

Dynamics of bubble-shaped Bose-Einstein condensates on two-dimensional cross-section in micro-gravity environment

Tie-Fu Zhang(张铁夫), Cheng-Xi Li(李成蹊), and Wu-Ming Liu(刘伍明)
Citation: Chin. Phys. B, 2023, 32 (**9**): 090501. DOI: 10.1088/1674-1056/acd922
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Ferromagnetic Heisenberg spin chain in a resonator

Yusong Cao(曹雨松), Junpeng Cao(曹俊鹏), and Heng Fan(范桁)
Chin. Phys. B, 2021, 30 (**9**): 090506. DOI: 10.1088/1674-1056/ac1334

Equilibrium dynamics of the sub-ohmic spin-boson model at finite temperature

Ke Yang(杨珂) and Ning-Hua Tong(同宁华)
Chin. Phys. B, 2021, 30 (**4**): 040501. DOI: 10.1088/1674-1056/abd393

Nonequilibrium reservoir engineering of a biased coherent conductor for hybrid energy transport in nanojunctions

"Bing-Zhong Hu(胡柄中), Lei-Lei Nian(年磊磊), and Jing-Tao Lü(吕京涛)
Chin. Phys. B, 2020, 29 (**12**): 120505. DOI: 10.1088/1674-1056/abb3ee

Localized characteristics of lump and interaction solutions to two extended Jimbo-Miwa equations

Yu-Hang Yin(尹宇航), Si-Jia Chen(陈思佳), and Xing Lü(吕兴)
Chin. Phys. B, 2020, 29 (**12**): 120502. DOI: 10.1088/1674-1056/aba9c4

Dynamical correlation functions of the quadratic coupling spin-Boson model

Da-Chuan Zheng(郑大川), Ning-Hua Tong(同宁华)
Chin. Phys. B, 2017, 26 (**6**): 060502. DOI: 10.1088/1674-1056/26/6/060502

Dynamics of bubble-shaped Bose–Einstein condensates on two-dimensional cross-section in micro-gravity environment

Tie-Fu Zhang(张铁夫)[1,2], Cheng-Xi Li(李成蹊)[1,2], and Wu-Ming Liu(刘伍明)[1,2,3,†]

[1] *Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing* 100190, *China*
[2] *School of Physical Sciences, University of Chinese Academy of Sciences, Beijing* 100049, *China*
[3] *Songshan Lake Materials Laboratory, Dongguan* 523808, *China*

(Received 25 March 2023; revised manuscript received 19 May 2023; accepted manuscript online 26 May 2023)

We investigated the dynamic evolution and interference phenomena of bubble-shaped Bose–Einstein condensates achievable in a micro-gravity environment. Using numerical solutions of the Gross–Pitaevskii equation describing the dynamic evolution of the bubble-shaped Bose–Einstein condensates, we plotted the evolution of the wave function density distribution on its two-dimensional (2D) cross-section and analysed the resulting patterns. We found that changes in the strength of atomic interactions and initial momentum can affect the dynamic evolution of the bubble-shaped Bose–Einstein condensates and their interference fringes. Notably, we have observed that when the initial momentum is sufficiently high, the thickness of the bubble-shaped Bose–Einstein condensate undergoes a counterintuitive thinning, which is a counterintuitive result that requires further investigation. Our findings are poised to advance our comprehension of the physical essence of bubble-shaped Bose–Einstein condensates and to facilitate the development of relevant experiments in micro-gravity environments.

Keywords: boson systems, ultracold gases, dynamic properties of condensates

PACS: 05.30.Jp, 67.85.–d, 03.75.Kk **DOI:** 10.1088/1674-1056/acd922

1. Introduction

The bubble-shaped Bose–Einstein condensates with a topological structure, that is almost unattainable through conventional experiments on the Earth's surface,[1] have been experimentally observed[2] in NASA Cold Atom Laboratory,[3] which provides the micro-gravity environment on the International Space Station. The seminal proposal of the matter–wave condensate bubbles was proposed by Zobay and Garraway,[4–6] and experimentally examined by several research groups.[7–10] The quantum bubble's zero-temperature density distribution and free expansion have been studied[11] by the simulation of Gross–Pitaevskii equation,[12–15] which provides important references for the study of the bubble-shaped Bose–Einstein condensates in micro-gravity environments.

