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Symmetric Brownian motor subjected to Lévy noise

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In the past few years, attention has mainly been focused on the symmetric Brownian motor (BM) with Gaussian noises, whose current and energy conversion efficiency are very low. Here, we investigate the operating performance of the symmetric BM subjected to Lévy noise. Through numerical simulations, it is found that the operating performance of the motor can be greatly improved in asymmetric Lévy noise. Without any load, the Lévy noises with smaller stable indexes can let the motor give rise to a much greater current. With a load, the energy conversion efficiency of the motor can be enhanced by adjusting the stable indexes of the Lévy noises with symmetry breaking. The results of this research are of great significance for opening up BM's intrinsic physical mechanism and promoting the development of nanotechnology.

Keywords: symmetric Brownian motor, average velocity, energy conversion efficiency, Lévy noise

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1. Introduction

Much attention has been paid to the energy problem to ensure continued development.[1] Since Smoluchowski and Feynman's Gedanken experiment about converting thermal energy into useful work,[2] Brownian motors (BMs) have been intensively investigated due to their wide range of applications in bioengineering[3] and nanotechnology, zeolite,[4] cerebral cell,[5] environmental fractal dimension,[6] and so on. A symmetric BM based on the Feynman ratchet has been proposed,[7,8] which performs like a Carnot engine. Although Gaussian white noise is involved in many aspects, such as binocular rivalry,[9–11] transport of coupled BM,[12] simulated molecular dynamics,[13] stochastic gene transcriptional dynamics[14] and so on, BMs work with small current and low energy conversion efficiency due to Gaussian white noise.

In the action of a periodic force, particles in periodic potentials behave with anomalous transport behaviors. There are some instances where the periodic force induces multiple current reversals and absolute negative mobility in periodic potential systems.[15–20] Furthermore, noise-induced phenomena have prompted researchers to investigate stochastic systems with anomalous diffusions in recent years.[21–23] Continuous-time random walk, fractional Fokker–Plank equation and Lévy noise are frequently used to describe systems with anomalous diffusions.[24–27] Stochastic Lévy processes, which serve as paradigms for description of many unusual transport processes, are characterized by the mean squared displacement of their state variables $\langle(x(t)-\langle x(t)\rangle)^2\rangle \propto t^{\nu}$ ($\nu = 2/\alpha$), where α is the stable index. The motion of a free Brownian particle driven by Lévy noise belongs to superdiffusion for $0 < \alpha < 2$, subdiffusion for $\alpha > 2$, and normal diffusion for $\alpha = 2$.[28] The Lévy noise with anomalous diffusion involves many complex dynamical behaviors. Lévy noise causes multiple absolute negative mobilities in periodic potential systems.[29,30] For ratchet systems, symmetry breaking of the Lévy noise also induces multiple current reversals.[31,32] During the region of superdiffusion, there is a current maximum in the asymmetric flashing systems.[33] The authors of Ref. [34] investigated heat conduction in low-dimensional nonlinear lattices subjected to Lévy noise, and found that the noise generates anomalous thermal conduction. In particular, Lévy noise induces a divergent thermal conductivity with the system size for $\alpha < 2$ or a convergent thermal conductivity (or an insulator in the limit case) for $\alpha > 2$. However, the characteristic dynamical behaviors caused by Lévy noise have not been completely understood. In this paper, it will be seen that the current and energy conversion efficiency of the symmetric BM can be greatly enhanced in the case of Lévy noise or the Lévy noise can improve the operating performance of the symmetric BM.

We aim to investigate the effects of the Lévy noise on average velocity of the symmetric BM without load, its energy conversion efficiency in the case of load, and the measures for optimizing its operating performance. The rest of this paper is structured as follows. In Section 2, the model and theoretical analysis are provided. In Section 3, the results and discussion are presented. Finally, the conclusion is given in Section 4.

