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Protected simultaneous quantum remote state preparation scheme by weak and reversal measurements in noisy environments

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We discuss a quantum remote state preparation protocol by which two parties, Alice and Candy, prepare a single-qubit and a two-qubit state, respectively, at the site of the receiver Bob. The single-qubit state is known to Alice while the two-qubit state which is a non-maximally entangled Bell state is known to Candy. The three parties are connected through a single entangled state which acts as a quantum channel. We first describe the protocol in the ideal case when the entangled channel under use is in a pure state. After that, we consider the effect of amplitude damping (AD) noise on the quantum channel and describe the protocol executed through the noisy channel. The decrement of the fidelity is shown to occur with the increment in the noise parameter. This is shown by numerical computation in specific examples of the states to be created. Finally, we show that it is possible to maintain the label of fidelity to some extent and hence to decrease the effect of noise by the application of weak and reversal measurements. We also present a scheme for the generation of the five-qubit entangled resource which we require as a quantum channel. The generation scheme is run on the IBMQ platform.

Keywords: multi-qubit entangled channel, quantum remote state preparation, noisy environments, weak and reversal measurements

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1. Introduction

In this paper, our basic program is to present a protocol for simultaneous remote preparation of a one-qubit and a two-qubit state at a distant location where the preparations are intended by two different parties. It is a remote state preparation protocol which belongs to a different category of quantum communication schemes compared to teleportation, the basic difference between the two is that in the former the states to be remotely created are known to the parties who intend to create them while in the latter the states intended for transfer are totally unknown to the senders in the protocol. The teleportation protocol was first proposed by Bennett *et al.*[1] in 1993. It is supposed that quantum communication science was initiated in this particular work. Remote state preparation protocol came into being through the work of Lo[2] in 2000. Both these types of protocols have several variations and are applied to very wide varieties of quantum states. References [3–14] are some instances from the literature on quantum teleportation while[15–22] are works on remote state preparation protocols. Particularly, in this paper we present a remote state preparation protocol in which there are two parties Alice and Candy whose intentions are to create a single-qubit and a two-qubit state respectively in the laboratory of Bob. The protocol is executed by utilizing a five-qubit entangled state shared between Alice, Bob and Candy where Bob holds three-qubits while Alice and Candy hold one-qubit each.

Our protocol is included in the program of multi-tasking quantum communications which is that through a single protocol, more than one quantum communication tasks are performed. These types of works have appeared in a good number of recent articles. Bidirectional teleportation in which the simultaneous exchange of two quantum states takes place was discussed by Jiang *et al.*[23] in 2021, Wang *et al.*[24] in 2021 and Verma[25] in 2021. Quantum conference protocols have appeared in works by Banerjee *et al.*[26] in 2018, Choudhury *et al.*[27] in 2022, etc. A hybrid protocol involving both remote state preparation and quantum teleportation was presented by Choudhury *et al.*[28] in 2023. In all these multi-tasking protocols, there is an unavoidable requirement for entanglement resources involving a high number of qubits shared amongst the parties participating in the protocol. This is a challenge for quantum communication science since multipartite entanglement is difficult to produce and, even if produced, they are susceptible to environmental noise due to which the communication protocol may develop unreliability to a certain extent. There are certain methods for minimizing the effect of such noises such as quantum error correction,[29–31] entanglement purification,[32,33] weak and reversal measurements,[34–41] etc. The ideal noiseless case is discussed, following which the effect of amplitude-damping noise is considered. It is shown by analysis of specific cases that the fidelity of the protocol decreases with the increase in the noise parameter. Further, by performing weak and reversal measurements, we observe that

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the effect of noise can be minimized.

The present protocol is a demonstration of performing two remote state preparation tasks through a single protocol by utilization of a single entangled resource. There can be different ways of performing the same tasks, for instance, two separate entangled channels of two-qubits and three-qubits can be utilized here. But that would be the case of exactly two separate protocols. Our objective here is to use a single integrated protocol where Bob finally executes unitary operations on the basis of the classical communications jointly received from Alice and Candy.

Quantum communication protocols have a vast range of applications. Its versatility extends to various fields within the realm of quantum information science, facilitating advancements in quantum communication and enabling new possibilities in quantum computation such as quantum key distribution,[42–52] quantum secure dense coding,[53,54] etc.

For experimental aspects of the implementation of weak and reversal measurements, we refer to Ref. [55] and references therein.

In the present case we use a five-qubit entangled state as a quantum channel. We describe the generation of this entanglement resource and execute the circuit algorithm on the IBMQ platform.[56,57] We consider the effect of amplitude-damping noise on the protocol and show that the effect of such noise can be minimized by executing weak and reversal measurements.

2. Remote state preparation with two senders and one receiver

Suppose there are three parties namely, Alice, Bob and Candy. Also, there are two quantum states

$$|M_1\rangle = (\alpha_0|0\rangle + \alpha_1|1\rangle), \quad |M_2\rangle = (\beta_0|00\rangle + \beta_1|11\rangle), \quad (1)$$

where the coefficients $\alpha_0, \alpha_1, \beta_0$ and β_1 are real and satisfy the normalization conditions $|\alpha_0|^2 + |\alpha_1|^2 = 1$ and $|\beta_0|^2 + |\beta_1|^2 = 1$.

In this scenario suppose Alice wants to create the quantum state $|M_1\rangle$ at Bob's laboratory and also at the same time Candy wants to create the quantum state $|M_2\rangle$ at Bob's site using a single entangled resource given in Eq. (2) (see below). The sender Alice knows the information about the state $|M_1\rangle$ and Candy knows the information about the state $|M_2\rangle$, that is, the coefficients (α_0, α_1) and (β_0, β_1) are known to Alice and Candy, respectively.

There is a maximally entangled state amongst three parties which serves as the quantum resource which is given by

$$|\tau\rangle = \frac{1}{2}(|00000\rangle - |01011\rangle + |10100\rangle + |11111\rangle)_{a_1c_1b_1b_2b_3}, \quad (2)$$

where the particle a_1 belongs to Alice, particle c_1 belongs to Candy and particles (b_1, b_2, b_3) are in the hands of Bob. The above-mentioned task of quantum information transfer is to be accomplished using the quantum channel Eq. (2). Here the state $|M_1\rangle$ and $|M_2\rangle$ intended for creation at Bob's site is possessed by none of the parties.

