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Robustness of community networks against cascading failures with heterogeneous redistribution strategies

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Network robustness is one of the core contents of complex network security research. This paper focuses on the robustness of community networks with respect to cascading failures, considering the nodes influence and community heterogeneity. A novel node influence ranking method, community-based Clustering–LeaderRank (CCL) algorithm, is first proposed to identify influential nodes in community networks. Simulation results show that the CCL method can effectively identify the influence of nodes. Based on node influence, a new cascading failure model with heterogeneous redistribution strategy is proposed to describe and analyze node fault propagation in community networks. Analytical and numerical simulation results on cascading failure show that the community attribute has an important influence on the cascading failure process. The network robustness against cascading failures increases when the load is more distributed to neighbors of the same community instead of different communities. When the initial load distribution and the load redistribution strategy based on the node influence are the same, the network shows better robustness against node failure.

Keywords: community networks, cascading failure model, network robustness, nodes influence identification

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1. Introduction

Cascading failures have attracted considerable attention due to their widespread occurrence in various real-world systems and their highly destructive effect on network robustness.[1–5] Triggered by the failure of a single node or subsystem, the load of the failing node spreads to other nodes of the system, which in turn further increases the possibility of system failures, resulting in a vicious cycle or snowball effect. Cascading failures can bring the whole service down in a short space of time, and it will gradually deteriorate. In particular, the increasingly complex structures and functions of real-world systems are having a greater impact on the cascading failure process.[6–9] For example, the heterogeneity of nodes and communities in networks has an important impact on the whole cascading failure process, and node load-redistribution strategies based on node's influence are continuously proposed.[3,4,10] Therefore, the impact of network structure on cascading failure has been one of the hot topics in research of complex network dynamics.

As a typical feature of complex systems, community attributes generally exist in various real systems,[11–16] such as social networks, energy networks and supply chain networks,[12–17] and have a strong impact on cascading failure processes.[18–20] Nakarmi *et al.*[18] studied the role of community structures in cascading failures in power grids. A centrality measure based on community structures is proposed to identify critical components of the system, and their protection can help in containing failures within a community and prevent the propagation of failures to large sections of the power grid. Pan *et al.*[19] analyzed the vulnerability of a cyber-physical power system under community attack styles to study cascading failure. Wen *et al.*[20] proposed a new entropy-based method combining the internal factors and external factors of community to measure the vulnerability of communities. These latest achievements have demonstrated the close interaction between community structure and cascading failure process.

Although the current research on cascading failures in community networks is relatively in-depth, it still faces many challenges as the structural and dynamic heterogeneity of real systems increases due to community structure. On the one hand, the influence of nodes becomes more granular as the importance, e.g., size, connections, locations and functions of the communities to which they belong, varies.[21] Therefore, the initial load, load capacity setting and load redistribution strategy associated with node selection are also more heterogeneous due to the diversity of nodes and communities. On the other hand, the allocation problems between nodes in the cascading failure process are not only related to their influence, but also to whether they belong to the same community or even to the influence of different communities. For exam-

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ple, node i in community A has two neighbors with the same influence in two different communities A and B. When i is attacked and fails, it will preferentially select nodes in the same community for load balancing due to node similarity. It is not easy to take node similarity into account when measuring node influence, as the importance of similarity is not suitable for sorting. However, a load distribution strategy that preferentially selects nodes within or outside the community can be proposed to account for node similarity in the cascading failure model.

Therefore, node influence identification in community networks is the key point of studying network robustness against cascading failures, and the accurate measurement of node influence in community networks is the prerequisite for effective control of cascading failures. During the past decade, many node ranking methods have been proposed and improved to identify node influence, such as degree centrality (DC), betweenness centrality (BC), closeness centrality (CC), ClusterRanking (CR), PageRank and LeaderRank (LR) algorithms.[21–24] Due to the classification and clustering of nodes,[25] node influence identification in the community network is a very challenging task. First, the community structure makes the clustering of nodes more heterogeneous. Connections within communities are often closer than those between communities, and the heterogeneity of connections leads to greater differences in node status. For example, the relationships between nodes within the community are close, which makes the information difficult to break out as local clustering plays a negative role in diffusion. Secondly, even if two nodes have the same status in their respective communities, their importance in the network would be different due to different kinds of importance of the communities.

This paper presents a new cascading failure model based on node's influence on community networks to analyze the impact of community structure on cascading failures. As a consequence of the new challenges, new derivations are necessary to extend the cascading failure model and evaluate the impact of community structure and node's influence on the network robustness:

(i) A community-based Clustering–LeaderRank (CCL) algorithm for identifying node influence is proposed in this paper. Simulation results show that our algorithm can effectively identify the nodes with high influence in the community network under the premise of ensuring low computational complexity.

