

Finesse measurement for high-power optical enhancement cavity

Xin-Yi Lu(陆心怡), Xing Liu(柳兴), Qi-Li Tian(田其立), Huan Wang(王焕), Jia-Jun Wang(汪嘉俊), and Li-Xin Yan(颜立新)

Citation: Chin. Phys. B, 2024, 33 (**1**): 014205. DOI: 10.1088/1674-1056/acd8ad

Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Dynamic light storage based on controllable electromagnetically induced transparency effect

Liu-Ying Zeng(曾柳莹), Jun-Fang Wu(吴俊芳), and Chao Li(李潮)
Chin. Phys. B, 2023, 32 (**6**): 064213. DOI: 10.1088/1674-1056/acd2bd

Continuous-wave optical enhancement cavity with 30-kW average power

Xing Liu(柳兴), Xin-Yi Lu(陆心怡), Huan Wang(王焕), Li-Xin Yan(颜立新), Ren-Kai Li(李任恺), Wen-Hui Huang(黄文会), Chuan-Xiang Tang(唐传祥), Ronic Chiche, and Fabian Zomer
Chin. Phys. B, 2023, 32 (**3**): 034206. DOI: 10.1088/1674-1056/ac873e

High-sensitivity methane monitoring based on quasi-fundamental mode matched continuous-wave cavity ring-down spectroscopy

Zhe Li(李哲), Shuang Yang(杨爽), Zhirong Zhang(张志荣), Hua Xia(夏滑), Tao Pang(庞涛),Bian Wu(吴边), Pengshuai Sun(孙鹏帅), Huadong Wang(王华东), and Runqing Yu(余润磬)
Chin. Phys. B, 2022, 31 (**9**): 094207. DOI: 10.1088/1674-1056/ac5d33

An apodized cubic phase mask used in a wavefront coding system to extend the depth of field

Lina Zhu(朱丽娜), Fei Li(李飞), Zeyu Huang(黄泽宇), and Tingyu Zhao(赵廷玉)
Chin. Phys. B, 2022, 31 (**5**): 054217. DOI: 10.1088/1674-1056/ac5979

Data point selection for weighted least square fitting of cavity decay time constant

Xing He(何星), Hu Yan(晏虎), Li-Zhi Dong(董理治), Ping Yang(杨平), Bing Xu(许冰)
Chin. Phys. B, 2016, 25 (**1**): 014211. DOI: 10.1088/1674-1056/25/1/014211

Finesse measurement for high-power optical enhancement cavity

Xin-Yi Lu(陆心怡)[1,2], Xing Liu(柳兴)[1,2,†], Qi-Li Tian(田其立)[1,2], Huan Wang(王焕)[1,2], Jia-Jun Wang(汪嘉俊)[1,2], and Li-Xin Yan(颜立新)[1,2,‡]

[1] *Department of Engineering Physics, Tsinghua University, Beijing* 100084, *China*
[2] *Key Laboratory of Particle & Radiation Imaging (Tsinghua University), Ministry of Education, Beijing* 100084, *China*

(Received 2 March 2023; revised manuscript received 23 May 2023; accepted manuscript online 25 May 2023)

Finesse is a critical parameter for describing the characteristics of an optical enhancement cavity (OEC). This paper first presents a review of finesse measurement techniques, including a comparative analysis of the advantages, disadvantages, and potential limitations of several main methods from both theoretical and practical perspectives. A variant of the existing method called the free spectral range (FSR) modulation method is proposed and compared with three other finesse measurement methods, i.e., the fast-switching cavity ring-down (CRD) method, the rapidly swept-frequency (SF) CRD method, and the ringing effect method. A high-power OEC platform with a high finesse of approximately 16000 is built and measured with the four methods. The performance of these methods is compared, and the results show that the FSR modulation method and the fast-switching CRD method are more suitable and accurate than the other two methods for high-finesse OEC measurements. The CRD method and the ringing effect method can be implemented in open loop using simple equipment and are easy to perform. Additionally, recommendations for selecting finesse measurement methods under different conditions are proposed, which benefit the development of OEC and its applications.

Keywords: optical enhancement cavity, finesse measurement, cavity ring-down, ringing effect

PACS: 42.60.Da, 42.60.–v, 42.60.Fc **DOI:** 10.1088/1674-1056/acd8ad

1. Introduction

An optical enhancement cavity (OEC) is a passive optical resonator that enables coherent superposition of laser light. A high-finesse OEC provides high-power density and high-frequency resolution, and has a wide range of applications, including high-order harmonic generation,[1,2] precision metrology,[3,4] quantum optics,[5,6] high-flux x/gamma-ray generation,[7–9] and gravitational wave detection.[10] In addition, the high-power laser stored in an OEC can serve as a modulator in steady-state microbunching (SSMB) experiments.[11] Recently, our team has made experimental progress in the study of high-power continuous-wave (CW) OEC for SSMB, with an average intracavity power of 30 kW and a finesse of 6402.[12]

There are a number of parameters that describe an OEC, including finesse, gain, dispersion, and cavity loss. These optical characteristics play an essential role in selecting the appropriate type of cavity for a specific application. In particular, finesse is considered to be a fundamental property of resonators and is commonly used as a measure of optical cavity performance. Higher finesse typically indicates sharper resonances, lower cavity loss, higher gain, and finer spectral resolution. Accurate knowledge of finesse is crucial for most applications that rely on OECs.

