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The (1+1)-dimensional nonlinear ion acoustic waves in multicomponent plasma containing kappa electrons

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Large amplitude (1+1)-dimensional nonlinear ion acoustic waves are theoretically studied in multicomponent plasma consisting of positively charged ions and negatively charged ions, ion beam, kappa-distributed electrons, and dust grains, respectively. By using the Sagdeev potential method, the dynamical system and the Sagdeev potential function are obtained. The important influences of system parameters on the phase diagram of this system are investigated. It is found that the linear waves, the nonlinear waves and the solitary waves are coexistent in the multicomponent plasma system. Meanwhile, the variations of Sagdeev potential with parameter can also be obtained. Finally, it seems that the propagating characteristics of (1+1)-dimensional nonlinear ion acoustic solitary waves and ion acoustic nonlinear shock wave can be influenced by different parameters of this system.

Keywords: multicomponent plasma, nonlinear ion acoustic waves, Sagdeev potential method

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1. Introduction

Recently, the nonlinear waves in multicomponent plasma have been investigated in various fields of physics, such as the laboratory plasma, the earth atmosphere space plasma exploration and others.[1–5] Owing to the balance between nonlinearity and dissipation, a dissipative effect leading to the formation of nonlinear structures is created. The study of nonlinear waves in complex plasmas has become one of the hot topics in plasma physics.[6] Ion acoustic wave has become one of the most important aspects of nonlinear phenomena in plasma physics. Since Nakamua studied the ion plasma physics,[6] ion acoustic wave has become one of the most important aspects of nonlinear phenomena in plasma physics. Since Nakamua studied the ion acoustic solitary wave in multicomponent plasma, many scholars have done a lot of theoretical researches on this topic.[7] The propagation of arbitrary amplitude ion acoustic solitary wave in a two-component plasma has been investigated by using an alternative model.[8] The behavior of particle distribution in complex plasma is usually described by kappa distribution.[9] The diffusion of particles with kappa-distributed velocities is strongly influenced by statistical correlations.[10] A lot of observations have shown that ultra-hot particles with non-Maxwell distribution in the form of power law exist widely in space plasma. The plasma systems with superthermal electrons are well described by the kappa distribution.[11] It is necessary to point out that the Sagdeev potential method plays an important role in studying nonlinear waves.[12] Meanwhile, the propagation of nonlinear dust ion acoustic wave (DIAW) in unmagnetized collisionless dusty plasma consisting of ions, nonthermal electrons, and negatively charged dust grains has been investigated by using the Sagdeev potential method.[13] In a collisionless magnetized plasma in the presence of negative ions, the solitary kinetic Alfven wave has been theoretically investigated by the Sagdeev potential approach.[14]

In this paper, the large amplitude nonlinear ion acoustic waves in a multicomponent plasma composed of positively charged ions, negative ions, ion beam, and kappa-distributed electrons are investigated by the Sagdeev potential method. The rest of this paper is organized as follows: In Section 2, the basic equations of theoretical model are presented. Meanwhile, the Sagdeev potential equation is obtained. In Section 3, the effects of different parameters on the phase diagramare discussed. In Section 4, the variations of the Sagdeev potential function and the characters of solitary waves are studied in detail. In Section 5, the main conclusions are drawn from the present study.

2. Basic equations

In order to study the nonlinear ion acoustic wave in a multicomponent plasma, we assume that a collisionless and unmagnetized plasma consists of five components: positively charged ions, negatively charged ions, ion beam, kappa-distributed electrons, and dust particles.[15] The dimensionless equations are as follows:

$$\frac{\partial n_{i+}}{\partial t}+\frac{\partial\left(n_{i+}u_{i+}\right)}{\partial x}=0, \tag{1}$$

$$\frac{\partial u_{i+}}{\partial t}+u_{i+}\frac{\partial u_{i+}}{\partial x}=-\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial n_{i-}}{\partial t}+\frac{\partial\left(n_{i-}u_{i-}\right)}{\partial x}=0, \tag{3}$$