(ii) Based on the load-capacity model, a novel cascading failure model in a community network is proposed by considering the node and community influence. Heterogeneous load redistribution strategies are designed and analyzed.

(iii) Theoretical analysis and simulation on the robustness of community networks against cascading failures are carried out in this study, and the results show that network heterogeneity has a great impact on the cascading failure process. When the load share allocated to the same community neighbors becomes higher than that of the cross-community neighbors, the network shows better robustness.

The rest of this paper is organized as follows. In Section 2, we present the CCL algorithm proposed for node influence ranking. In Section 3, a novel cascading failure model in community networks is proposed, followed by the theoretical analysis of network robustness in different cases in Section 4. Numerical results are presented in Section 5, followed by conclusions in Section 6.

2. Community-based clustering-LeaderRank algorithm

Considering the local attributes, the connectivity of nodes and the influence of their community, a community-based clustering-LeaderRank (CCL) algorithm is proposed to identify influential nodes in community networks. Consider a network $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_N\}$ represents node set and $E = \{e_1, e_2, \ldots, e_l\}$ represents link set, N and l are the numbers of nodes and links, respectively. The nodes in G are divided into m ($2 \leqslant m < N$) communities, where the nodes within the community are closely connected, while the links between the communities are sparse. The $\mathrm{CCL}_i(t)$ of node i at time t can be defined as

$$\mathrm{CCL}_i(t) = I_{\mathrm{C}}(t-1)\Big[\sum_{j=1}^{N+1} f(c_j)\frac{A_{ji}(t)}{k_j}\mathrm{CCL}_j(t-1)\Big], \quad (1)$$

where c_j represents the clustering coefficient of node j and $f(c_j)$ represents the function of c_j; $f(c_j)$ and c_j are negatively correlated. $I_{\mathrm{C}}(t)$ represents the influence of community C at time t. $A_{ij}(t)$ is the adjacency matrix at time t, with the values being the weights between nodes,

$$A_{ij}(t) = \begin{cases} w_{ij}, & \text{if } i \text{ and } j \text{ are neighbors,} \\ 0, & \text{else.} \end{cases} \quad (2)$$

Since the influence of a community is positively related to the influence of its internal nodes, $I_{\mathrm{C}}(t)$ is defined as the sum of the influence of the internal nodes,

$$I_{\mathrm{C}}(t) = \frac{\sum_{i\in C}\mathrm{CCL}_i(t)}{\sum_{k=1}^{N}\mathrm{CCL}_k(t)}. \quad (3)$$

Here, an unweighted and bidirectional network with $N = 11$ nodes and $E = 15$ links is used to compare the effectiveness of the CCL algorithm proposed in this paper and some typical indicators to measure the influence of nodes. As shown in

Fig. 1, the network has been divided into two communities by the method in Ref. [26], where nodes I, J, K, and L are in the same community and other nodes are in another community.

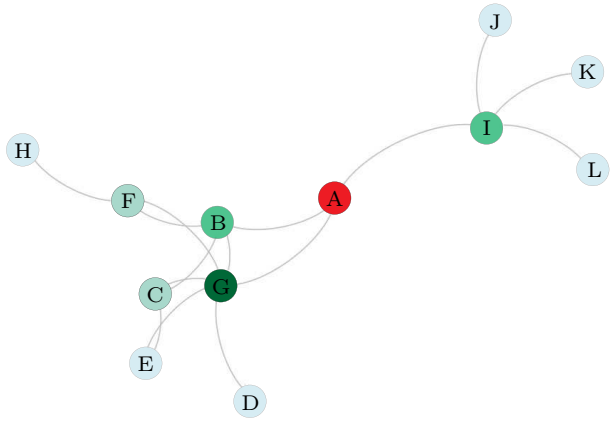


Fig. 1. A simple community network model.

The top 5 nodes according to the different methods are listed in Table 1. Using the CCL method, node A, which can be seen as a 'bridge' between two communities, has the highest influence. Both CCL and BC methods take into account the information transfer capacity of nodes in the network when measuring node influence. Nodes G and B are the second and third most important nodes in the CCL method due to the larger number of neighbors and the higher ranking of the community to which they belong. An interesting result is that in the CCL method, the influence of node B is higher than that of node I, while in the other methods it is the opposite. This is because, although the number of neighbors is the same, we can see from Fig. 1 that the neighbors of node B have a higher influence than those of node I, and the community to which node B belongs is ranked higher than the one to which node I belongs. The results clearly show that our method integrates more comprehensive information.

Table 1. Top 5 nodes ranked by different methods in the network shown in Fig 1.