Alice prepares a basis for a possible measurement on her qubit a_1 given by

$$|\phi_0\rangle = (\alpha_0|0\rangle + \alpha_1|1\rangle)_{a_1}, \quad |\phi_1\rangle = (\alpha_1|0\rangle - \alpha_0|1\rangle)_{a_1}. \quad (3)$$

Candy prepares a basis for a possible measurement on her qubit c_1 given by

$$|\psi_0\rangle = (\beta_0|0\rangle + \beta_1|1\rangle)_{c_1}, \quad |\psi_1\rangle = (\beta_1|0\rangle - \beta_0|1\rangle)_{c_1}. \quad (4)$$

The choices of such measurement bases by Alice and Candy are possible since the coefficients (α_0, α_1) and (β_0, β_1) are known to Alice and Candy, respectively.

We can write the above quantum channel given in Eq. (2) using the bases Eqs. (3) and (4) as

$$\begin{aligned}
|\tau\rangle \;=\; & \frac{1}{2}[|\phi_0\rangle_{a_1} \otimes |\psi_0\rangle_{c_1} \otimes (\alpha_0\beta_0|000\rangle - \alpha_0\beta_1|011\rangle \\
& + \alpha_1\beta_0|100\rangle + \alpha_1\beta_1|111\rangle)_{b_1b_2b_3} \\
& + |\phi_0\rangle_{a_1} \otimes |\psi_1\rangle_{c_1} \otimes (\alpha_0\beta_1|000\rangle + \alpha_0\beta_0|011\rangle \\
& + \alpha_1\beta_1|100\rangle - \alpha_1\beta_0|111\rangle)_{b_1b_2b_3} \\
& + |\phi_1\rangle_{a_1} \otimes |\psi_0\rangle_{c_1} \otimes (\alpha_1\beta_0|000\rangle - \alpha_1\beta_1|011\rangle \\
& - \alpha_0\beta_0|100\rangle - \alpha_0\beta_1|111\rangle)_{b_1b_2b_3} \\
& + |\phi_1\rangle_{a_1} \otimes |\psi_1\rangle_{c_1} \otimes (\alpha_1\beta_1|000\rangle + \alpha_1\beta_0|011\rangle \\
& - \alpha_0\beta_1|100\rangle + \alpha_0\beta_0|111\rangle)_{b_1b_2b_3}].
\end{aligned}$$

That is,

$$\begin{aligned}
|\tau\rangle \;=\; & \frac{1}{2}\sum_{i=0}^{1}\sum_{j=0}^{1} |\phi_i\rangle \otimes |\psi_j\rangle \otimes \Big(\alpha_{(i \bmod 2)}\beta_{(j \bmod 2)}|000\rangle \\
& - (-1)^j \alpha_{(i \bmod 2)}\beta_{(j+1 \bmod 2)}|011\rangle \\
& + (-1)^i \alpha_{(i+1 \bmod 2)}\beta_{(j \bmod 2)}|100\rangle \\
& + (-1)^{i+j} \alpha_{(i+1 \bmod 2)}\beta_{(j+1 \bmod 2)}|111\rangle\Big).
\end{aligned}$$

Alice and Candy perform measurements on the qubits a_1 and c_1, respectively. After the measurements of Alice and Candy, they transmit their outcomes to Bob utilizing two separate 1-bit classical communication channels. According to the measurement results, Bob applies a suitable sequence of unitary operations on his three particles to get the intended states. The details of the sender's measurement outcomes and corresponding unitary operations are given in Table 1. After using unitary operations all the possible reduced states can be written as $(\alpha_0|0\rangle + \alpha_1|1\rangle)_{b_1} \otimes (\beta_0|00\rangle + \beta_1|11\rangle)_{b_2b_3}$. This is the end of the protocol.

Table 1. Outcome results and corresponding unitary operators.

Alice's and Candy's measurement results $(\|\phi_i\rangle, \|\psi_j\rangle)$	Bob's first unitary operator $(U^{ij})_{b_1b_2b_3}$	Bob's second unitary operator $(V^{ij})_{b_1b_2b_3}$
$(\|\phi_0\rangle, \|\psi_0\rangle)$	$\|000\rangle\langle 000\| - \|011\rangle\langle 011\|$ $+\|100\rangle\langle 100\| + \|001\rangle\langle 001\|$ $+\|101\rangle\langle 101\| + \|010\rangle\langle 010\|$ $+\|110\rangle\langle 110\| - \|111\rangle\langle 111\|$	$I\otimes I\otimes I$
$(\|\phi_0\rangle, \|\psi_1\rangle)$	$\|000\rangle\langle 000\| + \|011\rangle\langle 011\|$ $+\|100\rangle\langle 100\| + \|001\rangle\langle 001\|$ $+\|101\rangle\langle 101\| + \|010\rangle\langle 010\|$ $+\|110\rangle\langle 110\| + \|111\rangle\langle 111\|$	$I\otimes\sigma_x\otimes\sigma_x$
$(\|\phi_1\rangle, \|\psi_0\rangle)$	$\|000\rangle\langle 000\| - \|011\rangle\langle 011\|$ $-\|100\rangle\langle 100\| + \|001\rangle\langle 001\|$ $+\|101\rangle\langle 101\| + \|010\rangle\langle 010\|$ $+\|110\rangle\langle 110\| - \|111\rangle\langle 111\|$	$\sigma_x\otimes I\otimes I$
$(\|\phi_1\rangle, \|\psi_1\rangle)$	$\|000\rangle\langle 000\| + \|011\rangle\langle 011\|$ $-\|100\rangle\langle 100\| + \|001\rangle\langle 001\|$ $+\|101\rangle\langle 101\| + \|010\rangle\langle 010\|$ $+\|110\rangle\langle 110\| + \|111\rangle\langle 111\|$	$\sigma_x\otimes\sigma_x\otimes\sigma_x$

