

High-order effect on the transmission of two optical solitons

Houhui Yi(伊厚会), Yanli Yao(姚延立), Xin Zhang(张鑫), and Guoli Ma(马国利)
Citation: Chin. Phys. B, 2023, 32 (**10**): 100509. DOI: 10.1088/1674-1056/aceeec
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Influence of the initial parameters on soliton interaction in nonlinear optical systems

Xinyi Zhang(张昕仪) and Ye Wu(吴晔)
Chin. Phys. B, 2023, 32 (**7**): 070505. DOI: 10.1088/1674-1056/ace1da

All-optical switches based on three-soliton inelastic interaction and its application in optical communication systems

Shubin Wang(王树斌), Xin Zhang(张鑫), Guoli Ma(马国利), and Daiyin Zhu(朱岱寅)
Chin. Phys. B, 2023, 32 (**3**): 030506. DOI: 10.1088/1674-1056/acad6c

Soliton fusion and fission for the high-order coupled nonlinear Schrödinger system in fiber lasers

Tian-Yi Wang(王天一), Qin Zhou(周勤), and Wen-Jun Liu(刘文军)
Chin. Phys. B, 2022, 31 (**2**): 020501. DOI: 10.1088/1674-1056/ac2d22

Periodic solitons in dispersion decreasingfibers with a cosine profile

Jia Ren-Xu (贾仁需), Yan Hong-Li (闫宏丽), Liu Wen-Jun (刘文军), Lei Ming (雷鸣)
Chin. Phys. B, 2014, 23 (**10**): 100502. DOI: 10.1088/1674-1056/23/10/100502

Propagation of electromagnetic soliton in anisotropic biquadratic ferromagnetic medium

L. Kavitha, M. Saravanan, D. Gopi
Chin. Phys. B, 2013, 22 (**3**): 030512. DOI: 10.1088/1674-1056/22/3/030512

High-order effect on the transmission of two optical solitons

Houhui Yi(伊厚会)[1], Yanli Yao(姚延立)[2], Xin Zhang(张鑫)[2], and Guoli Ma(马国利)[2,†]

[1] *School of Intelligent Manufacturing, Weifang University of Science and Technology, Weifang* 262700, *China*
[2] *Institute of Aeronautical Engineering, Binzhou University, Binzhou* 256603, *China*

(Received 13 July 2023; revised manuscript received 31 July 2023; accepted manuscript online 10 August 2023)

For optical solitons with the pulse width in the subpicosecond and femtosecond scales in optical fibers, a modified model containing higher-order effects such as third-order dispersion and third-order nonlinearity is needed. In this paper, in order to study the dynamic mechanism of femtosecond solitons in different media, we take the nonlinear Schrödinger equation considering higher-order effects as the theoretical model, discuss the propagation of solitons in single-mode fibers, and explore the third-order dispersion and third-order nonlinear effects on the generation of optical solitons. The exact solution of the theoretical model is obtained through the bilinear method, and the transmission characteristics of two solitons with exact soliton solutions in actual fiber systems are analyzed and studied. The influence of various conditions on the transmission and interaction of optical solitons is explored. Methods for optimizing the transmission characteristics of optical solitons in optical communication systems are suggested. The relevant conclusions of this paper have guiding significance for improving the quality of fiber optic communication and increasing bit rates.

Keywords: nonlinear optics, optical communication system, optical solitons, solitons interaction

PACS: 05.45.Yv, 42.65.Tg, 42.81.Dp **DOI:** 10.1088/1674-1056/aceeec

1. Introduction

The transmission characteristics of solitons in optical fibers have always been a hot research direction.[1–4] Due to the balance between dispersion and nonlinear effects, optical pulses can be transmitted in soliton form without distortion in the anomalous dispersion region of the optical fiber.[5,6] In this case, the nonlinear Schrödinger (NLS) equation is generally used as the mathematical model, which can be used to describe the transmission of optical pulses in the fiber, and the mathematical model only includes group velocity dispersion and self-phase modulation effects.[7–12] However, with the development of ultrashort pulse technology, when the pulse width of solitons is in the order of subpicosecond or femtosecond, the higher-order nonlinear effect and higher-order dispersion effect of optical fibers have a greater impact on solitons.[13,14] At this time, the transmission equation needs to be modified to the higher-order NLS equation.[15,16] Therefore, all kinds of NLS equations are the nonlinear dynamic basis for studying the propagation characteristics of solitons.[17,18] Among them, this type of equation has a first-order soliton solution in hyperbolic secant form, which maintains its shape and frequency spectrum unchanged during transmission. When the input energy of solitons is large, they will evolve into higher-order solitons, which in turn will involve the interaction between solitons.[19–22]