Node index	CCL	DC	BC	LR	Community ranking
A	1	3	1	5	1
G	2	1	3	1	1
B	3	2	5	3	1
I	4	2	2	2	2
C	5	3	6	6	1

In order to further verify the effectiveness of our proposed method, two real networks with significant community characteristics are used in our paper. One is the widely used Dolphin social network, and the other is an aviation network we constructed. An aviation network is generated by intercepting the flights of 104 international airports distributed in different continents on 1 May 2022 from flightaware,[27] a global flight traffic query website. The nodes in the aviation network are international airports, and the links represent routes between the airports, while the number of routes is considered as the weight of the links. The basic information of the two networks is given in Table 2.

Table 2. Basic information of two networks.

	N	l	k	C
Dolphin network	62	215	6.935	0.612
Aviation network	104	921	17.7	0.696

In this way, we can draw graphs with N nodes and l edges to describe the two networks. The adjacency matrix of graph G is $\{a_{ij}\}$. If there is a connection from node$i$ to node j, $a_{ij} = w$, otherwise $a_{ij} = 0$. Using CCL and other classic node ranking methods, the influence of each node can be ranked and the effectiveness of different ranking methods can be analyzed by comparing the ranking results.

In addition, the susceptible-infected (SI) epidemic model is used in our paper to study the propagation influence of top nodes.[24,28,29] In the SI model, each node has two discrete states, susceptible $S(t)$ represents the number of individuals susceptible to the disease (not yet infected), and infected $I(t)$ denotes the number of individuals who have been infected and are able to spread the disease to susceptible individuals. At each step, for each infected node, a randomly susceptible neighbor is infected with probability γ (for uniformity, we set $\gamma = aw/w_{\max}$ here). For epidemic spread on networks, γ determines the range or scale over which a node can exert influence.

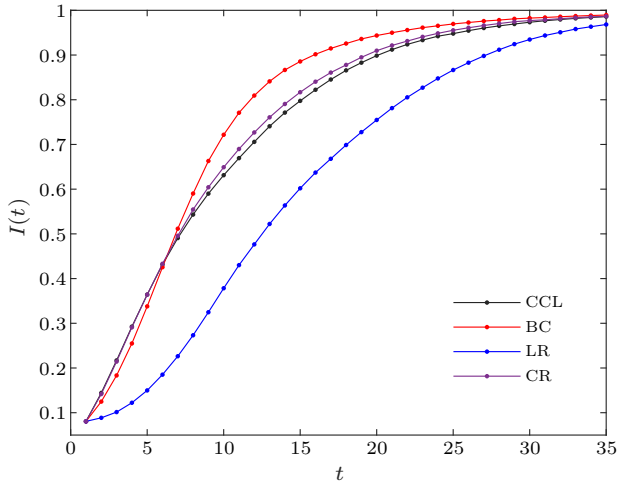


Fig. 2. Propagation processes in the Dolphin network with the top 5 nodes under different ranking methods as initial sources.

First, in the Dolphin network, the top 5 nodes under different node ranking methods are shown in Table 3. We can see that the ranking results of top 5 nodes under the CCL and CR methods are almost the same, and also very similar to the result of BC ranking methods. By using the SI epidemic model, with the top 5 nodes under different methods as the propagation sources, we show the propagation processes in Fig. 2 under the same conditions. With the global information of the network, the performance of the BC method is the best. The performance of the CCL method is similar to that of the CR method, and is better than the LR method.

Table 3. Top 5 nodes in Dolphin network.

Node ranking	CCL	LR	BC	CR
1	37	12	36	37
2	14	35	1	14
3	45	58	40	36
4	20	4	37	20
5	40	11	7	45

In Aviation network, we show the top 20 nodes under the CCL method through Table 4, and also give the ranking performance of these nodes under other ranking methods. The results in Table 4 show that the top 20 nodes ranked by the CCL method perform well in the other three ranking methods, especially in the LR and BC methods.

Table 4. Top 20 nodes ranking by different methods in the Aviation network.

Node	CCL	LR	BC	CR	Community
New York	1	1	1	4	2
London	2	2	7	3	1
Paris	3	3	2	1	1
Amsterdam	4	5	9	2	1
Sydney	5	9	5	64	5
Chicago	6	4	24	10	2
Tokyo	7	7	12	14	3
Toronto	8	6	15	6	2
Singapore	9	10	3	39	3
Los Angeles	10	8	10	8	2
Port Vila	11	11	27	9	5
Barcelona	12	16	25	72	1
Melbourne	13	14	4	5	5
Istanbul	14	12	50	35	3
Boston	15	13	28	19	2
Atlanta	16	18	13	46	2
Panama City	17	17	8	42	3
Bangkok	18	15	19	16	3
San Francisco	19	19	36	27	2
Milan	20	25	22	52	1

