

Influence of the initial parameters on soliton interaction in nonlinear optical systems

Xinyi Zhang(张昕仪) and Ye Wu(吴晔)

Citation: Chin. Phys. B, 2023, 32 (**7**): 070505. DOI: 10.1088/1674-1056/ace1da
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

---

reather and its interaction with rogue wave of the coupled modified nonlinear Schrödinger equation

Ming Wang(王明), Tao Xu(徐涛), Guoliang He(何国亮), and Yu Tian(田雨)
Chin. Phys. B, 2023, 32 (**5**): 050503. DOI: 10.1088/1674-1056/ac833d

Lie symmetry analysis and invariant solutions for the (3+1)-dimensional Virasoro integrable model

Hengchun Hu(胡恒春) and Yaqi Li(李雅琦)
Chin. Phys. B, 2023, 32 (**4**): 040503. DOI: 10.1088/1674-1056/ac9de6

Dynamics of lump chains for the BKP equation describing propagation of nonlinear waves

Zhonglong Zhao(赵忠龙), Lingchao He(和玲超), and Abdul-Majid Wazwaz
Chin. Phys. B, 2023, 32 (**4**): 040501. DOI: 10.1088/1674-1056/acb0c1

All-optical switches based on three-soliton inelastic interaction and its application in optical communication systems

Shubin Wang(王树斌), Xin Zhang(张鑫), Guoli Ma(马国利), and Daiyin Zhu(朱岱寅)
Chin. Phys. B, 2023, 32 (**3**): 030506. DOI: 10.1088/1674-1056/acad6c

Quantitative analysis of soliton interactions based on the exact solutions of the nonlinear Schrödinger equation

Xuefeng Zhang(张雪峰), Tao Xu(许韬), Min Li(李敏), and Yue Meng(孟悦)
Chin. Phys. B, 2023, 32 (**1**): 010505. DOI: 10.1088/1674-1056/ac9822

Influence of the initial parameters on soliton interaction in nonlinear optical systems

Xinyi Zhang(张昕仪)[1] and Ye Wu(吴晔)[2,†]

[1] *School of Statistics, Beijing Normal University, Beijing* 100875, *China*
[2] *School of Journalism and Communication, Beijing Normal University, Beijing* 100875, *China*

(Received 6 June 2023; revised manuscript received 22 June 2023; accepted manuscript online 27 June 2023)

In nonlinear optical systems, optical solitons have the transmission properties of reducing error rate, improving system security and stability, and have important research significance in future research on all optical communication. This paper uses the bilinear method to obtain the two-soliton solutions of the nonlinear Schrödinger equation. By analyzing the relevant physical parameters in the obtained solutions, the interaction between optical solitons is optimized. The influence of the initial conditions on the interactions of the optical solitons is analyzed in detail, the reason why the interaction of the optical solitons is sensitive to the initial condition is discussed, and the interactions of the optical solitons are effectively weakened. The relevant results are beneficial for reducing the error rate and promoting the communication quality of the system.

Keywords: optical solitons, nonlinear Schrödinger equation, soliton interactions

PACS: 05.45.Yv, 42.65.Tg, 42.81.Dp **DOI:** 10.1088/1674-1056/ace1da

1. Introduction

In the field of nonlinear optics, researchers have shifted their attention to studying the propagation characteristics of individual optical solitons while also studying the transmission of multiple optical solitons in optical fibers and fiber lasers.[1–7] Research has shown that when there are two or more optical solitons in a fiber, they will interact with each other, attracting or repelling each other like particles.[8–10] Under certain initial conditions, complex interactions between optical solitons can lead to a series of interesting nonlinear phenomena.[11–13] Therefore, studying the interaction between optical solitons is of great significance.[14–17] In the study of the interaction between optical solitons, the interaction between two optical solitons is the most fundamental one.[18–20]