2. Model and theoretical analysis

In this section, we will consider the symmetric BM subjected to Lévy noises, which consists of two compartments connected by a spring, each of which contains blades and wheels, see Fig. 1 from Ref. [7]. The blades are driven by two independent temperature sources. The BM is characterized by a Carrot heat engine with two constant temperature heat baths. In this sense, the BM plays a role in transforming thermal energy into useful work. The transport of particle and

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energy in biological organisms and nanomachines can be realized via the BM. Previous research regarded the two heat baths as the Gaussian white noises, which resulted in a extremely low energy conversion efficiency. In contrast, here they are considered to be two independent Lévy noises, and therefore the case can be considerably improved. The stochastic differential equations describing the symmetric BM read

$$\frac{\partial\theta_1}{\mathrm{d}t} = -\partial_{\theta_1}V + \xi_1(t), \tag{1}$$

$$\frac{\partial\theta_2}{\mathrm{d}t} = -\partial_{\theta_2}V + \xi_2(t), \tag{2}$$

where θ_i $(i=1,2)$ is the state variable, and V is the potential of the system with the form

$$V(\theta_1,\theta_2) = \tau\theta_1 + V_{\mathrm{R}}(\theta_1) + V_{\mathrm{R}}(-\theta_2) + \frac{1}{2}k(\theta_1+\phi-\theta_2)^2, \tag{3}$$

where τ is the load of the BM, k is the spring constant, and ϕ is the phase shift. The ratchet potential $V_{\mathrm{R}}(\theta)$ is given by

$$V_{\mathrm{R}}(\theta) = -\frac{V_0}{2.23}v(\theta), \tag{4}$$

with

$$v(\theta) = \sin(d\theta) + 0.275\sin(2d\theta) + 0.0533\sin(3d\theta), \tag{5}$$

where V_0 controls the height of the potential, and d is the number of teeth per cycle 2π.

The $\xi_i(t)$ $(i=1,2)$ in Eqs. (1) and (2) are the Lévy noises. There are many molecular BMs in biological systems, where the noises are usually assumed to be Lévy noises due to their complexity.[7,15,22] The statistical properties of Lévy noise can be specified according to the following characteristic function:

$$\psi(k) = \exp\left[-D^{\alpha}|k|^{\alpha}\left(1-\mathrm{i}\beta\,\mathrm{sign}(k)\tan\frac{\pi\alpha}{2}\right)\right] \tag{6}$$

for $\alpha\neq 1$, while

$$\psi(k) = \exp\left[-D|k|\left(1+\mathrm{i}\beta\frac{2}{\pi}\,\mathrm{sign}(k)\ln|k|\right)\right] \tag{7}$$

for $\alpha = 1$, where D, α and β are the noise intensity, the stability index, and the symmetry parameter, respectively. The three main parameters vary within the regimes $\alpha\in(0,2)$, $\beta\in[-1,1]$, and $D\in(0,\infty)$.

Thus, the probability distribution L of the Lévy noise is given by

$$L_{\alpha,\beta}(D,\xi) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}k\xi}\psi(k)\,\mathrm{d}k, \tag{8}$$

where ξ represents the Lévy random variable. The δ-associated Gaussian white noise occurs only when $\alpha=2$, which is a special case of the Lévy noise.

It is difficult to find an analytical expression of the system. Therefore, we will use numerical calculations to solve Eqs. (1) and (2). The numerical algorithm is obtained with a discrete time step h

$$\theta_1(t+h) = \theta_1(t) - h\partial_{\theta_1}V + h^{\frac{1}{\alpha_1}}\xi_1, \tag{9}$$

$$\theta_2(t+h) = \theta_2(t) - h\partial_{\theta_2}V + h^{\frac{1}{\alpha_2}}\xi_2, \tag{10}$$

where α_i $(i=1,2)$ is the stability index of the Lévy noise and ξ_i $(i=1,2)$ is the Lévy random variable.

The time step h is chosen to be so small (e.g., $h=0.001$) that the errors brought by the above Euler algorithm can be accepted. The random variable ξ_i corresponding to the characteristic functions (6) and (7) can be generated using the Janicki–Weron algorithm. For $\alpha_i\neq 1$, ξ_i can be realized by the following method:[28]

$$\xi_i = D_{\alpha_i,\beta,D_i}\frac{\sin(\alpha_i(V+C_{\alpha_i,\beta}))}{(\cos(V))^{\frac{1}{\alpha_i}}} \times\left[\frac{\cos(V-\alpha_i(V+C_{\alpha_i,\beta}))}{W}\right]^{\frac{1-\alpha_i}{\alpha_i}}, \tag{11}$$

the constants C and D are given by

$$C_{\alpha_i,\beta} = \frac{\arctan(\beta(\tan(\alpha_i\pi/2)))}{\alpha_i},$$
$$D_{\alpha_i,\beta,D_i} = D_i[\cos(\arctan(\beta(\tan(\alpha_i\pi/2))))]^{-1/\alpha_i}. \tag{12}$$

