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Citation: Chin. Phys. B, 2023, 32 (**11**): 118901. DOI: 10.1088/1674-1056/ace682
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Assessing edge-coupled interdependent network disintegration via rank aggregation and elite enumeration

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(Received 4 April 2023; revised manuscript received 3 July 2023; accepted manuscript online 12 July 2023)

The disintegration of networks is a widely researched topic with significant applications in fields such as counter-terrorism and infectious disease control. While the traditional approaches for achieving network disintegration involve identifying critical sets of nodes or edges, limited research has been carried out on edge-based disintegration strategies. We propose a novel algorithm, i.e., a rank aggregation elite enumeration algorithm based on edge-coupled networks (RAEEC), which aims to implement tiling for edge-coupled networks by finding important sets of edges in the network while balancing effectiveness and efficiency. Our algorithm is based on a two-layer edge-coupled network model with one-to-one links, and utilizes three advanced edge importance metrics to rank the edges separately. A comprehensive ranking of edges is obtained using a rank aggregation approach proposed in this study. The top few edges from the ranking set obtained by RAEEC are then used to generate an enumeration set, which is continuously iteratively updated to identify the set of elite attack edges. We conduct extensive experiments on synthetic networks to evaluate the performance of our proposed method, and the results indicate that RAEEC achieves a satisfactory balance between efficiency and effectiveness. Our approach represents a significant contribution to the field of network disintegration, particularly for edge-based strategies.

Keywords: edged-coupled, rank aggregation, interdependent networks, elite enumeration

PACS: 89.75.Fb **DOI:** 10.1088/1674-1056/ace682

1. Introduction

Complex networks are ubiquitous in modern society,[1] encompassing a wide range of systems such as power grids, communication networks, and social networks. As technology advances, these networks are becoming increasingly intertwined and their structures are growing more complex.[2–5] Therefore, gaining a comprehensive understanding of the intricate association patterns within and between these networks is crucial for in-depth analysis.

Extensive research has been conducted on robustness of single-layer networks, including the identification of key nodes[6–12] and edges.[13–15] However, with the increasing interconnectivity of networks, single-layer models are no longer sufficient to meet practical needs. In reality, interdependent networks are more prevalent, such as cyber-physical power systems (CPPSs) and financial networks composed of different regions. Although current research on interdependent networks is based on point-to-point coupled networks,[16,17] which establish links between layers through individual nodes, many real-world interdependent networks rely on edge-to-edge coupling. For instance, city networks comprise many cities connected by transportation routes. Another example is the edge-coupled network of the transportation network and economic trade network, where the trade relations between economies depend on the routes in the transportation network. As well as the edge-coupled network of production supply chain network and logistics and transportation network where suppliers need logistics services for distribution and transportation of goods, etc. Despite the prevalence of such edge-coupled interdependent networks, there is a lack of in-depth research on their robustness, resulting in limited understanding of the impact of edge failures in one network on another network. Therefore, studying the robustness of interdependent networks based on edge-coupled links is critical.

Real-world networks exhibit heterogeneity, where a few nodes and edges play crucial roles in determining network structure and function, while the majority of nodes and edges are less significant. Numerous methods have been developed to identify key nodes, such as degree centrality, betweenness, and k-core decomposition. For instance, Xu *et al.*[18] proposed an information entropy-based algorithm that considers two-hop information to identify key nodes. Wang *et al.*[19] developed a multi-scale information importance method that integrates local and global information to identify critical nodes more effectively. However, few studies have focused on identifying significant edges in networks, but it is also critical to network disintegration and protection. For instance, in CPPSs,[20] identifying critical transmission lines can help prevent possible attacks and reduce the likelihood of cascading failures in interdependent networks. Similarly, in infectious disease networks, breaking the most damaging transmission chains by identifying critical edges can prevent further disease spread. Consequently, this paper investigates the robustness of edge-coupled interdependent networks by examining the critical edges in the network.

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Identifying the significant edge sets in the network has been proven to be NP-hard (non-deterministic polynomial), indicating that it is a non-deterministic polynomial-time problem. Therefore, the resolution of this combinatorial optimization problem necessitates the development of innovative and efficient algorithms. The time cost can be high when choosing methods such as intelligent optimization to search for significant edge sets. Therefore, our goal is to make the proposed algorithms achieve a satisfactory balance between effectiveness and efficiency as much as possible. Compared with point-coupled networks, edge-coupled networks are more robust and less prone to disintegration, so how to effectively disintegrate edge-coupled networks is novel and well worth considering. Therefore, edge-coupled networks under one-to-one links are chosen as the object of the present study.

To improve efficiency and effectiveness, a novel rank aggregation elite enumeration algorithm based on edge-coupled networks (RAEEC) is proposed. Our algorithm performs rank aggregation of multiple edge importance metrics, avoiding the one-sidedness of single metrics and the high time cost of intelligent optimization algorithms. The conversion of large enumeration to small enumeration in selecting the set of elite attack edges saves time and cost significantly. Extensive experiments are conducted on synthetic networks, and results show that our new method outperforms existing state-of-the-art algorithms. The main contributions of this study are as follows: (1) Model one-to-one interdependent edge-coupled networks. (2) Implement to combine metrics with different layers of edge importance into one. (3) The RAEEC algorithm achieves a satisfactory balance between effectiveness and efficiency.

The rest of this paper is organized as follows. Section 2 presents recent advances in the study of the importance of edge nodes. The basic definition of complex networks is given in Section 3, and advanced metrics for studying edge importance are presented. Section 4 gives the model of edge-coupled interdependent networks under one-to-one links and the process by which cascading occurs. In Section 5, we propose the RAEEC algorithm for identifying the significant edges in the network. Section 6 presents the results of simulation experiments and compares the RAEEC algorithm with four state-of-the-art algorithms. Finally, in Section 7, conclusions are drawn based on our findings.

2. Related works

In recent years, there has been an increasing interest among researchers in identifying important edges in a network due to their significant practical applications. Most of the current methods for measuring the importance of edges rely on the structural information of the network.