Several methods have been proposed to measure the finesse of cavities. The classical approach involves measuring the linewidth of the cavity by directly scanning the frequency of the laser around the resonance condition and comparing it with the free spectral range (FSR).[13] However, this unmodulated frequency scanned method has several limitations. The measured result is a convolution of the laser linewidth with the cavity linewidth, leading to a wider measured linewidth. Moreover, laser frequency instability results in fluctuations in the measured transmission coefficient of the cavity. To obtain a more comprehensive measurement of the cavity response, more advanced measurement schemes have been proposed using various combinations of phase modulation and wavelength modulation,[14–17] including the FSR modulation method. In the FSR modulation method, the drawbacks of the unmodulated method can be avoided by phase modulation and cavity locking. Due to the strong correlation between finesse and cavity loss, the cavity ring-down (CRD) technique is commonly used for finesse measurements by rapidly modifying the amplitude or frequency of the incident laser while resonating with the cavity mode.[18] One advantage of the CRD technique is that it measures the decay time of the radiation in the cavity rather than the transmitted light intensity, which makes it insensitive to the amplitude fluctuations of the laser. The CRD scheme has evolved from pulsed light to continuous light, and the interruption scheme includes the optical switch, rapidly swept-frequency (SF) strategy, and rapidly swept-cavity strategy.[19–21] During the study of the swept frequency or swept cavity, the ringing effects between the Airy peak and the exponential decay curve were fully investigated

†Corresponding author. E-mail: xingliu@mail.tsinghua.edu.cn
‡Corresponding author. E-mail: yanlx@mail.tsinghua.edu.cn

© 2024 Chinese Physical Society and IOP Publishing Ltd

and used for finesse measurements.[22] In addition to these methods, several new methods have been proposed to obtain a more complete cavity characterization.[23–25] Although there have been numerous studies on finesse measurements, few have attempted to systematically sort and compare different methods. Given the diversity of optical cavity parameters and available devices, there is a lack of guidance on the selection of finesse measurement methods for specific research or engineering applications.

In this paper, we present our latest progress in finesse measurement methods, and construct a high-finesse and high-power CW OEC system. A variant method for finesse measurement is proposed and compared with three other methods. Long-term stable locking of the high-power OEC is achieved by using a digital slow loop and an analog fast loop for feedback control. To obtain accurate finesse, we compare the measurement results of the four methods and conduct a detailed study of their application range and error sources. As a result, the final measured finesse of the OEC is approximately 16000. Although this high-power OEC is oriented to SSMB, the OEC control techniques and finesse measurement methods that are investigated in this study have the potential for wider applications in various fields.

2. Methodology

The cavity finesse $\mathcal{F}$ is a parameter that quantifies the narrowness of the resonances of the optical cavity relative to the frequency distance. It is defined as the ratio of the FSR, which is the frequency spacing between cavity resonances, to the linewidth of the cavity resonance peak (defined as the full width at half maximum (FWHM) bandwidth of the resonances, $\Delta\nu_{\mathrm{FWHM}}$)

$$\mathcal{F} = \frac{\mathrm{FSR}}{\Delta\nu_{\mathrm{FWHM}}}, \tag{1}$$

where $\mathrm{FSR} = c/L$, here c is the speed of light in vacuum and L is the cavity round-trip length. Define $R = \prod_i R_i$, where R_i denotes the power reflectivity of the i-th mirror. Mathematically, the finesse can also be expressed as the product of the mirror reflectivity, according to the following relation:[26]

$$\mathcal{F} = \frac{\pi\sqrt[4]{R}}{1-\sqrt{R}}. \tag{2}$$

When the total loss of the cavity is low, the finesse can be approximated using a simple formula $\mathcal{F} \approx 2\pi/\mathrm{RTL}$, where RTL is the total round-trip loss. The finesse is a significant quantity for describing an OEC and it is important to know it exactly. Here, we introduce four methods and principles for measuring finesse.

2.1. The FSR modulation method

According to the definition of finesse, the finesse value can be directly obtained if the optical cavity FSR and linewidth can be measured. The intensity transmission function of an optical cavity versus frequency is known to consist of a series of narrow transmission peaks described by the Airy function and separated by an FSR.[27]

A simple method to measure the finesse is to record the cavity transmission intensity profile by phase-modulating the laser near the cavity FSR frequency, as described in Ref. [16]. Here, we propose a new variant of this method called the FSR modulation method. Consider an incident phase-modulated input laser field with second-order expansion, such as

$$\begin{aligned}
&\tilde{E}_{\mathrm{in}}(t) = E_{\mathrm{in}}\,\mathrm{e}^{\mathrm{i}[\omega_{\mathrm{c}}t+\beta\sin(\omega_{\mathrm{m}}t)]} \\
&\approx E_{\mathrm{in}}\Big[J_0(\beta)\,\mathrm{e}^{\mathrm{i}\omega_{\mathrm{c}}t} + J_1(\beta)\left(\mathrm{e}^{\mathrm{i}(\omega_{\mathrm{c}}+\omega_{\mathrm{m}})t} - \mathrm{e}^{\mathrm{i}(\omega_{\mathrm{c}}-\omega_{\mathrm{m}})t}\right) \\
&\quad + J_2(\beta)\left(\mathrm{e}^{\mathrm{i}(\omega_{\mathrm{c}}+2\omega_{\mathrm{m}})t} + \mathrm{e}^{\mathrm{i}(\omega_{\mathrm{c}}-2\omega_{\mathrm{m}})t}\right)\Big],
\end{aligned} \tag{3}$$

where ω_{c} is the angular carrier frequency, ω_{m} is the angular phase modulation frequency, β is the phase modulation depth and J_i is the Bessel function of order i. In this method, $f_{\mathrm{m}} = \omega_{\mathrm{m}}/(2\pi)$ is tuned around the FSR, while the carrier frequency is locked to the cavity. Considering the direct current (DC) terms, the expression for the average transmitted power $|\tilde{E}_t|^2$ with small loss and perfect modal matching is