As an illustration, using the basis given in Eq. (3) the quantum channel $|\tau\rangle$ in Eq. (2) can be written as

$$\begin{aligned}|\tau\rangle \;=\; & \frac{1}{\sqrt{2}}|\phi_0\rangle_{a_1}\otimes\big(\alpha_0|0000\rangle-\alpha_0|1011\rangle\\ & +\alpha_1|0100\rangle+\alpha_1|1111\rangle\big)_{c_1b_1b_2b_3}\\ & +\frac{1}{\sqrt{2}}|\phi_1\rangle_{a_1}\otimes\big(\alpha_1|0000\rangle-\alpha_1|1011\rangle\\ & -\alpha_0|0100\rangle-\alpha_0|1111\rangle\big)_{c_1b_1b_2b_3}.\end{aligned}$$

Now, Alice makes a measurement on her single qubit a_1 in the basis given in Eq. (3). Suppose her measurement result is $|\phi_0\rangle_{a_1}$ then the reduced state is given by

$$|\xi_0\rangle=\frac{1}{2}(\alpha_0|0000\rangle-\alpha_0|1011\rangle+\alpha_1|0100\rangle+\alpha_1|1111\rangle)_{c_1b_1b_2b_3}.$$

The above reduced state $|\xi_0\rangle$ can be written using the basis given in Eq. (4) as

$$\begin{aligned}|\xi_0\rangle \;=\; & \frac{1}{2}|\psi_0\rangle_{c_1}\otimes\big(\alpha_0\beta_0|000\rangle-\alpha_0\beta_1|011\rangle\\ & +\alpha_1\beta_0|100\rangle+\alpha_1\beta_1|111\rangle\big)_{b_1b_2b_3}\\ & +\frac{1}{2}|\psi_1\rangle_{c_1}\otimes\big(\alpha_0\beta_1|000\rangle+\alpha_0\beta_0|011\rangle\\ & +\alpha_1\beta_1|100\rangle-\alpha_1\beta_0|111\rangle\big)_{b_1b_2b_3}.\end{aligned}$$

Now, Candy performs a measurement on his single qubit c_1 in the basis given by Eq. (4). If Candy's measurement outcome is $|\psi_1\rangle_{c_1}$ then the reduced state is given by

$$\begin{aligned}|\Xi_{0,1}\rangle \;=\; & (\alpha_0\beta_1|000\rangle+\alpha_0\beta_0|011\rangle\\ & +\alpha_1\beta_1|100\rangle-\alpha_1\beta_0|111\rangle)_{b_1b_2b_3}.\end{aligned}$$

Now Bob first applies a unitary operator $(|000\rangle\langle 000| + |011\rangle\langle 100| + |100\rangle\langle 100| + |001\rangle\langle 001| + |101\rangle\langle 101| + |010\rangle\langle 010| + |110\rangle\langle 110| - |111\rangle\langle 111|)$ on the state of the qubits b_1, b_2, and b_3. After that Bob applies another unitary operator $I\otimes\sigma_x\otimes\sigma_x$ on his qubits b_1, b_2, and b_3. Then the final state of the qubits b_1, b_2, and b_3 becomes $(\alpha_0\beta_1|011\rangle+\alpha_0\beta_0|000\rangle+\alpha_1\beta_1|111\rangle+\alpha_1\beta_0|100\rangle)_{b_1b_2b_3} = (\alpha_0|0\rangle+\alpha_1|1\rangle)_{b_1}\otimes(\beta_0|00\rangle+\beta_1|11\rangle)_{b_2b_3}$. This is the end of the protocol.

3. Preparation of entangled channel

The circuit for generating the quantum state $|\tau\rangle$ in Eq. (2) is given in Fig. 1. It is generated by utilizing two Hadamard gates, four CNOT gates and one Z-gate.

Initially, a five-qubit state is prepared with five zero initial states as

$$|X_1\rangle=|0\rangle_1\otimes|0\rangle_2\otimes|0\rangle_3\otimes|0\rangle_4\otimes|0\rangle_5.$$

Now, first, one Hadamard gate is applied on qubit 1 and then a CNOT gate is applied with qubit 1 as a control qubit and qubit 2 as the target qubit. Then the initial state $|X_1\rangle$ is converted to

$$|X_2\rangle=\frac{1}{\sqrt{2}}\Big(|00000\rangle+|11000\rangle\Big)_{12345}.$$

Again one Hadamard gate is applied on qubit 2, then the state $|X_2\rangle$ of five-qubits becomes

$$|X_3\rangle=\frac{1}{2}\Big(|00000\rangle+|01000\rangle+|10000\rangle-|11000\rangle\Big)_{12345}.$$

In the next step, one CNOT gate is applied with qubit 1 as the control qubit and qubit 3 as the target. Again two CNOT gates are applied with qubit 2 as the control qubit for each of qubits 4 and 5, respectively. Lastly, one Z-gate is applied on the qubit 2. Then the state $|X_3\rangle$ of five-qubits is transferred to gain one Hadamard gate is applied on qubit 2, then the state $|X_2\rangle$ of five-qubits becomes

$$|\tau\rangle=\frac{1}{2}\Big(|00000\rangle-|01011\rangle+|10100\rangle+|11111\rangle\Big)_{12345}.$$

We have executed this scheme of constructing the entangled state $|\tau\rangle$ on IBM Quantum Composer and run over ibmq_qasm_simulator of 32 qubits. The explicit circuit and output results are given in Figs. 1–3, respectively.

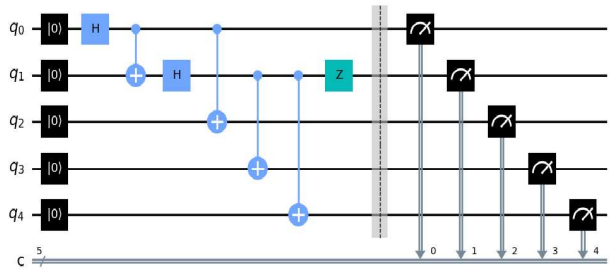


Fig. 1. Quantum circuit for the generation of the entanglement resource $|\tau\rangle$.

