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Citation: Chin. Phys. B, 2023, 32 (**8**): 087306. DOI: 10.1088/1674-1056/acd9c5
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Nonlinear current response and electric quantum oscillations in the Dirac semimetal Cd_3As_2

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(Received 13 April 2023; revised manuscript received 23 May 2023; accepted manuscript online 30 May 2023)

Chiral anomaly is a distinct quantum anomaly associated with chiral fermions in Dirac or Weyl semimetals. The use of negative magnetoresistance (negative MR) as a signature for this anomaly remains contentious, as trivial mechanisms such as current jetting and weak localization can also induce negative MR. In this study, we report a novel nonlinear behavior of the chiral anomaly in the longitudinal direction, which we observed by applying parallel current and magnetic field to the Dirac semimetal Cd_3As_2. This nonlinear characteristic peaks at an intermediate magnetic field of approximately 5 T, displaying a resistance-increasing property concomitant with strengthening of the current source. Through angle-dependence experiments, we were able to rule out trivial factors, such as thermal effects, geometric artifacts, and anisotropy. Furthermore, additional electric quantum oscillations were observed when the direct current (DC) was applied as high as 300 μA. Such an unusual phenomenon is ascribed to the formation of quantized levels due to Bloch oscillation in the high DC regime, suggesting that an oscillatory density distribution may arise as the electric field increases. The non-Ohmic electric quantum oscillations open a new avenue for exploring chiral anomaly and other nontrivial topological properties, which is also one of the salient features of nonequilibrium steady states in condensed matter physics.

Keywords: nonlinear response, electric quantum oscillations, chiral anomaly, topological semimetals

PACS: 73.50.Fq, 73.43.Qt, 71.55.Ak, 73.63.–b **DOI:** 10.1088/1674-1056/acd9c5

The Ohm's law, which describes the linear relationship between current and voltage, is a fundamental principle in circuitry. Exceptions arise in cases with non-resistive properties or resistance variations, such as gapped bands and p-n diodes where trivial excitation and depletion layers govern non-Ohmic characteristics. In nontrivial condensed matter systems, low-energy excited quasiparticles and significant Berry curvature near band extrema can yield a nonlinear dependence of I–V as well.[1,2] For instance, a non-zero Berry curvature dipole contributes to extra transverse response $j_H \propto (\boldsymbol{D}\cdot\boldsymbol{E})\hat{z}\times\boldsymbol{E}$ by breaking inversion symmetry and preserving lower symmetric point groups.[3,4] This orbital magnetization-related phenomenon introduces an E^2 quadratic term, violating the linear law in the Hall direction. In contrast, earlier studies on longitudinal current responses in semimetals have primarily focused on the chiral anomaly,[5,6] initially employed to elucidate the rapid decay of pions in the Adler–Bell–Jackiw model. The conservation law is violated by $\boldsymbol{B}\cdot\boldsymbol{E}$ due to the presence of two independent spin–momentum projections, and the charge pumping effect induces an imbalanced chiral chemical potential accompanied by a positive magnetoconductance contribution in the longitudinal direction. Analogous to the transverse counterpart, such an additional axial current might also spread non-monotonic current behavior based on Berry curvature as well.[7,8]

Here we direct our attention to the Dirac semimetal Cd_3As_2,[9,10] which exhibits linear energy dispersion at its double bulk Dirac points of $(0, 0, \pm k_D)$ with $k_D \sim 0.032$ Å^{-1}. The presence of an additional C_4 rotation symmetry assigns inverted bands to distinct irreducible representations, protecting them from gapping due to hybridization with spin–orbit coupling. Its topological properties have been investigated through various nontrivial transport phenomena, such as π-phase A-B oscillation,[11] quantum Hall effect modulated by symmetry,[12] etc. In addition, chiral anomaly with negative MR has also been widely observed with parallel current and magnetic field in Cd_3As_2, or through suppressed thermopower coefficient.[13–16] However, most studies concentrate on semiclassical transport theory, considering the weak-B limit of $\Delta\sigma \propto B^2$ and low magnitudes of alternating

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current (AC) response. The interpretation of negative MR as a signature of the chiral anomaly is still under controversy, since current jetting,[17] weak-localization[18] and special disorder potential/scattering[19] could trigger analogous magneto-dependence as well. Therefore, detecting DC responses at higher electric fields may offer alternative insights into the characteristic behavior of chiral anomaly.