$$\begin{aligned}
\left|\frac{\tilde{E}_t}{\tilde{E}_{\mathrm{in}}}\right|^2 &= T^2\Big[J_0(\beta)^2 + 2J_1(\beta)^2\frac{\mathrm{RTL}^2}{4{f_{\mathrm{s}}}^2+\mathrm{RTL}^2} \\
&\quad + 2J_2(\beta)^2\frac{\mathrm{RTL}^2}{16{f_{\mathrm{s}}}^2+\mathrm{RTL}^2}\Big],
\end{aligned} \tag{4}$$

where T is field transmittance and $f_{\mathrm{s}} = 2\pi\,(f_{\mathrm{m}} - \mathrm{FSR})\,/\mathrm{FSR}$ is the angular frequency detuning normalized to the FSR. The resonance condition is still satisfied when the optical frequency is shifted by FSR because the spacing of the resonant frequencies of the optical cavities is equal to FSR. Typically, this modulation is performed using an electro-optical modulator (EOM). By scanning the laser over one of the resonances of the cavity and monitoring the optical field by photodetectors, we can obtain an estimate of the finesse. Compared to the unmodulated frequency scanned method, the parameters of the transfer function of this phase modulation method are not the optical frequency detuning but the difference between the modulation frequency and the FSR of the cavity, which is beneficial for improving the measurement accuracy and stability. The FSR modulation method that we propose differs from the method of Ref. [16] in two aspects: the choice of the phase modulation depth and the way of parameter fitting. The method in Ref. [16] uses the transmission and reflection curves for full cavity parameter fitting and calculates finesse from measured total cavity loss. This method has no restriction on the choice of modulation depth, so as the modulation

depth increases it leads to a lineshape change, which makes the measured lineshape different from the cavity resonance. In contrast, the FSR modulation method is oriented to the finesse measurement and tries to choose a small phase modulation depth that does not affect the Lorentzian lineshape. Due to the small modulation depth, the influence of the second-order Bessel function of the lineshape can be neglected so that measured transmission bandwidth is consistent with cavity linewidth. An appropriate small modulation parameter can be determined by comparing actual measurement results. Therefore, by scanning the laser frequency over one of the resonances, we can reconstruct the Airy function of cavity resonance. In data processing, we only need to use the transmission curve. The FSR of the cavity can be obtained by finding the modulation frequency that maximizes the transmission and the cavity linewidth can be obtained by Lorentzian fitting. The ratio of the two is the finesse. Our method eliminates the need to monitor reflected optical fields and complex data processing, making it simpler and faster for finesse measurements.

2.2. The fast-switching CRD method

The CRD method is a technique that is commonly used to measure cavity finesse by observing the decay rate of the optical field inside the cavity. The decay time, τ, also known as the ring-down time, is determined by the round-trip loss of the cavity and the round-trip optical path length. When the laser is resonant with the cavity and then the light source is quickly shut off, the output intensity of the resonant cavity will gradually decay due to the loss. If the cavity round-trip time $\tau_{\rm rt} = 1/\mathrm{FSR}$ is much smaller than τ, then the intensity of light decays exponentially with time t,

$$I(t) = I_0 \exp(-t/\tau), \tag{5}$$

where

$$\tau = \frac{\sqrt{R}}{2\left(1-\sqrt{R}\right)}\tau_{\rm rt}. \tag{6}$$

Once the decay time is obtained, the finesse can be calculated with $R \approx 1$, as given by

$$\mathcal{F} = 2\pi \cdot \mathrm{FSR} \cdot \tau. \tag{7}$$

In the pulsed CRD scheme, the termination of the injected pulse light naturally triggers a ring-down signal. However, in the CW CRD scheme, interrupting the interaction rapidly between the cavity and the injection CW laser after building up a resonant optical field in the cavity is essential to obtain a usable decay curve. A common approach to achieve this is to use a fast optical switch, such as an acousto–optic modulator (AOM). Turning off the AOM quickly diverts the laser path to the beam dump, effectively cutting off the light incident on the cavity. The light shut-off time must be significantly shorter than the decay time to obtain an accurate decay curve. In the unlocked CW CRD scheme, the optical cavity system operates in an open-loop configuration. Initially, when the shutter is opened, the laser frequency drifts slowly until it resonates with the cavity and a resonance optical field is rapidly built up in the cavity. When the cavity transmission signal reaches a certain threshold, the shutter is closed, and the decay of the field is measured as a function of time using a fast-response photodiode (PD).

If the laser linewidth is narrower than the optical cavity linewidth, the cavity-locked CW CRD can be employed to lock the laser frequency and cavity resonance together through a feedback system. This closed-loop operation ensures that a single transverse mode of the cavity is excited and remains resonant, thereby reducing errors and fluctuations in the decay profile, and maximizing the dynamic range of the decay signal.[28]