The interaction of solitons to some extent can affect the transmission quality of optical communication. So as to achieve distortionless transmission of ultra-short pulses and truly meet the requirements of high bandwidth and high transmission capacity, overcoming the influence of high-order dispersion and nonlinear effects has become a hot research topic.[23–25] In addition, in fiber optical communication, reducing the distance between adjacent solitons can increase the bit rate, but the interaction between solitons is more severe during this time. Therefore, it is necessary to study its action law and propose suppression methods to improve communication quality.[26–29]

In this article, based on the higher-order NLS equation, the transmission characteristics of femtosecond solitons in fibers and the interaction between them are analyzed and studied. The intuitive transmission form and interaction law of femtosecond solitons in fibers are obtained when considering different higher-order effects. The specific form of this transfer equation is shown as[30]

$$\mathrm{i}u_t + u_{xx} + \alpha|u|^2 u + \mathrm{i}\left(\gamma_1 u_{xxx} + \gamma_2|u|^2 u_x\right) = 0. \tag{1}$$

For Eq. (1), the modulation instability has been analysed.[30] However, the soliton interactions have not been reported before. We will discuss the influences of the higher-order effects on their interactions to optimize the transmission characteristics of optical solitons in optical communication systems.

In this paper, the bilinear form and the two soliton solutions of the equation are obtained. Then, the two soliton solutions are analyzed and the evolution characteristics of the two solitons under various conditions are studied. Finally, a summary is provided for this article.

†Corresponding author. E-mail: bz_mgl@163.com

© 2023 Chinese Physical Society and IOP Publishing Ltd

2. Two-soliton solutions

For Eq. (1), we first assume that $u(x,t)=h(x,t)/f(x,t)$, and the forms of $h(x,t)$ and $f(x,t)$ can be written as

$$\begin{aligned} h(x,t) &= \varepsilon h_1(x,t)+\varepsilon^3 h_3(x,t),\\ f(x,t) &= 1+\varepsilon^2 f_2(x,t)+\varepsilon^4 f_4(x,t). \end{aligned} \tag{2}$$

By substituting the assumed form of $u(x,t)$ into Eq. (1), we can obtain the bilinear forms of it as

$$\left[\mathrm{i}D_t+D_x^2+\mathrm{i}\gamma_1 D_x^3\right]h\cdot f=0,\ \ D_x^2 f\cdot f-\gamma_2/(3\gamma_1)|h|^2=0, \tag{3}$$

with the coefficient constraint relationship being $\alpha=\gamma_2/(3\gamma_1)$. Assuming that

$$\begin{aligned} h_1(x,t) &= A_1\mathrm{e}^{\phi_1}+A_2\mathrm{e}^{\phi_2},\\ f_2(x,t) &= B_1\mathrm{e}^{\phi_1+\phi_1^*}+B_2\mathrm{e}^{\phi_1+\phi_2^*}+B_3\mathrm{e}^{\phi_2+\phi_1^*}+B_4\mathrm{e}^{\phi_2+\phi_1^*}, \end{aligned} \tag{4}$$

among them,

$$\begin{aligned} \phi_1 &= (k_{11}+\mathrm{i}k_{12})x+(\omega_{11}t+\mathrm{i}\omega_{12})t+\xi_{11}+\mathrm{i}\xi_{12},\\ \phi_2 &= (k_{21}+\mathrm{i}k_{22})x+(\omega_{21}t+\mathrm{i}\omega_{22})t+\xi_{21}+\mathrm{i}\xi_{22}. \end{aligned} \tag{5}$$