In this work, we report a nonlinear I–V behavior at high electric fields when an external magnetic field is aligned parallel to the longitudinal current in Cd_3As_2 nanowires. By subtracting the linear background around zero points, the nonlinear components peak at approximately 5 T under 150 μA, corresponding to a resistance-rising process. Furthermore, the nonlinear behavior exhibits electric quantum oscillations upon increasing the DC to 300 μA. Such nonlinear and oscillatory current response can be ascribed to the formation of Wannier–Stark ladders by considering Bloch oscillations in the chiral anomaly. The resonance between different Landau–Stark eigenstates could contribute to varying tunneling current, analogous to oscillations in Landau-fan diagrams. This work highlights unprecedented nonlinear electric quantum oscillations of Dirac semimetals in the longitudinal direction, which could serve as a unique signature of the chiral anomaly. From a broader perspective, the electric quantum oscillations observed in this work also represent a prominent feature of nonequilibrium steady states in condensed matter physics.

The Cd_3As_2 nanowires were synthesized by chemical vapor deposition (CVD), achieving a large surface-to-volume ratio. Earlier studies have established the $I4_1/acd$ lattice structure and [112] growth direction for our nanowires.[13] The nanowires were transferred to silicon wafers with 285 nm SiO_2 for gate control using needle tips. Electron beam lithography was employed to pattern a four-terminal configuration on the nanowires, and Ar^+ etching was performed prior to evaporating Ti/Au electrodes to reduce interface barriers, as illustrated in Fig. 1(a). All transport measurements were conducted by using a commercial Oxford cryostat at 1.6 K. Here we applied the AC resistance measurement with a lock-in technique using a Stanford SR830 at a frequency of $\omega = 17.777$ Hz.

Initially, we adjusted the gate voltage to $V_g = -140$ V to tune the Fermi level approaching the Dirac point in sample A, albeit the electron-doping is heavy. The complete gate dependence of resistance is shown in Fig. S1. A pronounced longitudinal negative MR was observed when the magnetic field was parallel to the current, as depicted in Fig. 1(b). Geometry artifacts can be excluded as an origin of negative MR because our sample is a nanowire, which implies that inhomogeneous current distribution does not exist with electrodes extending over.[17] A noticeable dip in the negative MR curve is evident at 4 T–5 T, followed by positive MR dominating the high-magnetic-field region. Typically, the natural crystal direction of nanowires grown by CVD is [112] and Cd_3As_2 has topological protection through C_4 rotation symmetry along the [001] direction. One interpretation is that a large magnetic field deviating from [001] direction could break C_4 symmetry and induce an extra mass gap around monopoles,[13,20] which might obscure the negative MR contribution of the chiral anomaly. Besides, some small oscillations could be seen along the magnetoresistance curve, as shown in the inset of Fig. 1(b). These oscillations are associated with the periodic A-B oscillations reported in nanowires.[11,21] By calculating the quantum flux h/e, a radius of 72.6 nm is extracted, consistent with the estimated diameter of 150 nm for our nanowires.

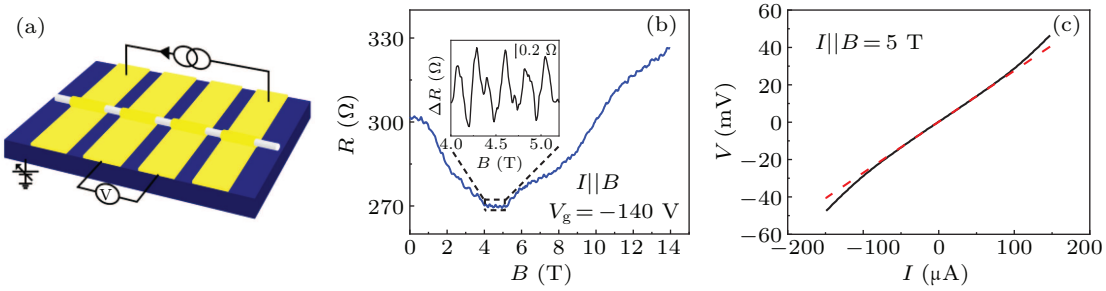


Fig. 1. (a) The schematic of four-terminal configuration of a Cd_3As_2 nanowire with 10 nm Ti and 200 nm Au as electrodes. 285 nm SiO_2/Si is employed to control back-voltage. (b) The negative MR of chiral anomaly when B is parallel to I at $V_g = -140$ V. The inset shows locally amplified A-B oscillations after subtracting background. (c) The nonlinear I–V response of chiral anomaly at $B = 5$ T. The red dashed line marks the linear fitting around the zero point. All transport measurements are conducted at 1.6 K.