2.3. The rapidly SF CRD method

The rapidly SF CRD[29] is another CRD scheme that does not require an active optical switch or frequency modulator. In this approach, the resonance is disrupted by quickly sweeping the frequency of the laser source. As the laser frequency is scanned rapidly across a cavity resonance, the optical field is built up quickly in the cavity and the measured transient begins with a rapid increase in intensity. The cavity mode is then scanned out of resonance and the ring-down starts. The transmitted intensity signal consists of a short accumulation period and a long decay period, in which there are significant oscillations in the early part of the decay. These oscillations can be described by[18]

$$I(t) = I_0 \exp(-t/\tau)\frac{2\exp\left[-\frac{1}{2}\left(\frac{\tau_{\rm rt}}{\tau}+\frac{2\omega_0}{\tau\dot{\omega}}\right)\right]}{\dot{\omega}\left(\sqrt{{\tau_{\rm rt}}^2+16\tau^2}-\tau_{\rm rt}\right)}\left|\mathrm{erfc}\left[\Lambda(t)\right]\right|^2, \tag{8}$$

where I_0 is the incident optical intensity, $\omega(t) = \dot{\omega}t + \omega_0$ is the laser angular frequency, and "erfc" is the complementary error function with the argument

$$\begin{aligned}\Lambda(t) \;=\; & -\frac{1+\mathrm{i}}{2}\sqrt{\dot{\omega}}t \\ & +\frac{1-2\tau\omega_0+\tau\tau_{\rm rt}\dot{\omega}-\mathrm{i}\left(1+2\tau\omega_0-\tau\tau_{\rm rt}\dot{\omega}\right)}{4\tau\sqrt{\dot{\omega}}}.\end{aligned} \tag{9}$$

The oscillations are caused by the jumps between the laser frequency and the cavity mode due to the mutual interference of the intracavity field and a small portion of the transient incident optical field.[15] As the oscillation period and modulation depth decrease over time, the decay time is commonly determined by fitting an exponential function to the relatively smooth tail of the transmitted intensity profile.[30] Indeed, these oscillations can also provide valuable information about the cavity, and several methods have been developed

to analyze and interpret the oscillations to obtain additional information.[21]

To obtain a ring-down signal from the optical cavity, it is necessary to rapidly scan the laser frequency to avoid the steady-state case described by the Airy formula. This requires that the time scale of the laser frequency shift is much shorter than the decay time of the optical cavity, which reduces the impact of oscillations. Typically, fast scanning requires $v \gg 15\lambda\Delta\nu_{\mathrm{FWHM}}/\mathcal{F}$, where v is the mirror movement velocity and λ is the laser wavelength.[22]

2.4. The ringing effect method

As mentioned above, when the laser frequency or optical cavity length is swept rapidly, the cavity transmission curve becomes asymmetric, oscillating, and distinct from the Airy peak. This phenomenon is known as the ringing effect and is observed when the time swept through the optical resonance is less than or equal to the cavity decay time. In this case, the optical cavity does not have enough time to fill itself as it approaches resonance, leading to the observed oscillations.[31]

A semianalytical expression for the ringing effect is as follows:[22]

$$\frac{I_1}{I_2} = 4\pi\frac{\mathrm{FSR}}{\mathcal{F}}\Delta t + e - 2, \tag{10}$$

where I_1 and I_2 are the intensities of the first and second peaks due to the ringing effect, and Δt is the time difference between the first two maxima. From this equation, the finesse of the cavity can be obtained by measuring the peak parameters of the decay curve. Moreover, the ratio I_1/I_2 is linearly related to Δt, and the slope of the line is uniquely determined by the finesse of the cavity for a fixed cavity length. As the scanning speed increases, the peak intensity of the cavity decay signals I_1 and I_2 decreases and the time interval Δt becomes smaller. To determine a definite finesse value, the decay signal can be measured at a range of different scanning speeds.

3. Experimental setup

The OEC is designed as a four-mirror planar bow-tie structure due to its larger optically stable region size and better mechanical stability. The distances of L_1, L_2, L_3, and L_4 are 882 mm, 947 mm, 1006 mm, and 947 mm, respectively, corresponding to the FSR of 79.32 MHz, and the incidence angle is $\theta = 2.42^\circ$. Mirror M_1, as the coupling mirror, and M_2 are plane mirrors, while M_3 and M_4 are spherical mirrors with curvature $\rho = 1000$ mm. The reflectivity of M_1 is higher than 99.9%, and the substrate is made of fused silica. The reflectivity of M_2, M_3 and M_4 is greater than 99.999%, and the substrate is made of ultra-low expansion (ULE) glass to minimize the deformation caused by temperature rise. The transmissions of four cavity mirrors are calibrated to be $T_1 = 262.70$ ppm, $T_2 = 6.42$ ppm, $T_3 = 6.13$ ppm, and $T_4 = 5.95$ ppm, respectively. However, the exact finesse cannot be accurately calculated because the exact loss of the cavity mirrors is unknown. Assuming an estimated absorption and scattering of 25 ppm per mirror in the class 1000 environment, the estimated total loss RTL is 381.20 ppm, corresponding to a theoretical finesse of 16483 and a cavity linewidth of 4.81 kHz. The entire OEC is placed in a vacuum chamber to provide temperature control and vibration isolation.