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$$\frac{\partial u_{i-}}{\partial t}+u_{i-}\frac{\partial u_{i-}}{\partial x}=\alpha\frac{\partial \phi}{\partial x}, \tag{4}$$

$$\frac{\partial n_b}{\partial t}+\frac{\partial (n_b u_b)}{\partial x}=0, \tag{5}$$

$$\frac{\partial n_b}{\partial t}+u_b\frac{\partial u_b}{\partial x}=-\beta\frac{\partial \phi}{\partial x}, \tag{6}$$

$$\frac{\partial^2 \phi}{\partial x^2}=n_e+\mu_d+\mu_{i-}n_{i-}-\mu_b n_b-\mu_{i+}n_{i+}, \tag{7}$$

where n_{i+}, n_{i-}, and n_b denote the dimensionless number density of positively charged ions, negatively charged ions, and ion beam, which are normalized by their equilibrium values n_{i+0}, n_{i-0}, n_{b0}, respectively, u_{i+}, u_{i-}, u_b denote the velocity of positive ions, velocity of negative ions, and velocity of ion beam, which are normalized by the positively charged ion sound velocity $C_S=(k_B T_e/m_{i+})$, with k_B being the Boltzmann constant, T_e the temperature of electrons and m_{i+} the mass of positive ions. The potential ϕ is normalized by $k_B T_e/e$, where e is the number of electronic charges. The parameters in this system are as follows: $\alpha=m_{i+}/m_{i-}$, $\beta=m_{i+}/m_b$, $\mu_d=Z_d n_{d0}/n_{e0}$, $\mu_{i+}=n_{i+0}/n_{e0}$, $\mu_{i-}=n_{i-0}/n_{e0}$, and $\mu_b=n_{b0}/n_{e0}$, where m_{i-} is the mass of negative ion and m_b is the mass of ion beam, Z_d is the charge number of dust grain, n_{d0} is the equilibrium number density of dust grains and n_{e0} is the equilibrium number density of electrons. The kappa distribution of electrons is

$$n_e=\left(1-\frac{\phi}{\kappa-3/2}\right)^{-\kappa+1/2}, \tag{8}$$

where κ is the kappa number of electrons.

The plane-wave solutions for the particle number density, particle propagation velocity, and electric potential have the following form:

$$n_{i+}=1+\varepsilon n_{i+}^{(1)}\mathrm{e}^{\mathrm{i}(kx-\omega t)}, \tag{9}$$

$$n_{i-}=1+\varepsilon n_{i-}^{(1)}\mathrm{e}^{\mathrm{i}(kx-\omega t)}, \tag{10}$$

$$n_b=1+\varepsilon n_b^{(1)}\mathrm{e}^{\mathrm{i}(kx-\omega t)}, \tag{11}$$

$$u_{i+}=\varepsilon u_{i+}^{(1)}\mathrm{e}^{\mathrm{i}(kx-\omega t)}, \tag{12}$$

$$u_{i-}=\varepsilon u_{i-}^{(1)}\mathrm{e}^{\mathrm{i}(kx-\omega t)}, \tag{13}$$

$$u_b=\varepsilon u_b^{(1)}\mathrm{e}^{\mathrm{i}(kx-\omega t)}, \tag{14}$$

$$\phi=\varepsilon\phi_1\mathrm{e}^{\mathrm{i}(kx-\omega t)}, \tag{15}$$

$$n_e=\frac{(-1+2\kappa)}{(2\kappa-3)}\varepsilon\phi_1\mathrm{e}^{\mathrm{i}(kx-\omega t)}. \tag{16}$$

The linear wave dispersion relation can be expressed as

$$\omega^2=\frac{k^2(\alpha\mu_{i-}+\beta\mu_b+\mu_{i+})}{k^2+\frac{(2\kappa-1)}{(2\kappa-3)}}. \tag{17}$$

Figure 1 shows the influence of wave number k on the dispersion relation for linear equation for different values of μ_{i+}, μ_{i-}, μ_b, and κ, indicating that when $0<k<1$, the ω^2 increases with wave number k increasing. The parameters in the system are expressed as $\mu_{i+}=n_{i+0}/n_{e0}$, $\mu_{i-}=n_{i-0}/n_{e0}$, and $\mu_b=n_{b0}/n_{e0}$, respectively. It means that for the multicomponent plasma with kappa-distributed electrons, the variation of the equilibrium number density for positive ions, negative ions, ion beam, and kappa-distributed electrons in this system will change the frequency and the dispersion relation for linear equation.