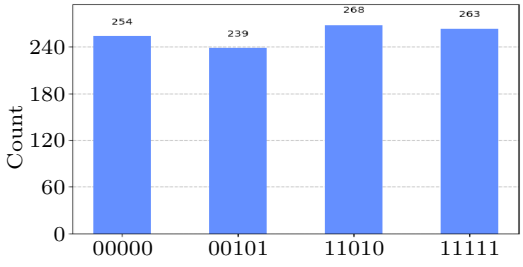


Fig. 2. Output of the quantum circuit in Fig. 1 on IBMQ.

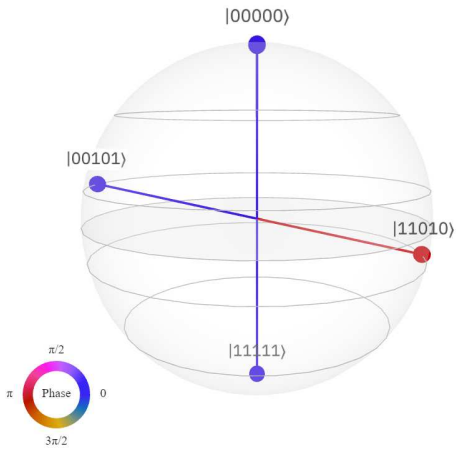


Fig. 3. Bloch sphere representation of the output of the circuit in Fig. 1.

4. Effect of noise in the protocol

From the experimental point of view, a real quantum state will couple inevitably with its surrounding environment which disturbs the coherence of the entangled channel and produces an environmental noise. The quantum entangled channel which is the fundamental ingredient for the purpose of a remote state preparation scheme between two distant parties is completely affected by the environmental noise during the distribution of qubits amongst the entangled parties. As a result, the performance of the present remote state preparation protocol is decreased. In this section, we discuss the effect of noise in our proposed remote state preparation scheme.

To minimize the influence of quantum noise, as much as possible, we suppose that the receiver Bob creates the five-qubit entangled state $|\tau\rangle_{a_1c_1b_1b_2b_3}$ in his laboratory and distributes qubit a_1 to Alice and qubit c_1 to Candy. Also, we assume that in the process of distribution of qubits, only two qubits go through amplitude damping noisy channel and qubits b_1, b_2, b_3 are not affected by this noise since these are not transmitted.

The density matrix of the corresponding entangled channel (2) can be written as $\rho = |\tau\rangle_{a_1c_1b_1b_2b_3}\langle\tau|$ and, after passing through the noisy environment, the corresponding density matrix evolves as

$$\epsilon(\rho) = \sum_{i,j} \left(K_i \otimes K_j \otimes I \otimes I \otimes I\right) \rho \left(K_i \otimes K_j \otimes I \otimes I \otimes I\right)^{\dagger}, \quad (5)$$

where K_i s are the Kraus operators satisfying $\sum_i K_i^{\dagger} K_i = I$ and $\dagger$ stands for the conjugate transpose of a matrix.

After that, Alice, Bob and Candy perform their duties as discussed in the previous section. Then the final output state ρ_{ij}^{out}, where $i, j \in \{0, 1\}$, can be evaluated as

$$\rho_{ij}^{\text{out}} = \text{Tr}_{a_1c_1}\left[R_{ij}\left(\epsilon(\rho)\right)R_{ij}^{\dagger}\right], \quad (6)$$

where partial trace is taken over the qubits a_1 and c_1 and R_{ij} is given by

$$\begin{aligned} R_{ij} \;=\; & \{I_{a_1} \otimes I_{c_1} \otimes (U^{ij})_{b_1b_2b_3} \otimes (V^{ij})_{b_1b_2b_3}\} \\ & \times \{I_{a_1} \otimes |\psi_j\rangle_{c_1}\langle\psi_j| \otimes I_{b_1b_2b_3}\} \\ & \times \{|\phi_i\rangle_{a_1}\langle\phi_i| \otimes I_{c_1} \otimes I_{b_1b_2b_3}\}, \end{aligned} \quad (7)$$

in which U^{ij} s and V^{ij} s are the unitary operators given in Table 1.

The fidelity corresponding to the final output state ρ_{ij}^{out} can be calculated as

$$F = \langle\chi|\rho_{ij}^{\text{out}}|\chi\rangle, \quad (8)$$

where $|\chi\rangle$ is the ideal output state. In the present protocol, it is given by gain one Hadamard gate is applied on qubit 2, then the state $|X_2\rangle$ of five-qubits becomes

$$|\chi\rangle = (\alpha_0|0\rangle + \alpha_1|1\rangle)_{b_1} \otimes (\beta_0|00\rangle + \beta_1|11\rangle)_{b_2b_3}.$$

4.1. Simultaneous remote state preparation protocol via amplitude damping noise

In this subsection, we particularly analyze the effect of amplitude damping (AD) noise in the simultaneous remote state preparation scheme. The Kraus operators of the AD noise are defined as[36]

$$K_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-k} \end{bmatrix}, \quad K_1 = \begin{bmatrix} 0 & \sqrt{k} \\ 0 & 0 \end{bmatrix},$$

where $k \in [0, 1]$ represents the decoherence rate of AD noise.

In this protocol, Bob prepares the 5-qubit entangled channel in his laboratory and distributes the qubits a_1 and c_1 to Alice and Candy respectively through AD noisy environment. Also, suppose that during the distribution process, these qubits pass through all possible AD noise. Then the corresponding density matrix of the transformed channel evolves in the following way:

$$\rho^{\text{AD}} = \frac{1}{4}\sum_{i=0}^{3} |P_i\rangle\langle P_i|,$$

where

$$\begin{aligned} |P_0\rangle &= |00000\rangle - (1-k)^{1/2}|01011\rangle + (1-k)^{1/2}|10100\rangle \\ &\quad + (1-k)|11111\rangle, \\ |P_1\rangle &= -(k)^{1/2}|00011\rangle + (k(1-k))^{1/2}|10111\rangle, \\ |P_2\rangle &= (k)^{1/2}|00100\rangle + (k(1-k))^{1/2}|01111\rangle, \\ |P_3\rangle &= k|00111\rangle. \end{aligned}$$

Now Alice and Candy perform their measurement with the basis given in Eqs. (3) and (4) and transmit the measurement results to Bob via 1-bit classical channels, respectively. After obtaining all the measurement results, Bob performs appropriate unitary operations with his qubits to get the desired state which both Alice and Candy want to create at Bob's laboratory.