Substituting them into Eq. (3), we can get that

$$\begin{aligned} \omega_{11} &= 3\gamma_1 k_{11}k_{12}^2-\gamma_1 k_{11}^3-2k_{11}k_{12},\\ \omega_{12} &= k_{11}^2-k_{12}^2-3\gamma_1 k_{11}^2 k_{12}+\gamma_1 k_{12}^3,\\ \omega_{21} &= 3\gamma_1 k_{21}k_{22}^2-\gamma_1 k_{21}^3-2k_{21}k_{22},\\ \omega_{22} &= k_{21}^2-k_{22}^2-3\gamma_1 k_{21}^2 k_{22}+\gamma_1 k_{22}^3, \end{aligned} \tag{6}$$

$$B_1=\frac{|A_1|^2\gamma_2}{24\gamma_1 k_{11}^2},\ \ B_2=\frac{A_1A_2^*\gamma_2}{6\gamma_1(k_{11}+\mathrm{i}k_{12}+k_{21}-\mathrm{i}k_{22})^2},$$

$$B_3=\frac{A_2A_1^*\gamma_2}{6\gamma_1(k_{11}-\mathrm{i}k_{12}+k_{21}+\mathrm{i}k_{22})^2},\ \ B_4=\frac{|A_2|^2\gamma_2}{24\gamma_1 k_{21}^2}. \tag{7}$$

For Eq. (2), we assume that

$$\begin{aligned} h_3(x,t) &= D_1\mathrm{e}^{\phi_1+\phi_2+\phi_1^*}+D_2\mathrm{e}^{\phi_1+\phi_2+\phi_2^*},\\ f_4(x,t) &= F_1\mathrm{e}^{\phi_1+\phi_2+\phi_1^*+\phi_2^*}, \end{aligned} \tag{8}$$

and then can obtain the coefficients as

$$\begin{aligned} D_1 &= \frac{|A_1|^2A_2\gamma_2(k_{11}+\mathrm{i}k_{12}-k_{21}-\mathrm{i}k_{22})^2}{24\gamma_1 k_{11}^2(k_{11}-\mathrm{i}k_{12}+k_{21}+\mathrm{i}k_{22})^2},\\ D_2 &= \frac{|A_2|^2A_1\gamma_2(k_{11}+\mathrm{i}k_{12}-k_{21}-\mathrm{i}k_{22})^2}{24\gamma_1 k_{21}^2(k_{11}+\mathrm{i}k_{12}+k_{21}-\mathrm{i}k_{22})^2},\\ F_1 &= \frac{|A_1|^2|A_2|^2\gamma_2^2((k_{11}-k_{21})^2+(k_{12}-k_{22})^2)^2}{576k_{11}^2k_{21}^2\gamma_1^2((k_{11}+k_{21})^2+(k_{12}-k_{22})^2)^2}. \end{aligned} \tag{9}$$

Thus, we set $\varepsilon=1$, and the two-soliton solutions can be presented as

$$u(x,t)=\frac{h_1(x,t)+h_3(x,t)}{1+f_2(x,t)+f_4(x,t)}. \tag{10}$$

3. Discussion

For solutions (10), we assume $k_{11}=1$, $k_{12}=0$, $k_{21}=2$, $k_{22}=0$, $\xi_{11}=1$, $\xi_{12}=0$, $\xi_{21}=3$, $\xi_{22}=0$, $A_1=1$, and $A_2=2$. At this point, we can obtain various cases of two soliton interactions when we take different values of γ_1 and γ_2, as shown in Fig. 1. When $\gamma_1=-0.53$ and $\gamma_2=-1.9$, we find that two optical solitons interact near $x=0$. During the interaction process, two optical solitons will be fused into one optical soliton, and the energy of this optical soliton will increase sharply. After the interaction, both optical solitons will undergo the phase shift and propagate along their original velocity and amplitude. By changing the values of γ_1 and γ_2, we can change the incident phase, propagation speed, and amplitude of optical solitons as shown in Figs. 1(b)–1(d). It should be emphasized that as the values of γ_1 and γ_2 increase in Fig. 1(d), the amplitude of optical solitons will significantly increase, and unstable phenomena may occur during the transmission due to high energy.