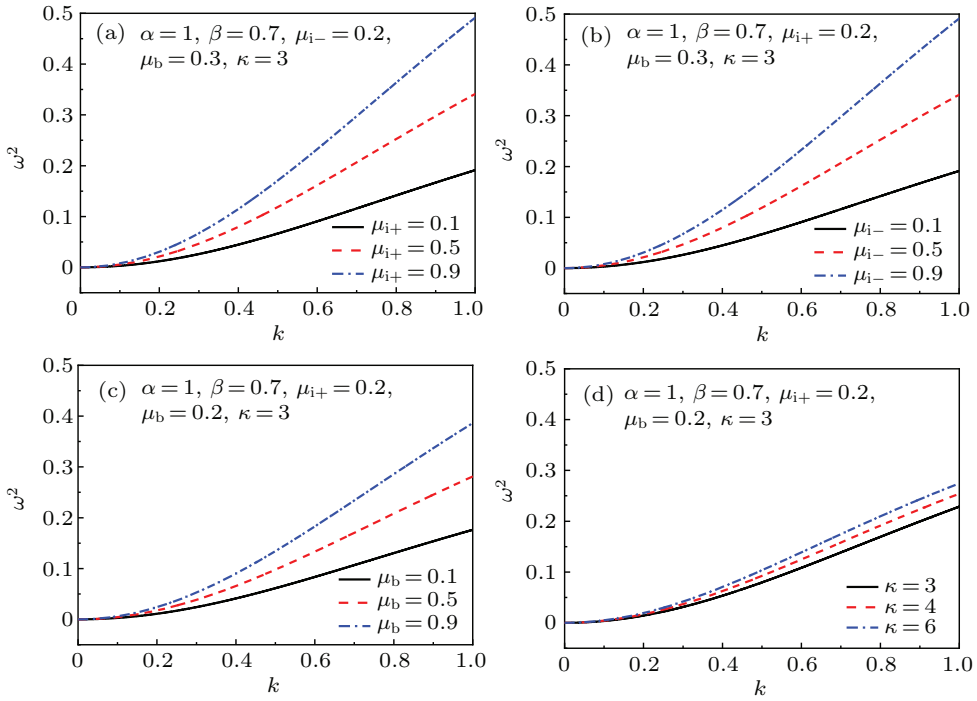


Fig. 1. Characteristics of dispersion relation at $\alpha=1$ and $\beta=0.7$, and various values of (a)–(c) μ_{i+}, μ_{i-}, and μ_b and (d) κ, respectively.

To investigate the ion acoustic waves in the multicomponent plasma system, the Sagdeev potential method is used to solve Eqs. (1)–(7). Now, a new coordinate ξ is introduced, which is given by $\xi = x - Mt$, where M is the Mach number. Then, The following boundary conditions should be noted: when $\xi \to \pm\infty$, the dimensionless number density $n_{\mathrm{i}+,\mathrm{i}-,\mathrm{b}} \to 1$, the dimensionless velocity $u_{\mathrm{i}+,\mathrm{i}-,\mathrm{b}} \to 0$, and the potential $\phi \to 0$.

With the help of Sagdeev potential method, the number densities can be obtained as shown below:

$$n_{\mathrm{i}+} = \frac{M}{\sqrt{M^2 - 2\phi}}, \quad n_{\mathrm{i}-} = \frac{M}{\sqrt{M^2 + 2\alpha\phi}}.$$