We then examined the DC response of the chiral anomaly. Figure 1(c) presents the primary finding of our work: a nonlinear electric response with a parallel external field $B = 5$ T and current in high electric fields. It is noteworthy that the nonlinear behavior only appears above a current threshold, beyond which the resistance increases rapidly. The linear fitting of the dashed line around the zero point clearly demonstrates the deviation feature of the I–V measurement.

To observe the nonlinear behavior clearly, we subtracted the linear fitting background around the zero point in Figs. 2(a) and 2(b). In the range from 1 T to 14 T, all curves exhibit significant nonlinear results. The deviation amplitude peaks around 5 T, which coincides with the dip location of the R–B curve in Fig. 1(b). It appears that the high DC response could

reflect the evolution of the chiral anomaly with magnetic field. Besides, we observed an increase in resistance above a current threshold, particularly in the negative MR region of 3 T–7 T. This resistance-increasing behavior is distinct from the nonlinear results reported by other groups for $Bi_{1-x}Sb_x$[7,22] and $Co_3Sn_2S_2$.[23] A subtle asymmetry between the negative and positive current directions suggests anisotropy in the Fermi velocity and possibly doping-related defects in our sample.

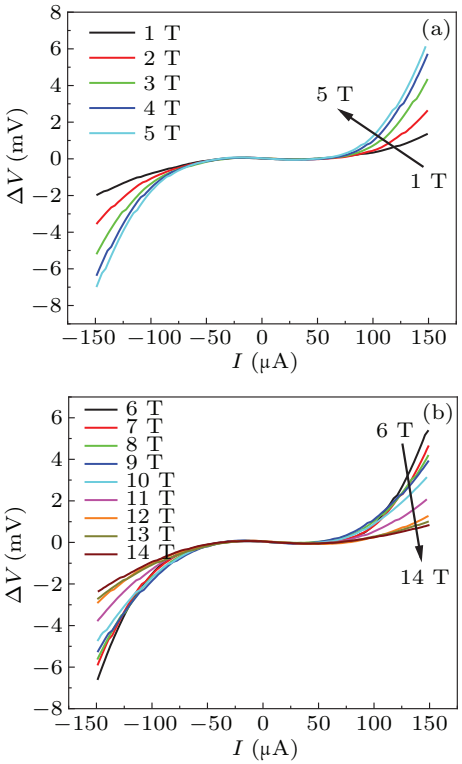


Fig. 2. (a)–(b) Nonlinear parts of *I*–*V* under parallel current and magnetic fields from 1 T to 14 T after subtracting the linear fitting background around the zero point.

This remarkable current response was consistently observed in another Cd_3As_2 nanowire, sample B. The analogous evolution of nonlinear *I*–*V* with magnetic field is summarized in Fig. S2. Besides, we also conducted a gate voltage contrast of nonlinear *I*–*V* in Fig. S3, which implies that the nonlinear result still persists even the Fermi level is not close to the Dirac point. Here we focus on the rotation comparison of *I* and *B* in Fig. 3 to further confirm the relationship among the chiral anomaly, negative MR and high DC response. For 0° and 15°, the MR calculated by $\frac{R(B)-R(0)}{R(0)} \times 100\%$ remains negative at -16% and -7% around 6 T, respectively. Simultaneously, the *I*–*V* counterparts exhibit nonlinear features. When *B* is gradually perpendicular to *I*, for instance, at 30°, the trivial positive MR dominates, and linear Ohm's law prevails. Factors such as thermal effect of resistance, inhomogeneous electrode contact, and structural anisotropy can be ruled out as sources of the nonlinear responses, since they are all irrelevant to the condition of parallel magnetic field and current. This controlled experiment highlights the fundamental connection between nonlinearity and the chiral anomaly of topological semimetals in our work.