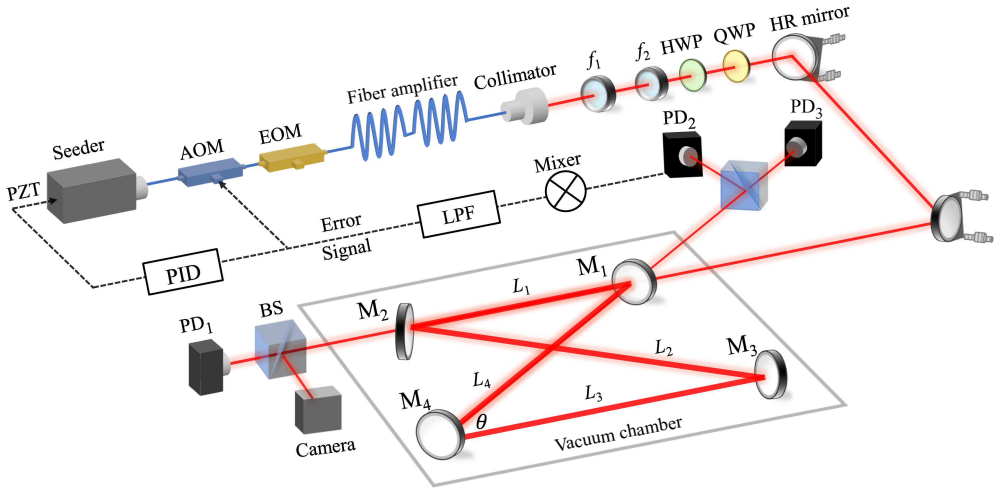


Fig. 1. Experiment setup. AOM: acousto-optic modulator. EOM: electro-optic modulator. f: mode-matching lens. HWP: half-wave plate. QWP: quarter-wave plate. M: cavity mirror. PD: photodiode. LPF: low-pass filter. PID: proportional-integral-derivative. PZT: piezoelectric transducer. BS: beam splitter.

The OEC experimental setup is shown in Fig. 1, which has been refined from a previous setup described in Ref. [12]. A CW low-noise single-frequency laser from NKT Photonics, with a wavelength of 1064 nm and a linewidth of 3 kHz, is used to seed a fiber amplifier with a maximum output of 120 W from Azurlight Systems. To reduce the coupling of higher-order modes, a pair of lenses is used to match the laser beam and cavity mode, and a half-wave plate (HWP) and a

quarter-wave plate (QWP) are used to match the polarization. The amplified laser is coupled into the OEC through the mode-matching lenses, the polarization-matching waveplate set, and the highly reflective (HR) mirror set in turn. The transmitted light from the OEC is divided into two parts by a beam splitter: one light beam is monitored by a camera for the resonance mode signal, and the other is detected by PD_1 (Thorlabs DET10A2) for the intensity signal. The Pound–Drever–Hall (PDH) method is used to lock the injection laser to the OEC.[32] An EOM provides phase modulation on the incident beam with a modulation frequency of $\Omega = 8.2$ MHz. The reflected laser of the OEC consists of two sidebands and a central carrier, the phase of which is determined by the frequency difference with the resonance of the OEC. The detected signal at PD_2 is processed by an operational amplifier to retain and amplify the single Ω frequency component, and is then demodulated and low-pass filtered to produce the PDH error signal. PD_3 measures the reflected light intensity to obtain the coupling efficiency. The system comprises two feedback loops to ensure long-term stable locking of the OEC. In the digital slow loop, the piezoelectric translator (PZT) of the seeder is regulated by a proportional-integral-derivative (PID) controller to compensate for low-frequency noise. In the analog fast loop, an AOM shifts the laser central frequency to compensate for high-frequency noise.

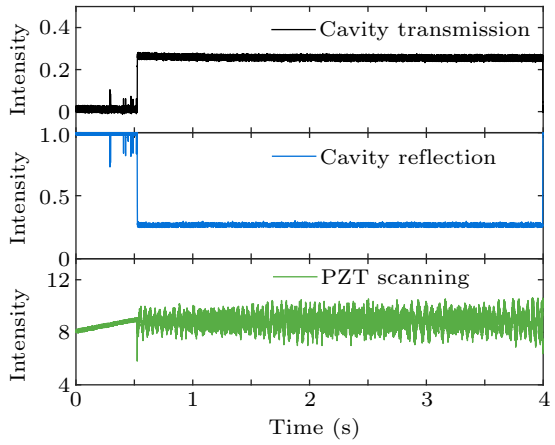


Fig. 2. Experimental measurements of the evolution of cavity transmission (black curve), cavity reflection (blue curve), and the scan signal applied to PZT (green curve) over 4 seconds.

The PZT of the seeder is used to slowly scan the laser frequency to find the resonance between the incident laser and the cavity modes. The distance between the mode-matching lenses is adjusted and the incident angle of the injection laser is optimized to suppress the higher-order modes and increase the intensity of fundamental modes. Finally, stable locking of the OEC in Fig. 2 is achieved with the help of two feedback loops. The OEC locking makes the cavity resonance remain locked for several minutes. The black curve represents the transmitted signal from the cavity and the blue curve is the reflected signal. The green curve represents the voltage signal applied to the PZT of the seed laser cavity for frequency tuning. When the cavity is locked, the transmitted signal remains at a maximum and the reflected signal remains at a minimum, with a coupling efficiency of about 75%, an effective power gain of over 4000, and an intra-cavity power of 30 kW.

4. Finesse measurement

In this study, we employ the four finesse measurement methods described above to compare and ensure the accuracy of the results. The first two methods are utilized in a closed loop, wherein the laser frequency is locked to the OEC, while the latter two approaches are employed in an open loop.

In the FSR modulation method, an EOM_2 is added after the seed laser to provide phase modulation around the FSR. A linear frequency sweep over a small range is performed to track the Lorentzian lineshape of the intensity transmitted through the cavity. The frequency sweep rate is set to be 1 kHz per second, which corresponds to the conversion rate from the oscilloscope time base to the optical frequency domain. In this scheme, the laser is locked to the cavity and scanned with modulation frequency ± 20 kHz around FSR. The light transmitted through the cavity is detected by PD_1, and directed to a high-speed oscilloscope for data acquisition and subsequent analysis. The measured Lorentzian-shaped curve is shown in Fig. 3. Based on the results of this method, an FSR of (78.7922 ± 0.0011) MHz and a linewidth of (4.77 ± 0.03) kHz are obtained, from which the finesse is calculated to be 16518 ± 103, where the uncertainty is given at the 1σ confidence interval.