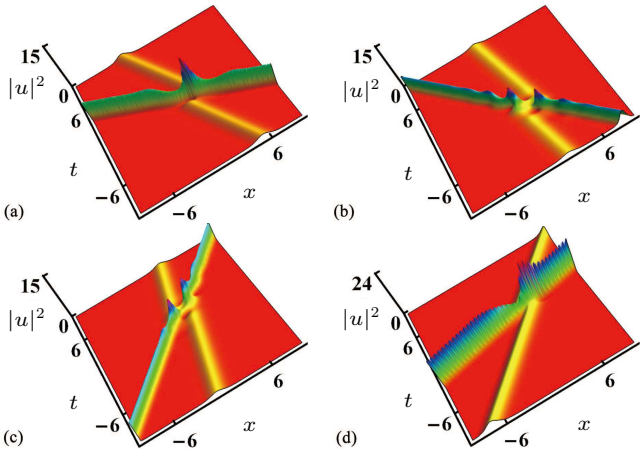


Fig. 1. The influence of γ_1 and γ_2 on two-soliton transmission. Parameters are $k_{11}=1$, $k_{12}=0$, $k_{21}=2$, $k_{22}=0$, $\xi_{11}=1$, $\xi_{12}=0$, $\xi_{21}=3$, $\xi_{22}=0$, $A_1=1$, $A_2=2$ with (a) $\gamma_1=-0.53$, $\gamma_2=-1.9$; (b) $\gamma_1=-0.22$, $\gamma_2=-1.3$; (c) $\gamma_1=0.25$, $\gamma_2=1.3$; (d) $\gamma_1=1$, $\gamma_2=1.4$.

Based on the values of Fig. 1(c), we discuss how different values of k_{11}, k_{12}, k_{21} and k_{22} affect the interaction of optical solitons. Unlike Fig. 1(c), in Fig. 2(a), the incident phase and amplitude of the optical soliton have changed when k_{11}, k_{12}, k_{21} and k_{22} take different values. By changing the values of k_{11}, k_{12}, k_{21} and k_{22}, we further change the interaction state of optical solitons. In Fig. 2(c), when the small amplitude optical soliton interacts with the large amplitude optical soliton, the modulation will occur at the interaction point, making the optical soliton transmission unstable. In Fig. 2(d), two optical solitons achieve parallel transmission, and even if their spacing is small, no interaction occurs.

In Fig. 3, based on the relevant parameter values in Fig. 2(b), we only change the values of ξ_{11}, ξ_{12}, ξ_{21} and ξ_{22}, and discuss their impact on the interaction of optical solitons. From Fig. 3, we can see that changing the values of them only has a certain impact on the phase of the optical solitons, and

does not affect the overall transmission state of the optical solitons. Based on Fig. 3(d), we simultaneously change the values of k_{11}, k_{21}, γ_1 and γ_2, and find that the interaction state of optical solitons has undergone a significant change. During the interaction process, optical solitons will oscillate. The severity of oscillation during interaction can be effectively controlled through those parameters. The closer the values of γ_1 and γ_2 are, the more intense their interaction is and the longer their interaction time is.

In Fig. 5, we mainly discuss the impact of changes in k_{11}, k_{21}, γ_1 and γ_2 on the interactions of optical solitons. We find that the interaction characteristics of optical solitons have not undergone fundamental changes. Their initial incident phase only has some small changes, but their intensity varies during the interaction, and the amplitude of optical solitons varies greatly.