Recently, a series of advances have been made in the study of Bose–Einstein condensates in micro-gravity environments.[2,3,11,16–19] The mean-field theory developed by Bogoliubov provided an effective method for studying dilute Bose gas with weak repulsive interaction.[15,20] Subsequently, the Gross–Pitaevskii equation, which is based on this theory, has become the fundamental paradigm for investigating Bose–Einstein condensates and their matter–wave interference phenomena.[21–24] Through the numerically solving of the Gross–Pitaevskii equation, interesting phenomena such as variations in the thickness of bubble-shaped Bose–Einstein condensates and changes in interference fringes between two bubble-shaped Bose–Einstein condensates can be studied. The physical nature of bubble-shaped Bose–Einstein condensates still deserves attention.

In the presented study, the dynamical evolution of the bubble-shaped Bose–Einstein condensate has been investigated by numerically solving the Gross–Pitaevskii equation. In order to more effectively illustrate the variations in the density distribution of the condensate, we have selected a two-dimensional (2D) cross-section of the three-dimensional (3D) bubble-shaped Bose–Einstein condensate as the focus of our study. Utilizing numerical solutions of the Gross–Pitaevskii equation, we generated the temporal density evolution plots for a singular bubble-shaped Bose–Einstein condensate, subsequently delving into the impact of the atomic interaction strength and the initial momentum on the density distribution. Employing the same methodology, we further examined the interference effects between the two bubble-shaped Bose–Einstein condensates.

2. Method: Gross–Pitaevskii equation

Consider the bubble-shaped Bose–Einstein condensate in micro-gravity environment where the gravitational force from the Earth is sufficiently weak to be negligible, under the de-

[†]Corresponding author. E-mail: wliu@iphy.ac.cn

© 2023 Chinese Physical Society and IOP Publishing Ltd

scription of the mean-field theory,[15,20] the time-evolution of the wave function can be described by the following Gross–Pitaevskii equation:[12–15]

$$\mathrm{i}\frac{\partial}{\partial\tau}\phi = -\nabla_{\xi}^{2}\phi + \eta|\phi|^{2}\phi, \tag{1}$$

where ϕ is the dimensionless wave function, τ is the dimensionless time, and

$$\begin{aligned}\nabla_{\xi}^{2} &= \frac{\partial^{2}}{\partial\xi_{1}^{2}} + \frac{\partial^{2}}{\partial\xi_{2}^{2}} + \frac{\partial^{2}}{\partial\xi_{3}^{2}} \\ &= \beta^{2}\left(\frac{\partial^{2}}{\partial x_{1}^{2}} + \frac{\partial^{2}}{\partial x_{2}^{2}} + \frac{\partial^{2}}{\partial x_{3}^{2}}\right) = \beta^{2}\nabla^{2}\end{aligned}$$

is the Laplace operator of the dimensionless space coordinates characterized by ξ, and the expression

$$\eta = \frac{8\pi N a_{\mathrm{s}}}{\beta}$$

is a dimensionless parameter, which characterizes the atomic interaction strength. The original variables (t, x, and ψ) are presented by the dimensionless variables (τ, ξ, and ϕ) as follows:

$$t = \tau\alpha, \quad x = \xi\beta, \quad \psi = \phi\gamma, \tag{2}$$

where α is the time unit, β is the space unit, and γ is the wave function unit. And t, x, ψ are the original time, space, wave function variables of the original free Gross–Pitaevskii equation

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^{2}}{2M}\nabla^{2}\psi + \frac{4\pi\hbar^{2}a_{\mathrm{s}}}{M}|\psi|^{2}\psi, \tag{3}$$

where $\hbar$ is the reduced Planck's constant, M and a_{s} are mass and s-wave scattering length of the atom,

$$\nabla^{2} = \frac{\partial^{2}}{\partial x_{1}^{2}} + \frac{\partial^{2}}{\partial x_{2}^{2}} + \frac{\partial^{2}}{\partial x_{3}^{2}}$$

is the Laplace operator. Under the condition for simplification:

$$\hbar\alpha/2M\beta^{2} = 1,$$

this equation can be simplified into a dimensionless form in Eq. (1), which is convenient for calculation.

Regarding the bubble structure of the Bose–Einstein condensate, consider the 2D cross-section of the 3D bubble in order to simplify the calculations and emphasize the key physical issues. In this case, the 2D cross-section of a 3D bubble can be treated as a ring. The related schematic diagram has been presented in Fig. 1.