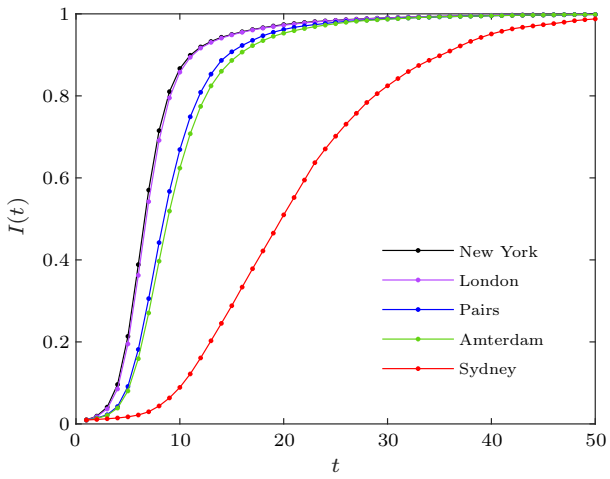


Fig. 3. The propagation process in the Aviation network with the top 5 nodes as a single infectious source, raked separately by the CCL method.

During verifying the effectiveness of our proposed CCL method, we also analyze and compare its time complexity. The BC method performs best on node's ranking due to its global attribute based approach but with the highest time complexity of $O(N^3)$, where N is the number of nodes. The computational complexity of the proposed CCL and LeaderRank algorithm can be written as $O(ml)$, where l is the number of iterations when the CCL algorithm converges and m is the number of edges. The CR method owns the lowest computational complexity of $O(N)$, but the worst effects on node ranking in our simulations.

The LeaderRank algorithm has been proven to have excellent convergence as a 'super node' is introduced into the PageRank algorithm. Bidirectional edges between the super node and each node in the network make the network highly connected.[30] The clustering coefficient of nodes introduced in the CCL method is a constant and does not change with the increase of iteration times. Therefore, as an improved LeaderRank algorithm, the CCL method can converges speedily. In our simulations, the CCL of each node will no longer change after 10 iterations.

The CCL method is used to rank nodes in two typical community networks, and the effectiveness of the CCL method is analyzed by comparing it with other typical node sorting methods. The results show that our method can effectively rank nodes in community networks. Furthermore, by using the SI propagation model, the role of the CCL method in identifying high-influence nodes is well verified.

3. A novel cascading failure model in community network

Consider a network $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_N\}$ represents node set and $E = \{e_1, e_2, \ldots, e_m\}$ represents link set, N and m are the number of nodes and links, respectively. The nodes in G are divided into n $(2 \leqslant n < N)$ communities, where the nodes within the community are closely connected, while the links between the communities are sparse. We assume that when $N \to \infty$, the number of nodes in each community is equal. Thus, the probability that a node and its neighbor belong to the same community is $1/n$.

At the initial stage, each node has an initial load $L(0)$ and a load capacity C. When $L(0) < C$, the network is in a steady state. When a node fails, e.g., poor capital turnover of firms, bankruptcy, or product backlog, the load of the failed node is redistributed to its neighbors. If the load capacity of the neighbor is not sufficient to handle the additional loads, the neighbor would collapse. The dynamic process of redistribution continues until it disappears or all nodes fail. Assuming that the potential cascading failure is caused by the removal of a single node, we look at the cascading failure process of the network. Our model is briefly described as follows:

(1) **Node initial load and load capacity** The nodes' load and load capacity are generally related to the node influence,

e.g., the position, the size, the scales. Therefore, the initial load of node i is defined as

$$L_i(0) = \lambda I_i^{\alpha},\ i = 1,2,\ldots,N, \tag{4}$$

where I_i is the influence of node i, which can be measured by different methods, e.g., different centralities, CCL method; α and λ are the adjustable parameters that control the strength of the initial loads, $\alpha \geqslant 0$ and $\lambda \geqslant 0$. The initial load of the node is proportional to its influence, which means that the nodes with higher influence can take more load. At the initial stage, the network is in a steady state, $L_i(0)$ is always lower than its load capability C_i, as given by

$$C_i = (1+\beta)L_i,\ i = 1,2,\ldots,N, \tag{5}$$

where β (β>0) is the tolerance parameter and suggests the upper bound of the ability that node i withstands risks, i.e., the network shows stronger tolerance against cascading failures as β grows.

(2) **Heterogeneous load redistribution strategies** First, the redistribution strategy is usually related to the node's capacity, which can be described to some extent by structural properties such as degree, betweenness and closeness, which are used to measure the position of nodes and their relationship with other nodes in the network. Since node capability in our setting is directly proportional to node influence, we assign it according to node influence.