As an illustration, suppose Alice's and Candy's measurement results be $|\phi_0\rangle_{a_1}$ and $|\psi_1\rangle_{c_1}$, respectively. Then, according to the Eq. (6), the density matrix of the final output state reduces to

$$\rho_{01}^{\text{out}} = \frac{1}{N}\sum_{i=0}^{3}|Q_i\rangle\langle Q_i|,$$

where

$$\begin{aligned}
|Q_0\rangle &= \alpha_0\beta_1|011\rangle + \alpha_0\beta_0(1-k)^{1/2}|000\rangle \\
&\quad + \alpha_1\beta_1(1-k)^{1/2}|111\rangle + \alpha_1\beta_0(1-k)|100\rangle, \\
|Q_1\rangle &= -\alpha_0\beta_1(k)^{1/2}|000\rangle - \alpha_1\beta_1(k(1-k))^{1/2}|100\rangle, \\
|Q_2\rangle &= \alpha_0\beta_1(k)^{1/2}|111\rangle + \alpha_0\beta_0(k(1-k))^{1/2}|100\rangle, \\
|Q_3\rangle &= -\alpha_0\beta_1 k|100\rangle,
\end{aligned}$$

and N is the normalization factor, which is given by

$$\begin{aligned}
N &= (1-k+k\alpha_0^2)(1-k\beta_0^2)+k(1-\beta_0^2)(1-k+k\alpha_0^2) \\
&\quad +k\alpha_0^2(1-k\beta_0^2)+k^2\alpha_0^2(1-\beta_0^2).
\end{aligned}$$

The fidelity of the final output state can be evaluated according to the formula (8) as

$$\begin{aligned}
F_1 &= \langle\chi|\rho_{01}^{\text{out}}|\chi\rangle \\
&= \frac{1}{N}\Big[\Big(\alpha_0^2\beta_0^2(1-k)^{1/2}+\alpha_0^2\beta_1^2 \\
&\quad +\alpha_1^2\beta_0^2(1-k)+\alpha_1^2\beta_1^2(1-k)^{1/2}\Big)^2 \\
&\quad +\Big(-\alpha_0^2\beta_0\beta_1 k^{1/2}-\alpha_1^2\beta_0\beta_1(k(1-k))^{1/2}\Big)^2 \\
&\quad +\Big(\alpha_0\alpha_1\beta_1^2 k^{1/2}+\alpha_0\alpha_1\beta_0^2(k(1-k))^{1/2}\Big)^2 \\
&\quad +\big(-\alpha_0\alpha_1\beta_0\beta_1 k\big)^2\Big]. \qquad (9)
\end{aligned}$$

In Fig. 4, we examine the changes in fidelity F_1 as we vary different parameters. Figure 4(a) demonstrates that when the noise strength $k = 0.5$, fidelity decreases as α_0 increases, but it increases with the increment of β_0. Figures 4(b)–4(d) illustrate that fidelity consistently decreases as the noise strength increases. In the upcoming section, we will explore the incorporation of weak and reversal measurements to enhance the fidelity of our scheme.

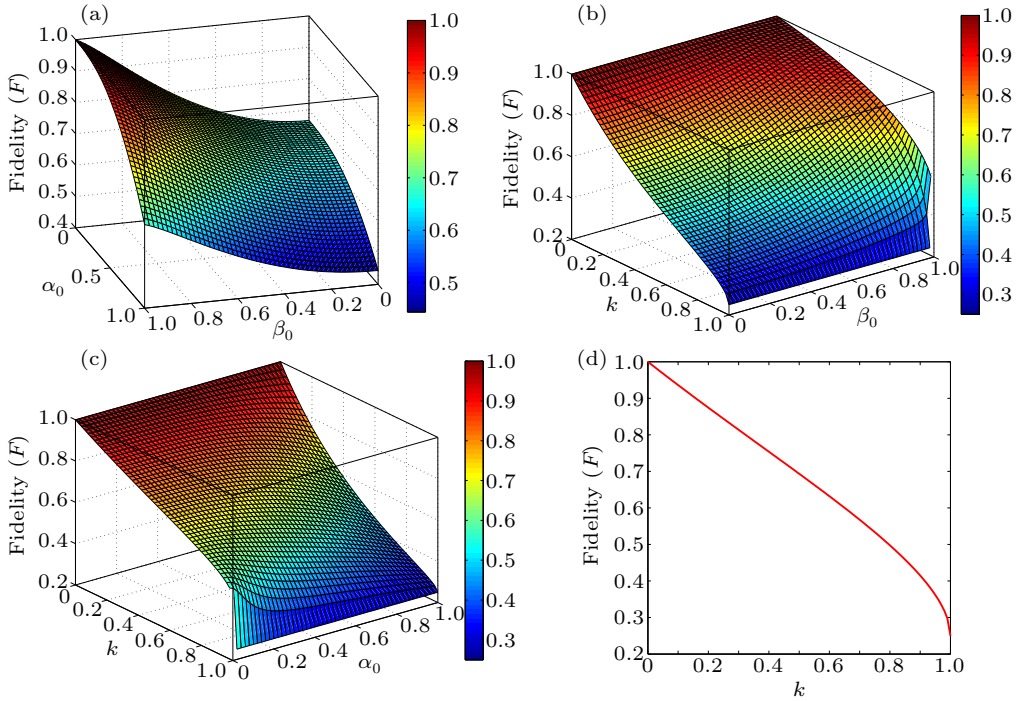


Fig. 4. (a) Variation of fidelity F_1 with α_0 and β_0 when $k = 0.5$. (b) Variation of fidelity F_1 with β_0 and k when $|M_1\rangle = (\sqrt{0.4}|0\rangle + \sqrt{0.6}|1\rangle)$. (c) Variation of fidelity F_1 with α_0 and k when $|M_2\rangle = (\sqrt{0.3}|000\rangle + \sqrt{0.7}|111\rangle)$. (d) Variation of fidelity F_1 with k when $|M_1\rangle = (\sqrt{0.4}|0\rangle + \sqrt{0.6}|1\rangle)$ and $|M_2\rangle = (\sqrt{0.3}|000\rangle + \sqrt{0.7}|111\rangle)$.