On the other hand, the nonlinear Schrödinger (NLS) equation is an important equation for describing the propagation of optical solitons.[21–23] It is widely used in fields such as optical communication, optical computing, and optical storage. In the field of nonlinear optics, the research significance of the NLS equation is particularly prominent.[24–26] For example, the self-phase modulation is one of the most basic phenomena in nonlinear optics, which is caused by the nonlinear effect of optical solitons. When optical solitons propagate in a fiber, they interact with each other, causing local phase changes in the fiber, thereby altering the frequency and waveform of the optical solitons.[27–29] Therefore, the NLS equation can not only help us better understand the nonlinear effects in the transmission and interaction of optical solitons, but also provide new solutions and technical means to improve the performance and reliability of nonlinear optical systems.[30–32]

In this paper, we will study the interaction between two optical solitons based on the NLS equation. By studying the interaction between two optical solitons, the problem of signal interference in nonlinear optical systems can be solved. The forms of the NLS equation studied here are as follows:[33]

$$\mathrm{i}u_x+\alpha_2(u_{tt}+2u|u|^2)-\mathrm{i}\alpha_3(u_{ttt}+6u_t|u|^2)=0, \tag{1}$$

where u is the optical soliton envelope, α_2 is the second-order effects, and α_3 is the third-order effects. For Eq. (1), the periodic soliton interactions have been analyzed in Ref. [33]. However, the influence of the initial parameters on soliton interactions is not reported in nonlinear optical systems. Here, we will investigate the effects of the initial phase, the initial spacing, and other parameters on the interaction of optical solitons.

In Section 2, we will get the two-soliton solutions for Eq. (1). In Section 3, we will discuss the influences of the relevant physical parameters in the obtained solutions on soliton interactions. In Section 4, we will give the conclusion.

2. Two-soliton solutions

Assuming that $u=g(x,t)/f(x,t)$, and substituting it into Eq. (1), we have

$$\begin{aligned}&\mathrm{i}\left(\frac{g}{f}\right)_x+\alpha_2\left[\left(\frac{g}{f}\right)_{tt}+2\frac{g}{f}\left|\frac{g}{f}\right|^2\right]\\&-\mathrm{i}\alpha_3\left[\left(\frac{g}{f}\right)_{ttt}+6\left(\frac{g}{f}\right)_t\left|\frac{g}{f}\right|^2\right]=0,\end{aligned} \tag{2}$$

†Corresponding author. E-mail: wuye@bnu.edu.cn

© 2023 Chinese Physical Society and IOP Publishing Ltd

and then, Eq. (2) can be expanded into the following form:

$$\mathrm{i}\frac{D_x g\cdot f}{f^2}+\alpha_2\left[\left(\frac{D_t^2 g\cdot f}{f^2}-\frac{g}{f}\frac{D_t^2 f\cdot f}{f^2}\right)+2\frac{g}{f}\left|\frac{g}{f}\right|^2\right]$$
$$-\mathrm{i}\alpha_3\left[\left(\frac{D_t^3 g\cdot f}{f^2}-3\frac{D_t g\cdot f}{f^2}\frac{D_x^2 f\cdot f}{f^2}\right)+6\frac{D_t g\cdot f}{f^2}\left|\frac{g}{f}\right|^2\right]=0.$$

Here, D_x, D_t^2 and D_t^3 are the bilinear operators, which are defined as[33]

$$D_x^n D_t^m g(x,t)\cdot f(x,t)$$
$$=\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^n\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^m\times g(x,t)\cdot f(x',t')|_{x'=x,t'=t}.$$

Thus, the bilinear forms of Eq. (1) can be derived as

$$\left(\mathrm{i}D_x+\alpha_2 D_t^2-\mathrm{i}\alpha_3 D_t^3\right)g\cdot f=0,$$
$$D_t^2(f\cdot f)-2|g|^2=0. \tag{3}$$