Taking into account the timeliness of the redistribution strategies and the updating of information, the probability that the failed node i transfers the load to its neighbor j is given as

$$p_{i\to j} = \begin{cases} \sigma \dfrac{I_j^{\theta}}{\sum_{i'\in \mathrm{Neigbor}(i)} I_{i'}^{\theta}}, & \text{if } i \text{ and } j \text{ are in the same community,} \\ (1-\sigma)\dfrac{I_j^{\theta}}{\sum_{i'\in \mathrm{Neigbor}(i)} I_{i'}^{\theta}}, & \text{otherwise,} \end{cases} \tag{6}$$

where σ is the parameter controlling the distribution share of cross-community nodes and intra-community nodes, $0 \leqslant \sigma \leqslant 1$. When $\sigma > 0.5$, the share of load allocated to neighbors in the same municipality is higher than that of neighbors in different municipalities, and when $\sigma < 0.5$, the share of load allocated to neighbors in the same municipality is lower than that allocated to neighbors in different municipalities. Specially, when $\sigma = 0.5$, the probability of transmission, $p_{i\to j}$, has nothing to do with the community. Therefore, one of the key points of our research is the influence of the value of σ on cascading failure. At time t, the load distributed from node i to j can be written as

$$\Delta L_{i\to j}(t) = L_i(t-1)p_{i\to j} = \begin{cases} L_i(t-1)\sigma \dfrac{I_j^{\theta}}{\sum_{i'\in \mathrm{Neigbor}(i)} I_{i'}^{\theta}}, & \text{if } i \text{ and } j \text{ are in the same community,} \\ L_i(t-1)(1-\sigma)\dfrac{I_j^{\theta}}{\sum_{i'\in \mathrm{Neigbor}(i)} I_{i'}^{\theta}}, & \text{otherwise.} \end{cases} \tag{7}$$

For the network G with N nodes and n communities, the load of node i changes when its neighbor fails. Assuming the neighbor j failed at the initial time, then the load of node i at time $T = 1$ changes as

$$L_i(1) = L_i(0) + \Delta L_{j\to i}(1). \tag{8}$$

Cascading failure can be completely blocked if the load of any node i at time $T = 1$ is less than its load capacity, i.e.,

$$L_i(1) = L_i(0) + \Delta L_{j\to i}(1) < C_i. \tag{9}$$

Based on the redistribution strategy in Eq. (7), the model should satisfy the following condition to ensure global robustness

$$\begin{aligned} L_i(1) =& L_i(0) + \frac{1}{n}\sigma \frac{I_i^{\theta}}{\sum_{j'\in \mathrm{Neigbor}(j)} I_{j'}^{\theta}} L_j(0) \\ &+ \frac{n-1}{n}(1-\sigma)\frac{I_i^{\theta}}{\sum_{j'\in \mathrm{Neigbor}(j)} I_{j'}^{\theta}} L_j(0) \\ =& L_i(0) + \frac{\sigma + (n-1)(1-\sigma)}{n} \\ &\times \frac{I_i^{\theta}}{\sum_{j'\in \mathrm{Neigbor}(j)} I_{j'}^{\theta}} L_j(0) < C_i. \end{aligned} \tag{10}$$

As a special case, if the network has no community attribute, i.e., $n = 1$, Eq. (10) degenerates to

$$L_i(1) = L_i(0) + \sigma \frac{I_i^{\theta}}{\sum_{j'\in \mathrm{Neigbor}(j)} I_{j'}^{\theta}} L_j(0) < C_i. \tag{11}$$

Based on Eq. (10), from the cascading failure condition formula considering the community structure, we can derive the network robustness against cascading failures under different cases. In particular, when the nodes in the network are uniformly mixed, such as in the ER network or regular network, we assume that the importance of each node is basically the same, i.e., $I_i = I_j$, Eq. (10) can be rewritten as

$$\begin{aligned} L_i(1) =& L_i(0) + \frac{\sigma + (n-1)(1-\sigma)}{n} \frac{I_i^{\theta}}{\sum_{j'\in \mathrm{Neigbor}(j)} I_{j'}^{\theta}} L_j(0) \\ =& L_i(0) + \frac{\sigma + (n-1)(1-\sigma)}{n} \frac{1}{k} L_j(0) < C_i. \end{aligned} \tag{12}$$

Keeping the initial load distribution scheme and taking node

initial load and load capacity in Eq. (12), we can reach

$$\lambda I_i^{\alpha} + \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{1}{k}\lambda I_j^{\alpha} < (1+\beta)\lambda I_i^{\alpha}.$$

By a simple transformation, we can obtain the critical condition of node load capacity to ensure network robustness:

$$\beta > \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{1}{k}\left(\frac{I_j}{I_i}\right)^{\alpha} = \frac{\sigma+(n-1)(1-\sigma)}{kn}. \quad (13)$$

We introduce the critical threshold of β, denoted by β^*, to further discuss the robustness of different networks facing cascading failures. When $\beta \geqslant \beta^*$, each node in the network is capable of handling additional loads so that there are no cascading failures and the system can continue to function as it used to. Otherwise, there are failed nodes in the network, which can lead to partial or even complete network failure, and the cascading failure occurs. Here, β^* is the minimum value of the network's ability to avoid global cascading failures. We can see from Eq. (13) that as σ grows, β^* becomes smaller, and the network shows greater robustness against cascading failures.