The concept of edge meson, which is a generalization of the point meson, has been proposed, but it requires a large amount of computational resources. Similar to the betweenness of nodes, the edge betweenness[21] was proposed by Newman *et al.* Yu *et al.*[22] proposed the BCCMOD algorithm based on the number of edge networks, which takes into account the topology and information dissemination capability of the network. However, all of these algorithms are based on global information and perform better on small-scale networks, but are not suitable for large-scale networks. To address this issue, Holme *et al.*[23] introduced the concept of degree product (DP), where the DP of an edge is the product of the degrees of the two endpoints of the edge. Chen *et al.*[24] proposed a semi-local centrality metric (SLC) that considers information from one-, two-, three-, and four-hop neighbors and works better than some global centrality indices on most real networks.

Cheng *et al.*[25] have proposed the Bridgeness metric to quantify the importance of edges in maintaining network links. Liu *et al.*[26] have introduced diffusion intensity (DI) as a measure to explain the diffusion of edges in dynamics, and Zhao *et al.*[27] have proposed a second-order neighborhood (SN) index that accounts for the topological overlap (TO)[28] of the second-order neighbors of the two endpoints of an edge. Additionally, some methods have investigated the importance of edges, including eigenvalues,[29] nearest neighbor links,[30] and so on.

Intelligent optimization algorithms, such as evolutionary algorithms[31] and tabu-search algorithms,[32] can also be used to find significant edges in the network. However, while these algorithms may yield better results compared to centrality algorithms, their time complexity is generally very high. Therefore, achieving a balance between efficiency and effectiveness is a critical issue.

While previous algorithms for identifying significant edges in networks are based on single-layer networks, for edge-coupled interdependent networks, considering only intra-layer neighborhoods can result in a loss of significant information. Thus, this paper improves previous algorithms for edge-coupled interdependent networks and proposes a novel algorithm that balances effectiveness and efficiency to achieve a satisfactory trade-off between the them.

3. Preliminaries

The interdependent networks consisting of two networks are abstracted as an undirected, unweighted graph $G_A := (V_A, E_A)$, $G_B := (V_B, E_B)$, respectively. $N_A = |V_A|$ and $N_B = |V_B|$ are the numbers of nodes in networks G_A and G_B; E_A and E_B is the sets of edges in the networks. We describe the networks G_A and G_B by adjacency matrices A and B, respectively, where $a_{ij} = 1$ and $b_{ij} = 1$ indicate the direct existence of links between nodes i and j, otherwise $a_{ij} = 0$ and $b_{ij} = 0$. In this

section, four advanced metrics used to measure edge importance are presented.

3.1. Degree product

The degree product (DP) is expressed as the product of the degrees of nodes i and j at the ends of edge e_{ij},

$$D(i,j) = k_i k_j, \tag{1}$$

where k_i and k_j are the intra-layer degrees of nodes i and j, respectively.

3.2. Topological overlap

To measure the topological overlap (TO) of two vertices in an edge, we introduce the concept of topological overlap. A smaller value of topological overlap means that the edge is more important,

$$T(i,j) = \frac{n_{ij}}{k_i + k_j - n_{ij} - 2}, \tag{2}$$

where k_i and k_j denote the degrees of nodes i and j, n_{ij} denotes the number of common neighbors of nodes i and j. If nodes i and j do not have any common neighboring nodes, then $n_{ij} = 0$, implying that the value of $T(i,j)$ is also zero. If the common neighbors of nodes i and j are identical, $T(i,j) = 1$.

3.3. Diffusion intensity

Diffusion intensity (DI) is the average of the propagation intensity of nodes i and j,

$$I(i,j) = \frac{n_{i/j} + n_{j/i}}{2}, \tag{3}$$

where $n_{i/j}$ is all neighbors of node i minus those remaining neighbors associated with node j, and $n_{j/i}$ is all neighbors of node j minus those associated with node i. The $I(i,j)$ metric quantifies the average potential impact of edge e_{ij} in both directions ($i \to j, j \to i$).

3.4. Second-order neighborhood index

The second-order neighborhood (SN) index shows the topological overlap of the second-order neighbors of the two endpoints of an edge, and the index is considered an extension of the topological overlap index. Similar to topological overlap, the lower the SN value is, the more important the edge is,

$$S(i,j) = \frac{\left|p_{ij}^{(2)} \cap p_{ji}^{(2)}\right|}{\left|p_{ij}^{(2)} \cap p_{ji}^{(2)}\right|}, \tag{4}$$

where $p_{ij}^{(2)}$ denotes the number of vertices in the remaining network with distance 2 from node i after deleting edge ij in network G_{A}, and $p_{ji}^{(2)}$ denotes the number of vertices in the remaining network with distance 2 from node j after deleting edge ij in network G_{A}.

4. Edge-coupled one-to-one interdependent networks

4.1. Definition of edge-coupled interdependent networks

In this paper, we consider a two-layer edge-coupled interdependent network under one-to-one links, where the two layers are denoted as network G_{A} and network G_{B}, and assume that a node in network G_{A} can have at most one dependent edge in network G_{B}.

To visualize the edge-coupled network with one-to-one links, a specific example is shown in Fig. 1. The interdependent network in the figure consists of two networks, G_{A} and G_{B}, with eight nodes in G_{A} and eight nodes in G_{B}, respectively. G_{A} and G_{B} contain 9 and 8 edges, respectively, with at most one coupling edge existing at each node. Both G_{A} and G_{B} are maximal connected branches, so the edge-coupled network consisting of G_{A} and G_{B} also forms a giant component.

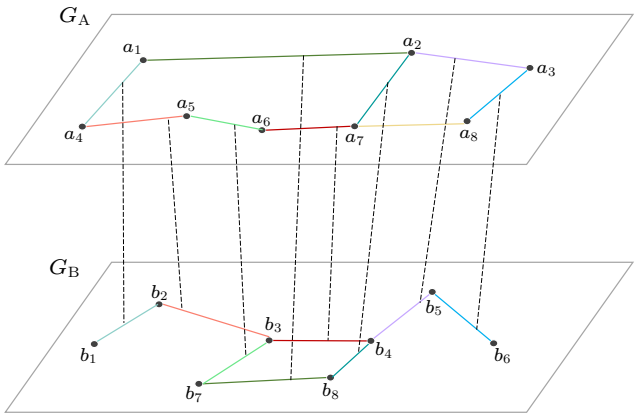


Fig. 1. Edge-coupled interdependent networks under one-to-one links.