Assuming that

$$g(x,t)=\varepsilon g_1(x,t)+\varepsilon^3 g_3(x,t),$$
$$f(x,t)=1+\varepsilon^2 f_2(x,t)+\varepsilon^4 f_4(x,t),$$

among them

$$g_1(x,t)=\mathrm{e}^{\theta_1}+\mathrm{e}^{\theta_2},$$
$$g_3(x,t)=A_1\mathrm{e}^{\theta_1+\theta_2+\theta_1^*}+A_2\mathrm{e}^{\theta_1+\theta_2+\theta_2^*},$$
$$f_2(x,t)=B_1\mathrm{e}^{\theta_1+\theta_1^*}+B_2\mathrm{e}^{\theta_1+\theta_2^*}+B_3\mathrm{e}^{\theta_2+\theta_1^*}+B_4\mathrm{e}^{\theta_2+\theta_2^*},$$
$$f_4(x,t)=C_1\mathrm{e}^{\theta_1+\theta_1^*+\theta_2+\theta_2^*}, \tag{4}$$

and

$$\theta_1=k_1(x)+m_1 t+\omega_1,$$
$$\theta_2=k_2(x)+m_2 t+\omega_2. \tag{5}$$

Substituting Eqs. (10) and (4) into Eq. (1), extracting the nth power coefficients, and setting them to zero, we can obtain

$$k_1(x)=(\mathrm{i}\alpha_2 m_1^2+\alpha_3 m_1^3)xk_2(x)=\left(\mathrm{i}\alpha_2 m_2^2+\alpha_3 m_2^3\right)x, \tag{6}$$

and

$$A_1=\frac{(m_2-m_1)^2}{(m_1+m_1^*)^2(m_2+m_1^*)^2},$$
$$A_2=\frac{(m_2-m_1)^2}{(m_1+m_2^*)^2(m_2+m_2^*)^2}, \tag{7}$$
$$B_1=\frac{1}{\left(m_1+m_1^*\right)^2},\quad B_2=\frac{1}{(m_2+m_1^*)^2},$$
$$B_3=\frac{1}{(m_1+m_2^*)^2},\quad B_4=\frac{1}{(m_2+m_2^*)^2}, \tag{8}$$
$$C_1=\frac{(m_1-m_2)^2(m_1^*-m_2^*)^2}{(m_1+m_1^*)^2(m_2+m_1^*)^2(m_1+m_2^*)^2(m_2+m_2^*)^2}. \tag{9}$$

Setting $\varepsilon=1$, we can get u as

$$u=\frac{\mathrm{e}^{\theta_1}+\mathrm{e}^{\theta_2}+A_1\mathrm{e}^{\theta_1+\theta_2+\theta_1^*}+A_2\mathrm{e}^{\theta_1+\theta_2+\theta_2^*}}{1+B_1\mathrm{e}^{\theta_1+\theta_1^*}+B_2\mathrm{e}^{\theta_1+\theta_2^*}+B_3\mathrm{e}^{\theta_2+\theta_1^*}+B_4\mathrm{e}^{\theta_2+\theta_2^*}+C_1\mathrm{e}^{\theta_1+\theta_2+\theta_1^*+\theta_2^*}}. \tag{10}$$

Expression (10) is the two-soliton solutions of Eq. (1).

3. Discussion

For two-soliton solutions (10), there are six parameters m_1, m_2, α_2,α_3, ω_1 and ω_2. Next, we will mainly discuss the influence of these parameters on the interaction of optical solitons. At first, we assume that $\alpha_2=1$, $\alpha_3=0.002$, $\omega_1=0$, $\omega_2=0$, and analyze the influences of m_1 and m_2. In Fig. 1(a), $m_1=-0.88$, $m_2=-0.34$, we can find that periodic interaction between two optical solitons occurs. During the interaction process, optical solitons exhibit the periodic oscillation behavior. In Fig. 1(a), two solitons first exhibit mutual repulsion, then attract each other, and finally interact with each other, resulting in the pulse splitting. Changing the values of m_1 and m_2, we can adjust the period of the optical soliton interactions. In Fig. 1(b), the optical soliton interaction increases. While the optical soliton interaction decreases in Figs. 1(c) and 1(d). In Fig. 1, the change of the values of m_1 and m_2 can affect the interaction period of the optical solitons when they are input in the same phase.