4. Theoretical analysis on network robustness against cascading failures

In most cases, the network structure shows various heterogeneities, which lead to the different influences of nodes. Based on the influences, load redistribution strategies can be made to control cascading failures. Substituting Eqs. (1) and (2) into Eq. (10), we obtain the condition to ensure network robustness as follows:

$$I_i^{\alpha} + \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{I_i^{\theta}}{\sum_{j'\in \mathrm{Neigbor}(j)} I_{j'}^{\theta}} I_j^{\alpha} < (1+\beta) I_i^{\alpha}. \quad (14)$$

After a simple adjustment, β should satisfy

$$\beta > \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{I_i^{(\theta-\alpha)} I_j^{\alpha}}{\sum_{j'\in \mathrm{Neigbor}(j)} I_{j'}^{\theta}}. \quad (15)$$

Assuming that the influence of the node is irrelevant, e.g., the degree of nodes, as proved in Ref. [31], we can rewrite Eq. (15) as

$$\begin{aligned}
\beta &> \frac{\sigma+(n-1)(1-\sigma)}{n} I_j^{\alpha}\frac{I_i^{(\theta-\alpha)}}{\sum_{j'\in \mathrm{Neigbor}(j)} I_{j'}^{\theta}} \\
&= \frac{\sigma+(n-1)(1-\sigma)}{n} I_j^{\alpha}\frac{I_i^{(\theta-\alpha)}}{\sum_{I'=I_{\min}}^{I_{\max}} I_j P(I'|I_j) I'^{\theta}} \\
&= \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{I_j^{\alpha-1} I_i^{(\theta-\alpha)}}{\sum_{I'=I_{\min}}^{I_{\max}} P(I'|I_j) I'^{\theta}} \\
&= \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{I_j^{\alpha-1} I_i^{(\theta-\alpha)}\langle I\rangle}{\langle I^{\theta+1}\rangle}.
\end{aligned} \quad (16)$$

The parameters show great impact on the load capacity threshold β^* of cascading failure, when $\theta = \alpha$, β^* satisfies

$$\beta^* = \begin{cases} \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{I_{\min}^{\alpha-1}\langle I\rangle}{\langle I^{\theta+1}\rangle}, & \theta=\alpha<1, \\ \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{\langle I\rangle}{\langle I^2\rangle}, & \theta=\alpha=1, \\ \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{I_{\max}^{\alpha-1}\langle I\rangle}{\langle I^{\theta+1}\rangle}, & \theta=\alpha>1, \end{cases} \quad (17)$$

where $I_{\min}$ and $I_{\max}$ are the minimum and maximum of node's influence in the network, respectively.

When $\theta < \alpha$, the critical threshold satisfies

$$\beta^* = \begin{cases} \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{I_{\min}^{\theta-1}\langle I\rangle}{\langle I^{\theta+1}\rangle}, & \alpha\leqslant 1, \\ \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{I_{\max}^{\alpha-1} I_{\min}^{(\theta-\alpha)}\langle I\rangle}{\langle I^{\theta+1}\rangle}, & \alpha>1. \end{cases} \quad (18)$$

When $\theta > \alpha$, the critical threshold satisfies

$$\beta^* = \begin{cases} \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{I_{\min}^{\alpha-1} I_{\max}^{(\theta-\alpha)}\langle I\rangle}{\langle I^{\theta+1}\rangle}, & \alpha<1, \\ \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{I_{\max}^{(\theta-1)}\langle I\rangle}{\langle I^{\theta+1}\rangle}, & \alpha\geqslant 1. \end{cases} \quad (19)$$

From Eqs. (17)–(19) we can see that β^* becomes smaller as σ grows. That is, when more loads are allocated to neighbors who are in the same community, the network becomes more robust against cascading failures under the heterogeneous load-redistribution strategies based on node influence. We can also derive the relationship between β^* and the load-redistribution parameter θ by referring to Ref. [30].

When $\alpha < 1$,

$$\frac{I_{\min}^{\theta-1}\langle I\rangle}{\langle I^{\theta+1}\rangle} = \frac{\langle I\rangle}{\frac{1}{N}\sum_{i=1}^{N} I_i^2\left(\frac{I_i}{I_{\min}}\right)^{\theta-1}} \geqslant \frac{\langle I\rangle}{\frac{1}{N}\sum_{i=1}^{N} I_i^2} = \frac{\langle I\rangle}{\langle I^2\rangle}. \quad (20)$$

When $\alpha > 1$,

$$\frac{I_{\max}^{\theta-1}\langle I\rangle}{\langle I^{\theta+1}\rangle} = \frac{\langle I\rangle}{\frac{1}{N}\sum_{i=1}^{N} I_i^2\left(\frac{I_i}{I_{\max}}\right)^{\theta-1}} \geqslant \frac{\langle I\rangle}{\frac{1}{N}\sum_{i=1}^{N} I_i^2} = \frac{\langle I\rangle}{\langle I^2\rangle}. \quad (21)$$