Since the coupling relationship between the layers is edge-coupled, using a symmetric adjacency matrix to record the coupling edges is not reasonable. For edge-coupled networks, we need to record the edges of G_{A} and G_{B} corresponding to the coupling edges in two separate matrices. For convenience, we define the matrix D to describe the inter-layer coupling edges. Its dimension is n-by-4, where n represents the number of coupling edges and $n \leq \min(|E_{\mathrm{A}}|, |E_{\mathrm{B}}|)$. The first two numbers in each row of matrix D represent the node numbers i and j of the edges in G_{A} for which the coupling edge is coupled, and the last two numbers represent the node numbers i' and j' of the edges in G_{B} corresponding to the coupling edge.

4.2. The propagation process of cascading failures

In this paper, we presume that only edges in layer G_{A} are attacked and that layers G_{A} and G_{B} have the same cascade pattern after a failure. When an edge in layer G_{A} is attacked, that edge is immediately disconnected, and at the same time, if there is a coupled edge in that edge, the edge in layer G_{B}

to which it is coupled is also disconnected. Next, a cascade failure may occur in the interdependent network, simply by finding the maximum connected branches of layers $G_{\rm A}$ and $G_{\rm B}$ respectively, and then continuously deleting edges and nodes until the cascade failure ends.

We presume the ratio of the number of nodes belonging to the giant component in the network at the end of the cascade failure to the total number of initial nodes is used as a measure of the robustness of the edge-coupled interdependent network, denoted by $S(Q)$, where $S(Q) = (N'_{\rm A} + N'_{\rm B})/(N_{\rm A} + N_{\rm B})$. Q is the edge set of the attack, and $N'_{\rm A}$ and $N'_{\rm B}$ are the numbers of nodes remaining in the network $G_{\rm A}$ and $G_{\rm B}$, respectively, after the attack and the cascade occurs.

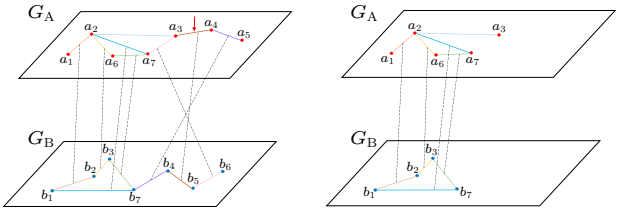


Fig. 2. A concrete example of a cascading failure in an edge-coupled network. Layer $G_{\rm A}$ and $G_{\rm B}$ are a pair of one-sided couplings, with solid black lines indicating edges connecting nodes belonging to the giant component of each layer and vertical dashed lines indicating coupled edges between layers. Attack edge a_3–a_4 in layer $G_{\rm A}$, the final percentage of nodes remaining in the interdependent network is $S(Q) = (5+4)/(7+7) = 0.64$.

- First stage: In layer $G_{\rm A}$, edges a_3–a_4 are invalidated due to deliberate attacks, and since layers, $G_{\rm A}$ and $G_{\rm B}$ are one-to-one coupled, edges b_4–b_5 are removed from layer $G_{\rm B}$, while the coupled edges linking a_3–a_4 and b_4–b_5 are removed.
- Second stage: Find the giant component in the $G_{\rm A}$ layer and delete the edge a_4–a_5. Next, find the edge $b_4 - b_7$ coupled to the edge a_4–a_5 in the $G_{\rm B}$ layer and delete it.
- Third stage: Similar to the second stage, find the edge in layer $G_{\rm B}$ that does not belong to the giant component and delete the edge b_5–b_6, while finding the edge a_3–a_7 coupled with the edge b_5–b_6 in layer $G_{\rm A}$ and delete it.
- Fourth stage: Repeat the steps in the second and third stages to delete nodes and edges in $G_{\rm A}$ and $G_{\rm B}$ that do not belong to the giant component until all nodes and edges belong to the giant component, ending the cascade.

Our goal is to maximize the degree of disintegration of the interdependent network by attacking edges in network $G_{\rm A}$. In this paper, we consider an edge-coupled network measured by the proportion of remaining nodes in the network denoted by $S(Q)$. Therefore, the problem of minimizing the maximum number of giant component (MIN-GC) of the edge-coupled networks can be formulated as follows:

$$\begin{aligned} &\min\ S(v_1, v_2, \ldots, v_{|E_{\rm A}|}) \\ &\text{s.t.} \sum_{k=1}^{|E_{\rm A}|} v_k \le |E_{\rm A}| \cdot f,\ k = 1,2,\ldots,|E_{\rm A}|, \\ &\quad v_k \in \{0,1\},\ k = 1,2,\ldots,|E_{\rm A}|, \end{aligned} \tag{5}$$

where v_k indicates whether the k-th edge is selected, $v_k = 1$ means that the k-th edge is selected into the attack edge set, $v_k = 0$ means that it is not selected, and f $(0 \le f \le 1)$ indicates the deliberate attack strength.

5. Rank aggregation elite enumeration algorithm for disintegrating edge-coupled networks

5.1. Definition of rank aggregation method

To facilitate the study, this paper chooses to attack only the edges in network $G_{\rm A}$. For the importance of edges in network $G_{\rm A}$, we need to consider also the possible coupled edges between network $G_{\rm A}$ and network $G_{\rm B}$. Taking the edge diffusion strength as an example of a measure of edge importance, the idea of this paper is to assume that any edge e_{ij} is chosen in-network $G_{\rm A}$ and if there is a coupling edge e_{ij} between $e_{i'j'}$ and network $G_{\rm B}$, then the overall diffusion strength of that edge e_{ij} is

$$I(e_{ij}) = \begin{cases} I_{\rm A}(e_{ij}) + I_{\rm B}(e_{i'j'}), & \text{if coupling edge exists between} \\ & e_{ij} \text{ and } e_{i'j'}, \\ I_{\rm A}(e_{ij}), & \text{otherwise}, \end{cases} \tag{6}$$

where $I_{\rm A}(e_{ij})$ and $I_{\rm B}(e_{i'j'})$ are the diffusion intensity values of $G_{\rm A}$ and $G_{\rm B}$ at the network layer.

Since the coupling between networks $G_{\rm A}$ and $G_{\rm B}$ is one-to-one when an edge e_{ij} in network $G_{\rm A}$ is attacked, if a coupling edge $e_{i'j'}$ exists for that edge, then edge $e_{i'j'}$ will also be removed from network $G_{\rm B}$. Therefore, the importance of an edge in network $G_{\rm A}$ should be measured by adding the edge in network $G_{\rm A}$ that is coupled to that edge.