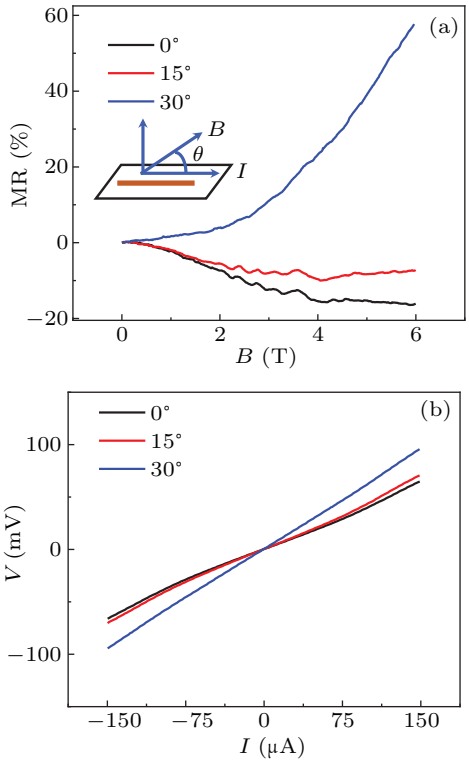


Fig. 3. (a) Angle dependence of the MR from negative (0°, 15°) to positive (30°). The schematic exhibits specific rotation angle θ between magnetic field and current. (b) The nonlinear to linear *I*–*V* curves with rotating the relative orientation of *I* and *B* correspondingly.

Chiral anomaly-dependent non-Ohm behavior has been reported by several groups,[7,22,23] ranging from the doped semimetal $Bi_{1-x}Sb_x$ to the magnetic shandite compound $Co_3Sn_2S_2$. Its origin is ascribed to high-order contribution via considering semiclassical Boltzmann equation.[8] Iteration of the distribution function introduces high-order terms of $\dot{p}^{\chi}$ to the current density, and an abnormal term E^2 is derived in the formula of conductivity, distinguishing from the classical Drude model. However, this conductance-rising feature is not applicable to our work, as our samples consistently exhibit resistance-increasing behavior up to 150 μA in contrast. Besides, this type of nonlinear property in high electric fields has been observed in graphene and carbon nanotube without extra magnetic field, which is related to scattering and quasiparticle emission.[24,25] Our rotation contrasts, however, confirm that the condition is strictly limited to parallel current and magnetic field as previously described, excluding trivial inter-band transition. Actually, a strong magnetic field could break chiral levels by opening gaps when the inverse magnetic length is comparable to the momentum separation of Weyl nodes,[26,27] accompanied by the breakdown of the chiral anomaly. A debate arises as to whether band excitation or tunneling incorporating nontrivial chiral modes could make a difference,[28,29] since the breakdown of chiral anomaly might cause resistance-

increasing behavior due to trivially dissipative states, or in reference to Landau–Zener tunneling.[30] However, unlike photon excitation with selection rules, the Berry connection between different Landau levels (LLs) is zero, implying that the transition might be forbidden by only applying an electric field along with the dispersion of LLs.[31]

The relativistic gapless structure, characterized by spin–momentum locking, can suppress backscattering to a certain extent. Consequently, Bloch particles may traverse Brillouin zone boundaries and carriers' motion becomes localized in real space,[31,32] manifesting as Bloch oscillations. This phenomenon generally arises in optical or semiconductor superlattice with large lattice constants or small Brillouin zones.[33] Similar to SdH oscillations and LLs related to cyclotron frequency, the frequency of the Bloch oscillation can be used to interpret electric quantum oscillations and quantized Wannier–Stark ladders (WSL).[33,34] Combing LLs with WSL, a quantum version of the chiral anomaly is predicted to induce oscillatory current distribution in the presence of an electric field.[31] To probe oscillations at higher DC bias, the amplitude of the current source is doubled from 150 μA to 300 μA, as shown in Figs. 4(a) and 4(b). All the related raw data of *I*–*V* is summarized in Fig. S4. Here, the $\Delta V_{\rm oscillation}$ is extracted after subtracting the linear fitting background from −300 μA to 300 μA. By applying parallel current and magnetic fields from 3 T to 14 T, the nonlinearity at 150 μA evolves into electric quantum oscillations at 300 μA, which is further summarized in Fig. 4(c). The positions of peaks and dips gradually shift towards higher DC beyond 6–7 T, consistent with the theory of electric quantum oscillations.[31] This observation may be related to the formation of discrete LLs that require a substantial magnetic field, especially considering the Cd_3As_2 mobility of $\sim 10^3$ $cm^2{\cdot}V^{-1}{\cdot}s^{-1}$ reported in previous work.[13] Besides, the linear *I*–*V* relation without magnetic field in Fig. 4(d) confirms that the oscillation feature does not come from Joule heating. The contrasting outcomes of nonlinearity in $Bi_{1-x}Sb_x$[7,22] and our work under 150 μA could also be explained by the diversity of initial oscillation or phase, which might be related to the band structure, Fermi level, carrier density, etc.