Taking the more complex case further, i.e., $\theta > \alpha$ and $\alpha < 1$, we obtain

$$\begin{aligned}
\frac{I_{\min}^{\alpha-1} I_{\max}^{\theta-\alpha}\langle I\rangle}{\langle I^{\theta+1}\rangle} &= \frac{I_{\min}^{\alpha-1} I_{\max}^{\theta-\alpha}\langle I\rangle}{\frac{1}{N}\sum_{i=1}^{N} I_i^{\theta+1}} \\
&= \frac{\langle I\rangle}{\frac{1}{N}\sum_{i=1}^{N} I_i^2\left(\frac{I_i}{I_{\min}}\right)^{\alpha-1}\left(\frac{I_i}{I_{\max}}\right)^{\theta-\alpha}} \\
&\geqslant \frac{\langle I\rangle}{\frac{1}{N}\sum_{i=1}^{N} I_i^2\left(\frac{I_i}{I_{\min}}\right)^{\alpha-1}},
\end{aligned} \quad (22)$$

where $\left(\frac{I_i}{I_{\min}}\right)^{\alpha-1}$ monotonically decreases, i.e., $\left(\frac{I_i}{I_{\min}}\right)^{\alpha-1} \leqslant \left(\frac{I_i}{I_{\min}}\right)^{1-1} = 1$. Therefore,

$$\frac{I_{\min}^{\alpha-1} I_{\max}^{\theta-\alpha}\langle I\rangle}{\langle I^{\theta+1}\rangle} \geqslant \frac{\langle I\rangle}{\langle I^2\rangle}. \quad (23)$$

When $\theta < \alpha$ and $\alpha > 1$,

$$\begin{aligned}\frac{I_{\max}^{\alpha-1} I_{\min}^{\theta-\alpha}\langle I\rangle}{\langle I^{\theta+1}\rangle} &= \frac{I_{\max}^{\alpha-1} I_{\min}^{\theta-\alpha}\langle I\rangle}{\frac{1}{N}\sum_{i=1}^{N} I_i^{\theta+1}} \\ &= \frac{\langle I\rangle}{\frac{1}{N}\sum_{i=1}^{N} I_i^2 \left(\frac{I_i}{I_{\max}}\right)^{\alpha-1}\left(\frac{I_i}{I_{\min}}\right)^{\theta-\alpha}} \\ &\geqslant \frac{\langle I\rangle}{\frac{1}{N}\sum_{i=1}^{N} I_i^2 \left(\frac{I_i}{I_{\min}}\right)^{\theta-\alpha}},\end{aligned} \tag{24}$$

where $\left(\frac{I_i}{I_{\min}}\right)^{\theta-\alpha}$ monotonically decreases, i.e., $\left(\frac{I_i}{I_{\min}}\right)^{\theta-\alpha} \leqslant \left(\frac{I_i}{I_{\min}}\right)^{0} = 1$. Therefore,

$$\frac{I_{\max}^{\alpha-1} I_{\min}^{\theta-\alpha}\langle I\rangle}{\langle I^{\theta+1}\rangle} \geqslant \frac{\langle I\rangle}{\langle I^2\rangle}. \tag{25}$$

From the results of the theoretical analysis, we can conclude that when the initial load distribution and load redistribution strategy of the network are both linear correlation with node influence I, i.e., $\theta = \alpha = 1$, we can obtain $\beta^* = \frac{\sigma+(n-1)(1-\sigma)}{n}\frac{\langle I\rangle}{\langle I^2\rangle}$, which is smaller than other cases we have analyzed, and the network shows the strongest robustness against cascading failures. Further interesting results have been obtained by applying numerical simulations in artificial and real networks.

5. Simulations

Numerical simulation results are provided to validate our proposed model and theoretical analysis in Section 4. First, we validate our model on 200 different BA networks.[32] Each network consists of 2000 nodes and the average degree is 6. The simulation results are the average of 200 simulation experiments in BA networks. As discussed in Section 4, when $\theta = \alpha$, we can obtain the minimum of β^*, i.e., the network is most robust. Figure 4 shows the relations between robustness threshold β^* and θ, with different values of α. The simulation results in Fig. 4 show intuitively that the value of β^* is smallest when $\theta = \alpha$ in each case. It is also shown that when $\theta = \alpha = 1$, the value of β^* is smallest in all 3 cases.

Furthermore, we also simulate and analyze the robustness with respect to load redistribution strategy on nodes in different communities or in the same community, i.e., the impact of σ on the robustness threshold β^*. Figure 5 shows β^* changing as θ grows under different values of σ. With the same θ, β^* becomes smaller as σ grows, which means that, when the load proportion allocated to the same community neighbors becomes higher than that of the cross-community ones, the network shows better robustness.