In order to maximize the disintegration of the network, we need to find the optimal set of attacking edges such that the combination of this set of attacking edges hurts the coupled edge network the most. Previous studies on the importance of nodes have many measures, including degree, betweenness, and so on. In fact, if we use only one criterion to measure node importance, we are likely to miss some potentially critical nodes. This is true for nodes as well as for edges. For single-layer networks, the graph-based rank aggregation method[33] aggregates all individual rankings into a single consensus ranking. Therefore, in this study, by combining several more advanced methods for studying the importance of edges in networks, we propose an RAEEC algorithm based on edge-coupled interdependent networks.

Consider N representatives $\{r_1, r_2, \ldots, r_N\}$ ranking $|E_{\rm A}|$ edges in network $G_{\rm A}$ sequentially, and the obtained ranking

matrix is set to $R=(r_{ij})_{N\cdot|E_{\rm A}|}$, where r_{ij} denotes the ranking after the scoring given to the j-th edge by representative r_i. A concrete example of rank aggregation is given in the following:

$$R=\begin{bmatrix}4&3&5&1&2\\3&4&2&5&1\\5&4&2&3&1\\5&3&2&4&1\\3&4&5&1&2\\5&2&3&4&1\end{bmatrix}, \tag{7}$$

where each row in R denotes the ranking of each edge after the representative person r_i scores, respectively. Matrix (7) gives the ranking of the six representatives scoring the five edges. Take the first row of the R matrix as an example, the five edges are ranked as 4, 3, 5, 1, and 2.

A transition matrix $T^i=(t_{jk})_{|E_{\rm A}|\cdot|E_{\rm A}|}$ $(1\le i\le N)$ is obtained below from each row of R separately. If r_{ij} is smaller than r_{ik}, it means that the i-th representative considers the edge numbered j more important than the edge numbered k, i.e., $t_{jk}=1$, otherwise, $t_{jk}=0$.

$$T^1=\begin{bmatrix}0&0&1&0&0\\1&0&1&0&0\\0&0&0&0&0\\1&1&1&0&1\\1&1&1&0&0\end{bmatrix},\quad T^2=\begin{bmatrix}0&1&0&1&0\\0&0&0&1&0\\1&1&0&1&0\\0&0&0&0&0\\1&1&1&1&0\end{bmatrix},$$
$$T^3=\begin{bmatrix}0&0&0&0&0\\1&0&0&0&0\\1&1&0&1&0\\1&1&0&0&0\\1&1&1&1&0\end{bmatrix},\quad T^4=\begin{bmatrix}0&0&0&0&0\\1&0&0&1&0\\1&1&0&1&0\\1&0&0&0&0\\1&1&1&1&0\end{bmatrix},$$
$$T^5=\begin{bmatrix}0&1&1&0&0\\0&0&1&0&0\\0&0&0&0&0\\1&1&1&0&1\\1&1&1&0&0\end{bmatrix},\quad T^6=\begin{bmatrix}0&0&0&0&0\\1&0&1&1&0\\1&0&0&1&0\\1&0&0&0&0\\1&1&1&1&0\end{bmatrix}. \tag{8}$$

Next, based on the transmission matrix T^i, we can obtain the score matrix $U=(u_{ij})_{|E_{\rm A}|\cdot|E_{\rm A}|}$, where $u_{ij}=\sum_{n=1}^{N}t_{ij}^n$. Thus, from matrix (8), we can obtain the score matrix

$$U=\begin{bmatrix}0&2&2&1&0\\4&0&3&3&0\\4&3&0&4&0\\5&3&2&0&2\\6&6&6&4&0\end{bmatrix}. \tag{9}$$

Based on the score matrix, we can find the "in-degree" and "out-degree" values of each edge in network $G_{\rm A}$,

$$r_j^{\rm in}=\sum_{j=1}^{|E_{\rm A}|}u_{jk}, \tag{10}$$

$$r_j^{\rm out}=\sum_{j=1}^{|E_{\rm A}|}u_{kj}. \tag{11}$$

Finally, we can define a measure of the importance of edges in network $G_{\rm A}$ by calculating the ratio of incoming and outgoing edges (RIOE) as follows:

$$\gamma(j)=\frac{r_j^{\rm in}+1}{r_j^{\rm out}+1}. \tag{12}$$

The three parameters and the final ranking of each edge are listed in Table 1. The overall ranking of the edge set from front to back is e_5,e_4,e_3,e_2,e_1. The final rank aggregation based on edge coupling is ranked as $\hat{R}=[e_5,e_4,e_3,e_2,e_1]$.

Table 1. Final ranking results of the edge set based on the rank aggregation method.

Edge	$r_j^{\rm in}$	$r_j^{\rm out}$	$\gamma(j)$	Rank
e_1	5	19	0.3	5
e_2	10	14	0.733	4
e_3	11	13	0.923	3
e_4	12	12	1	2
e_5	22	2	7.667	1

5.2. Rank aggregation elite enumeration algorithm

In the following, we will propose a novel rank aggregation enumeration algorithm. First, the rank aggregation is performed for $E_{\rm A}$ edges in the network layer $G_{\rm A}$ according to the previously proposed method. Based on the extensive experimental analysis, it is found that the best result is achieved when $N=3$. Too few or too many indicators will affect the effect of aggregation. The metrics chosen in this paper are HDI,[26] HTO,[28] and HSN.[27]

The HDI, HTO, and HSN methods differ from the DI, TO, and SN methods in that each time. The most significant edge in the current network is determined, that edge is removed from the network. The above steps are repeated until all edges in the network are sorted. It is not difficult to infer that the HDI, HTO and HSN methods can dismantle the network to a greater extent under the same attack strength because it avoids repeated attacks and improves the attack efficiency.

Our approach is to select the optimal set of edges I among the three methods HDI, HTO and HSN and to take the highest ranked part of edges in the rank aggregation and merge them. Suppose that the attack strength is f, i.e., the number of edges attacked at this point is $f\cdot|E_{\rm A}|$, and then the optimal set of edges among the three methods and the top-ranked set in the rank aggregation method are selected and merged, respectively, to obtain $2\cdot f\cdot|E_{\rm A}|$ $(|I|=2\cdot f\cdot|E_{\rm A}|)$ edges as the candidate edge set. Next, the set of candidate edges is enumerated, but if we perform a simple one-by-one enumeration of the set I, i.e., take $C_{2\cdot f\cdot|E_{\rm A}|}^{f\cdot|E_{\rm A}|}$, it is not difficult to find that the time cost must be very high.

