as an integrated optical isolator. And this work demonstrates how the 4D quantum Hall effect is characterized in the 3D lattice of the resonator. The synthetic dimension provides a new way to study DTP.[93]

4. Realization platforms on dynamic topological photonics

The dynamic topological photonics is an important direction in photonics. In the previous discussion, we introduced the physical mechanism for DTP in photonics. Next, we will mainly introduce the photonic realization platforms for studying DTP. With the development of photonics, the nano-photonics devices provide a great boost for the research of the novel properties of photonic systems in the time domain, and different photonic devices provide possibilities for the study of DTP in different research systems. Below we focus on several major nano-photonics platforms and briefly introduce the applications of the DTP.

4.1. Optical waveguide arrays

The waveguide array is an ideal platform to study the time domain variation. Since the waveguide system follows the evolution form of the Schrodinger equation, the evolution property can be reflected in the process of beam transmission along the waveguide. Therefore, the waveguide system is an excellent quantum simulation platform in topological photonics, which provides a feasible experimental idea for carrying out time-dependent evolution in optical systems. For example, the non-Abelian braiding on-chip can be realized based on the waveguide system, and the beam evolution process in the braiding process can be also studied on the waveguide system.[88] Researchers can also construct the artificial gauge fields based on the waveguide arrays to study the novel physical effects or achieve some novel functions.[94,95]

As one of the most basic micro and nano optical devices, waveguide has excellent properties in the study of the optical time domain properties. On the one hand, the characteristics of beam transmission along the waveguide provide researchers with the possibility of dynamic observation of light field changes along the beam transmission direction; on the other hand, the idea that the waveguide system can be combined with the reconstructed system or the synthetic dimension provides us with a time-domain research platform based on the waveguide itself. Existing studies have used the Floquet waveguide system of helically bent waveguides to study influence of defects with time-dependent coupling on the robustness of the transport along the edge.[96] For the equilibrium photonic systems, edge modes in topological insulators are robust to defects. This work proves that "dynamic defects" hardly affect the robustness of the system when the time dimension is added so that the system becomes a time-dependent dynamic system. It is mainly implemented by the Floquet waveguide array. Figure 4(a) shows the dynamic defect diagram of the waveguide array. Since the dynamic transmission process of the beam can be directly observed as one of the important advantages of the waveguide system, we can also directly study the dynamic evolution of the beam in the transmission process based on the waveguide system, as shown in Fig. 4(b).[97] By comparing the transmission state of the beam in the topological system and the traditional waveguide system, it can be found that the topological structure has the characteristics of robustness and broadband, which is of great significance for the future photon integration and robust information transmission. In addition, the waveguide system can also be combined with the idea of synthetic dimension, providing a new perspective for the study of DTP.[98]

4.2. Microring resonator/cavity systems

The coupled-cavity and micro-ring resonators are important photonic devices in the micro–nano optics. The study of topological photonics with the coupled-cavity and micro-ring resonators is a traditional research direction. Currently, topological systems can be constructed based on the photonic crystal defect cavities, micro-ring resonators and the other kinds of coupled-cavities. Because the modulation of coupling parameters in resonator and micro-ring resonator systems is flexible, it provides a perfect platform for the study of topological photonics, especially for non-Hermitic topological systems. For example, coupled micro-ring resonators can be used to construct non-Hermitian one-dimensional SSH arrays or quantum spin Hall effect systems to study novel physical effects in topological photonics,[99,100] and even the physical mechanism of topological phase transitions in non-Hermitian topological systems can be studied based on resonant ring arrays.[101] Micro-ring resonators can also build time-domain reconfigurable topological photonics systems. For example, a traditional work on non-Hermitian topologies based on microring

resonators has mentioned that non-Hermitian controlled reconfigurable optical transmission schemes are inherently topologically robust against defects, and the light path is easily reconfigured along any designed shape.[77] The coupled-cavity and micro-ring resonators have important applications in the study of topological photonics.