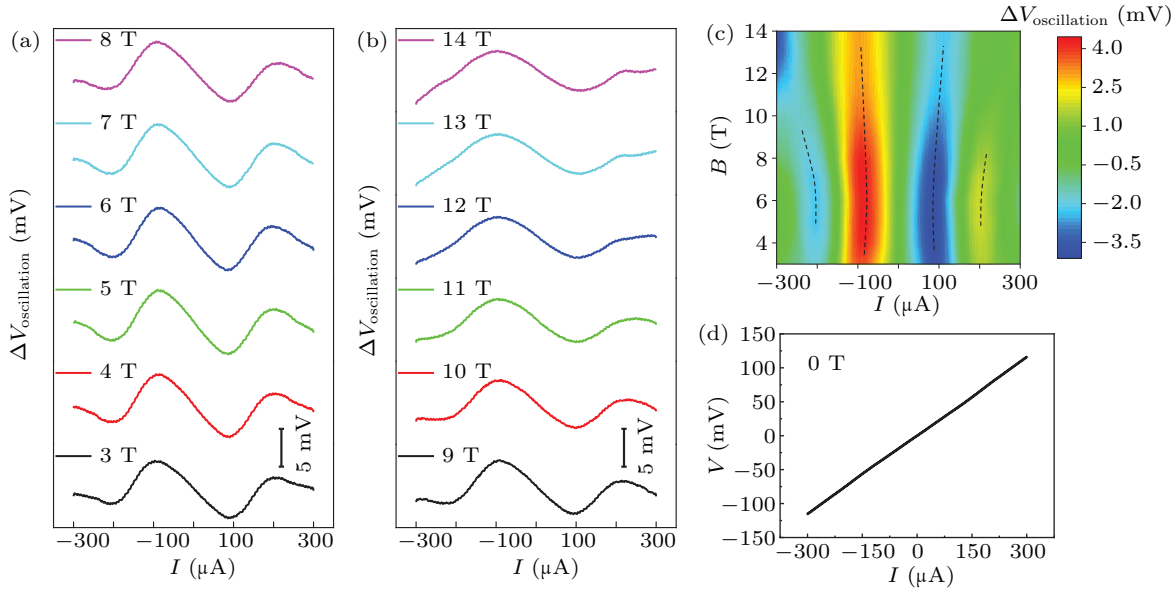


Fig. 4. (a) and (b) Electric quantum oscillations from 3 T to 14 T after subtracting the linear background of *I*–*V*. The scale bar is 5 mV. (c) The $\Delta V_{\rm oscillation}$ map of magnetic field and current. Dashed lines mark the positions of oscillation peaks and dips. (d) The *I*–*V* relation up to 300 μA when magnetic field is 0 T.

In summary, we have uncovered a newfangled phenomenon involving nonlinear *I*–*V* and electric quantum oscillations under high electric fields in the Dirac semimetal Cd_3As_2. At a moderate magnetic field of approximately 5 T, nonlinearity under 150 μA and the negative MR of chiral anomaly both reach their maximums simultaneously, exhibiting a strong connection as demonstrated by the angle-dependent control experiments. Such nonlinearity further evolves into electric quantum oscillations up to 300 μA when considering quantized levels of LLs and WSL, potentially serving as an additional signature of chiral anomaly beyond the negative MR. Our work also introduces a new transport method for exploring the interplay between relativistic quasiparticles and the classical Bloch band structure, which helps to understand the nonequilibrium steady states in condensed matter physics.

Acknowledgements

Ben-Chuan Lin thanks Jeehoon Kim, Heon-Jung Kim, Wei Zhang for fruitful discussions. This work was supported by the National Natural Science Foundation of China (Grant Nos. 12074162, 12004158, and 91964201), the National Key Research and Development Program of China (Grant Nos. 2022YFA1403700 and 2020YFA0309300), the

Key-Area Research and Development Program of Guangdong Province (Grant No. 2018B030327001), Guangdong Provincial Key Laboratory (Grant No. 2019B121203002), and Guangdong Basic and Applied Basic Research Foundation (Grant No. 2022B1515130005).

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