Substituting the above expressions into Eq. (7), it is found that

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\xi^2} = A_1\phi + A_2\phi^2 + A_3\phi^3 + A_4\phi^4, \tag{18}$$

where

$$\begin{aligned}
A_1 &= \frac{(1-2\kappa)}{(2\kappa-3)} - \frac{\alpha}{M^2}\mu_{\mathrm{i}-} - \frac{\beta}{M^2}\mu_{\mathrm{b}} - \frac{1}{M^2}\mu_{\mathrm{i}+}, \\
A_2 &= -\frac{(1-2\kappa)(1+2\kappa)}{2(2\kappa-3)^2} + \frac{3\alpha^2}{2M^4}\mu_{\mathrm{i}-} \\
&\quad - \frac{3\beta^2}{2M^4}\mu_{\mathrm{b}} - \frac{3}{2M^4}\mu_{\mathrm{i}+}, \\
A_3 &= \frac{(1-2\kappa)(1+2\kappa)(3+2\kappa)}{6(2\kappa-3)^3} - \frac{5\alpha^3}{2M^6}\mu_{\mathrm{i}-} \\
&\quad - \frac{5\beta^3}{2M^6}\mu_{\mathrm{b}} - \frac{5}{2M^6}\mu_{\mathrm{i}+}, \\
A_4 &= -\frac{(1-2\kappa)(1+2\kappa)(3+2\kappa)(5+2\kappa)}{24(2\kappa-3)^4} \\
&\quad + \frac{35\alpha^4}{8M^8}\mu_{\mathrm{i}-} - \frac{35\beta^4}{8M^8}\mu_{\mathrm{b}} - \frac{35}{8M^8}\mu_{\mathrm{i}+}.
\end{aligned}$$

The above equations can be transformed into a two-dimensional dynamical system as follows:

$$\begin{cases}
\dfrac{\mathrm{d}\phi}{\mathrm{d}\xi} = \psi, \\
\dfrac{\mathrm{d}\psi}{\mathrm{d}\xi} = A_1\phi + A_2\phi^2 + A_3\phi^3 + A_4\phi^4.
\end{cases} \tag{19}$$

Meanwhile, the Sagdeev potential equation can be expressed as

$$\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}\xi}\right)^2 + V(\phi) = 0, \tag{20}$$

where the Sagdeev potential function is

$$V(\phi) = -\frac{1}{2}A_1\phi^2 - \frac{1}{3}A_2\phi^3 - \frac{1}{4}A_3\phi^4 - \frac{1}{5}A_4\phi^5. \tag{21}$$

As is well known, the form of Sagdeev potential $V(\phi)$ can determine whether a soliton exists in this system, if the following condition can be satisfied:

$$\text{(i)} \quad V(\phi_{\mathrm{m}}) = 0, \tag{22}$$

$$\text{(ii)} \quad \left.\frac{\mathrm{d}V}{\mathrm{d}\phi}\right|_{\phi=\phi_{\mathrm{m}}} = 0, \tag{23}$$

$$\text{(iii)} \quad \left.\frac{\mathrm{d}^2V}{\mathrm{d}\phi^2}\right|_{\phi=\phi_{\mathrm{m}}} < 0. \tag{24}$$

It is noted that there is an analytical soliton solution for ion acoustic solitary wave in this system as given below:[17]

$$\phi = \frac{2A_1}{\left(\dfrac{4}{9}A_2^2 + 2A_1A_3\right)\left(2\cosh^2\xi - 1\right) - \dfrac{2}{3}A_2}. \tag{25}$$

At the same time, an analytical soliton solution for ion acoustic nonlinear shock wave in this system can be given as follows:

$$\sqrt{\frac{2A_1}{A_3}}\arctan\left[\frac{2A_2 + \sqrt{4A_2^2 + 18A_1A_3}}{3\sqrt{2A_1A_3}}\tanh(\xi)\right]. \tag{26}$$

3. Analysis of dynamical system

In order to investigate the different types of waves in the multicomponent plasma with kappa-distributed electrons, the various effects of different values of parameters κ, $\mu_{\mathrm{i}+}$, $\mu_{\mathrm{i}-}$, and μ_{b} are analyzed and the results are plotted in Figs. 2–5.