The research platform of DTP can also be built based on the micro-ring resonator and the resonator system. In 2023, a study discussed the topological properties of coupled cavity arrays corresponding to the Su–Schrieffer–Heeger model and the kagome model in the time domain. In the work, the 1D SSH coupled chain and kagome model are constructed by the arrangement of cavity arrays. The Kerr-like nonlinear effect is introduced into the 1D SSH model, and the topological phase transition caused by this effect can be described by the time evolution of an edge cavity stimulated initially. Therefore, the time domain properties of the two boundary atoms at the end of the coupling chain are discussed in the time domain. This discussion has also been applied to the 2D topological kagome model. It has been found that phase transitions occur when nonlinearities are enhanced in metallic kagome models, while it is difficult for nonlinearities to change the topological properties of the trivial kagome model.[102] Figure 4(c) shows the time-domain variation of the wave function of boundary atoms in the kagome model. In addition, microcavity systems can also be combined with photonic time crystals. An experimental observation of continuous time crystals was reported in 2022, using high-precision optical cavities and Bose–Einstein condensates to form experimental devices, as shown in Fig. 4(d). The determined time-crystalline regime is also shown in Fig. 4(d). This work experimentally observes the continuous time crystals and provides a theoretical understanding. This class of dynamical many-body states expands the concepts of long-range order and spontaneous symmetry breaking into the time domain, which have fundamental significance.[103] As a common photonic platform, the resonators and the micro-ring resonators have important applications in the future research of time domain photonics.

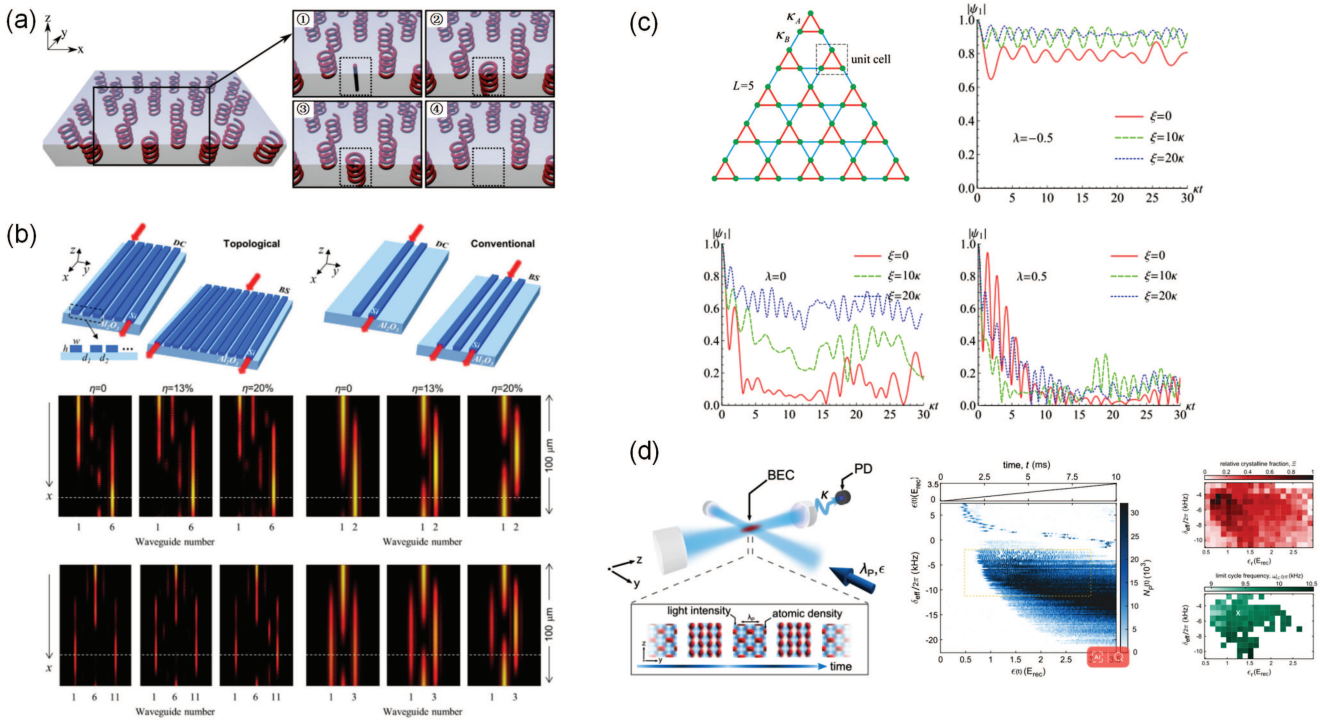


Fig. 4. (a) Different kinds of dynamic defects in the Floquet waveguide array.[96] (b) The dynamic evolution of the beam in the transmission process based on the waveguide system.[97] (c) The time-domain variation of the wave function of boundary atoms in the kagome model.[102] (d) The continuous time crystal in an atom-cavity system and determining the time-crystalline regime.[103] Reprinted with permission from Ref. [97]. Copyright 2020 Laser Photonics Reviews/Wiley-VCH. Reference [102] is an open access article distributed under the terms of the Creative Commons CC-BY license. Reprinted with permission from Ref. [103]. Copyright 2022 AAAS.