For $\alpha_i = 1$,

$$\xi_i = \frac{2D_i}{\pi}\left[\left(\frac{\pi}{2}+\beta V\right)\tan(V) - \beta\ln\left(\frac{\frac{\pi}{2}W\cos(V)}{\frac{\pi}{2}+\beta V}\right)\right]. \tag{13}$$

In these equations, V and W denote independent random variables; namely, V being uniformly distributed in the interval $(-\pi/2,\pi/2)$ and W being exponentially distributed with a unit mean. Here, let $\beta_1=\beta_2=\beta$, and the noise is symmetric for $\beta=0$, while it is asymmetric for $\beta\neq 0$. D_i is the intensity of the Lévy noise.

Thus, the mean velocity (or current) of the BM can be obtained numerically via Eqs. (1)–(12) according to its definition

$$\langle v\rangle = \lim_{t\longrightarrow\infty}[\theta_1(t)-\theta_1(0)]/t, \tag{14}$$

where $\langle\,\rangle$ denotes the ensemble averaging. In the hypothesis of ergodicity, the time averaging is equal to the ensemble averaging. In the process of computing $\langle v\rangle$, 500 different trajectories were run, and each trajectory was evolved for $t=2\times10^4$.

The physical mechanism of the BM is similar to that of the Carrot heat engine, where its input total energy from the compartment with a higher noise intensity is passed to the other compartment with a lower noise intensity through the coupling spring. Some of them are then used to lift the load τ or do useful work, and the others are dissipated. The useful work is given by

$$\langle W\rangle = \tau\langle\theta_1(t_{\mathrm{f}})-\theta_1(t_{\mathrm{i}})\rangle, \tag{15}$$

where $t_{\rm i}$ and $t_{\rm f}$ are the initial time and the final time, respectively. The input total energy can be obtained by

$$R = \int_{t_{\rm i}}^{t_{\rm f}} \left[\frac{\partial V(\theta_1(t),\theta_2(t))}{\partial \theta_2}\right] {\rm d}\theta_2(t) = \int_{t_{\rm i}}^{t_{\rm f}} \left[-k(\theta_1(t)+\phi-\theta_2(t))+V_{\rm R}'(-\theta_2(t))\right] {\rm d}\theta_2(t). \quad (16)$$

Therefore, energy conversion efficiency is expressed by

$$\eta = \frac{\langle W\rangle}{\langle R\rangle} = \frac{\tau\langle\theta_1((t_{\rm f}))-\theta_1((t_{\rm i}))\rangle}{-k\left\langle\int_{t_{\rm i}}^{t_{\rm f}}[\theta_1(t)+\phi-\theta_2(t)]\,{\rm d}\theta_2(t)\right\rangle}, \quad (17)$$

which can reflect the operating performance of the BM. By adjusting the parameters of the Lévy noise, the energy conversion efficiency can be maximized to optimize the BM.

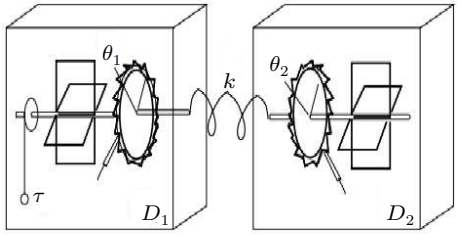


Fig. 1. Schematic diagram of a symmetric Brownian motor.

3. Results and discussion

The current and the energy conversion efficiency of the symmetric BM can be computed numerically at different parameters of the Lévy noise via Eqs. (1)–(17). The results are plotted in Figs. 2–6. The operating performance of the BM will be discussed in the following two cases.