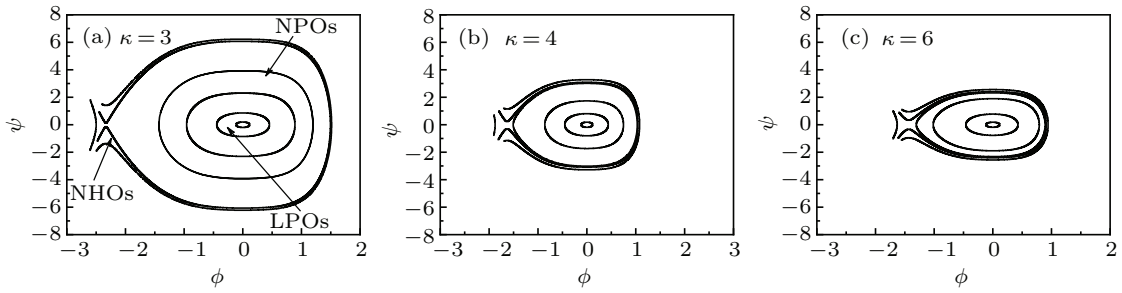


Fig. 2. Variations of phase diagram with ϕ for three different values of κ.

Figure 2 shows the variations of phase diagram with ϕ for kappa number $\kappa = 3$, 4, and 6, and at $\alpha = 1$, $\beta = 0.7$, $\mu_{\mathrm{i}+} = 0.4$, $\mu_{\mathrm{i}-} = 0.2$, and $\mu_{\mathrm{b}} = 0.2$.[18,19] It is shown that three kinds of orbits coexist in the multicomponent plasma system, which have the linear periodic orbits (LPOs), the nonlinear periodic orbits (NPOs) and the nonlinear homoclinic or-

bits (NHOs). It means that the linear periodic wave solution, the nonlinear periodic wave solution and the solitary wave solution are coexistent in this multicomponent plasma. Meanwhile, it is found from Fig. 2 that the variation of the kappa number in the complex plasma system also affects the existence range of different types of orbits in this dynamical system.

Figures 3–5 show the influence of μ_{i+}, μ_{i-}, and μ_b on the phase diagram for this dynamical system. As is well known, the parameters in the multicomponent plasma are expressed as $\mu_{i+} = n_{i+0}/n_{e0}$, $\mu_{i-} = n_{i-0}/n_{e0}$, and $\mu_b = n_{b0}/n_{e0}$, indicating that for the multicomponent plasma with kappa-distributed electron, the various values of the equilibrium number densities for positive ions, negative ions, ion beam, and electrons in this system will change the existence ranges of different types in the phase diagram for this system. However, the linear periodic wave solution, the nonlinear periodic wave solution and the solitary waves solution are all coexistent in this system.

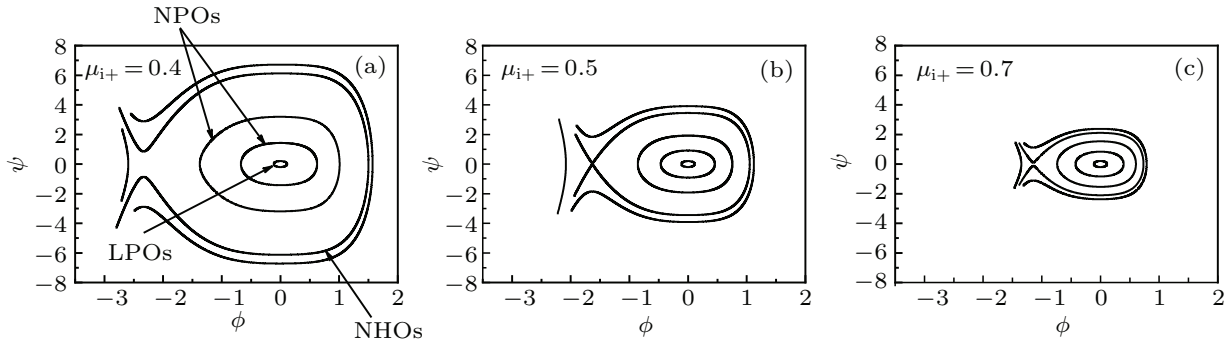


Fig. 3. Variations of phase diagram with ϕ for three different values of μ_{i+} at $\kappa = 3$, $\mu_{i-} = 0.2$, and $\mu_b = 0.2$.