4.3. Photonic crystals and lattices

The photonic crystal system is the most basic system in the traditional study of topological photonics. Because photonic crystal generally has a wonderful photonic band structure, photonic crystal system is one of the most basic experimental platforms in topological photonics. Common topological photonic lattices include the SSH model,[104] the kagome model,[105] and valley-Hall photonic topological system.[106] In addition to the periodic characteristics of photonic crystals, which can be used as the topological photonics platforms, the optical lattice with spatial periodicity is also a method to study topological systems. Existing studies have shown that regions with periodic refractive index changes can be prepared in media through photo-refractive effect to form specific lattice

configurations. Based on 1D non-Hermite-SSH chains, a reconfigurable experimental platform was built based on photorefractive effect.[107] Since reconfigurable systems have the characteristics of dynamic control in time domain, such experimental platforms have the potential to develop DTP in time domain.

For the DTP, periodic media represented by the photonic crystals and the optical lattices can be a useful tool. For the photonic crystal system of traditional dielectric columns, the topological properties of the system can be regulated by methods of the temperature control or the electrical modulation. As the dielectric column array shown in Fig. 5(a), when temperature changes, it can cause the expansion or contraction of those dielectric columns, thus causing phase transition of the entire topological system.[108] Figure 5(b) shows a liquid crystal system (LC), which allows to modify the refractive index with external electric field, and the topological edge state can be dynamically controlled by modifying the refractive index of LC background medium.[1] The DTP can also be studied in the momentum space. A work reported in 2023 demonstrated the dynamic evolution of polarization singularities by dynamically changing the geometric parameters of the photonic crystal plate itself.[109] Its structure is shown in Fig. 5(c), and the polarization singularities evolve with the distance between the photonic crystal plate and the substrate. During the evolution, all BICs are protected from splitting owing to the preserved structural symmetry. As a classical periodic structure, the photonic crystal system can be characterized by dynamic evolution through dynamic regulation, thus providing a platform for DTP research.