Therefore, reducing the running time becomes the most fundamental problem to be solved. Our idea is to construct a queue containing a total of $\lceil(f\cdot|E_{\rm A}|)/5\rceil$ groups of edges, with 5 edges in each group except the last group, which contains the

number of edges $f\cdot|E_{\rm A}|-5\cdot(\lceil(f\cdot|E_{\rm A}|)/5\rceil-1)$. This converts the $C^{f\cdot|E_{\rm A}|}_{2\cdot f\cdot|E_{\rm A}|}$ enumeration into a $\lceil(f\cdot|E_{\rm A}|)/5\rceil\cdot C^{k'}_{j}$ enumeration (j is the total number of edges at the intersection of the top five edges in the current three methods' optimal sum rank aggregate ranking; $k'=k$ or q; $j\leq 10, k'\leq 5$), which can greatly reduce runtime. Figure 3 shows the process of entering the queue with edge sets. After each group of edges enters the queue, it is removed from the network, and the next time it continues to select the top 5 edges from the remaining edges to enter the queue until all $f\cdot|E_{\rm A}|$ edges enter the queue.

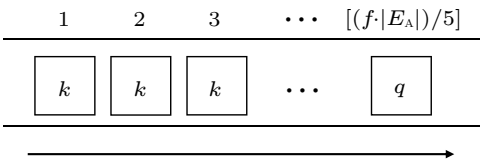


Fig. 3. The process of entering the queue of attack edge sets.

The RAEEC algorithm based on edge-coupled interdependent networks for solving the set of attacking edges can be summarized in the following five steps:

Step I Calculate the HDI, HTO, and HSN values of all edges in the network layers $G_{\rm A}$ and $G_{\rm B}$ corresponding to the three metrics.

Step II If edge i in network $G_{\rm A}$ has coupling edge j in network $G_{\rm B}$, the importance value corresponding to edge i is equal to the importance value of edge i plus edge j in the previous step.

Step III Using the three indicators HDI, HTO, and HSN as representatives, the score matrix U is calculated using the rank aggregation method, and the importance ranking of all edges in network $G_{\rm A}$ is obtained.

Step IV Under the condition that the attack intensity is f, perform a queuing operation on $f\cdot|E_{\rm A}|$ edges, with each group of five edges. Enumerate the union sets of the top 5 edges obtained from the best of the three algorithms and rank aggregation, queue the best solution in $C^{k'}_{j}$, and delete the queued edges in the network.

Step V Repeat step IV until all the $f\cdot|E_{\rm A}|$ edges are in the queue, the $f\cdot|E_{\rm A}|$ edges are the set of elite attack edges.

6. Simulation results and analysis

We conducted extensive numerical experiments on synthetic and real networks and compared the RAEEC algorithm proposed in this paper with four classical methods for computing edge importance to verify the advantages of the RAEEC algorithm. The four compared algorithms are the high degree product (HDP),[23] the high topological overlap (HTO),[28] the high diffusion intensity (HDI),[26] and the high second-order neighborhood index (HSN).[27]

All the experiments listed in this paper were performed on a Windows 11 computer with an Intel i7-12700 CPU and 16 GB of RAM, implemented under MATLAB 2020b.

6.1. Benchmark datasets

In this section, three representative artificial networks, i.e., BA scale-free networks,[34] ER random networks,[35] and small-world (WS) networks[36] are chosen for comparisons. The edge coupling between networks $G_{\rm A}$ and $G_{\rm B}$ is chosen in this paper to be a random one-to-one link, with at most one dependent edge in the network that exists in the other network. The details are as follows.

ER networks Each layer in the network is defined by two parameters: N and p, i.e., p is the probability of connecting each pair of nodes and N is the number of nodes in the network. In this paper, we set $N_{\rm A}, N_{\rm B}=50$, 80, 100, 200 and $p=0.12$, 0.08, 0.06, 0.03. The final edge-coupled ER-ER network includes 100, 160, 200 and 400 nodes.

BA networks Generating a scale-free network requires three input parameters: m_0, m, and N. The initial number of vertices in each layer of the network is m_0, and m edges are added for each additional y vertices until the number of nodes in the network is N. For extensive experiments, we set $N_{\rm A}=N_{\rm B}=50$, 80, 100, 200, $m_0=5$, and $m=3$. The final edge-coupled interdependent network includes 100, 160, 200, and 400 nodes.

WS networks Three parameters are required to generate the WS network, N is the number of points, K represents the total number of neighbors per node, where the left side has $K/2$ neighbors and the right side $K/2$ neighbors, and p represents the reconnection of each edge in the network with probability p. For extensive experiments, we set $N_{\rm A}=N_{\rm B}=50$, 80, 100, 200, $p=0.02$, and $K=6$.

Although the above three networks are artificially generated, they contain many features of complex networks and can be studied as the topology of real social relationships. BA scale-free networks are highly heterogeneous, as evidenced by the preferential connection mechanism of their nodes. Real-life social, biological, and trade networks all have the characteristics of scale-free networks. ER networks are homogeneous, and small-world graphs reflect a characteristic of friend networks. To diversify the experiment, we coupled different networks for comparative study.

6.2. Experiments and results

In this paper, we assume that the network has achieved disintegration when the proportion of remaining nodes in the coupled network is less than 20%. For comparative analysis, the intra-layer average degree of BA, ER and WS is set to 6, while networks $G_{\rm A}$ and $G_{\rm B}$ are generated separately.

Figure 4 shows the total number of nodes in the network for 100, 160, 200, and 400, respectively. It can be seen that

all RAEEC algorithms outperform the four advanced methods mentioned above in identifying the significant edges of the network. In the first half of the curve, the RAEEC algorithm does not differ much from HDI, HTO and HSN, but as the percentage of attacks increases, the advantage of RAEEC becomes more obvious.

The slope of the curve in the figure becomes steeper and steeper. When the attack intensity is small, the overall damage to the edge-coupled network is small, and the curve falls more gently, attacking roughly 40% of the nodes in the network. However, as the attack intensity increases, the edges connected to the vertices are removed one after another, and cascade failures occur continuously within the network layer. If there are inter-layer coupled edges, the cascade will also occur within another layer, which leads to the proportion of remaining nodes dropping directly from 60% to less than 20%.