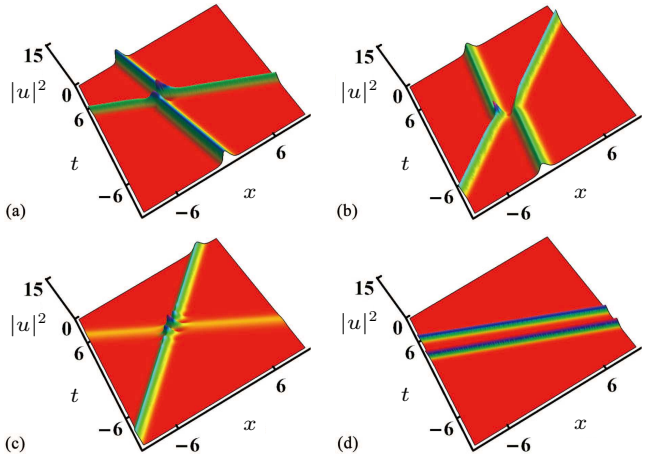


Fig. 2. The influence of k_{11}, k_{12}, k_{21} and k_{22} on two-soliton transmission. Parameters are $\xi_{11}=1$, $\xi_{12}=0$, $\xi_{21}=3$, $\xi_{22}=0$, $A_1=1$, $A_2=2$, $\gamma_1=-0.22$, $\gamma_2=-1.3$ with (a) $k_{11}=-1.7$, $k_{12}=-0.97$, $k_{21}=2$, $k_{22}=0.28$; (b) $k_{11}=-1.7$, $k_{12}=0.34$, $k_{21}=-1.9$, $k_{22}=0.81$; (c) $k_{11}=-0.75$, $k_{12}=-1.9$, $k_{21}=1.5$, $k_{22}=0.59$; (d) $k_{11}=-1.6$, $k_{12}=-1.4$, $k_{21}=-1.4$, $k_{22}=-1.5$.

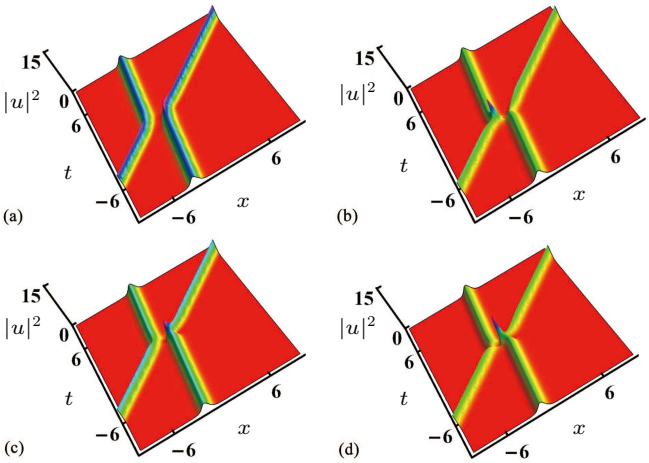


Fig. 3. The influence of ξ_{11}, ξ_{12}, ξ_{21} and ξ_{22} on two-soliton transmission. Parameters are $k_{11}=-1.7$, $k_{12}=0.34$, $k_{21}=-1.9$, $k_{22}=0.81$, $A_1=1$, $A_2=2$, $\gamma_1=-0.22$, $\gamma_2=-1.3$ with (a) $\xi_{11}=-2$, $\xi_{12}=-1.8$, $\xi_{21}=-2$, $\xi_{22}=0.8$; (b) $\xi_{11}=-0.41$, $\xi_{12}=-0.63$, $\xi_{21}=0.88$, $\xi_{22}=-0.91$; (c) $\xi_{11}=0.34$, $\xi_{12}=0$, $\xi_{21}=-1.8$, $\xi_{22}=1.7$; (d) $\xi_{11}=1$, $\xi_{12}=1.5$, $\xi_{21}=-0.16$, $\xi_{22}=-1.9$.