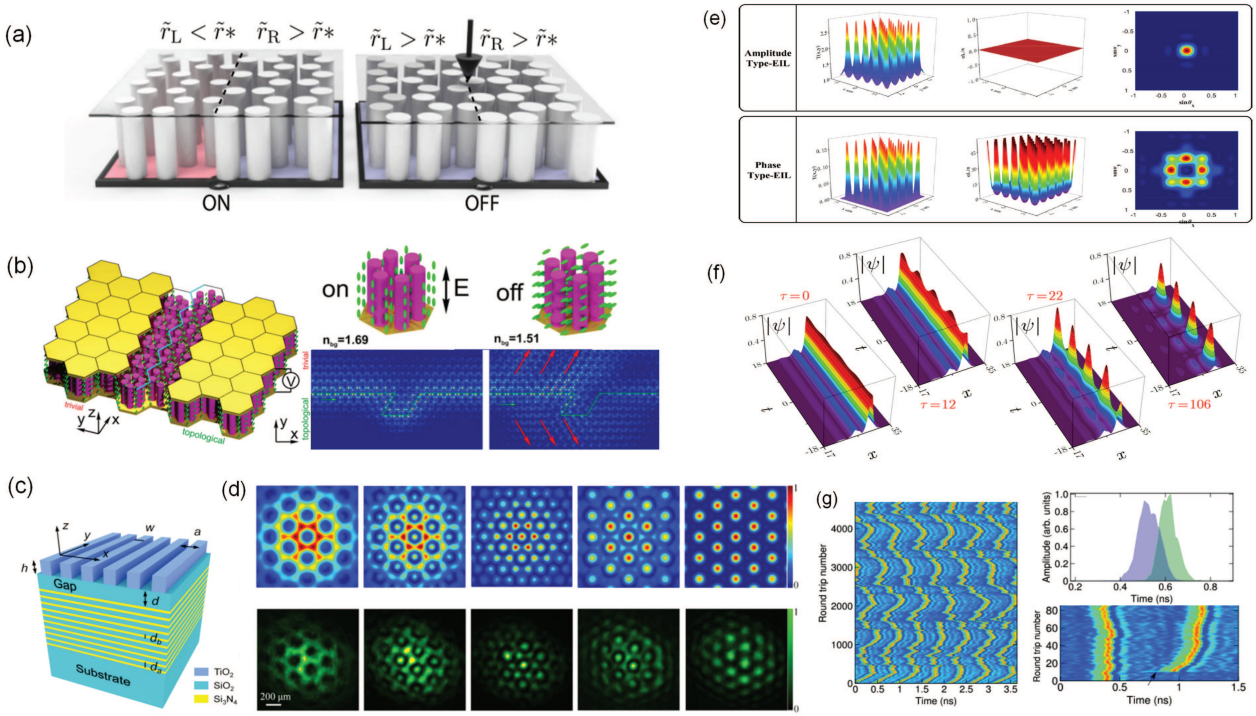


Fig. 5. (a) The photonic crystal system induced topological phase transition based on temperature control.[108] (b) Topological boundary states of liquid crystal systems are dynamically controlled by changing refractive index based on the external electric field.[1] (c) The schematic diagram of a photonic crystal plate and the substrate. This structure is used to study the evolution of polarization singularities in the momentum space.[109] (d) The spatial light field modulation based on graphene. The field can be dynamically changed with system parameters, and it has reconfigurability.[110] (e) The optical lattice is constructed by nonlinear effect. This method showed that a type conversion is realized from amplitude electromagnetically induced optical lattices to synthesized electromagnetically induced optical lattices by adding a detuning term.[111] (f) Spatiotemporal intensity profiles emerging in the course of modulational instability development in the mode-locked lasers.[112] (g) The space–time pattern of soliton repulsion.[113] Reprinted with permission from Ref. [108]. Copyright 2022 American Physical Society. Reprinted with permission from Ref. [109]. Copyright 2023 American Physical Society. Reprinted with permission from Ref. [110]. Copyright 2023 Optical Society of America. Reprinted with permission from Ref. [112]. Copyright 2021 Optical Society of America. Reprinted with permission from Ref. [113]. Copyright 2017 AIP Publishing.

In addition to the photonic crystal systems mentioned above, the electromagnetically induced optical lattices are also a great platform for the DTP. The electromagnetically induced optical lattice is a new kind of artificial periodic dielectric structure induced by applying standing wave coupled optical field on the basis of the electromagnetically induced transparency effect. It is therefore possible to introduce time domain variations on an electromagnetically induced optical lattice to study DTP. The reconfigurable experimental platform for photo-refractive effects mentioned above is a promising time domain regulatory system. Figure 5(d) shows a graphene-based spatial light field modulation that can construct a periodic light field distribution in space to achieve the optical lattice construction. Since the refractive index of photonic

graphene dynamically evolves with system parameters, such a design provides the possibility to study the DTP in the optical lattice systems.[110] The nonlinear effect can also be applied to the construction of the optical lattice, as shown in Fig. 5(e). This work proposes a new method for quantifying lattice using nonlinear optically induced periodic lattice, which has a striking characteristic of adjustable refractive index. And the method showed that a type conversion is realized from amplitude electromagnetically induced optical lattices to synthesized electromagnetically induced optical lattices by adding a detuning term.[111] As a reconfigurable optical platform, electromagnetically induced optical lattices have great potential for DTP research in time domain.

4.4. Optical solitons

With the development of photonics, laser has become an indispensable important equipment for the optical research today. A laser of high quality should have a stable laser pulse output when working normally. Therefore, the normal working state of the laser can be considered as a photon equilibrium state. However, the generation of the laser pulse is not a transient process. The laser system must experience a non-equilibrium evolution process to produce stable output. Non-equilibrium state evolution is inevitable during the operation of laser. Therefore, the time domain characteristics and transient behavior of the laser are particularly important for the performance of the laser. DTP has important reference value in laser manufacturing and performance improvement. Understanding and utilizing pulsed nonlinear dynamics are an important way to improve fiber laser performance. The mode-locked pulses formed in fiber lasers can be called solitons, and the transient behavior of solitons has important research value. Studies on the transient behavior of solitons have emerged in recent years,[114–116] which will be described in detail below.

In Subsection 2.2, we take the establishment of normal dispersive solitons as an example to show that the establishment of solitons is a non-equilibrium process. The transient behavior in soliton systems has important value in time domain research, including mutually ignited soliton explosions in multi-soliton systems,[114] dynamic trapping of a polarization rotation vector soliton,[115] visualizing the invisible soliton pulsation[117] and so on. Building lasers based on the topological photonics systems is also a classic research direction.[118,119] We can introduce the soliton time-domain dynamics into the topological soliton laser, analyzing the characteristics of the laser in the time-domain and optimizing it. For example, a recent work reported topological solitons in mode-locked laser arrays based on SSH model, and studied the characteristics of the system in time and space domains. Figure 5(f) shows the spatiotemporal intensity profiles emerging in the course of modulational instability development.[112] The dynamic behavior of solitons can also be studied on the photon platform of the laser, and different types of dynamic interactions of solitons can be studied under different laser parameters, such as repulsion, annihilation, or formation of soliton bound states. Figure 5(g) shows the space–time pattern of soliton repulsion.[113] The time-domain interaction based on soliton is helpful for the simulation of multi-body interaction. Solitons in lasers have potential to play an important role in the study of DTP in the future because of the rich temporal dynamics of solitons.