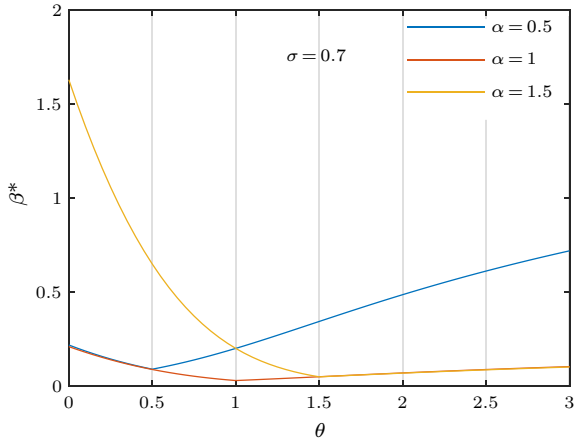


Fig. 4. Relations between robustness threshold β^* and θ with different values of α in BA networks.

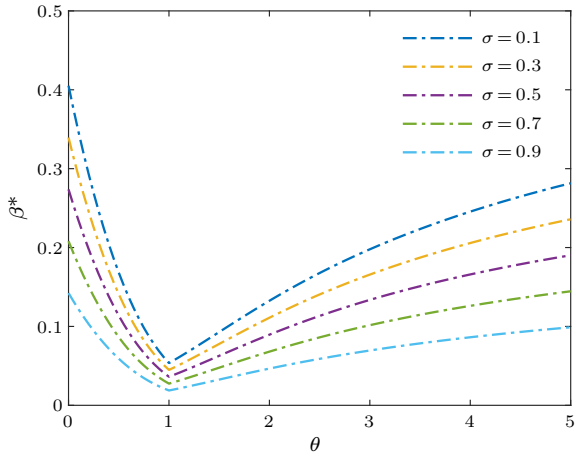


Fig. 5. Relations between robustness threshold β^* and θ with different values of σ in BA networks.

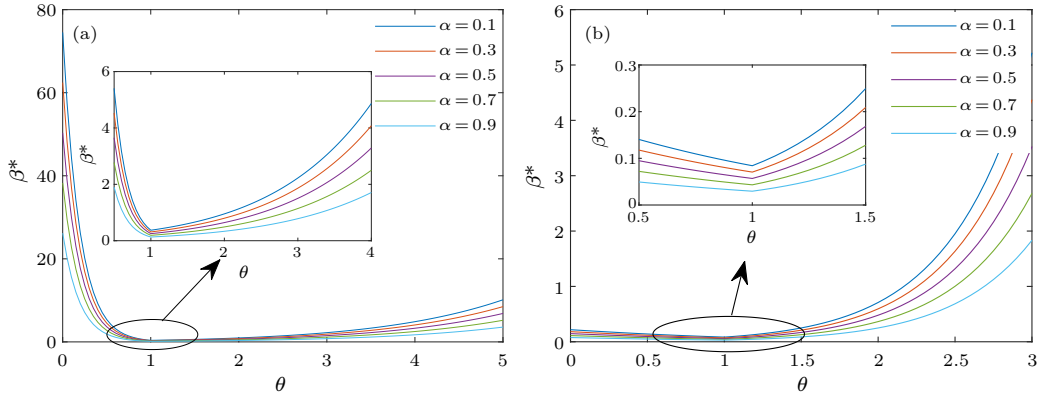


Fig. 6. Relations between robustness threshold β^* and θ with different values of α in the Dolphin social network (a) and the Aviation network (b).

In Section 2, the Dolphin social network and the Aviation network are used to verify the effectiveness of the CCL method on node ranking. Based on the node ranking results, here we proceed to perform numerical simulations on both the Dolphin social network and the Aviation network to analyze the network robustness against cascading failures, as shown in Fig. 6. We can see that the value of critical threshold β^* is smallest when $\theta = \alpha$ in both networks. When $\theta = 1$, the value of β^* is smaller than the other 2 cases.

6. Discussion

In this paper, we focus on the robustness of community networks against cascading failures with heterogeneous redistribution strategies. First, an effective node influence ranking method, the CCL algorithm, is introduced and analyzed. Simulation results show that the CCL method can effectively identify the influence of nodes in community networks. Then, a novel cascading failure model based on node influence in community networks is proposed and the theoretical analysis on network robustness against cascading failures is carried out in our paper. The results show that the community attribute has an important influence on the cascading failure process. The network robustness against cascading failures increases when the load is more distributed to neighbors of the same community instead of different communities. When the initial load distribution and the load redistribution strategy based on the node influence in our paper are the same, the network shows better robustness against node failure. Simulation results show that keeping the transfer rate of load redistribution linearly proportional to the node influence leads to the strongest network robustness under the same conditions.

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