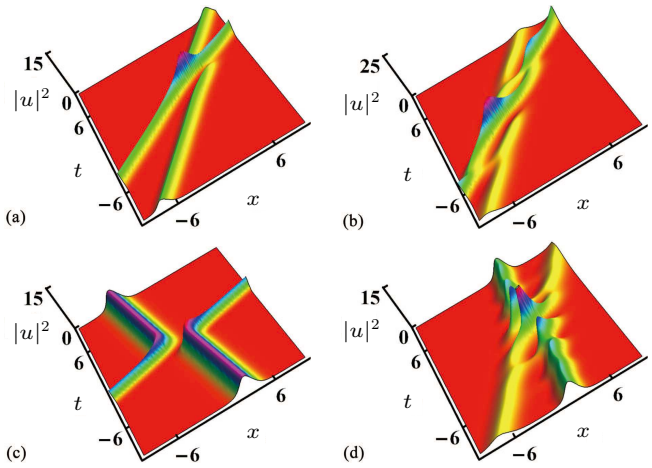


Fig. 4. The influence of k_{11}, k_{21}, γ_1 and γ_2 on two-soliton transmission. Parameters are $k_{12}=0.34$, $k_{22}=0.81$, $\xi_{11}=1$, $\xi_{12}=1.5$, $\xi_{21}=-0.16$, $\xi_{22}=-1.9$, $A_1=1$, $A_2=2$ with (a) $k_{11}=1.2$, $k_{21}=-1.1$, $\gamma_1=0.88$, $\gamma_2=1.9$; (b) $k_{11}=-0.75$, $k_{21}=-1.2$, $\gamma_1=0.84$, $\gamma_2=1$; (c) $k_{11}=0.91$, $k_{21}=-0.75$, $\gamma_1=-1.9$, $\gamma_2=-1.6$; (d) $k_{11}=0.63$, $k_{21}=1$, $\gamma_1=1.5$, $\gamma_2=1.4$.

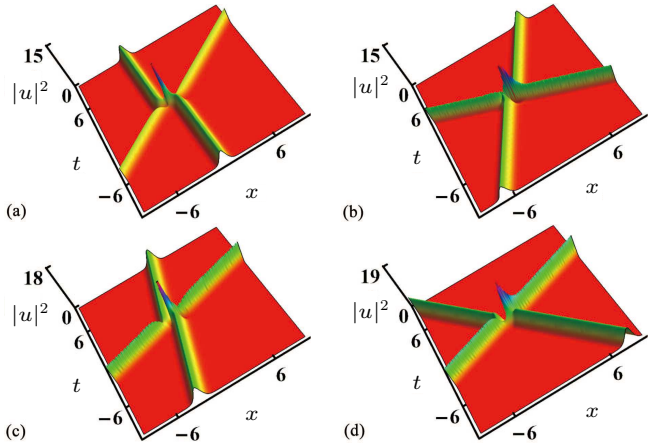


Fig. 5. The influence of k_{12}, k_{22}, γ_1 and γ_2 on two-soliton transmission. Parameters are $k_{11}=-1.7$, $k_{21}=-1.9$, $\xi_{11}=1$, $\xi_{12}=1.5$, $\xi_{21}=-0.16$, $\xi_{22}=-1.9$, $A_1=1$, $A_2=2$ with (a) $k_{12}=0.84$, $k_{22}=0.41$, $\gamma_1=-0.28$, $\gamma_2=-1.3$; (b) $k_{12}=-2$, $k_{22}=-0.38$, $\gamma_1=-0.5$, $\gamma_2=-2$; (c) $k_{12}=0.38$, $k_{22}=1.4$, $\gamma_1=-0.2$, $\gamma_2=-0.56$; (d) $k_{12}=-0.78$, $k_{22}=0.28$, $\gamma_1=0.63$, $\gamma_2=1.6$.

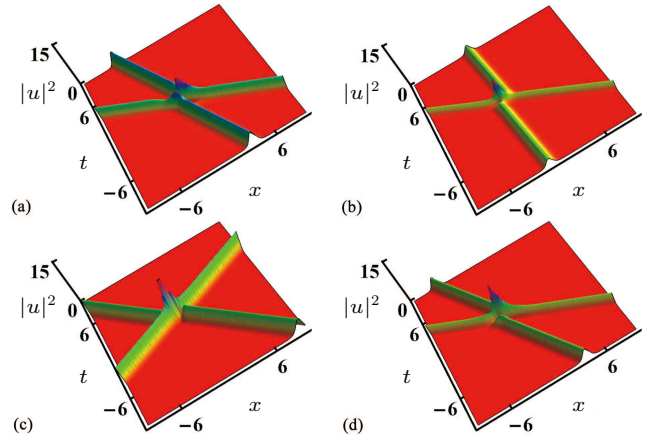


Fig. 6. The influence of ξ_{11}, ξ_{21}, γ_1 and γ_2 on two-soliton transmission. Parameters are $k_{11}=-1.7$, $k_{12}=-0.78$, $k_{21}=-1.9$, $k_{22}=0.28$, $\xi_{12}=1.5$, $\xi_{22}=-1.9$, $A_1=1$, $A_2=2$ with (a) $\xi_{11}=0$, $\xi_{21}=0.44$, $\gamma_1=-0.31$, $\gamma_2=-1.4$; (b) $\xi_{11}=1.3$, $\xi_{21}=-0.84$, $\gamma_1=-0.2$, $\gamma_2=-1.6$; (c) $\xi_{11}=0.44$, $\xi_{21}=1.2$, $\gamma_1=0.53$, $\gamma_2=1.8$; (d) $\xi_{11}=0.53$, $\xi_{21}=-0.81$, $\gamma_1=-0.34$, $\gamma_2=-1.9$.