Let us first discuss the case where the load is not suspended under the BM, i.e., $\tau = 0$. Figure 2 shows the dependence of the average velocity $\langle v\rangle$ (or current) on the phase shift ϕ at different symmetry parameters: $\beta = -1, 0, 1$. The other parameters are $\alpha_1 = \alpha_2 = 1.2$, $V_0 = 3$, $d = 4$, $k = 100$, $D_1 = 1$, and $D_2/D_1 = 2$. Here, let k take a great value (e.g., 100), which means that there is a rigid coupling between the two compartments of the BM. From Fig. 2, the current oscillates periodically with the phase shift and its amplitude is greatly affected by the symmetry parameter of the Lévy noise. As $\beta = 0, -1$, the $\langle v\rangle$ is both very small and negative. This means that the system is not used to lift the load and does not play a role of BM for $\alpha_1 = \alpha_2 = 1.2$. However, for $\beta = 1$, the amplitude of the $\langle v\rangle$ is the greatest and its direction is always greater than zero. Therefore, the asymmetric Lévy noise with $\beta = 1$ can enhance the current of the BM for $\alpha_1 = \alpha_2 = 1.2$ if the phase shift takes an optimal value. In what follows, the noise symmetry breaking with $\beta = 1$ will be addressed.

The factors causing symmetry breaking in the system are the ratchet potential V, the noise intensity difference, the load, and the noise symmetry parameter β. Therefore, the direction of the system's current is mainly determined by a combination of them. It is generally thought that adjusting the phase shift ϕ can let the $\langle v\rangle$ be positive for $D_2/D_1 = 2$ and $\beta = 0$. Nevertheless, it is worth noting that in Fig. 2 the average velocity for $\beta = 0$ is always negative, even though $D_2/D_1 > 1$. This means the stable state index of the noise also can change the symmetry of the system, and induce current reversal of the system.

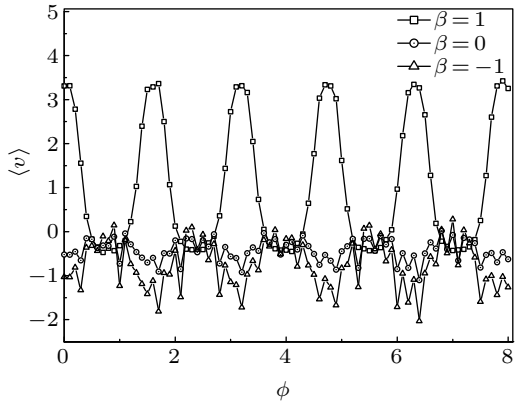


Fig. 2. The dependence of the average velocity $\langle v\rangle$ on the phase shift ϕ at different symmetry parameters: $\beta = -1, 0, 1$. The other parameters are $\alpha_1 = \alpha_2 = 1.2$, $V_0 = 3$, $d = 4$, $k = 100$, $D_2/D_1 = 2$, $D_1 = 1$, and $\tau = 0$.

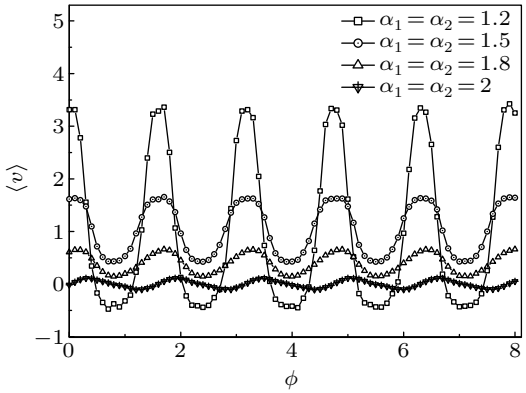


Fig. 3. The $\langle v\rangle$ versus ϕ at different stability indexes: $\alpha_1 = \alpha_2 = 1.2$, 1.5, 1.8, 2, with $\beta = 1$, $\phi = 1.7$. The other parameters are the same as those in Fig. 2.

To illustrate the effect of the stability indexes of the Lévy noises, the $\langle v\rangle$ versus ϕ at different stability indexes, i.e., $\alpha_1 = \alpha_2 = 1.2, 1.5, 1.8, 2$, in the asymmetric case $\beta = 1$ is shown in Fig. 3. In the computing process, it was found that when the stability index $\alpha_i < 1$ $(i = 1, 2)$, the state variables of the system fluctuate very seriously and have heavier tails.[35] Thus, the region of $\alpha_i \geq 1$ will be discussed here. Figure 3 shows that the smaller the stability indexes, the greater the amplitude of the $\langle v\rangle$ versus ϕ. For smaller stability indexes (e.g., $\alpha_1 = \alpha_2 = 1.2$), choosing an optimal phase shift can make the BM possess a greater positive current. It is well-known that the Lévy noise causes superdiffusion of free overdamped particle for $\alpha_i \in (0, 2)$, subdiffusion for $\alpha_i \in (2, \infty)$, and normal diffusion for $\alpha_i = 2$.[28] In other words, the Lévy noises with anomalous diffusions can enhance the current of the BM.