5. Protected remote state preparation protocol via weak and reversal measurement

Here we discuss the performance of weak and reversal measurements regarding the fidelity of our protocol. To protect the entangled resource, first weak measurements are carried out on the qubits before they are transmitted and reversal measurements are performed after the distribution of qubits to the respective parties.

The weak measurement and the corresponding reversal measurement operators on a single-qubit system can be expressed as[35]

$$W_{\rm m}=\begin{bmatrix}1 & 0\\ 0 & \sqrt{1-k_{\rm w}}\end{bmatrix}\quad\text{(weak measurement operator)},\tag{10}$$

$$R_{\rm m}=\begin{bmatrix}\sqrt{1-k_{\rm r}} & 0\\ 0 & 1\end{bmatrix}\quad\text{(reversal measurement operator)},\tag{11}$$

where $k_{\rm w}$ and $k_{\rm r}$ are the strengths of the weak and reversal measurement, respectively.

At the time of entanglement distribution only qubits a_1 and c_1 undergo AD noise to the respective parties Alice and Candy. Bob makes weak measurements given in Eq. (10) on the qubits a_1 and c_1 before they are distributed. After that, the quantum state (2) is transformed into the state (ignoring the constant factor)

$$\begin{aligned}|\tau^{\rm w}\rangle \;=\; &\Big(|00000\rangle-(1-k_{\rm w})^{1/2}|01011\rangle+(1-k_{\rm w})^{1/2}|10100\rangle\\ &+(1-k_{\rm w})|11111\rangle\Big)_{a_1c_1b_1b_2b_3}.\end{aligned}\tag{12}$$

Now Bob distributes the qubits a_1 and c_1 to Alice and Candy respectively and during this transmission qubits a_1 and c_1 pass through AD noisy environment. Then according to Eq. (5), the density matrix of the state (12) is evolved into

$$\rho_{\rm w}^{\rm AD}=\frac{1}{4}\sum_{i=0}^{3}|P_i'\rangle\langle P_i'|,$$

where

$$\begin{aligned}|P_0'\rangle &= |00000\rangle-(1-k)^{1/2}(1-k_{\rm w})^{1/2}|01011\rangle\\ &\quad+(1-k)^{1/2}(1-k_{\rm w})^{1/2}|10100\rangle+(1-k)(1-k_{\rm w})|11111\rangle,\\ |P_1'\rangle &= -k^{1/2}(1-k_{\rm w})^{1/2}|00011\rangle+(k(1-k))^{1/2}(1-k_{\rm w})|10111\rangle,\\ |P_2'\rangle &= k^{1/2}(1-k_{\rm w})^{1/2}|00100\rangle+(k(1-k))^{1/2}(1-k_{\rm w})|01111\rangle,\\ |P_3'\rangle &= k(1-k_{\rm w})|00111\rangle.\end{aligned}$$

After the above processes, Alice and Candy receive the respective qubits and then both the parties execute reversal measurements given in Eq. (11). Then the above quantum state is changed into the state

$$\rho_{\rm w-r}^{\rm AD}=\frac{1}{4}\sum_{i=0}^{3}|Q_i'\rangle\langle Q_i'|,$$

where

$$\begin{aligned}|Q_0'\rangle \;=\; &(1-k_{\rm r})|00000\rangle\\ &-(1-k)^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})^{1/2}|01011\rangle\\ &+(1-k)^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})^{1/2}|10100\rangle\\ &+(1-k)(1-k_{\rm w})|11111\rangle,\\ |Q_1'\rangle \;=\; &-k^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})|00011\rangle\\ &+(k(1-k))^{1/2}(1-k_{\rm w})(1-k_{\rm r})^{1/2}|10111\rangle,\\ |Q_2'\rangle \;=\; &k^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})|00100\rangle\\ &+(k(1-k))^{1/2}(1-k_{\rm w})(1-k_{\rm r})^{1/2}|01111\rangle,\\ |Q_3'\rangle \;=\; &k(1-k_{\rm w})(1-k_{\rm r})|00111\rangle.\end{aligned}$$

After that, Alice and Candy execute their measurement with the basis given in Eqs. (3) and (4) and transmit the measurement outcomes to Bob via 1-bit classical channels, respectively. After receiving all the measurement outcomes, Bob performs appropriate unitary operations with his qubits to get the original states which both Alice and Candy want to prepare at Bob's laboratory.

As an illustration, suppose Alice's and Candy's measurement results be $|\phi_0\rangle_{a_1}$ and $|\psi_1\rangle_{c_1}$ respectively. Then, according to Eq. (6), the density matrix of the final output state reduces to

$$\rho_{\rm w-r-01}^{\rm out}=\frac{1}{M}\sum_{i=0}^{3}|R_i'\rangle\langle R_i'|,$$

where

$$\begin{aligned}|R_0'\rangle \;=\; &\alpha_0\beta_1(1-k_{\rm r})|011\rangle\\ &+\alpha_0\beta_0(1-k)^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})^{1/2}|000\rangle\\ &+\alpha_1\beta_1(1-k)^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})^{1/2}|111\rangle\\ &+\alpha_1\beta_0(1-k)(1-k_{\rm w})|100\rangle,\\ |R_1'\rangle \;=\; &-\alpha_0\beta_1k^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})|000\rangle\\ &-\alpha_1\beta_1(k(1-k))^{1/2}(1-k_{\rm w})(1-k_{\rm r})^{1/2}|100\rangle,\\ |R_2'\rangle \;=\; &\alpha_0\beta_1k^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})|111\rangle\\ &+\alpha_0\beta_0(k(1-k))^{1/2}(1-k_{\rm w})(1-k_{\rm r})^{1/2}|100\rangle,\\ |R_3'\rangle \;=\; &-\alpha_0\beta_1k(1-k_{\rm w})(1-k_{\rm r})|100\rangle,\end{aligned}$$

and the normalization constant M is given by

$$\begin{aligned}M \;=\; &(1-k)^2(1-k_{\rm w})^2+k(1-k_{\rm w})^3(1-k)^2(\alpha_0^2+\beta_1^2)\\ &+\alpha_0^2\alpha_1^2\beta_0^2\beta_1^2k^2(1-k)^2(1-k_{\rm w})^4.\end{aligned}$$