5. Conclusion and outlook

In this review, we introduced the recent progress and realization platforms of DTP from the mechanism to experiential setups. As a novel field, time-domain DTP provides us with a new perspective to understand the topological behavior of optics and realize topological photonic devices. In the future researches, there can be broad prospects and many interesting research directions. In experiments, despite the introduced measurement methods, other methods with different instruments, materials and optical configurations can be further studied. In physics, the evolution process of topological edge states with direct observations have not been common, and it is of great significance to study the time-domain topology, which can be very different from that in frequency domain. Moreover, the DTP can also be used to study the optical quantum properties in time domain. In applications, time-domain DTP can be developed in optical sensing and topological photonic devices. By utilizing the protection properties and dynamic regulation of light, high sensitivity and stability optical sensors can be achieved. More important, DTP has great application prospects in the fields of optical transmission, optical control and information processing. Different from the traditional study of frequency-space domain, DTP introduces the concept of time domain to make the system dynamically adjustable. For example, the DTP theory can be applied in the reconfigurable system with diverse functions. When a system evolves from one steady state to another, it completes a reconfiguration process. Furthermore, dynamic processes involving topological phase transitions can establish robust optical switches, and those involving synthetic dimensions can be used for high-dimensional quantum simulations. DTP provides a way for researchers to control light, transmit light, reduce light response relaxation time, and design highly sensitive optical devices. In addition, it also provides novel ideas and methods for the design and fabrication of topological photonic devices from the perspective of time domain. In the future, the performance and integration of topological photonic devices can be further studied and optimized to promote their applications in photonics and information technology.