According to the theoretical analysis, the BA-BA network should be the easiest to disintegrate because its heterogeneity is the strongest. However, in Fig. 4, it is not difficult to find that the WS-WS edge-coupled network is the easiest to disintegrate. We analyze it because in this paper only the WS-WS network fully implements one-to-one inter-layer coupling, i.e., attacking the edges in the network G_{A}, it must be possible to find coupled edges connected in the network G_{B}, making its attack effect be better. The average degree of the remaining two coupled networks is only approximately equal to 6, i.e., the neighboring edges of networks G_{A} and G_{B} are not equal, which leads to the WS-WS network being easier to disintegrate compared to ER-ER and BA-BA. Secondly, the BA-BA network is more prone to disintegration with a percolation threshold equal to 0.7 less than the ER-ER network. The ER-ER edge-coupled network is the lowest degree of disintegration among the three coupled networks, and its percolation threshold f_{c} remains around 0.8.

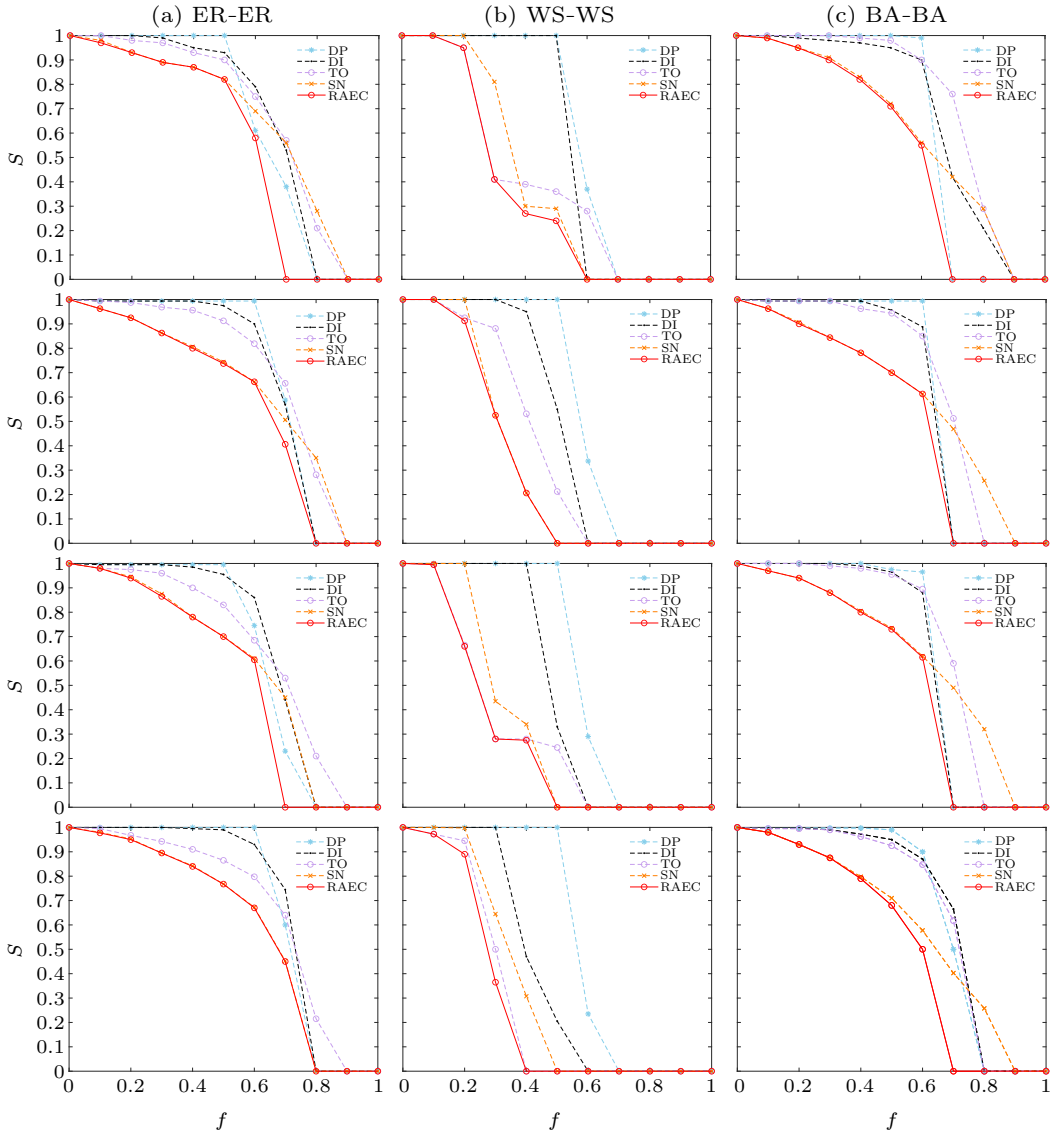


Fig. 4. Comparisons of different methods under edge-coupled interdependent networks. The four rows indicate the total number of nodes in the network as 100 ($N_{\mathrm{A}} = N_{\mathrm{B}} = 50$), 160 ($N_{\mathrm{A}} = N_{\mathrm{B}} = 80$), 200 ($N_{\mathrm{A}} = N_{\mathrm{B}} = 100$), 400 ($N_{\mathrm{A}} = N_{\mathrm{B}} = 200$).