The fidelity of the final output state can be calculated according to the formula (8) as

$$\begin{aligned}F_2 \;=\; &\langle\chi|\rho_{\rm w-r-01}^{\rm out}|\chi\rangle\\ =\; &\frac{1}{M}\Big[\big(\alpha_0^2\beta_0^2(1-k)^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})^{1/2}\\ &+\alpha_0^2\beta_1^2(1-k_{\rm r})+\alpha_1^2\beta_0^2(1-k)(1-k_{\rm w})\\ &+\alpha_1^2\beta_1^2(1-k)^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})^{1/2}\big)^2\\ &+\big(-\alpha_0^2\beta_0\beta_1k^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})\\ &-\alpha_1^2\beta_0\beta_1(k(1-k))^{1/2}(1-k_{\rm w})(1-k_{\rm r})^{1/2}\big)^2\\ &+\big(\alpha_0\alpha_1\beta_1^2k^{1/2}(1-k_{\rm w})^{1/2}(1-k_{\rm r})\end{aligned}$$

$$+\alpha_0\alpha_1\beta_0^2(k(1-k))^{1/2}(1-k_{\rm w})(1-k_{\rm r})^{1/2})^2$$
$$+(-\alpha_0\alpha_1\beta_0\beta_1 k(1-k_{\rm w})(1-k_{\rm r}))^2\Big]. \quad (13)$$

In the optimal reversal measurement condition $k_{\rm r}=k_{\rm w}+k(1-k_{\rm w})$,[34–36,38] the optimal fidelity can be derived from Eq. (13) which is

$$F_2^{\rm OP}=\frac{1+(\alpha_0^2\alpha_1^2+\beta_0^2\beta_1^2)k(1-k_{\rm w})+(\alpha_0\alpha_1\beta_0\beta_1)^2k^2(1-k_{\rm w})^2}{1+(\alpha_0^2+\beta_1^2)k(1-k_{\rm w})+(\alpha_0\alpha_1\beta_0\beta_1)^2k^2(1-k_{\rm w})^2}. \quad (14)$$

In Fig. 5, we represent the fidelity with and without weak and reversal measurements for specific values of the parameters. The figures show improvements in fidelity after executions of weak and reversal measurements in all the considered cases.

The total success probability for the protection of the scheme against noise can be calculated by taking the trace of the entangled channel after entanglement sharing is completed.[58]

In the ideal noiseless condition, the total success probability of our scheme is

$$p_{\rm (ideal)}={\rm Trace}\,(|\tau\rangle\langle\tau|)=1.$$

Also, in AD noisy environment (without weak and reversal measurements), the total success probability of our scheme is

$$p_{\rm (AD)}={\rm Trace}\,(\epsilon(\rho))=1.$$

But in the case when weak and reversal measurement is performed to increase the fidelity of our scheme, the total success probability is

$$\begin{aligned}p_{\rm (W-AD-R)} &= {\rm Trace}\,(\epsilon(\rho_{\rm w-r}^{\rm AD}))\\ &= \frac{1}{4}\Big[(1-k_{\rm r})^2+2(1-k)(1-k_{\rm w})(1-k_{\rm r})\\ &\quad +(1-k)^2(1-k_{\rm w})^2+2k(1-k_{\rm w})(1-k_{\rm r})^2\\ &\quad +2k(1-k)(1-k_{\rm w})^2(1-k_{\rm r})\\ &\quad +k^2(1-k_{\rm w})^2(1-k_{\rm r})^2\Big]. \quad (15)\end{aligned}$$

In the optimal reversal measurement condition $k_{\rm r}=k_{\rm w}+k(1-k_{\rm w})$, the total success probability becomes

$$p_{\rm (W-AD-R-O)}=\frac{1}{4}(1-k)(1-k_{\rm w})(2+k(1-k_{\rm w}))^2. \quad (16)$$

Figure 5 shows that the application of weak and reversal measurements improves the fidelity of our protocol. The utilization of weak and reversal measurements enhances the fidelity of our protocol compared to scenarios affected solely by AD noise. A comparison between Figs. 6(a) and 6(b) reveals the interesting feature that in the case when $k=0$, the fidelity is always of value 1 while success probability decreases with an increase in $k_{\rm w}$. But with an increase in the value of k, for fixed $k_{\rm w}$, the fidelity decreases while the success probability also decreases. Thus there is an inverse trade-off between fidelity and our failure to control the environmental noise.