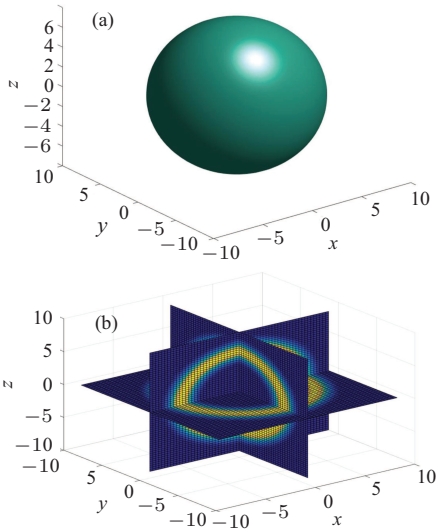


Fig. 1. The schematic diagram of a bubble-shaped Bose–Einstein condensate. (a) The contour of a bubble-shaped Bose–Einstein condensate in three-dimensional (3D) space. (b) The related 2D cross-sectional of the 3D bubble-shaped Bose–Einstein condensate.

Based on the density profiles of the bubble-shaped Bose–Einstein condensates in Refs. [1,2,25], and the long-established tradition of postulating Gaussian wave packets as the initial wave functions of the Bose–Einstein condensates,[24,26,27] it is natural to assume that the condensate's initial ground-state wave function satisfies the ring-shaped Gaussian distribution

$$\phi_{\mathrm{dis}} = \sqrt{\left(\frac{1}{\sqrt{2\pi\sigma^{2}}}\right)^{2}\exp\left[-\frac{(|\xi|-\rho_{0})^{2}}{2\sigma^{2}}\right]}, \tag{4}$$

where $|\phi_{\mathrm{dis}}|^{2}$ satisfies the normalization condition

$$1 = \int|\phi_{\mathrm{dis}}|^{2}\,\mathrm{d}\xi_{1}\,\mathrm{d}\xi_{2},$$

and $|\xi| = \sqrt{\xi_{1}^{2}+\xi_{2}^{2}}$ is the distance from the center, ρ_{0} is the radius of the ring, σ is the inflection point of the Gaussian distribution. Consider the single particle's wave function which contains the initial momentum

$$\phi_{\mathrm{single}} = \exp\left[\frac{\mathrm{i}\beta}{\hbar}p_{\mathrm{ini}}(|\xi|-\rho_{0})\right], \tag{5}$$

where p_{ini} is the initial momentum and β is the space unit. This momentum is related to the kinetic energy, which is characterized by the zero-point motion.[28] Let the macroscopic wave function be the product of the distribution function and the single particle wave function, the initial wave function can be presented as

$$\phi_{\mathrm{ini}} = \phi_{\mathrm{dis}}\cdot\phi_{\mathrm{single}} = \sqrt{\left(\frac{1}{\sqrt{2\pi\sigma^{2}}}\right)^{2}\exp\left[-\frac{(|\xi|-\rho_{0})^{2}}{2\sigma^{2}}\right]}\cdot\exp\left[\frac{\mathrm{i}\beta}{\hbar}p_{\mathrm{ini}}(|\xi|-\rho_{0})\right], \tag{6}$$

where $|\phi_{\rm ini}|^2$ also satisfies the normalization condition as given before

$$1 = \int |\phi_{\rm ini}|^2 \, \mathrm{d}\xi_1 \, \mathrm{d}\xi_2.$$

The dynamic evolution of the wave function of the ring-shaped Bose–Einstein condensate as the 2D cross-section of the 3D bubble can be characterized by numerically solving the dimensionless Gross–Pitaevskii equation (Eq. (1)) with the initial condition (Eq. (6)).

3. Dynamics of individual bubble-shaped Bose–Einstein condensate undergoing free diffusion

Consider free diffusion of the bubble-shaped Bose–Einstein condensate in micro-gravity. To observe the time-evolution of the wave function of the 2D cross-section of a 3D bubble, the Gross–Pitaevskii equation in Eq. (1) was numerically solved with the initial condition in Eq. (6).

The time-evolution of the density distribution of the wave function for a bubble's 2D cross-section is presented in Fig. 2. This bubble is constructed by Bose–Einstein condensate with weakly repulsive interaction. It shows that, as the time increases, the outer surface of the bubble gradually spreads outward, and at the same time, the inner surface of the bubble also spreads inward. In other words, the bubble will gradually expand, while the thickness increases.