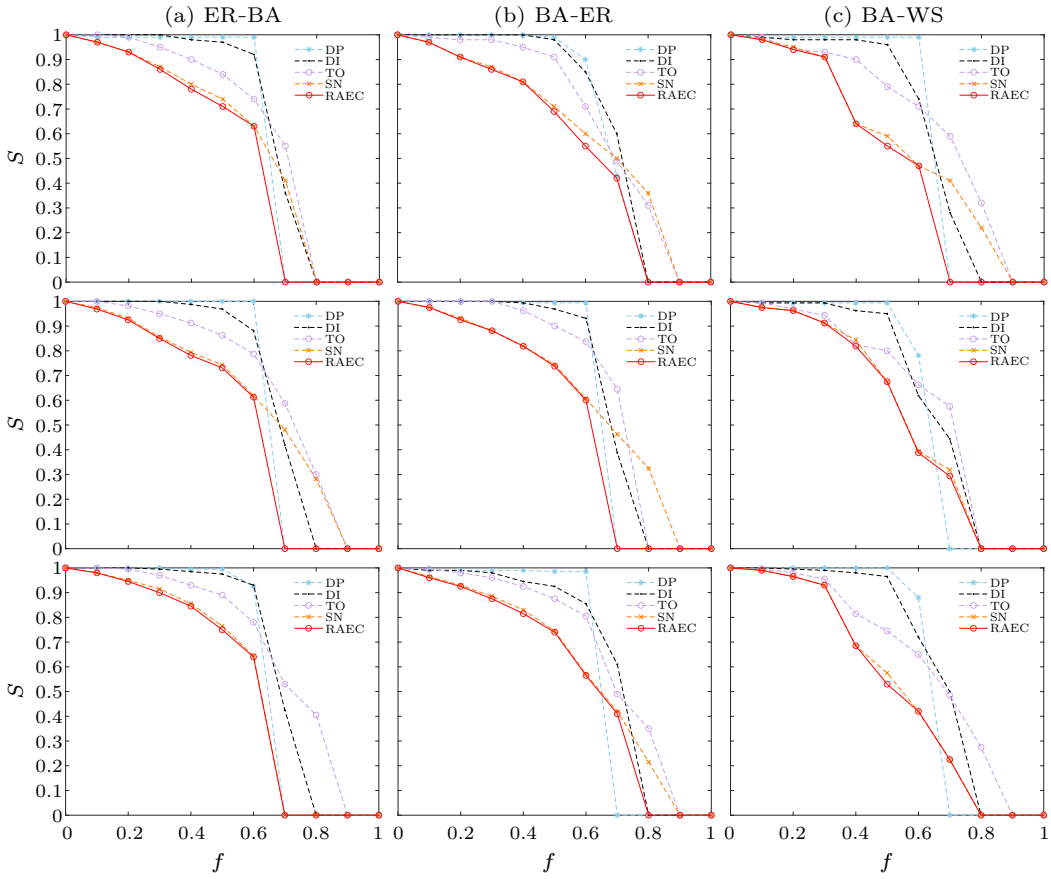


Fig. 5. Comparisons of different methods under ER-BA, BA-ER, and BA-WS edge-coupled interdependent networks. The three rows indicate the total number of nodes in the network as 100 ($N_A = N_B = 50$), 160 ($N_A = N_B = 80$), 200 ($N_A = N_B = 100$).

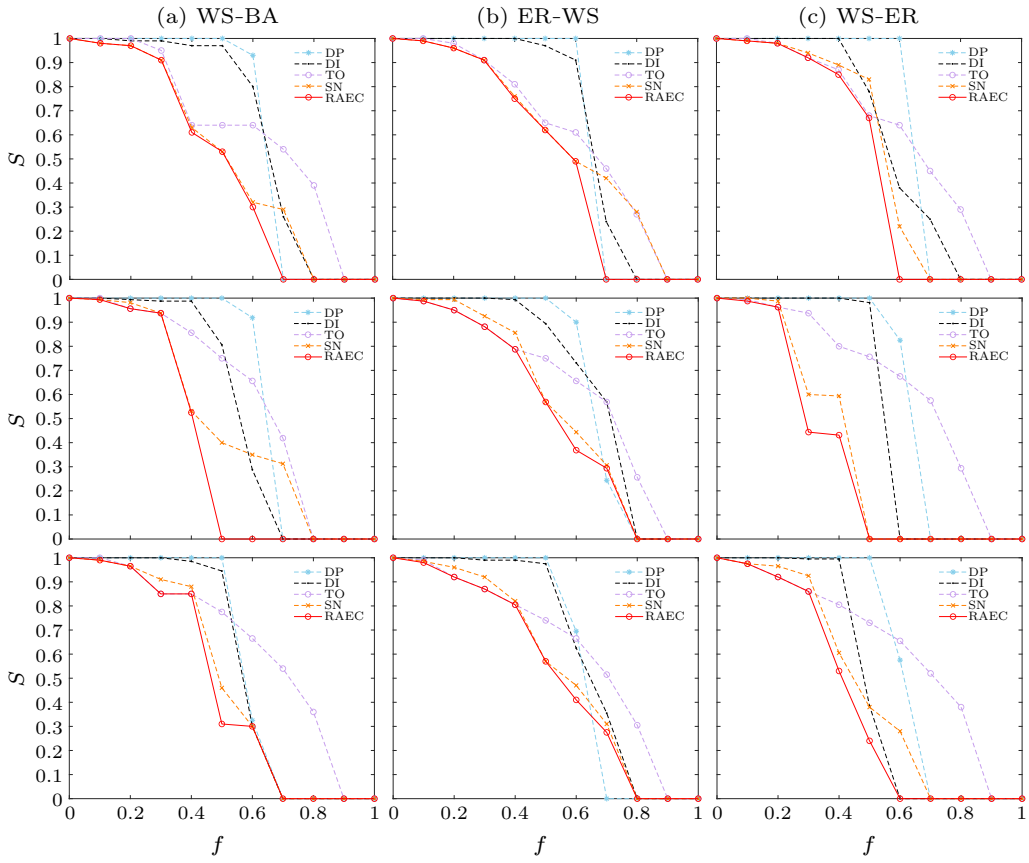


Fig. 6. Comparisons of different methods under WS-BA, ER-WS, and WS-ER edge-coupled interdependent networks. The three rows indicate the total number of nodes in the network as 100 ($N_A = N_B = 50$), 160 ($N_A = N_B = 80$), 200 ($N_A = N_B = 100$).

From Figs. 5 and 6, it can be seen that the RAEEC algorithm combines the advantages of HDI, HTO and HSN and outperforms all four compared algorithms. The shapes of the curves are roughly similar as the number of nodes in the network increases. Meanwhile, when the number of nodes in the network is the same, the ER-BA edge-coupled network is less robust, and its percolation threshold is all 0.7, which is more easily disintegrated compared with the BA-ER coupled network. Therefore, when the disintegration cost is limited to choosing one of BA and ER to construct the edge-coupled network, we choose the network G_{A} as the ER network. This is because the edge-coupled network can be disintegrated more efficiently for the same attack ratio.