With the increment of α_i (e.g., $\alpha_1 = \alpha_2 = 2$), the $\langle v \rangle$ oscillates slightly near the horizontal axis of ϕ. It is very difficult to pick out contribution of the asymmetry of the Lévy noise $\beta = 1$ from the curve of $\alpha_1 = \alpha_2 = 2$ in Fig. 3. Thus, it is deduced that the amplitude of $\langle v \rangle$ versus ϕ for $\alpha_i > 2$ will be smaller than that for $\alpha_i = 2$. Therefore, an interesting conclusion can be made: the Lévy noises with smaller stable indexes in the serious symmetry breaking $\beta = 1$ can enhance the positive current of the BM.

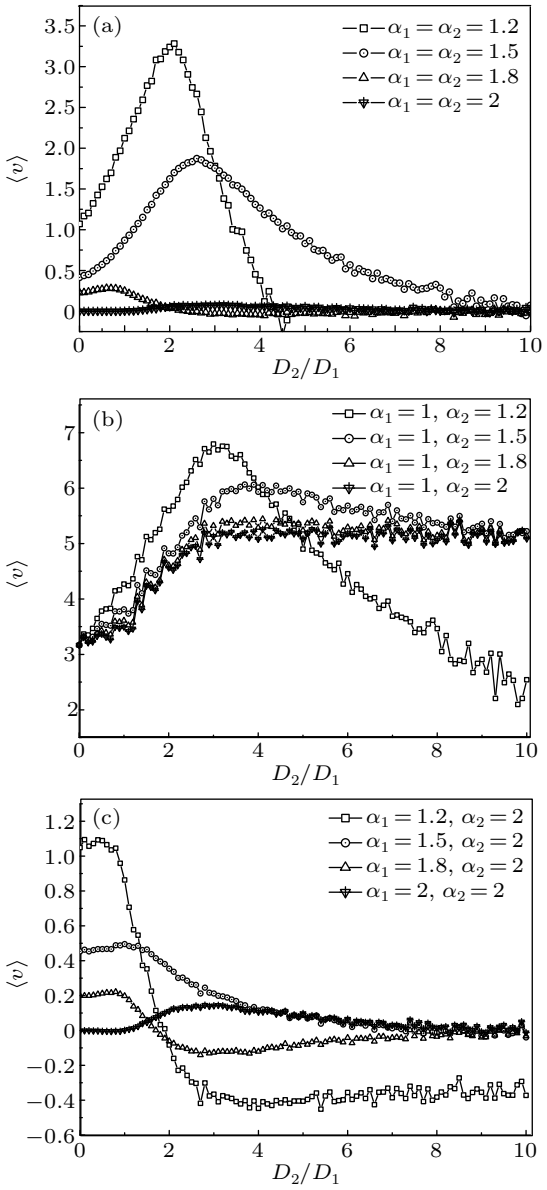


Fig. 4. The $\langle v \rangle$ versus D_2/D_1 at different stability indexes: $\alpha_1 = \alpha_2 = 1.2$, 1.5, 1.8, 2 for (a); $\alpha_1 = 1, \alpha_2 = 1.2$, 1.5, 1.8, 2 for (b); $\alpha_1 = 1.2$, 1.5, 1.8, 2, $\alpha_2 = 2$ for (c). The other parameters are the same as those in Fig. 3.

Since the stability indexes of the two noise sources are equal to each other, the $\langle v \rangle$ versus the ratio of noise intensity D_2/D_1 is drawn in Fig. 4(a). From Fig. 4(a) we can see that the stability indexes also affect the curves of $\langle v \rangle$ versus D_2/D_1. For $\alpha_1 = \alpha_2 = 2$, i.e., the noises with normal diffusions, $\langle v \rangle$ versus D_2/D_1 exhibits a very low peak. This result agrees with that in Ref. [7]. With the decrement of the stability indexes (e.g., $\alpha_1 = \alpha_2 = 1.5$), the peak becomes higher and gradually shifts to the negative direction of the D_2/D_1 axis. The $\langle v \rangle$ is still greater than zero as $D_2/D_1 \leq 10$ due to the symmetry breakings by the phase shift and the noise symmetry parameter $\beta = 1$. The left-hand shift of the peak happens because the smaller stability indexes can bring about greater fluctuations in the system. As the stability indexes are decreased further, e.g., $\alpha_1 = \alpha_2 = 1.2$, the direction of $\langle v \rangle$ changes from positive to negative with D_2/D_1 increasing, namely, a current reversal phenomenon. It is inferred that the reversal point becomes smaller with the decrement of the stability indexes. The current reversal phenomenon should be attributed to nonlinearity of the real noise intensity D_{α_i,β,D_i} in Eq. (12) with α_i.