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References

[1] Shalaev M I, Desnavi S, Walasik W and Litchinitser N M 2018 *New J. Phys.* **20** 023040
[2] Shalaev M I, Walasik W and Litchinitser N M 2019 *Optica* **6** 839
[3] Cao T, Fang L, Cao Y, Li N, Fan Z and Tao Z 2019 *Science Bulletin* **64** 814
[4] Tang G J, He X T, Shi F L, Liu J W, Chen X D and Dong J W 2022 *Laser Photon. Rev.* **16** 2100300
[5] Ota Y, Katsumi R, Watanabe K, Iwamoto S and Arakawa Y 2018 *Commun. Phys.* **1** 86
[6] Del Campo A and Zurek W H 2014 *Int. J. Mod. Phys. A* **29** 1430018
[7] Hagymási I, Hubig C, Legeza Ö and Schollwöck U 2019 *Phys. Rev. Lett.* **122** 250601
[8] Fläschner N, Vogel D, Tarnowski M, Rem B, Lühmann D S, Heyl M, Budich J, Mathey L, Sengstock K and Weitenberg C 2018 *Nat. Phys.* **14** 265
[9] Smirnova D, Leykam D, Chong Y and Kivshar Y 2020 *Appl. Phys. Rev.* **7** 021306
[10] Chaunsali R, Xu H, Yang J, Kevrekidis P G and Theocharis G 2021 *Phys. Rev. B* **103** 024106
[11] Hadad Y, Soric J C, Khanikaev A B and Alu A 2018 *Nat. Electron.* **1** 178
[12] Dobrykh D, Yulin A, Slobozhanyuk A, Poddubny A and Kivshar Y S 2018 *Phys. Rev. Lett.* **121** 163901
[13] Pal R K, Vila J, Leamy M and Ruzzene M 2018 *Phys. Rev. E* **97** 032209
[14] Hajlaoui M, Papalazarou E, Mauchain J, Lantz G, Moisan N, Boschetto D, Jiang Z, Miotkowski I, Chen Y and Taleb-Ibrahimi A 2012 *Nano Lett.* **12** 3532
[15] Yan Q, Cao E, Sun Q, Ao Y, Hu X, Shi X, Gong Q and Misawa H 2021 *Nano Lett.* **21** 9270
[16] Juge R, Sisodia N, Larrañaga J U, Zhang Q, Pham V T, Rana K G, Sarpi B, Mille N, Stanescu S and Belkhou R 2022 *Nat. Commun.* **13** 4807
[17] Faddeev L D and Tirkkonen O 1995 *Nucl. Phys. B* **453** 647
[18] Bender C M, Boettcher S and Meisinger P N 1999 *J. Math. Phys.* **40** 2201
[19] Bender C M and Boettcher S 1998 *Phys. Rev. Lett.* **80** 5243
[20] El-Ganainy R, Makris K, Christodoulides D and Musslimani Z H 2007 *Opt. Lett.* **32** 2632
[21] Makris K G, El-Ganainy R, Christodoulides D and Musslimani Z H 2008 *Phys. Rev. Lett.* **100** 103904
[22] Hafezi M, Demler E A, Lukin M D and Taylor J M 2011 *Nat. Phys.* **7** 907
[23] Hafezi M, Mittal S, Fan J, Migdall A and Taylor J 2013 *Nat. Photon.* **7** 1001
[24] Wu L H, Hu X 2015 *Phys. Rev. Lett.* **114** 223901
[25] Chen X D, Deng W M, Lu J C and Dong J W 2018 *Phys. Rev. B* **97**
[26] Slobozhanyuk A, Mousavi S H, Ni X, Smirnova D, Kivshar Y S and Khanikaev A B 2017 *Nat. Photon.* **11** 130
[27] Lindner N H, Refael G and Galitski V 2011 *Nat. Phys.* **7** 490
[28] Oka T and Aoki H 2009 *Phys. Rev. B* **79** 081406
[29] Kitagawa T, Berg E, Rudner M and Demler E 2010 *Phys. Rev. B* **82** 235114
[30] Rechtsman M C, Zeuner J M, Plotnik Y, Lumer Y, Podolsky D, Dreisow F, Nolte S, Segev M and Szameit A 2013 *Nature* **496** 196
[31] Thouless D 1983 *Phys. Rev. B* **27** 6083
[32] Kraus Y E, Lahini Y, Ringel Z, Verbin M and Zilberberg O 2012 *Phys. Rev. Lett.* **109** 106402
[33] Chen H J, Liu M, Yao J, Hu S, He J B, Luo A P, Xu W C and Luo Z C 2018 *Opt. Express* **26** 2972
[34] Chen H J, Liu M, Yao J, Hu S, He J B, Luo A P, Luo Z C and Xu W C 2018 *IEEE Photon. Journal* **10** 1
[35] Zhao J, Zhao W, Du W, Su R and Xiong Q 2020 *Nano Res.* **13** 1399
[36] Tang H, Wang T Y, Shi Z Y, Feng Z, Wang Y, Shang X W, Gao J, Jiao Z Q, Li Z M, Chang Y J 2022 *Photon. Res.* **10** 1430
[37] Yuan L, Dutt A and Fan S 2021 *APL Photon.* **6** 071102
[38] Li G, Zheng Y, Dutt A, Yu D, Shan Q, Liu S, Yuan L, Fan S and Chen X 2021 *Sci. Adv.* **7** eabe4335
[39] Yuan L, Lin Q, Xiao M and Fan S 2018 *Optica* **5** 1396
[40] Hey D and Li E 2018 *Roy. Soc. Open. Sci.* **5** 172447