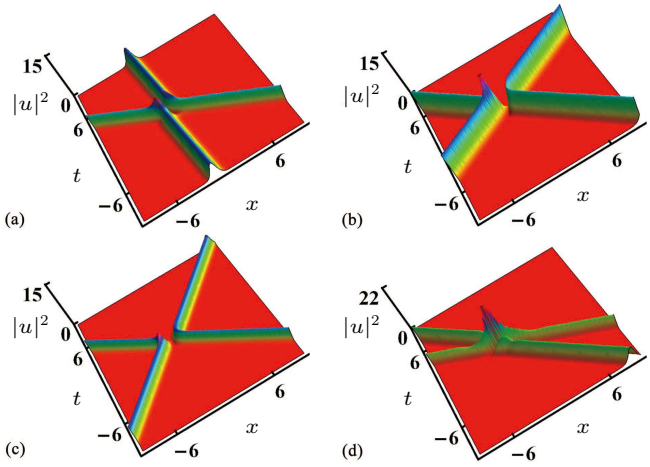


Fig. 7. The influence of ξ_{12}, ξ_{22}, γ_1 and γ_2 on two-soliton transmission. Parameters are $k_{11} = -1.7$, $k_{12} = -0.78$, $k_{21} = -1.9$, $k_{22} = 0.28$, $\xi_{11} = 0.53$, $\xi_{21} = -0.81$, $A_1 = 1$, $A_2 = 2$ with (a) $\xi_{12} = 1.3$, $\xi_{22} = 1.4$, $\gamma_1 = -0.2$, $\gamma_2 = -0.78$; (b) $\xi_{12} = 0.97$, $\xi_{22} = 0.8$, $\gamma_1 = 0.4$, $\gamma_2 = 0.9$; (c) $\xi_{12} = 0.41$, $\xi_{22} = 0.5$, $\gamma_1 = 0.13$, $\gamma_2 = 0.69$; (d) $\xi_{12} = 1.5$, $\xi_{22} = -1.5$, $\gamma_1 = -0.5$, $\gamma_2 = -1.5$.

In Fig. 6, we discuss how the values of ξ_{11}, ξ_{21}, γ_1 and γ_2 change the transmission characteristics of two optical solitons. We find that the interaction characteristics of optical solitons are similar to those in Fig. 5, except that the intensity of the interaction varies at different initial incident phases. In Fig. 7, the values of ξ_{12}, ξ_{22}, γ_1 and γ_2 are changed, and the interaction characteristics of optical solitons are different from those in Fig. 6. Two optical solitons exhibit repulsive properties at the interaction point in Figs. 7(b) and 7(c). Thus, we can find that γ_1 and γ_2 can influence the soliton incident phase, propagation speed and amplitude. k_{11}, k_{12}, k_{21} and k_{22} can change the interaction state, and the modulation and parallel transmission can occur. ξ_{11}, ξ_{12}, ξ_{21} and ξ_{22} have a certain impact on the soliton phases, and do not affect the overall soliton transmission state.

4. Conclusion

The research work in this paper mainly bases on the high-order NLS Eq. (1). By solving the equation analytically, the transmission characteristics of a single femtosecond soliton in the optical fiber and the interaction of two optical solitons have been studied under the condition of considering various high-order effects. The transmission characteristics and laws of their interactions in optical fibers have been analytically studied under different initial inputs and different high-order effects. The changes in the values of γ_1 and γ_2 have a significant impact on the interaction of optical solitons, leading to significant changes in the phase, velocity, and amplitude of optical solitons. At the same time, at the interaction point, different initial values and different higher-order effects have led to different degrees of interaction, and have also led to attraction and repulsion at the interaction point. The conclusion of this paper will provide a theoretical reference for further obtaining ultra-short and ultra-high energy optical solitons.