The symmetry breaking between the stability indexes of the two Lévy noises also influences the current of the system. Figure 4(b) shows the curves of $\langle v \rangle$ versus D_2/D_1 at different $\alpha_2 = 1.2, 1.5, 1.8, 2$ with the fixed $\alpha_1 = 1$. Figure 4(b) shows that the peak of $\langle v \rangle$ versus D_2/D_1 gradually goes up with α_2 decreasing, and its value is much greater than that in the case of $\alpha_1 = \alpha_2$. The Lévy noise for $\alpha_i = 1$ causes ballistic diffusion of free overdamped particles. In other words, the Lévy noises with ballistic diffusions can enhance the positive current of the BM to a large degree. Let $\alpha_2 = 2$ unchanged (see Fig. 4(c)). However, no matter how much α_1 takes, the positive current of the system is much smaller than that in Fig. 4(b). This means that the Lévy noises with $\alpha_i = 2$ can weaken the positive current of the system.

The case of $\tau \neq 0$ will be addressed next. Figure 5 shows the dependence of $\langle v \rangle$ on the load τ at some stability indexes: $\alpha_1 = \alpha_2 = 1.2, 1.5, 1.8, 2$. From Fig. 5, we can see that the $\langle v \rangle$ decreases as the load increases. The lighter load can only be elevated by the BM subjected to the Lévy noises with normal diffusions $\alpha_i = 2$, while the heavier load can only be elevated by the BM with the Lévy noises with $\alpha_i < 2$. Thus, energy conversion efficiency of the BM driven by the Lévy noises with smaller stable indexes is obviously higher than that of the BM by the Lévy noises with $\alpha_i = 2$, see Fig. 6. In particular, adjusting the stable indexes of Lévy noises can enhance the energy conversion efficiency of the symmetric BM. Note that as $D_2/D_1 = 2$, the curve of the efficiency η with τ for $\alpha_1 = \alpha_2 = 1.2$ is much lower than that for $\alpha_1 = \alpha_2 = 1.5$. However, the $\langle v \rangle$ for $\alpha_1 = \alpha_2 = 1.2$ is greater than that for $\alpha_1 = \alpha_2 = 1.5$ as $\tau = 0$, see Fig. 4(a). Therefore, in the region of superdiffusion, the Lévy noises with optimal stability indexes can let the BM achieve the highest energy conversion efficiency. In addition, it is deduced that ballistic diffusion can make the symmetric BM possess greater current but lower energy conversion efficiency.

In the following, the intrinsic physical mechanism of the symmetric BM will be discussed. Compared with the system considered in Ref. [7], the difference is that the noise sources

used here are two Lévy noises, not Gaussian white noises. The last term $h^{1/\alpha_i}\eta_i(t)$ in Eqs. (9) and (10), which has the same expression as the free Brownian particle by the Lévy noise, plays a very important role in determining the dynamical behaviors of the system. The stable index of the Lévy noise is smaller than 2, the free Brownian particle will make a superdiffusive motion, while it will make a normal diffusive motion for $\alpha_i = 2$. This is equivalent to increments of the noise intensities D_i in Eq. (12) in contrast to the case of $\alpha_i = 2$. Therefore, the greater current of the BM can only be achieved in the case of symmetry breaking (e.g., $\beta = 1$). Of course, the symmetry breaking and the smaller stable indexes also enhance the energy conversion efficiency of the BM. The nonlinearity of the energy conversion efficiency with the stable indexes in Eq. (17) brings about the nonmonotonicity of η. It is worth noting that the very low efficiencies in Fig. 6 result from the ratchet potential described by Eq. (3). However, the efficiencies are close to 1 if the potential of system is chosen as a flashing ratchet potential.