In Fig. 2, we study the effect of the values of m_1 and m_2 on the interaction between optical solitons when they have a certain spacing in the early stage of incidence. In Fig. 2(a), $m_1=-1.3$, $m_2=2$, the spacing between optical solitons is small, and they will always interact during the transmission process. Due to the larger amplitude of the optical soliton on the right, the influence will be greater during the interaction process, and the amplitude of the optical soliton will change more violently. On the contrary, in Fig. 2(b), due to the large value of m_1, the amplitude of the left optical soliton is greater than that of the right optical soliton, resulting in a more drastic change in the amplitude of the left optical soliton. In Fig. 2(c), the spacing between the optical solitons is very small, and they transmit periodically like breathing solitons. In Fig. 2(d), increasing the distance between the optical solitons can prevent them from interacting during transmission. In this case, although it is beneficial to improve the transmission quality of optical solitons in nonlinear optical systems and avoid signal crosstalk, it will reduce the communication capacity of the sys-

tem.

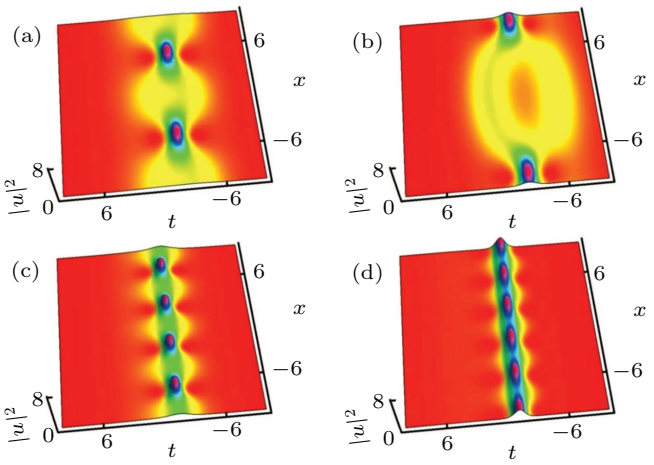


Fig. 1. The influence of initial parameters on soliton interactions. The parameters are $\alpha_2 = 1$, $\alpha_3 = 0.002$, $\omega_1 = 0$, $\omega_2 = 0$ with (a) $m_1 = -0.88$, $m_2 = -0.34$; (b) $m_1 = -0.41$, $m_2 = -0.72$; (c) $m_1 = -0.34$, $m_2 = -1.2$; (d) $m_1 = -0.22$, $m_2 = -1.3$.

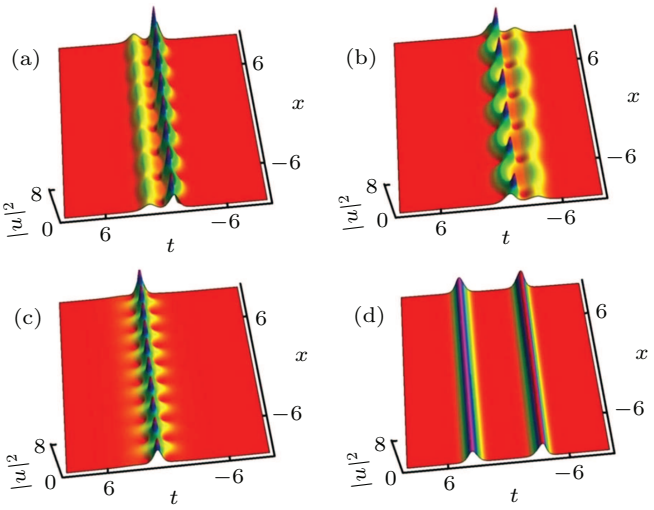


Fig. 2. The influence of initial parameters on soliton interactions. The parameters are $\alpha_2 = 1$, $\alpha_3 = 0.002$, $\omega_1 = 0$, $\omega_2 = 0$ with (a) $m_1 = -1.3$, $m_2 = 2$; (b) $m_1 = -1.7$, $m_2 = -1.1$; (c) $m_1 = 0.56$, $m_2 = 1.8$; (d) $m_1 = -1.7$, $m_2 = 1.8$.