[41] Liu Q, Liu W, Ziegler K, Chen F 2023 *Phys. Rev. Lett.* **130** 103801
[42] Burla M, Wang X, Li M, Chrostowski L and Azaña J 2016 *Nat. Commun.* **7** 13004
[43] Zhang X L, Jiang T and Chan C T 2019 *Light Sci. Appl.* **8** 88
[44] Sacher W D and Poon J K 2008 *Opt. Express* **16** 15741
[45] Lee K, Park J, Lee S, Baek S, Park J, Rotermund F and Min B 2022 *Nanophotonics* **11** 2045
[46] Fryett T K, Dodson C M and Majumdar A 2015 *Opt. Express* **23** 16246
[47] Xu Z M, Li C and Wu J F 2020 *New J. Phys.* **22** 063030
[48] Kim S H, Choi J H, Lee S K, Kim S H, Yang S M, Lee Y H, Seassal C, Regrency P and Viktorovitch P 2008 *Opt. Express* **16** 6515
[49] Bruck R, Liu C, Muskens O L, Fratalocchi A and Di Falco A 2016 *Laser Photon. Rev.* **10** 688
[50] Liu M, Wei Z W, Li H, Li T J, Luo A P, Xu W C and Luo Z C 2020 *Laser Photon. Rev.* **14** 1900317
[51] Liu M, Luo A P, Luo Z C and Xu W C 2017 *Opt. Lett.* **42** 330
[52] Yu Y, Luo Z C, Kang J Q and Wong K K Y 2018 *Opt. Lett.* **43** 4132
[53] Callou T P, Garcia R, Mukai A, Giacomin N T, de Souza R G and Bechara S J 2016 *Clinical Ophthalmology* **10** 697
[54] Kymionis G D, Kankariya V P, Plaka A D and Reinstein D Z 2012 *Journal of Refractive Surgery* **28** 912
[55] Abahussin M, Hayes S, Cartwright N E K, Kamma-Lorger C S, Khan Y, Marshall J and Meek K M 2009 *Investigative ophthalmology & visual science* **50** 5159
[56] Wang X, Yu H, Li P, Zhang Y, Wen Y, Qiu Y, Liu Z, Li Y and Liu L 2021 *Optics & Laser Technology* **135** 106687
[57] Niedzwiedzki D M, Sullivan J O, Polívka T, Birge R R and Frank H A 2006 *The Journal of Physical Chemistry B* **110** 22872
[58] Ohkita H and Ito S 2011 *Polymer* **52** 4397
[59] Lorenc M, Ziolek M, Naskrecki R, Karolczak J, Kubicki J and Maciejewski A 2002 *Appl. Phys. B* **74** 19
[60] Berera R, van Grondelle R and Kennis J T 2009 *Photosynthesis research* **101** 105
[61] Bauer E, Mundschau M, Swiech W and Telieps W 1989 *Ultramicroscopy* **31** 49
[62] Cinchetti M, Gloskovskii A, Nepjiko S, Schönhense G, Rochholz H and Kreiter M 2005 *Phys. Rev. Lett.* **95** 047601
[63] Dabrowski M, Dai Y and Petek H 2020 *Chem. Rev.* **120** 6247
[64] Schönhense G, Elmers H, Nepijko S and Schneider C 2006 *Advances in Imaging and Electron Physics* **142** 159
[65] Schmidt O, Bauer M, Wiemann C, Porath R, Scharte M, Andreyev O, Schönhense G and Aeschlimann M 2002 *Appl. Phys. B* **74** 223
[66] Yan Q, Cao E, Hu X, Du Z, Ao Y, Chu S, Sun Q, Shi X, Chan C and Gong Q 2022 *Nanophotonics* **11** 3495
[67] Sun Q, Ueno K, Yu H, Kubo A, Matsuo Y and Misawa H 2013 *Light Sci. Appl.* **2** e118
[68] Yang J, Sun Q, Ueno K, Shi X, Oshikiri T, Misawa H and Gong Q 2018 *Nat. Commun.* **9** 4858
[69] Tsai W Y, Sun Q, Hu G, Wu P C, Lin R J, Qiu C W, Ueno K, Misawa H and Tsai D P 2019 *Adv. Opt. Mater.* **7** 1801060
[70] Zewail A H and Thomas J M 2009 *4D electron microscopy: imaging in space and time* (World Scientific) p. 123
[71] Flannigan D J and Zewail A H 2012 *Accounts of chemical research* **45** 1828
[72] Lagarde D, Bouet L, Marie X, Zhu C, Liu B, Amand T, Tan P and Urbaszek B 2014 *Phys. Rev. Lett.* **112** 047401
[73] Millar D P 1996 *Current opinion in structural biology* **6** 637
[74] Diamandis E P 1988 *Clinical biochemistry* **21** 139
[75] Cross F, Al-Dhahir R, Dyer P and MacRobert A 1987 *Appl. Phys. Lett.* **50** 1019
[76] Leytner S and Hupp J T 2000 *Chemical Physics Letters* **330** 231
[77] Zhao H, Qiao X, Wu T, Midya B, Longhi S and Feng L 2019 *Science* **365** 1163
[78] Feng L, El-Ganainy R and Ge L 2017 *Nat. Photon.* **11** 752
[79] Dai T, Ao Y, Bao J, Mao J, Chi Y, Fu Z, You Y, Chen X, Zhai C and Tang B 2022 *Nat. Photon.* **16** 248