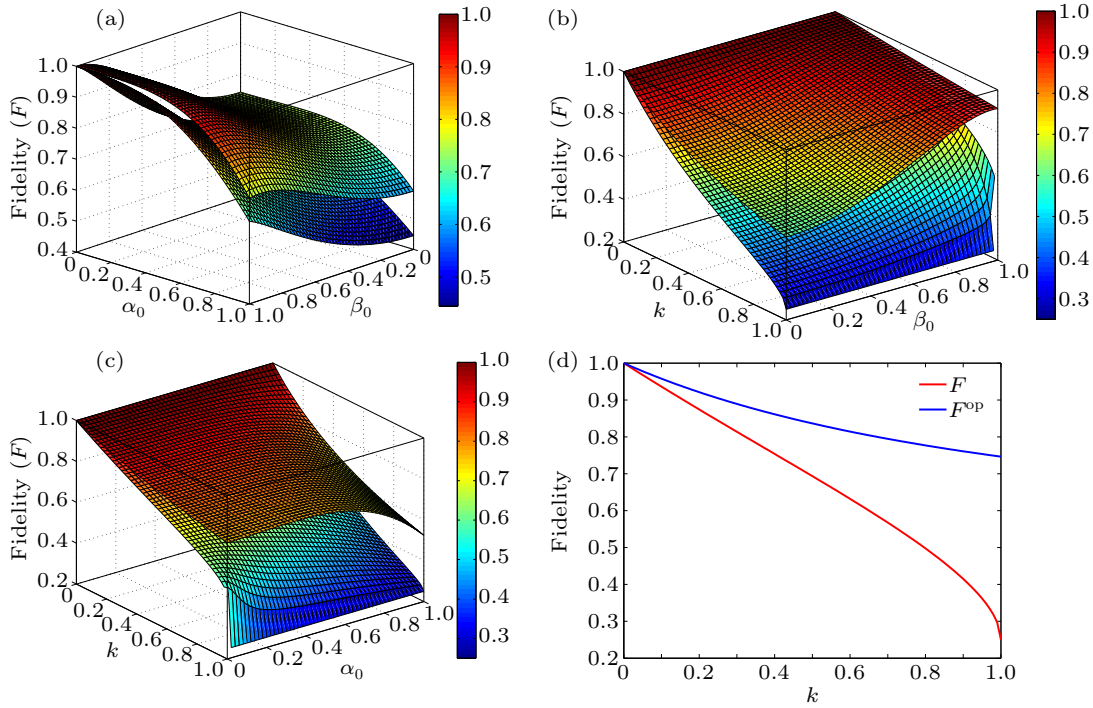


Fig. 5. (a) Variation of fidelity F_1 and $F_2^{\rm OP}$ with α_0 and β_0 when $k=0.5$, $k_{\rm w}=0.3$. (b) Variation of fidelity F_1 and $F_2^{\rm OP}$ with β_0 and k when $|M_1\rangle=(\sqrt{0.4}|0\rangle+\sqrt{0.6}|1\rangle)$ and $k_{\rm w}=0.3$. (c) Variation of fidelity F_1 and $F_2^{\rm OP}$ with α_0 and k when $|M_2\rangle=(\sqrt{0.3}|000\rangle+\sqrt{0.7}|111\rangle)$ and $k_{\rm w}=0.3$. (d) Variation of fidelity F_1 and $F_2^{\rm OP}$ with k when $|M_1\rangle=(\sqrt{0.4}|0\rangle+\sqrt{0.6}|1\rangle)$, $|M_2\rangle=(\sqrt{0.3}|000\rangle+\sqrt{0.7}|111\rangle)$ and $k_{\rm w}=0.3$. Here all the upper curves represent optimal fidelity ($F_2^{\rm OP}$) with weak and reversal measurements while lower curves represent fidelity F_1.

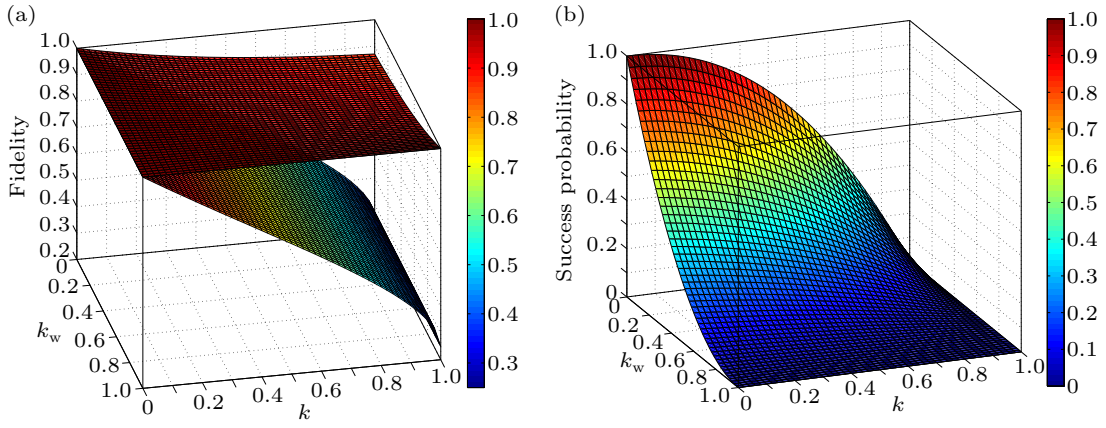


Fig. 6. (a) Variation of fidelity F_1 and F_2^{OP} with k_w and k when $|M_1\rangle = (\sqrt{0.4}|0\rangle + \sqrt{0.6}|1\rangle)$, $|M_2\rangle = (\sqrt{0.3}|000\rangle + \sqrt{0.7}|111\rangle)$. (b) Variation of success probability with respect to k and k_w.

6. Discussion and conclusion

We observe the fact that more deviation from the pure entangled resource has an adverse effect on fidelity. This has a physical implication that pure entangled states are better suitable for teleportation purposes. The noise may have different physical origins, which should bear a complex relationship with the performance of the protocol. In quantum systems, noise can arise due to random interactions with the environment and cause the system to lose coherence, leading to the degradation of quantum states. As the noise parameter increases, the rate of decoherence also increases, resulting in a decrease in fidelity.

In this paper, our final observation through Fig. 5 is that amplitude damping noise can be controlled successfully to some extent by employing weak and reversal measurements. Noise is unavoidable in a communication system. Any realistic picture of communication protocol must involve the effect of noise. For a reliable system of communication, the analysis and control of noise is very important. Here we have considered amplitude damping noise but there are several other types of noises like phase-flip, bit-flip, bit-phase flip, phase damping, etc. which are also modeled with the help of appropriate Kraus operators. A particular feature of the noise analysis, as it appears in Fig. 4, shows that the fidelity tends to 1 as the noise parameter goes to zero, which is what should be expected. Remarkably, the fidelity, albeit low, is never zero, even if the noise parameter is at its maximum.

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