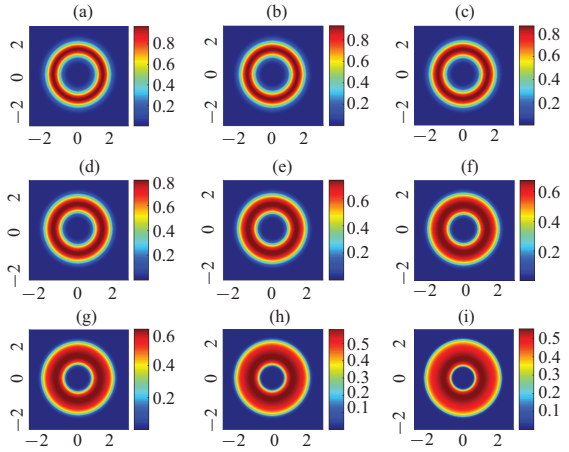


Fig. 2. The time-evolution of the 2D cross-section of a 3D bubble-shaped Bose–Einstein condensate. Here, the parameters are set as $\eta = 3000$, $\rho_0 = 2$, $\sigma = 0.4$, $\beta p_{\rm ini}/\hbar = 1$. The sub-figures present the distributions of $|\phi|^2$ from $\tau = 0.001$ in panel (a) to $\tau = 0.009$ in panel (i), the time interval between the neighbor sub-figures is 0.001.

The dynamics of the bubble may be influenced by the parameters in Gross–Pitaevskii equation and initial state. In order to explore the influences of the parameters, one of the parameters can be adjusted individually. First we adjust the interaction parameter η and numerically solve the Gross–Pitaevskii equation with corresponding different cases to observe the influences of the time-evolution of the wave function.

A comparison of the cross-section density distribution of the bubble-shaped Bose–Einstein condensates with different interaction parameter η is presented in Fig. 3. From these results, it seems that the bigger interaction parameter may cause the faster evolution of the wave function, and in this case, the faster diffusion of the atoms of this bubble-shaped Bose–Einstein condensate. When the interaction parameter is sufficiently large and the time is long enough, the outer surface of the bubble will gradually expand and the density of the condensed matter in close proximity to the external surface will decrease. At the same time, the inner surface of the bubble will gradually contract inward, and the density of the condensed matter in close proximity to the inner surface will increase significantly.

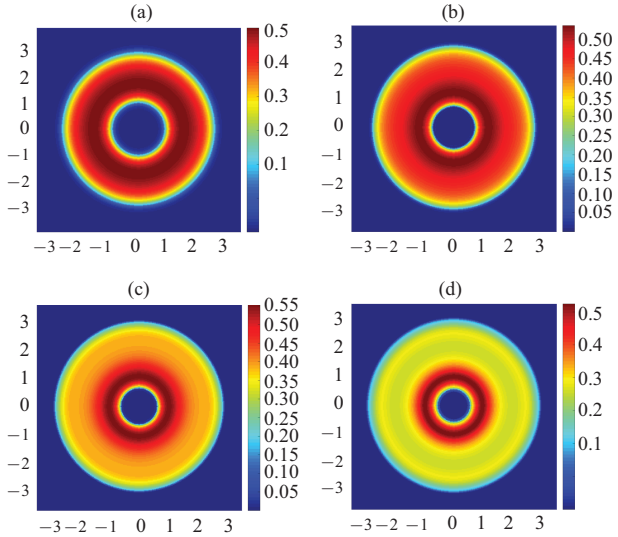


Fig. 3. A comparison of the time-evolutions of the 2D cross-section of a 3D bubble-shaped Bose–Einstein condensate with different interaction parameters. Here, the common parameters are set as $\rho_0 = 2$, $\sigma = 0.4$, $\beta p_{\rm ini}/\hbar = 1$. The interaction parameters are set as $\eta = 2000$ in panel (a), $\eta = 4000$ in panel (b), $\eta = 6000$ in panel (c), and $\eta = 8000$ in panel (d). The time variable τ is fixed at $\tau = 0.009$.

In order to discover the deeper physical mechanisms behind this, the effects of the varying of other parameters should be considered. The initial momentum of the condensate is related to the kinetic energy which can be determined by the zero-point motion.[28] Choose different initial momentum parameters and solve the Gross–Pitaevskii equation, the results can be shown in the following figures.

In Fig. 4, comparisons of the bubble's density distribution in a 2D cross-section are shown at a specific time. In contrast to the findings presented in Fig. 3, it is observed that augmenting the initial momentum parameter does not result in bubble thickening. Conversely, it is possible to observe bubble thinning when the initial momentum parameter is sufficiently large. This appears to be a counterintuitive result, and its physical essence necessitates further experimental and theoretical exploration to unveil.