In Fig. 3, we will keep the value of m_1 unchanged, and mainly consider the impact of the changes of the values of m_2, ω_1 and ω_2 on the interaction of optical solitons. At first, in Fig. 3(a), we assume $m_2 = -1.9$, $\omega_1 = -3.8$, $\omega_2 = 2.4$, and can find that the solitons may undergo subtle interactions during the parallel transmission. When $m_2 = 1.7$, the amplitude of the left optical soliton increases, which has a significant impact on the interaction process in Fig. 3(b). By increasing the spacing between optical solitons, we can also achieve non-interactive transmission between optical solitons in Figs. 3(c) and 3(d). By further reducing the distance between them, we can achieve intense interactions between optical solitons. No matter how they interact with each other, optical solitons with larger amplitudes undergo greater changes, as shown in Fig. 4. Changing the values of α_2 and α_3, the two optical solitons are incident at different phases at this point, and they only interact in a certain region during transmission and then propagate forward along the original path in Fig. 5. α_2 and α_3 take different

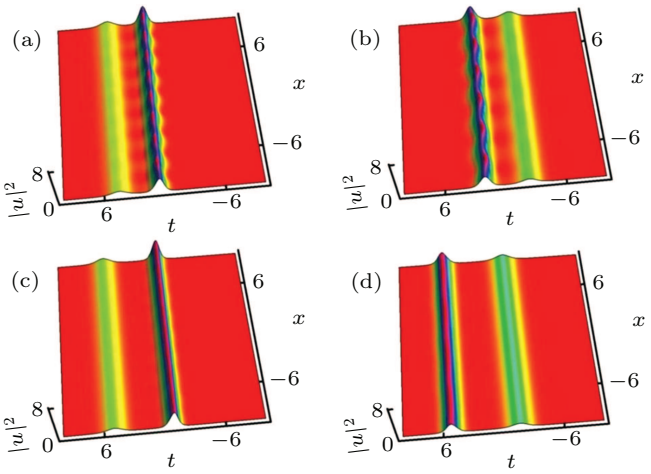


Fig. 3. The influence of initial parameters on soliton interactions. The parameters are $\alpha_2 = 1$, $\alpha_3 = 0.002$, $m_1 = 1$ with (a) $m_2 = -1.9$, $\omega_1 = -3.8$, $\omega_2 = 2.4$; (b) $m_2 = 1.7$, $\omega_1 = 3$, $\omega_2 = 0.7$; (c) $m_2 = 1.9$, $\omega_1 = -1.9$, $\omega_2 = 3$; (d) $m_2 = 1.5$, $\omega_1 = 2.3$, $\omega_2 = -3.3$.

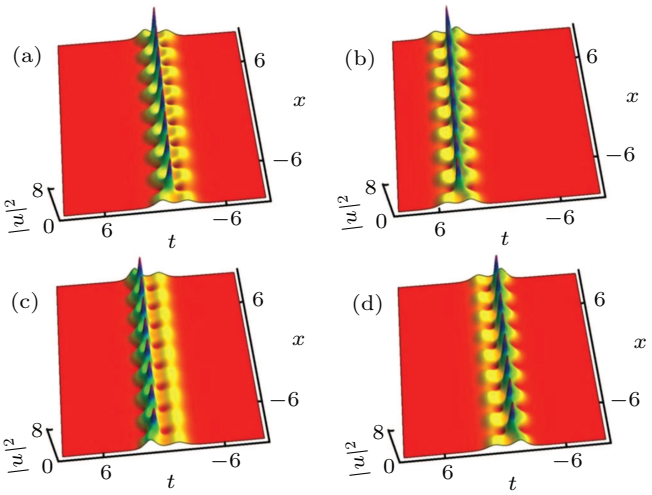


Fig. 4. The influence of initial parameters on soliton interactions. The parameters are $\alpha_2 = 1$, $\alpha_3 = 0.002$, $m_1 = 1$, $m_2 = 1.9$ with (a) $\omega_1 = 2.8$, $\omega_2 = 3.8$; (b) $\omega_1 = -2.2$, $\omega_2 = -4.8$; (c) $\omega_1 = 2.1$, $\omega_2 = 1.6$; (d) $\omega_1 = 2$, $\omega_2 = 3.6$.