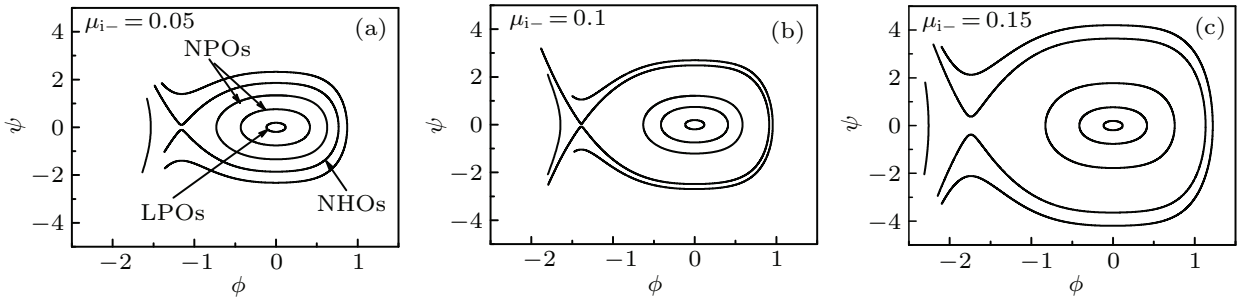


Fig. 4. Variations of phase with ϕ for three different values of μ_{i-} at $\kappa = 3$, $\mu_{i+} = 0.4$, and $\mu_b = 0.2$.

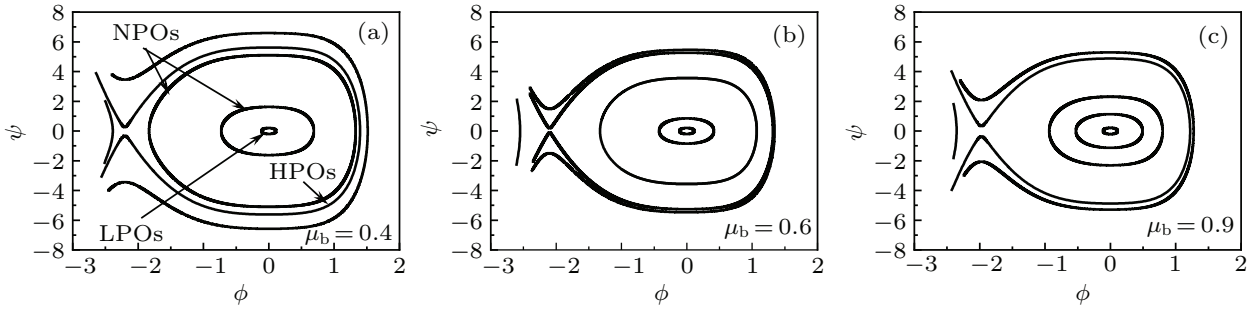


Fig. 5. Variations of phase with ϕ for different values of μ_b at $\kappa = 3$, $\mu_{i+} = 0.4$, and $\mu_{i-} = 0.2$.

4. Discussion

In order to discuss the behaviors of the nonlinear ion acoustic solitary waves, the characteristics of the Sagdeev potential $V(\phi)$ versus ϕ with various parameters is investigated in Fig. 6 by the numeral simulation method. Figure 6(a) shows that there exists ϕ_m on the Sagdeev potential curves, where $V(\phi_m) = 0$. It can also be found that $\phi_m < 0$ when $\mu_{i+} = 0.4$, 0.5, 0.7, respectively. It indicates that there exist only rarefactive solitary waves in the multicomponent plasma. In additional, figure 6(a) also indicates that the amplitude of ion acoustic solitary wave will decrease as the value of μ_{i+} increases. Because of the parameter $\mu_{i+} = n_{i+0}/n_{e0}$, it can be concluded that the amplitude of rarefactive solitary waves will decrease with equilibrium number density of positive ions increasing and equilibrium number density of electrons lowering. It means that both the equilibrium number density of positive ions and the equilibrium number density of electrons can have considerable influence on the solitary wave. Figure 6(b) shows the influence of μ_{i-} on the Sagdeev potential $V(\phi)$ and

only the rarefactive solitary wave existing in this system. Because $\mu_{i-} = n_{i-0}/n_{e0}$, the amplitude of solitary wave will increase if the multicomponent plasma has a higher equilibrium number density of negative ions. Figure 6(c) shows the variations of $V(\phi)$ versus ϕ for three different values of μ_b. It is shown that the influences of μ_b on the characteristics of solitary wave are similar to those in Fig. 6(a). The variations of $V(\phi)$ versus ϕ with different values of κ are plotted in Fig. 6(d). It is found that the amplitude of solitary wave decreases with the augment of the kappa number of kappa-distributed electrons in the multicomponent plasma. As stated above, it can be concluded that the variations of $V(\phi)$ versus ϕ can be influenced by many parameters of multicomponent plasma.