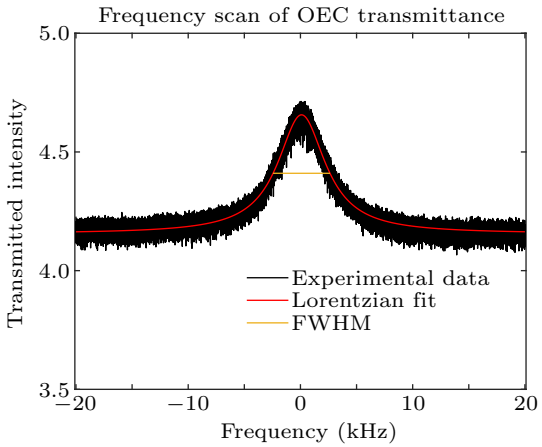


Fig. 3. Measurement data of cavity transmission (black curve), fitted Lorentzian function (red curve), and FWHM of the profile (yellow curve). The horizontal axis is the frequency difference from the FSR. The cavity FSR and linewidth are calculated to be (78.7922 ± 0.0011) MHz and (4.77 ± 0.03) kHz, corresponding to a finesse of 16518 ± 103.

The fast-switching CRD method is also performed to characterize the cavity finesse. To generate a reliable decay curve, the light shut-off time must be considerably shorter than the cavity decay time. To achieve the required shut-off time,

an AOM is introduced into the experimental setup. The cut-off time of the AOM is in the nanosecond range, while the decay time of a high-finesse cavity is a few microseconds. After locking to the resonance, turning off the AOM instantly cuts off the light incident to the cavity and obtains exponential attenuation of the cavity transmission through the fast-response PD with a rise time of 1 ns. The result is presented in Fig. 4. The decay time is fitted to be (33.01 ± 0.22) μs, corresponding to a cavity finesse of 16342 ± 109.

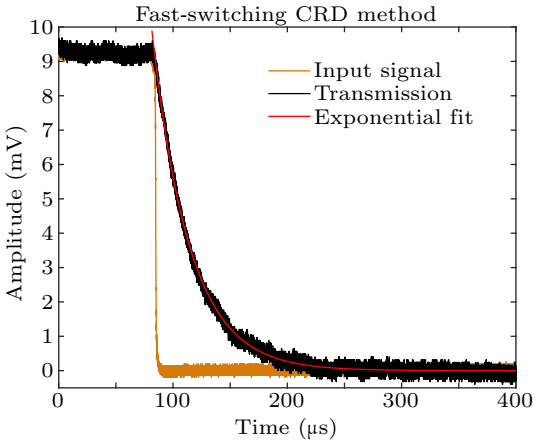


Fig. 4. Measurement of decay time by fast switching, showing the incident signal (brown curve), the cavity transmission (black curve), and the exponential fit (red curve). The measured decay time is (33.01 ± 0.22) μs and the cavity finesse is 16342 ± 109.

In the rapidly SF CRD method, the laser is scanned linearly across the cavity resonance instead of being shut off. As the laser frequency is rapidly swept, the optical power accumulates in the cavity and then undergoes ring-down decay as the frequency traverses the cavity's optical bandpass. The shape of the cavity decay curve is influenced by the scanning speed: slower scanning speeds result in more pronounced oscillations in the curve, while faster speeds lead to a smoother and quasi-exponential decay. The decay time can be extracted directly by fitting a single exponential function to the latter part of the light transmission curve. The starting point of the fit is the location of the curve that is relatively smooth and the best exponential fit. Figure 5 is fitted to obtain a decay time of (35.17 ± 0.78) μs and a finesse of 17412 ± 386.

Finally, the finesse is obtained using the ringing effect method. The peak I_1/I_2 of the oscillation curve in the above equation is linearly related to the time difference Δt, and the slope can uniquely determine the finesse of the cavity. Therefore, one can obtain finesse by varying the PZT sweep speed and measuring the decay signal at different sweep speeds. It is important to note that this method does not require prior knowledge of the sweep speed. Instead, the method involves recording several profiles, such as the one shown in Fig. 6, and observing the evolution of I_1/I_2 versus Δt. Only the cavity length L needs to be measured. The experimental fit yields a finesse of 18182 ± 517.

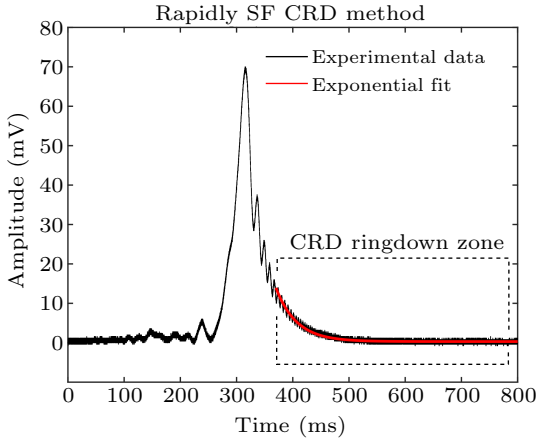


Fig. 5. Measurement of decay time using the rapidly SF CRD method. The cavity transmission (black curve) and the exponential fit (red curve). The box in the figure designates a zone that is used experimentally for CRD fitting purposes. The measured decay time is (35.17 ± 0.78) μs and the cavity finesse is 17412 ± 386.