[80] Zhiqiang Z, Lee C H, Kumar R, Arnold K, Masson S J, Parkins A and Barrett M 2017 *Optica* **4** 424
[81] Ma X, Kartashov Y V, Ferrando A and Schumacher S 2020 *Opt. Lett.* **45** 5311
[82] Chen B, Li S, Hou X, Ge F, Zhou F, Qian P, Mei F, Jia S, Xu N and Shen H 2021 *Photon. Res.* **9** 81
[83] Lyubarov M, Lumer Y, Dikopoltsev A, Lustig E, Sharabi Y and Segev M 2022 *Science* **377** 425
[84] Lustig E, Sharabi Y and Segev M 2018 *Optica* **5** 1390
[85] Li H, Yin S and Alù A 2022 *Phys. Rev. Lett.* **128** 173901
[86] Wang B, Quan J, Han J, Shen X, Wu H and Pan Y 2022 *Laser Photon. Rev.* **16** 2100469
[87] Sun Y K, Zhang X L, Yu F, Tian Z N, Chen Q D and Sun H B 2022 *Nat. Phys.* **18** 1080
[88] Zhang X L, Yu F, Chen Z G, Tian Z N, Chen Q D, Sun H B and Ma G 2022 *Nat. Photon.* **16** 390
[89] Ozawa T and Price H M 2019 *Nat. Rev. Phys.* **1** 349
[90] Yuan L, Shi Y and Fan S 2016 *Opt. Lett.* **41** 741
[91] Lustig E and Segev M 2021 *Adv. Opt. Photon.* **13** 426
[92] Dutt A, Lin Q, Yuan L, Minkov M, Xiao M and Fan S 2020 *Science* **367** 59
[93] Ozawa T, Price H M, Goldman N, Zilberberg O and Carusotto I 2016 *Phys. Rev. A* **93** 043827
[94] Lumer Y, Bandres M A, Heinrich M, Maczewsky L J, Herzig-Sheinfux H, Szameit A and Segev M 2019 *Nat. Photon.* **13** 339
[95] Song W, Li T, Wu S, Wang Z, Chen C, Chen Y, Huang C, Qiu K, Zhu S and Zou Y 2022 *Phys. Rev. Lett.* **129** 053901
[96] Jörg C, Letscher F, Fleischhauer M and von Freymann G 2017 *New J. Phys.* **19** 083003
[97] Song W, Sun W, Chen C, Song Q, Xiao S, Zhu S and Li T 2020 *Laser Photon. Rev.* **14** 1900193
[98] Lustig E, Weimann S, Plotnik Y, Lumer Y, Bandres M A, Szameit A and Segev M 2019 *Nature* **567** 356
[99] Song A Y, Sun X Q, Dutt A, Minkov M, Wojcik C, Wang H, Williamson I A, Orenstein M and Fan S 2020 *Phys. Rev. Lett.* **125** 033603
[100] Longhi S 2018 *Ann. Phys.* **530** 1800023
[101] Ao Y, Hu X, You Y, Lu C, Fu Y, Wang X and Gong Q 2020 *Phys. Rev. Lett.* **125** 013902
[102] Wei M S, Liao M J, Wang C, Zhu C, Xu J and Yang Y 2023 *Results in Physics* **45** 106232
[103] Kongkhambut P, Skulte J, Mathey L, Cosme J G, Hemmerich A and Keßler H 2022 *Science* **377** 670
[104] Chen Y, Lan Z and Zhu J 2022 *Nanophotonics* **11** 1345
[105] Zhong H, Kartashov Y V, Szameit A, Li Y, Liu C and Zhang Y 2021 *APL Photon.* **6** 040802
[106] Xiong L, Liu Y, Zhang Y, Zheng Y and Jiang X 2022 *ACS Photon.* **9** 2448
[107] Xia S, Song D, Wang N, Liu X, Ma J, Tang L, Buljan H and Chen Z 2021 *Opt. Mater. Express* **11** 1292
[108] Pirie H, Sadhuka S, Wang J, Andrei R and Hoffman J E 2022 *Phys. Rev. Lett.* **128** 015501
[109] Luo H, Liu L, Xi Z, Lu Y and Wang P 2023 *Phys. Rev. A* **107** 013504
[110] Zhang H, Yuan J, Xiao L, Jia S and Wang L 2023 *Opt. Express* **31** 11335
[111] Hui S, Wen F, Yu X, Dai Z, Ahmed I, Su Y, Zhang Y and Wang H 2020 *Sci. Rep.* **10** 13396
[112] Zykin A, Skryabin D and Kartashov Y 2021 *Opt. Lett.* **46** 2123
[113] Garbin B, Javaloyes J, Barland S and Tissoni G 2017 *Chaos* **27** 114308
[114] Yu Y, Luo Z C, Kang J and Wong K K 2018 *Opt. Lett.* **43** 4132
[115] Liu M, Luo A P, Luo Z C and Xu W C 2017 *Opt. Lett.* **42** 330
[116] Runge A F, Hudson D D, Tam K K, de Sterke C M and Blanco-Redondo A 2020 *Nat. Photon.* **14** 492
[117] Liu M, Wei Z W, Li H, Li T J, Luo A P, Xu W C and Luo Z C 2020 *Laser Photon. Rev.* **14** 1900317
[118] Zhang W, Xie X, Hao H, Dang J, Xiao S, Shi S, Ni H, Niu Z, Wang C, Jin K 2020 *Light Sci. Appl.* **9** 109
[119] Gong Y, Wong S, Bennett A J, Huffaker D L and Oh S S 2020 *ACS Photon.* **7** 2089