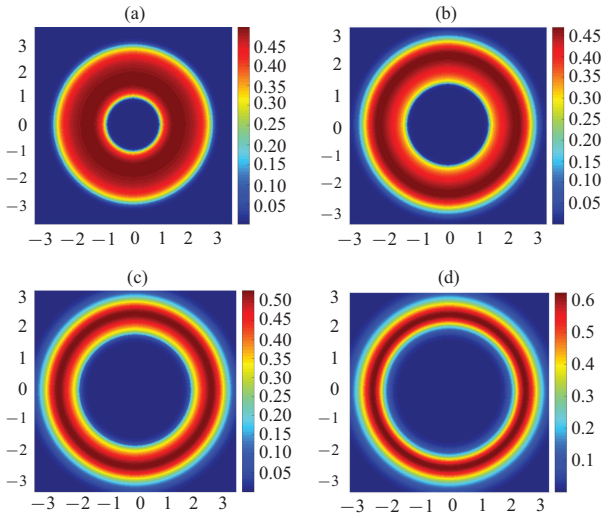


Fig. 4. A comparison of the time-evolutions of the 2D cross-section of a 3D bubble-shaped Bose–Einstein condensate with different initial momentum parameters. Here, the common parameters are set as $\rho_0 = 2$, $\sigma = 0.4$, $\eta = 3000$. The initial momentum parameters are set as $\beta p_{\rm ini}/\hbar = 10$ in panel (a), $\beta p_{\rm ini}/\hbar = 40$ in panel (b), $\beta p_{\rm ini}/\hbar = 70$ in panel (c), and $\beta p_{\rm ini}/\hbar = 100$ in panel (d). The time variable τ is fixed at $\tau = 0.009$.

4. Interference between two bubble-shaped Bose–Einstein condensates and the associated dynamic evolution

To investigate the matter–wave interference of the bubble-shaped Bose–Einstein condensates in micro-gravity, consider two of them which have been separated by a distance in space. The initial wave function can be revised into

$$\begin{aligned}\phi_0 &= \phi_{\rm ini}(\xi_1 + \delta_{\rm d},\, \xi_2) + \phi_{\rm ini}(\xi_1 - \delta_{\rm d},\, \xi_2) \\ &= \sqrt{\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^2 \exp\left[-\frac{(\xi_{\rm L}-\rho_0)^2}{2\sigma^2}\right]} \cdot \exp\left[\frac{{\rm i}\beta}{\hbar} p_{\rm ini}\xi_{\rm L}\right] \\ &\quad + \sqrt{\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^2 \exp\left[-\frac{(\xi_{\rm R}-\rho_0)^2}{2\sigma^2}\right]} \\ &\quad \times \exp\left[\frac{{\rm i}\beta}{\hbar} p_{\rm ini}\xi_{\rm R}\right], \end{aligned} \tag{7}$$

where

$$\xi_{\rm L} = \sqrt{(\xi_1 + \delta_{\rm d})^2 + \xi_2^2},$$
$$\xi_{\rm R} = \sqrt{(\xi_1 - \delta_{\rm d})^2 + \xi_2^2}$$

characterize the distance from the atoms in the left or right bubble condensate to the left or right center. Numerically solve Gross–Pitaevskii equation with this initial wave function of two bubble-shaped Bose–Einstein condensates in a 2D cross-section, the dynamical evolution of the distribution of the wave function can be drawn based on numerical results.

The time-evolution of two bubbles constructed by Bose–Einstein condensates in a 2D cross-section has been shown in Fig. 5. As their wave functions begin to overlap, interference patterns emerge between the wave functions. With the passage of time, the overlapping region of the wave functions gradually increases, resulting in an increase in the number and density of interference patterns.