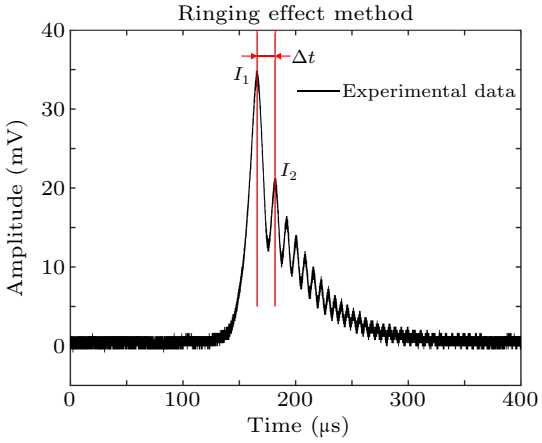


Fig. 6. Measurement of finesse by the ringing effect. The cavity transmission (black curve) and the auxiliary line (red curve). I_1 and I_2 are the intensities of the first and second peaks, and Δt is the time difference between the two peaks. The cavity finesse is calculated as 18182 ± 517.

5. Discussion

The following table summarizes the theoretical finesse of the OEC and the values obtained by each method in this study.

Table 1. Values of finesse measurement.

Method	Value
Theoretical estimation	16483
The FSR modulation method	16518 ± 103
The fast-switching CRD method	16342 ± 109
The rapidly SF CRD method	17412 ± 386
The ringing effect method	18182 ± 517

The FSR modulation method is considered to be reliable since it enables accurate scanning of the Airy function of the resonance by utilizing phase modulation around the FSR. One of the advantages of the FSR modulation method is that the detection equipment is simple, unlike the other three methods which require fast data acquisition with equipment whose response time is significantly shorter than the decay time. It

determines the modulation frequency with radio frequency accuracy and the absolute frequency instability of the FSR is typically lower than that of the cavity mode. As a result, this method provides greater measurement stability than direct scanning frequency measurements. In addition, this method can obtain comprehensive parameters of the optical cavity without additional experiments, including finesse, linewidth, FSR, mode-matching efficiency, mirror transmission, and so on. However, this method requires stable locking of the optical cavity to the laser frequency, which is challenging with a wide laser linewidth. To obtain a sufficient signal-to-noise ratio, this method requires the use of modulation devices and a slow scanning of the modulation frequency. The choice of sweeping frequency and modulation depth can impact fitting accuracy, and non-perfect locking and environmental noise can also reduce the accuracy of finesse and linewidth measurements. The FSR modulation method is suitable for measuring a wide range of finesse and is mainly limited by the available range of phase modulation devices. This method can even be performed when the frequency instability of the optical cavity or laser exceeds the cavity linewidth. A measurement precision of 5% has been obtained using a laser with a frequency instability twice as large as the linewidth itself.[16]

The CRD method measures the decay time instead of the light intensity, which makes it independent of laser linewidth, light intensity fluctuations, mechanical oscillations, and environmental disturbances. The decay time fluctuations mainly depend on optical noise in the decay waveform, electronic detection noise, and variations in experimental conditions.[33] In principle, a higher cavity finesse allows for a longer and more easily measurable decay time, which mitigates the effects of light shut off. Despite its common use for measuring high-finesse cavities, the CRD method still has several drawbacks. First, the FSR cannot be obtained directly during the measurement process. As a result, another measurement method is required to obtain FSR or cavity length, which in turn limits the accuracy of the finesse measurement due to the accuracy of the additional measurement. Second, when the finesse of the cavity is low, the decay time may be on the same order of magnitude as the response time of the measurement instrument, such as hundreds of nanoseconds, resulting in a significant decrease in measurement accuracy. Finally, different detector responses can also lead to different measurement results, which can affect the accuracy and reproducibility of the measurements.

For different interruption schemes, the fast-switching CRD method directly observes the loss of cavities and yields single-exponential decay without additional intervention, which makes the method more straightforward and less susceptible to additional sources of noise or error. Meanwhile, the rapidly SF CRD method has several advantages, including its simplicity, high speed, and low cost. It does not require an expensive AOM but the obtained signal has asymmetric oscillatory or ringing properties, resulting in a lower theoretical signal-to-noise ratio and measurement sensitivity than the fast-switching CRD method.[34] Moreover, the rapidly SF CRD method requires a very fast scanning speed, and PZT introduces unknown noise into the measurement, which is not considered in our experiment.

The ringing effect method does not require an extremely fast scanning speed like the rapidly SF CRD method, which makes it more convenient to measure finesse values in the intermediate range. However, the ringing effect method is subject to limitations imposed by the semianalytical model, resulting in a reduced space of available signals and low reproducibility. In addition, it is also susceptible to noise from the PZT, like the rapidly SF CRD method.

Based on the above analysis, both the fast-switching CRD method and the FSR modulation method are considered to be reliable in our experiments. In contrast, the rapidly SF CRD method's measurement results are less reliable due to unknown noise and inadequate scanning speed, resulting in larger results. Finally, the ringing effect method is considered the least reliable in our conditions due to its low reproducibility, the noise issue with the PZT, and the limitations of its semianalytical model.

To summarize, the four methods that we have discussed can all be used to measure the finesse of the OEC. In terms of finesse magnitude, the FSR modulation method applies to a wide range of values, whereas the CRD method is better suited for measuring high-finesse optical cavities, and the ringing effect method is appropriate for moderate finesse values. With respect to the experimental setup, the FSR modulation method does not necessitate a fast-response detection device or additional measurements, but it does require a cavity locking system to be in place. The fast-switching CRD method can be performed in either a closed- or open-loop configuration. Meanwhile, the rapidly SF CRD method and the ringing effect method are conducted in an open loop with simple equipment, which makes them cost-effective and easy to implement without requiring expensive modulation devices. Of the four methods, the FSR modulation method and the fast-switching CRD method exhibit superior measurement accuracy.