Acknowledgments

Project supported by the Scientific Research Foundation of Weifang University of Science and Technology (Grant No. KJRC2022002), the Shandong Province Higher Educational Science and Technology Program (Grant No. J18KB108), and the Research start-up fees for doctoral degree holders and senior professional title holders with master's degrees of Binzhou University (Grant No. 2022Y12).

References

[1] Yan Y Y and Liu W J 2021 *Chin. Phys. Lett.* **38** 094201
[2] Zhou Q 2022 *Chin. Phys. Lett.* **39** 010501
[3] Wang S B, Ma G L, Zhang X and Zhu D Y 2022 *Chin. Phys. Lett.* **39** 114202
[4] Zhou Q, Zhong Y, Triki H, Sun Y Z, Xu S L, Liu W J and Biswas A 2022 *Chin. Phys. Lett.* **39** 044202
[5] Zhang X M, Qin Y H, Ling L M and Zhao L C 2021 *Chin. Phys. Lett.* **38** 090201
[6] Wang T Y, Zhou Q and Liu W J 2022 *Chin. Phys. B* **31** 020501
[7] Li J, Yang Z J and Zhang S M 2023 *Appl. Math. Lett.* **140** 108588
[8] Song L M, Yang Z J, Li X L and Zhang S M 2020 *Appl. Math. Lett.* **102** 106114
[9] Zou Z F and Guo R 2023 *Commun. Nonlinear Sci.* **124** 107316
[10] Shen S, Yang Z J, Li X L and Zhang S M 2021 *Commun. Nonlinear Sci.* **103** 106005
[11] Han Y, Gao B, Huo J Y, Ma C Y, Wu G, Li Y Y, Chen B K, Guo Y B and Liu L 2022 *Chin. Phys. B* **31** 074208
[12] Wang S B, Zhang X, Ma G L and Zhu D Y 2023 *Chin. Phys. B* **32** 030506
[13] Liu X Y, Zhang H X, Yan Y Y and Liu W J 2023 *Phys. Lett. A* **457** 128568
[14] Wang H T, Li X, Zhou Q and Liu W J 2023 *Chaos Soliton. Fract.* **166** 112924
[15] Yu W T, Liu W J and Zhang H X 2022 *Chaos Soliton. Fract.* **159** 112132
[16] Wang H T, Zhou Q and Liu W J 2022 *J. Adv. Res.* **38** 179
[17] Yu W T, Luan Z T, Zhang H X and Liu W J 2022 *Chaos Soliton. Fract.* **157** 111816
[18] Liu X Y, Zhang H X and Liu W J 2022 *Appl. Math. Model.* **102** 305
[19] Ma G L, Zhou Q, Yu W T, Biswas A and Liu W J 2021 *Nonlinear Dyn.* **106** 2509
[20] Wang H T, Zhou Q, Biswas A and Liu W J 2021 *Nonlinear Dyn.* **106** 841
[21] Yu W T, Zhang H X, Wazwaz A M and Liu W J 2021 *Results Phys.* **28** 104618
[22] Triki H, Zhou Q, Biswas A, Liu W J, Yildirim Y, Alshehri H M and Belic M R 2021 *Phys. Lett. A* **417** 127698
[23] Veni S S and Rajan M S M 2021 *Chaos Soltion. Fract.* **152** 111390
[24] Wang L L, Luan Z T, Zhou Q, Biswas A, Alzahrani A K and Liu W J 2021 *Nonlinear Dyn.* **104** 629
[25] Liu X Y, Zhou Q, Biswas A, Alzahrani A K and Liu W J 2020 *J. Adv. Res.* **24** 167
[26] Wang L L and Liu W J 2020 *Chin. Phys. B* **29** 070502
[27] Chen J G, Luan Z T, Zhou Q, Alzahrani A K, Biswas A and Liu W J 2020 *Nonlinear Dyn.* **100** 2817
[28] Yan Y Y, Liu W J, Zhou Q and Biswas A 2020 *Nonlinear Dyn.* **99** 1313
[29] Yan Y Y and Liu W J 2019 *Appl. Math. Lett.* **98** 171
[30] Sonia A, Jamshad A, Shafqat U R, Shahzad S and Asghar A 2023 *Opt. Quantum Electron.* **55** 450