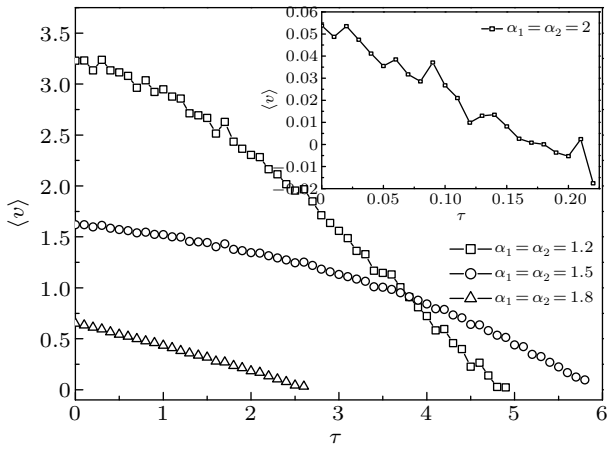


Fig. 5. The $\langle v\rangle$ versus the load τ at different stability indexes: $\alpha_1 = \alpha_2 =$ 1.2, 1.5, 1.8, 2 with $D_2/D_1 = 2$. The other parameters are the same as those in Fig. 3.

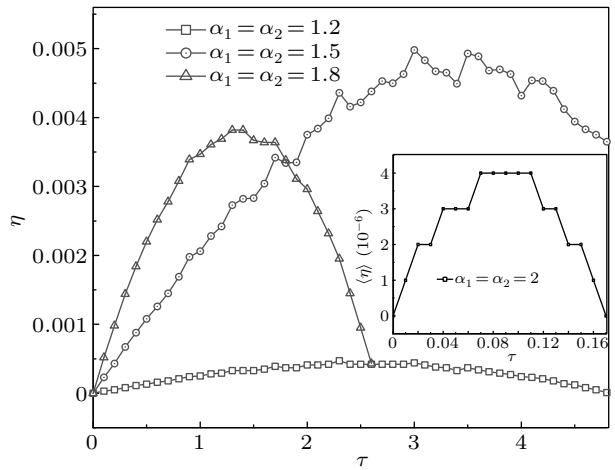


Fig. 6. The energy conversion efficiency η versus τ at different stability indexes: $\alpha_1 = \alpha_2 = 1.2, 1.5, 1.8, 2$ with $D_2/D_1 = 2$. The other parameters are the same as those in Fig. 5.

4. Conclusion

Most researchers previously studied the operating performance of the BM driven by the Gaussian noises. In this paper, we have investigated effects of the Lévy noises on the current and the energy conversion efficiency of the symmetric BM. Our results indicate that the BM subjected to the Lévy noises with smaller stable indexes can work in the greater current without the load and the higher energy conversion efficiency with the load than that subjected to the Lévy noises with $\alpha_i = 2$. This means that the noises with superdiffusions can better improve the operating performance of the BM. However, the prerequisite is that the Lévy noises must be those with symmetry breaking. For the ratchet potential of the system $V_R(\theta)$ (see Eq. (4)), the noise symmetry parameter β takes 1, while -1 for $-V_R(\theta)$. Since $\alpha_1 = \alpha_2 = 2$ (i.e., the Lévy noises with normal diffusion), the current of the BM is not sensitive to the noise symmetry parameter β. Yet, for the Lévy noises with superdiffusions, the symmetry breaking can let the BM be the best operating state. In symmetry breaking, the Lévy noises with $\alpha_i = 1$ can make the BM achieve a greater current but it does not acquire a better energy conversion efficiency. In addition, the nonlinearity of the real noise intensities of the Lévy noises with their stability indexes can induce the current reversal of the system about D_2/D_1. Of course, for the Lévy noises with $\alpha_i > 2$ the BM will lose its intrinsic essence due to the smallest fluctuations in it, and it has no potential research significance. Experimentally, we let a Gaussian white noise drive an RLC circuit and obtain a harmonic noise. Then, applying the harmonic noise to a system with ratchet periodic potential can generate an abnormal diffusion with diffusive index ν. Thus, according to $\alpha = 2/\nu$, control of the stability index is realized by adjusting the parameters of the RLC circuit.