In order to study the characteristics of the ion acoustic solitary waves, the variations of solitary wave described by Eq. (24) are plotted in Fig. 7 through the numerical simulation technique. Figure 7 shows that there exist only rarefactive solitary waves in the multicomponent plasma with kappa-distributed electrons, and that the amplitude of rarefactive solitary wave decreases as μ_{i+}, μ_b, and κ increase, but increases as μ_{i-} increases. This conclusion is consistent with that from Fig. 6. It is found that the effects of different parameters on the ion acoustic solitary waves are very important.

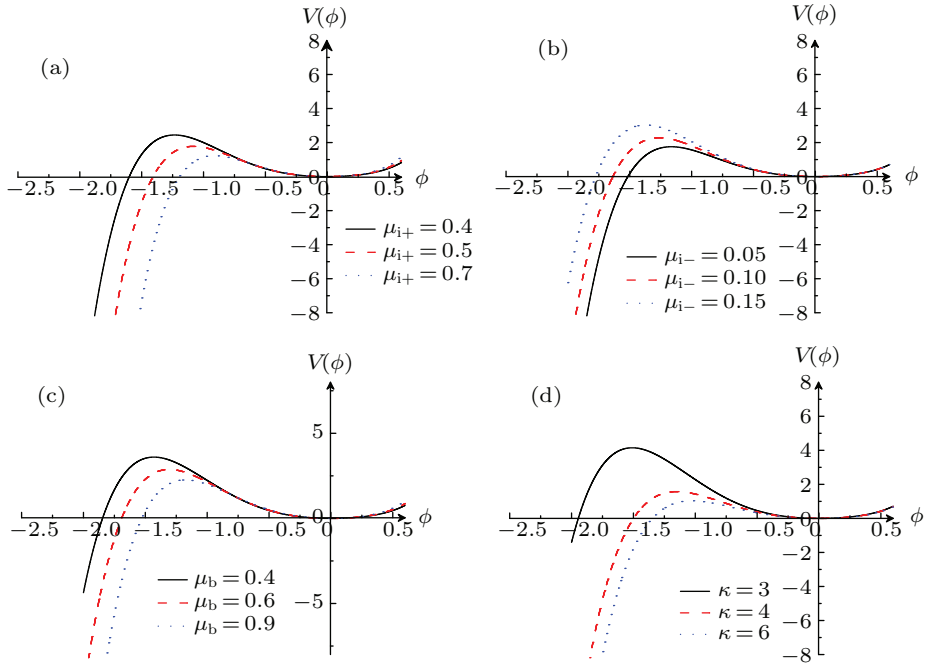


Fig. 6. Variations of Sagdeev potential $V(\phi)$ with ϕ at $\alpha = 0.8$ and $\beta = 0.7$.