6. Conclusion

In this paper, a new variant method for finesse measurement, the FSR modulation method, is proposed and studied in comparison with three other methods, i.e., the fast-switching CRD method, the rapidly SF CRD, and the ringing effect method. The measurement principles and the effects of systematic errors in the four methods are studied in detail. A high-power four-mirror OEC system injected by a single-frequency

CW laser is demonstrated, and four different finesse measurement methods are tested and optimized, producing similar results for each method. The final finesse measurement obtained is approximately 16000. Using this value and the measured cavity FSR, the cavity linewidth and intracavity loss are calculated to be 4.92 kHz and 393 ppm, respectively. It should be noted that the difference between the laser incidence angle and the nominal angles of the mirror coatings, as well as mirror defects and dust, could potentially alter the finesse values during measurement. Although all four methods are found to be valid, the results suggest that the FSR modulation method and the fast-switching CRD method are more suitable and accurate than the other two methods for high-finesse OEC measurements. However, the CRD method and the ringing effect method can be implemented in open loop at a lower cost. Finally, the benefits, drawbacks, and applicability of each method are discussed, along with recommendations for selecting a finesse measurement method. This work provides guidance for future OEC finesse measurements and research.

Acknowledgment

Project supported by National Key Research and Development Program of China (Grant No. 2022YFA1603403).

References

[1] Porat G, Heyl C M, Schoun S B, Benko C, Dorre N, Corwin K L and Ye J 2018 *Nat. Photonics* **12** 387
[2] Zhang J, Hua L Q, Yu S G, Chen Z and Liu X J 2014 *Chin. Phys. B* **23** 050502
[3] Pupeza I, Zhang C, Hogner M and Ye J 2021 *Nat. Photonics* **15** 175
[4] Zhou Y, Zhao G, Liu J L, Yan X, Li Z, Ma W and Jia S 2022 *Chin. Phys. B* **31** 064206
[5] Aspelmeyer M, Kippenberg T J and Marquardt F 2014 *Rev. Mod. Phys.* **86** 1391
[6] Yang P F, He H, Wang Z H, Han X, Li G, Zhang P F and Zhang T C 2019 *Chin. Phys. B* **28** 043701
[7] Huang Z and Ruth R D 1998 *Phys. Rev. Lett.* **80** 976
[8] Zomer F, Fedala Y, Pavloff N, Soskov V and Variola A 2009 *Appl. Opt.* **48** 6651
[9] Dupraz K, Alkadi M, Alves M, *et al.* 2020 *Phys. Open* **5** 100051
[10] Aasi J, Abbott B P, Abbott R, *et al.* 2015 *Class. Quantum Gravity* **32** 074001
[11] Deng X, Chao A, Feikes J, Hoehl A, Huang W, Klein R, Kruschinski A, Li J, Matveenko A, Petenev Y, Ries M, Tang C and Yan L 2021 *Nature* **590** 576
[12] Liu X, Lu X, Wang H, Yan L, Li R, Huang W, Tang C, Chiche R and Zomer F 2023 *Chin. Phys. B* **32** 034206
[13] Richard J P and Hamilton J J 1991 *Appl. Opt.* **30** 3560
[14] Uehara N and Ueda K 1995 *Appl. Phys. B Laser Opt.* **61** 9
[15] Lawrence M J, Willke B, Husman M E, Gustafson E K and Byer R L 1999 *J. Opt. Soc. Am. B* **16** 523
[16] Locke C R, Stuart D, Ivanov E N and Luiten A N 2009 *Opt. Express* **17** 21935
[17] Kuntz K B, Wheatley T A, Song H, Webb J G, Mabrok M A, Huntington E H and Yonezawa H 2017 *Opt. Express* **25** 573
[18] Morville J, Romanini D, Chenevier M and Kachanov A 2002 *Appl. Opt* **41** 6980
[19] O'Keefe A and Deacon D A G 1988 *Rev. Sci. Instrum.* **59** 2544
[20] Rempe G, Thompson R J, Kimble H J and Lalezari R 1992 *Opt. Lett.* **17** 3
[21] Orr B J and He Y 2011 *Chem. Phys. Lett.* **512** 1
[22] Poirson J, Bretenaker F, Vallet M and Floch A L 1997 *JOSA B* **14** 2811
[23] Tan S, Berceau P, Saraf S and Lipa J A 2017 *Opt. Express* **25** 7645
[24] He Y, Lu D and Zhao L 2019 *Opt. Express* **27** 17876
[25] Galzerano G, Suerra E, Giannotti D, Canella F, Vicentini E and Cialdi S 2020 *IEEE Trans. Instrum. Meas.* **69** 9119
[26] Suter M and Dietiker P 2014 *Appl. Opt.* **53** 7004
[27] Ismail N, Kores C C, Geskus D and Pollnau M 2016 *Opt. Express* **24** 16366
[28] Leeuwen N J van, Diettrich J C and Wilson A C 2003 *Appl. Opt.* **42** 3670
[29] An K, Yang C, Dasari R R and Feld M S 1995 *Opt. Lett.* **20** 1068
[30] He Y and Orr B J 2000 *Chem. Phys. Lett.* **319** 131
[31] Matone L, Barsuglia M, Bondu F, Cavalier F, Heitmann H and Man N 2000 *Phys. Lett. A* **271** 314
[32] Drever R W P, Hall J L, Kowalski F V, Hough J, Ford G M, Munley A J and Ward H 2022 *Appl. Phys. B* **31** 97
[33] Paldus B A, Harb C C, Spence T G, Wilke B, Xie J, Harris J S and Zare R N 1998 *J. Appl. Phys.* **83** 3991
[34] Huang H and Lehmann K K 2011 *J. Phys. Chem. A* **115** 9411