For the edge-coupled networks with different upper and lower layers consisting of BA and WS, the WS-BA network is less robust, and its percolation thresholds are all below 0.7. Meanwhile, compared with the BA-WS network, the RAEEC algorithm has more obvious advantages over the remaining four compared algorithms. For the edge-coupled network composed of ER and WS, the WS-ER edge-coupled network is less robust and has a larger drop in the curve compared to the ER-WS network.

Compared with the ER-ER edge coupling in Fig. 4, replacing the network G_{B} from ER network with the BA network reduces the robustness of the network and is more prone to disintegration. In contrast, replacing the network G_{A} with the BA network reduces its robustness but does not change much, and its percolation threshold remains around 0.8.

We selected three larger-scale edge-coupled networks, WS-WS ($N_{\mathrm{A}} = N_{\mathrm{B}} = 600$, $\langle k_{\mathrm{A}}\rangle = \langle k_{\mathrm{B}}\rangle = 4$), WS-WS ($N_{\mathrm{A}} = N_{\mathrm{B}} = 1000$, $\langle k_{\mathrm{A}}\rangle = \langle k_{\mathrm{B}}\rangle = 2$), and BA-BA ($N_{\mathrm{A}} = N_{\mathrm{B}} = 2000$, $\langle k_{\mathrm{A}}\rangle = \langle k_{\mathrm{B}}\rangle = 4$), for our experiments. From Fig. 7, it is easy to find that the RAEEC algorithm still outperforms the remaining four comparative algorithms on the larger-scale networks, indicating that the algorithm has effectiveness and stability.

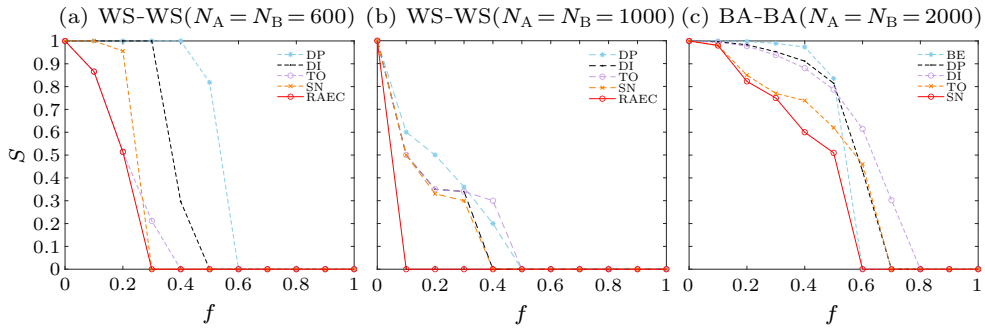


Fig. 7. Comparisons of different methods on larger scale edge-coupled networks. Three represent the total number of nodes in the network as 1200 ($N_{\mathrm{A}} = N_{\mathrm{B}} = 600$), 2000 ($N_{\mathrm{A}} = N_{\mathrm{B}} = 1000$), and 4000 ($N_{\mathrm{A}} = N_{\mathrm{B}} = 2000$).

6.3. Time complexity and runtime

In the following, we analyze the time complexity of the five algorithms. For the four methods DP, DI, TO, and SN, the time complexity is $O(|E_{\mathrm{A}}| \cdot N_{\mathrm{A}}^2)$.

Now we analyze the time complexity of the RAEEC algorithm proposed in this paper. First, it is necessary to calculate the edge importance ranking of the three algorithms DI, TO, and SN, respectively, whose time complexity is $O(|E_{\mathrm{A}}| \cdot N_{\mathrm{A}}^2)$. Next, ranking aggregation is performed to obtain the combined ranking of the set of edges of network G_{A} with time complexity $O(|E_{\mathrm{A}}|^2)$. Finally, elite enumeration is performed, with a time complexity of $O(\lceil (f \cdot |E_{\mathrm{A}}|)/5 \rceil \cdot f \cdot |E_{\mathrm{A}}| \cdot N_{\mathrm{A}}^2)$. In summary, the time complexity of the RAEEC algorithm is $O(|E_{\mathrm{A}}|^2 \cdot N_{\mathrm{A}}^2)$.

Although the time complexity of the RAEEC algorithm is slightly higher than the remaining four previous comparison algorithms, the results of RAEEC are significantly better than the four algorithms. Compared with intelligent algorithms, RAEEC does not need the repeated variation and crossover mechanisms in intelligent algorithms to maintain the improvement of results and has obvious efficiency advantages. Therefore, the RAEEC algorithm obtained by rank aggregation of three algorithms HDI, HTO and HSN, and elite enumeration achieves a balance in efficiency and effectiveness directly.

For the running time of these five algorithms, we conducted experiments on BA-BA edge-coupled network, and the results show that when the number of network nodes is 100, the running time of the DP algorithm is 1.29 s, the running time of the DI algorithm and TO algorithm are similar, which are 3.51 s and 3.39 s, respectively. For the SN algorithm it is 30.11 s, and for the RAEEC algorithm proposed in this paper it can be seen that the running time of the RREEC algorithm is close to that of the SN algorithm, but the effect is much better than that of the SN algorithm.

7. Conclusion

This paper presents a method to disintegrate edge-coupled interdependent networks by attacking the set of significant edges in the network G_{A}. A rank aggregation elite enumeration algorithm is proposed. First, the combined ranking of all edges in network G_{A} is obtained by the rank aggregation method. Next, the enumeration set is determined and the best of the three algorithms in HDI, HTO, HSN, and the

top-ranked edges obtained by the rank aggregation algorithm is selected. The enumeration set is enumerated one by one in the group, and the best edge set in the group is entered into the queue, and the cycle continues so that the edge set is entered into the queue until the condition is satisfied. Compared with direct enumeration, RAEEC can greatly reduce the time complexity. The set of attack edges obtained at the same time combines the advantages of all three methods, HDI, HTO, and HSN, under different attack strengths, and the results are better than all the three methods.

Extensive experiments have been conducted on synthetic networks, and the results show that the elite edge sets obtained by RAEEC are superior to the three methods of HDI, HTO, and HSN, and their time complexity is not significantly different. Moreover, the time complexity of RAEEC is greatly reduced compared to evolutionary algorithms, etc.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant Nos. 61877046, 12271419, and 62106186), the Natural Science Basic Research Program of Shaanxi (Program No. 2022JQ-620), and the Fundamental Research Funds for the Central Universities (Grant Nos. XJS220709, JB210701, and QTZX23002).

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