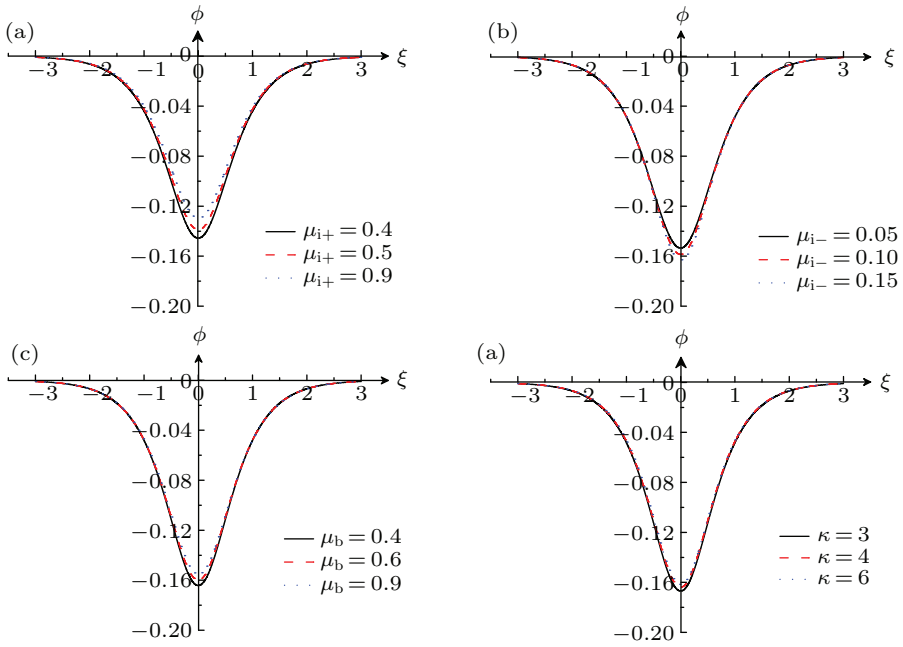


Fig. 7. Characteristics of ϕ versus ξ of solitary waves for three different values of (a)–(c) μ_{i+}, μ_{i-}, and μ_b and (d) κ, at $\alpha = 1$ and $\beta = 0.7$.

The propagating process of the shock wave is the same as that of sound waves, which is characterized by the heteroclinic orbits in a two-dimensional dynamical system. In some plasma studies, the electrostatic energy of the dust particles may affect the shock wave through its energy conversion.[20] The different values of Mach number can also affect the width of shock wave.[21]

In order to study the characteristics of ion acoustic nonlinear shock wave, the variations of shock wave in Eq. (25) are plotted in Fig. 8 by the numerical simulation technique. Figure 8 shows that there exist shock waves in the multicomponent plasma with kappa-distributed electrons. Moreover, it is shown that the amplitude of the nonlinear shock wave decreases as μ_{i+}, μ_b, and κ increase, but increases as μ_{i-} increases. It is found that the effects of kappa distribution properties and particle number density on the shock waves are also very important.

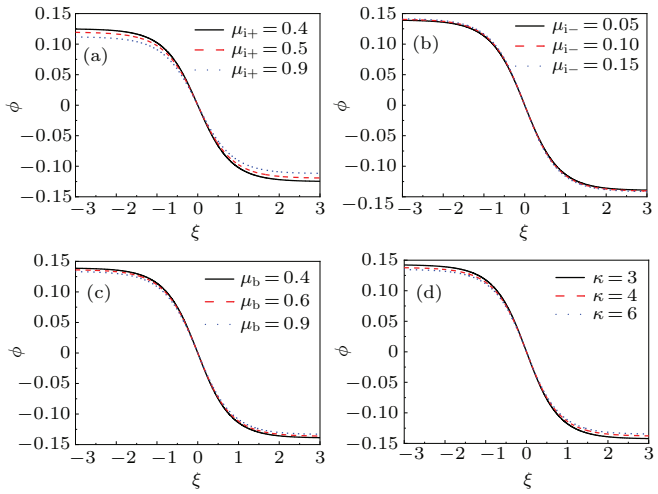


Fig. 8. Characteristics of ϕ versus ξ of shock waves for three different values of (a)–(c) μ_{i+}, μ_{i-}, μ_b and (d) κ, at $\alpha = 1$ and $\beta = 0.7$.

5. Conclusions

In this paper, the characteristics of large amplitude nonlinear ion acoustic solitary wave in collisionless, unmagnetized multicomponent plasma with kappa-distributed electrons are investigated by the Sagdeev potential method. It is found from the phase diagram for the dynamical system that the linear periodic orbits, the nonlinear periodic orbits, and the nonlinear homoclinic orbits are all exist in this system. Moreover, it can be concluded from the different values of $V(\phi)$ that only the rarefactive solitary wave exists, and different parameters exert important influence on the characteristics of ion acoustic solitary wave and ion acoustic nonlinear shock wave.

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