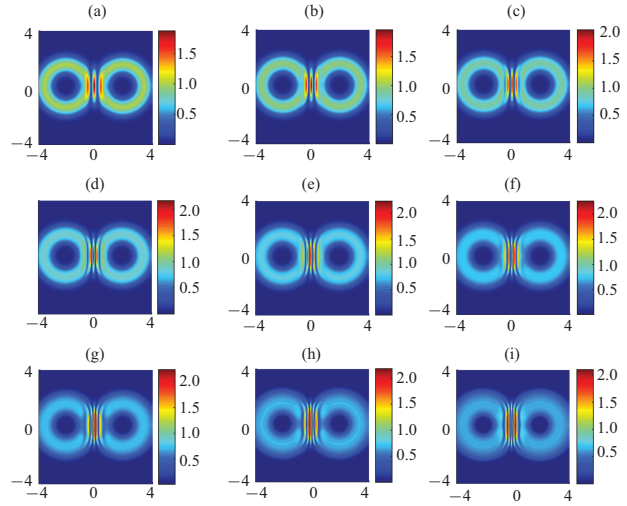


Fig. 5. The time-evolution of the interference between two bubble-shaped Bose–Einstein condensates in a 2D cross-section. Here, the parameters are set as $\eta = 1500$, $\rho_0 = 1.5$, $\sigma = 0.4$, $\beta p_{\rm ini}/\hbar = 5$. The sub-figures present the distribution of $|\phi|^2$ from $\tau = 0.001$ in panel (a) to $\tau = 0.009$ in panel (i), the time interval between the neighbor sub-figures is 0.001.

The time-evolution of the wave function may be influenced by the value of interaction parameter, to investigate the related effects, the Gross–Pitaevskii equation should be solved with the initial wave function characterized by different interaction parameters.

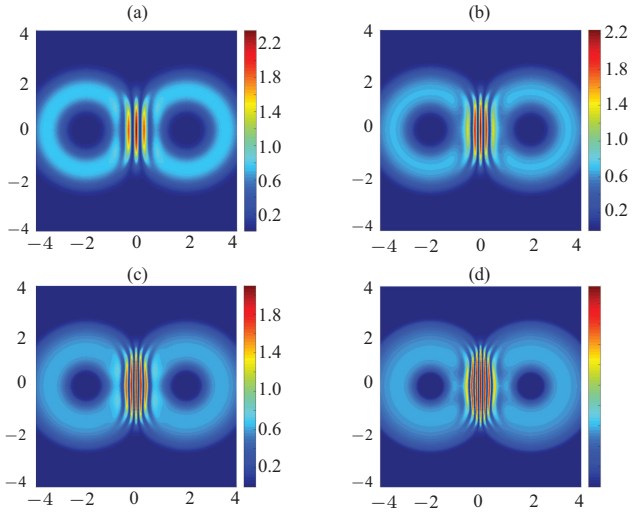


Fig. 6. The comparison of the time-evolutions of the interference between two bubble-shaped Bose–Einstein condensates in cases with different interaction parameters. Here, the common parameters are set as $\rho_0 = 1.5$, $\sigma = 0.4$, $\beta p_{\rm ini}/\hbar = 5$. The interaction parameters are set as $\eta = 500$ in panel (a), $\eta = 1000$ in panel (b), $\eta = 1500$ in panel (c), $\eta = 2000$ in panel (d). The time variable τ is fixed at $\tau = 0.009$.

In Fig. 6, the comparison of interference fringes of two bubbles constructed by Bose–Einstein condensates under different interaction parameters has been illustrated in a 2D cross-section. It can be observed that as the interaction parameter increases, the wave function of the bubbles expand more

rapidly. This results in an increase in the number of interference fringes between the two bubbles, as well as an increase in the density of the interference fringes.

The initial momentum of the bubble is equally important to the time-evolution of the wave function. Similarly, exploring its effects can be achieved by selecting different initial momentum parameters to solve the Gross–Pitaevskii equation and generate wave function distribution plots.

The contrast of interference fringes between two bubbles constructed by Bose–Einstein condensates, depicted in Fig. 7, has been shown based on the numerical solution of the Gross–Pitaevskii equation with different initial momentum parameters. It can be observed that, in this 2D cross-section, as the initial momentum parameter increases, the number of interference fringes between the bubbles increases, and the width of the interference fringes becomes narrower. Referring to the images of interference fringes under varying interaction parameters in Fig. 6, it can be observed that the increase in the initial momentum parameter and the increase in the interaction parameter have similar effects on the interference fringes.