The BM usually exists in some nanoscale machines and organisms. Greater current and higher energy conversion efficiency of the BM are very important for them to perform normally. Our research results not only have crucial significance in understanding performance mechanisms of nanomachines and organisms but they also provide a theoretical basis for experimentally improving the performance of the symmetric BM.

Acknowledgments

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References

[1] Maldovan M 2013 *Nature* **503** 209
[2] Smoluchowski M V 1912 *Phys. Z.* **13** 1069

[3] Vorotnikov D 2014 *J. Math. Bio.* **68** 1677
[4] Huang P, Yin Z J, Tian Y, Yang J, Zhong W, Li C Z, Lian C, Li Y and Liu H L 2021 *Chem. Eng. Sci.* **246** 116996
[5] Xiao F R, Hrabe J and Hrabetova S 2015 *J. Biophys.* **108** 2384
[6] Zhokh A, Trypolskyia A and Strizhak P 2018 *Chem. Phys.* **503** 71
[7] Gomez-Marin A and Sancho J M 2005 *Phys. Rev. E* **71** 021101
[8] Nie L R and Mei D C 2009 *Phys. Lett. A* **373** 3816
[9] Chen R Y, Xiong Y, Li Z K, He Z F, Hou F and Zhou J W 2022 *Chaos, Solitons and Fractals* **159** 112148
[10] Chen R Y, Xiong Y, Zhuge S Y, Li Z K, Chen Q T, He Z F, Dingqiang Wu D Q, Hou F and Zhou J W 2023 *Chaos, Solitons and Fractals* **172** 113564
[11] Chen R Y, Xiong Y, Jiang L L, He Z F and Zhou J W 2022 *Chaos, Solitons and Fractals* **152** 111439
[12] Wu W X, Zheng Z G, Song Y L, Han Y R, Sun Z C and Li C P 2022 *Chin. Phys. B* **29** 090503
[13] Wang L Y, Xu X P, Li Z G and Qian T Z 2020 *Chin. Phys. B* **29** 090501
[14] Feng Y L, Dong J M and Tang X L 2016 *Chin. Phys. Lett.* **33** 108701
[15] Chen R Y and Jiang L L 2020 *Chaos* **30** 123140
[16] Chen R Y, Wang C J and He Z F 2019 *Chaos, Solitons and Fractals* **126** 116
[17] Kong L W, Wan R Z and Fang H P 2016 *Chin. Phys. Lett.* **33** 020501
[18] Liu C H, Liu T Y, Huang R Z, Gao T F and Shu Y G 2019 *Acta Phys. Sin.* **68** 240501 (in Chinese)
[19] Chen R Y and Lv X N 2019 *Physica A* **514** 336
[20] Du W, Jia K, Shi Z L and Nie L R 2023 *Chin. Phys. B* **32** 020505
[21] Spiechowicz J and Luczka J 2015 *Phys. Rev. E* **91** 062104
[22] Chen R Y, Ruan X W, Wang C J and Jiang L L 2022 *Physica A* **593** 126929
[23] Ma Y T, Zhang F R and Li C P 2013 *Computers and Mathematics with Applications* **66** 682
[24] Tejedor V and Metzler R 2010 *J. Phys. A* **43** 082002
[25] Dos Santos M A F 2019 *Chaos, Solitons and Fractals* **124** 86
[26] Metzler R, Barkai E and Klafter J 1999 *Phys. Rev. Lett.* **82** 3563
[27] Mallick K and Marcq P 2002 *Phys. Rev. E* **66** 041113
[28] Dybiec B, Gudowska-Nowak E and Hanggi P 2006 *Phys. Rev. E* **73** 046104
[29] Chen R Y, Chen C Y and Nie L R 2017 *Int. J. Mod. Phys. B* **31** 1750259
[30] Chen R Y, Pan W L, Zhang J Q and Nie L R 2016 *Chaos* **26** 093113
[31] Dybiec B 2018 *Phys. Rev. E* **78** 061120
[32] Cubero D, Lebedev V and Renzoni F 2010 *Phys. Rev. E* **82** 041116
[33] Majee P, Goswami G, Sen M K and Bag B C 2009 *Eur. Phys. J. B* **72** 435
[34] Li B W, Wang J, Wang L and Zhang G 2005 *Chaos* **15** 015121
[35] Xu Y, Li J J, Feng J, Zhang H Q, Xu W and Duan J Q 2013 *Eur. Phys. J. B.* **86** 198