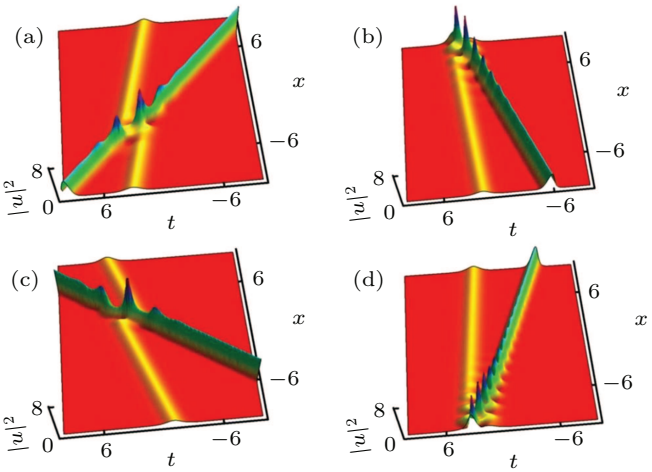


Fig. 5. The influence of initial parameters on soliton interactions. The parameters are $m_1 = 1$, $m_2 = -2$, $\omega_1 = -2.1$, $\omega_2 = -16$ with (a) $\alpha_2 = 1.1$, $\alpha_3 = 0.25$; (b) $\alpha_2 = 1.2$, $\alpha_3 = -0.1$; (c) $\alpha_2 = -1.4$, $\alpha_3 = -0.38$; (d) $\alpha_2 = 1.6$, $\alpha_3 = 0.1$.

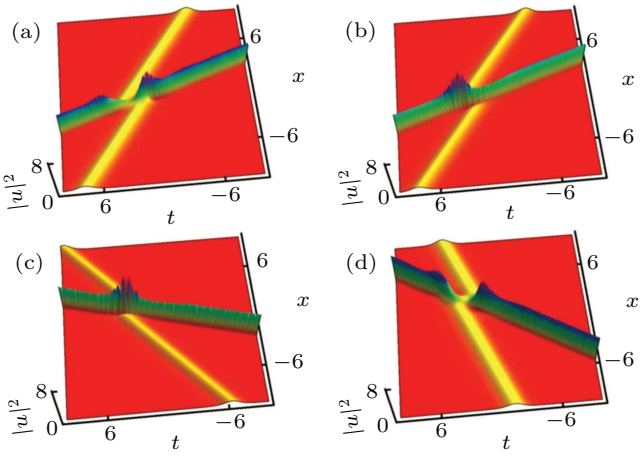


Fig. 6. The influence of initial parameters on soliton interactions. The parameters are $m_1 = 1$, $m_2 = -2$, $\omega_1 = -2.1$, $\omega_2 = -16$ with (a) $\alpha_2 = 1.4$, $\alpha_3 = 0.72$; (b) $\alpha_2 = 0.72$, $\alpha_3 = 0.75$; (c) $\alpha_2 = 0.94$, $\alpha_3 = -0.91$; (d) $\alpha_2 = 0.88$, $\alpha_3 = -0.41$.

values, and the regions of their interaction are also different. In Fig. 6, we can obtain different scenarios of soliton interactions. In Fig. 6(a), during the interaction between two optical solitons, the soliton amplitude will first decrease and then increase. In Fig. 6(b), the amplitude of optical solitons first increases and then decreases in the interaction region. In Figs. 6(c) and 6(d), we can achieve different interactions of different phases by changing the values of α_2 and α_3.

In the above analysis, different initial conditions lead to the interaction of optical solitons. The main reason is that the difference in the initial condition between adjacent optical solitons determines the difference of chirp caused by their nonlinear effects. When the frequency chirp is large, the transmission of optical solitons will become unstable. In addition, the refractive index has a strong dependence on the light intensity. During the interaction of optical solitons, the light intensity of the optical solitons increases sharply, leading to phase shift changes in the optical field itself during transmission in the fiber, thereby affecting its interaction.

4. Conclusion

This paper has mainly used the NLS equation (1) to study the interaction problem between two optical solitons. By analyzing the relevant parameters in the two-soliton solution (10), the factors affecting the interaction between optical solitons have been discussed. Changing the values of m_1 and m_2, we have adjusted the period of optical soliton interactions. By increasing the spacing between optical solitons, we have achieved the non-interactive transmission between optical solitons. The results of this article will provide important theoretical references for the optimization design of nonlinear optical systems.