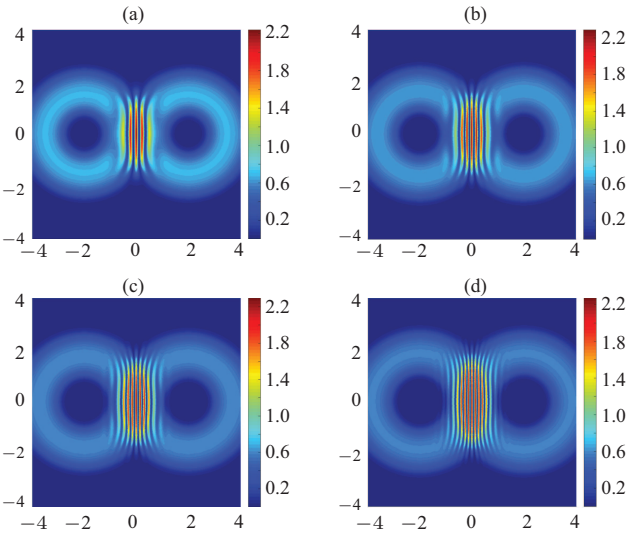


Fig. 7. The comparison of the time-evolutions of the interference between two bubble-shaped Bose–Einstein condensates in cases with different initial momentum parameters. Here, the common parameters are set as $\rho_0 = 1.5$, $\sigma = 0.4$, $\eta = 1000$. The initial momentum parameters are set as $\beta p_{\rm ini}/\hbar = 5$ in panel (a), $\beta p_{\rm ini}/\hbar = 10$ in panel (b), $\beta p_{\rm ini}/\hbar = 15$ in panel (c), $\beta p_{\rm ini}/\hbar = 20$ in panel (d). The time variable τ is fixed at $\tau = 0.009$.

5. Discussion

Through numerically solving the Gross–Pitaevskii equation with different initial conditions, the time-evolution of the density distribution of bubble-shaped Bose–Einstein condensates has been presented in the figures shown above.

Based on the above results, it can be observed that the increase of the atomic interaction strength can cause the faster time-evolution of the wave function. And in the case of the interference between two bubble-shaped Bose–Einstein condensates appears, the increase of the atomic interaction strength can cause the enhanced interference effect which characterized manifestly by the increased number of interference fringes and the increased fringe density. The interaction strength is related to the atomic s-wave scattering length, which can be modified by external magnetic field through the effects of the Fano–Feshbach resonance,[29–35] these results can be examined by further related experiments.

The initial momentum that is related to the zero-point motion[28] can also influence the time-evolution of the density distribution of the wave function. Increasing the initial momentum can enhance the rate of time-evolution of the wave function, which is consistent with our physical intuition, as a larger initial momentum implies that microscopic particles have higher initial velocities, leading to faster macroscopic wave function evolution. Furthermore, a larger initial momentum also implies a shorter de Broglie wavelength, corresponding to shorter period of interference fringes.[24,28] However, increasing the initial momentum can also result in a thinner bubble-shaped Bose–Einstein condensate, which appears counterintuitive and requires further theoretical and experimental exploration to verify.

6. Conclusion

The dynamical evolution of the bubble-shaped Bose–Einstein condensates has been investigated by numerically solving the Gross–Pitaevskii equation with several different initial conditions. We studied the effects of atomic interaction strength and initial momentum on the time evolution of single bubble and interference patterns between two bubbles by analyzing the evolution images based on numerical results. We have discovered several interesting physical phenomena, such as the fact that bubbles with larger initial momentum, i.e., stronger the zero-point motion, exhibit a thinning of their thicknesses after a certain period of evolution. This counterintuitive result requires further investigation to verify. Our findings will contribute to exploring the physical nature of bubble-shaped Bose–Einstein condensates and provide a valuable reference for future relevant experiments in micro-gravity.

Acknowledgements

Project supported by the National Key Research and Development Program of China (Grant Nos. 2021YFA1400900, 2021YFA0718300, and 2021YFA1402100), the National Natural Science Foundation of China (Grant Nos. 61835013, 12174461, 12234012, and 12334012), and the Space Application System of China Manned Space Program.

References

[1] Lundblad N, Aveline D C, Balaž A, Bentine E, Bigelow N P, Boegel P, Efremov M A, Gaaloul N, Meister M, Olshanii M, de Melo C A R