Acknowledgement

Project supported by the National Natural Science Foundation of China (Grant No. 11875005).

References

[1] Yan Y Y and Liu W J 2021 *Chin. Phys. Lett.* **38** 094201
[2] Zhou Q, Zhong Y, Triki H, *et al.* 2022 *Chin. Phys. Lett.* **39** 044202
[3] Govinden V, Prokhorenko S, Zhang Q, *et al.* 2023 *Nat. Mater.* **22** 553
[4] Wu X Q, Zhang Y, Peng J S, *et al.* 2022 *Nat. Commun.* **13** 5784
[5] Wu X Q, Peng J S, Boscolo S, *et al.* 2021 *Laser Photon. Rev.* **16** 2100191
[6] Peng J S, Boscolo S, Zhao Z H, *et al.* 2019 *Sci. Adv.* **5** eaax1110
[7] Wang Y, Zhou S J, Deng Y H and Chen Q 2023 *Chin. Phys. B* **32** 054203
[8] Zhang X M, Qin Y H, Ling L M and Zhao L C 2021 *Chin. Phys. Lett.* **38** 090201
[9] Wang T Y, Zhou Q and Liu W J 2022 *Chin. Phys. B* **31** 020501
[10] Anderson M H, Tikan A, Tusnin A, *et al.* 2023 *Phys. Rev. X* **13** 011040
[11] Dakova-Mollova A, Miteva P, Dakova D, *et al.* 2023 *Optik* **279** 170770
[12] Taylor J R 2023 *Opt. Commun.* **536** 129382
[13] Rehman S U and Ahmad J 2023 *Phys. Scripta* **98** 035216
[14] Yue Y Z, Li J, Zhang Z Q, *et al.* 2023 *Opt. Commun.* **529** 129082
[15] Soltani M, Triki H, Azzouzi F, *et al.* 2023 *Chaos Soliton. Fract.* **169** 113212
[16] Hu X, Guo J, Wang J, *et al.* 2023 *Light Sci. Appl.* **12** 38
[17] Fu L, Li J J, Yang H W, *et al.* 2023 *Front. Phys.* **11** 1108505
[18] Zhou Q 2022 *Chin. Phys. Lett.* **39** 010501
[19] Wang S B, Ma G L, Zhang X, *et al.* 2022 *Chin. Phys. Lett.* **39** 114202
[20] Zhou, Q, Huang Z H, Sun Y Z, *et al.* 2023 *Nonlinear Dyn.* **111** 5757
[21] Bezerra L J R, Morais D, Buarque A R C, *et al.* 2023 *Nonlinear Dyn.* **111** 6629
[22] Zhang X F, Xu T, Li M and Meng Y 2023 *Chin. Phys. B* **32** 010505
[23] Mou D S and Dai C Q 2022 *Appl. Math. Lett.* **133** 108230
[24] Ghosh N, Das A and Nath D 2023 *Nonlinear Dyn.* **111** 1589
[25] Geng K L, Mou D S and Dai C Q 2023 *Nonlinear Dyn.* **111** 603
[26] Wang S B, Zhang X, Ma G L and Zhu D Y 2023 *Chin. Phys. B* **32** 030506
[27] Bo W B, Wang R R, Liu W and Wang Y Y 2022 *Chaos* **32** 093104
[28] Seadawy A R, Rizvi S T, Sohail M and Ali K 2022 *Chaos Soliton.s Fract.* **163** 112558
[29] Wei B and Liang J 2022 *Nonlinear Dyn.* **109** 2969
[30] Zhou Q, Xu M Y, Sun Y Z, *et al.* 2022 *Nonlinear Dyn.* **110** 1747
[31] Zhao X H and Li S X 2022 *Appl. Math. Lett.* **132** 108159
[32] Paredes A, Salgueiro J R and Michinel H 2022 *Physica D* **437** 133340
[33] Liu X Y, Triki H, Zhou Q, *et al.* 2018 *Nonlinear Dyn.* **94** 703