S, Tononi A, Vishveshwara S, White A C, Wolf A and Garraway B M 2023 *Quantum Science and Technology* **8** 024003
[2] Carollo R A, Aveline D C, Rhyno B, Vishveshwara S, Lannert C, Murphree J D, Elliott E R, Williams J R, Thompson R J and Lundblad N 2022 *Nature* **606** 281
[3] Aveline D C, Williams J R, Elliott E R, Dutenhoffer C, Kellogg J R, Kohel J M, Lay N E, Oudrhiri K, Shotwell R F and Yu N 2020 *Nature* **582** 193
[4] Zobay O and Garraway B M 2001 *Phys. Rev. Lett.* **86** 1195
[5] Zobay O and Garraway B M 2004 *Phys. Rev. A* **69** 023605
[6] Garraway B M and Perrin H 2016 *J. Phys. B: At. Mol. Opt. Phys.* **49** 172001
[7] Lundblad N, Carollo R, Lannert C, Gold M, Jiang X, Paseltiner D, Sergay N and Aveline D 2019 *npj Microgravity* **5** 30
[8] Colombe Y, Knyazchyan E, Morizot O, Mercier B, Lorent V and Perrin H 2004 *Europhys. Lett.* **67** 593
[9] White M, Gao H, Pasienski M and DeMarco B 2006 *Phys. Rev. A* **74** 023616
[10] Harte T L, Bentine E, Luksch K, Barker A J, Trypogeorgos D, Yuen B and Foot C J 2018 *Phys. Rev. A* **97** 013616
[11] Tononi A, Cinti F and Salasnich L 2020 *Phys. Rev. Lett.* **125** 010402
[12] Gross E P 1961 *Il Nuovo Cimento* **20** 454
[13] Gross E P 1963 *J. Math. Phys.* **4** 195
[14] Pitaevskii L P 1961 *Sov. Phys. JETP* **13** 451
[15] Dalfovo F, Giorgini S, Pitaevskii L P and Stringari S 1999 *Rev. Mod. Phys.* **71** 463
[16] Liu L, Lü D S, Chen W B, Li T, Qu Q Z, Wang B, Li L, Ren W, Dong Z R and Zhao J B 2018 *Nat. Commun.* **9** 2760
[17] Lachmann M D, Ahlers H, Becker D, Dinkelaker A N, Grosse J, Hellmig O, Müntinga H, Schkolnik V, Seidel S T and Wendrich T 2021 *Nat. Commun.* **12** 1317
[18] Gaaloul N, Meister M, Corgier R, Pichery A, Boegel P, Herr W, Ahlers H, Charron E, Williams J R and Thompson R J 2022 *Nat. Commun.* **13** 7889
[19] Müntinga H, Ahlers H, Krutzik M, Wenzlawski A, Arnold S, Becker D, Bongs K, Dittus H, Duncker H, Gaaloul N, Gherasim C, Giese E, Grzeschik C, Hänsch T W, Hellmig O, Herr W, Herrmann S, Kajari E, Kleinert S, Lämmerzahl C, Lewoczko-Adamczyk W, Malcolm J, Meyer N, Nolte R, Peters A, Popp M, Reichel J, Roura A, Rudolph J, Schiemangk M, Schneider M, Seidel S T, Sengstock K, Tamma V, Valenzuela T, Vogel A, Walser R, Wendrich T, Windpassinger P, Zeller W, van Zoest T, Ertmer W, Schleich W P and Rasel E M 2013 *Phys. Rev. Lett.* **110** 093602
[20] Bogoliubov N 1947 *J. Phys.* **11** 23
[21] Li J, Yu Y M, Zhuang L and Liu W M 2017 *Phys. Rev. A* **95** 043633
[22] Han W, Zhang X F, Wang D S, Jiang H F, Zhang W and Zhang S G 2018 *Phys. Rev. Lett.* **121** 030404
[23] Han W, Zhang X F, Song S W, Saito H, Zhang W, Liu W M and Zhang S G 2016 *Phys. Rev. A* **94** 033629
[24] Liu W M, Wu B and Niu Q 2000 *Phys. Rev. Lett.* **84** 2294
[25] Wolf A, Boegel P, Meister M, Balaž A, Gaaloul N and Efremov M A 2022 *Phys. Rev. A* **106** 013309
[26] Wallis H, Röhrl A, Naraschewski M and Schenzle A 1997 *Phys. Rev. A* **55** 2109
[27] Röhrl A, Naraschewski M, Schenzle A and Wallis H 1997 *Phys. Rev. Lett.* **78** 4143
[28] Andrews M R, Townsend C G, Miesner H J, Durfee D S, Kurn D M and Ketterle W 1997 *Science* **275** 637
[29] Chin C, Grimm R, Julienne P and Tiesinga E 2010 *Rev. Mod. Phys.* **82** 1225
[30] Fano U 1935 *Il Nuovo Cimento* **12** 154
[31] Fano U 1961 *Phys. Rev.* **124** 1866
[32] Fano U, Pupillo G, Zannoni A and Clark C W 2005 *Journal of Research of the National Institute of Standards and Technology* **110** 583
[33] Feshbach H 1958 *Ann. Phys.* **5** 357
[34] Feshbach H 1962 *Annals of Physics* **19** 287
[35] Moerdijk A J, Verhaar B J and Axelsson A 1995 *Phys. Rev. A* **51** 4852