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Inertial effect on minimum magnetic field for magnetization reversal in ultrafast magnetism

Xue-Meng Nan(南雪萌)[1], Chuan Qu(屈川)[1], Peng-Bin He(贺鹏斌)[2], and Zai-Dong Li(李再东)[1,†]

[1] *Tianjin Key Laboratory of Quantum Optics and Intelligent Photonics, School of Science, Tianjin University of Technology, Tianjin* 300384, *China*
[2] *School of Physics and Electronics, Hunan University, Changsha* 410082, *China*

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In the field of ultrafast magnetism, i.e., subpicosecond or femtosecond time scales, the dynamics of magnetization can be described by the inertial Landau–Lifhitz–Gilbert equation. In terms of this equation, the intrinsic characteristics are investigated in detail for the theoretical limit of the magnetization reversal field. We can find that there is a critical value for the inertia parameter τ_c, which is affected by the damping and anisotropy parameter of the system. When the inertial parameter factor $\tau < \tau_c$, the limit value of the magnetization reversal field under the ultrafast magnetic mechanism is smaller than that of the fast magnetic mechanism. When $\tau > \tau_c$, the limit value of the magnetization reversal field will be larger than the limit value under the fast magnetic mechanism. Moreover, it is important to point out that the limit value of the magnetization reversal field under the ultrafast magnetic mechanism decreases with the increasing inertial factor, as $\tau < \tau_c/2$, which increases with inertial factor τ as $\tau > \tau_c/2$. Finally, with the joint action of damping and anisotropy, compared with fast magnetism, we find that the limit value of the magnetization reversal field has rich variation characteristics, i.e., there is not only a linear and proportional relationship, but also an inverse relationship, which is very significant for the study of ultrafast magnetism.

Keywords: inertial effect, minimum magnetic field, ultrafast magnetism

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1. Introduction

Understanding how magnetization moves and how it is manipulated on femtosecond time scales is of great significance for ultrafast and efficient data processing and storage applications. The magnetization dynamics in the writing process is described by the Landau–Lifhitz–Gilbert (LLG) equation, which correctly simulates the magnetization reversal on the nanosecond time scale.[1,2] The study of these fast magnetization reversals has great potential applications to the future development of high-speed information industry.[3] Until 20 years ago, it was believed that all relevant magnetization dynamics were included in this equation, and the optimization of storage devices was based on it alone. In recent years, the study of magnetization dynamics in the nanoscale systems has become very important. Hot topics include current-induced domain wall motion and the demagnetization effect of femtosecond laser pulses.[4–7]

However, the pioneering experiment of Bigot *et al.* in 1996 revealed the occurrence of spin dynamics on subpicosecond scales,[1,8] and Ciornai and Fähnle[9,10] *et al.*, and other theorists[4] also pointed out that in the magnetization dynamics of very short time scale, the lack of inertial is questionable.[11] Since then, many studies have also begun to focus on the inertial effect in the dynamics of ultrafast magnetization,[12–14] which was revealed by the ultrafast optical techniques[15] in 2016. The existence of the inertial effect will give birth to the field of ultrafast magnetism, which could not be described by the LLG equation. The LLG equation only describes precession and relaxation, but does not include nutation.[4] Recently, the LLG equation was reformulated to include a term to obtain a physically correct inertial response that does not exist in the original formula.[16–22] With the discovery of ultrafast magnetism,[23] people's interest in ultrafast magnetization on femtosecond scale has greatly increased.

In the future, the rapid growth of information will require rapid processing of information, which in principle depends on the ultrafast reversal of magnetization. The magnetization reversal is how to change the magnetization from one state to another, i.e., how to turn the magnetic moment from bit "1" to bit "0" (here, the bit "1" represents that the magnetic moment is upward and the bit "0" represents that the magnetic moment is down). The driven reversal force can be a laser, a spin-polarized current, or a magnetic field. Many reversal schemes have been proposed and tested in fast magnetism.[24–29] Also, for the issue of a minimal reversal field the classical result is the famous Stoner–Wohlfarth limit.[30] However, in ultrafast magnetism, the research on the magnetization reversal field has not been well explored. In this paper, the inertial LLG equation is analyzed in detail, and simulated numerically by the fourth-order Runge–Kutta algorithm. The limit of the minimum field for the magnetization reversal under inertial effect is obtained[16,17] by a tedious calculation. Compared with the magnetization reversal field in fast magnetism, a smaller value

†Corresponding author. E-mail: lizd@email.tjut.edu.cn

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of reversal field has the more advantages in ultrafast magnetism owing to the inertial effect.

2. Effective field form of the magnetic inertial effect

Until recently, with the discovery of ultrafast magnetization dynamics, the inertial effect has been added into the dimensionless LLG equation,[31–35] which takes the form

$$\dot{\boldsymbol{m}} = -\boldsymbol{m} \times \boldsymbol{h}_{\text{eff}} + \alpha \boldsymbol{m} \times (\dot{\boldsymbol{m}} + \tau \ddot{\boldsymbol{m}}), \tag{1}$$

where $\boldsymbol{m}$ is the normalized magnetization, $\boldsymbol{m} \equiv \boldsymbol{M}/M_{\text{s}}$, $\boldsymbol{M}$ is the localized magnetization, and M_{s} is the saturation magnetization. In Eq. (1), $\dot{\boldsymbol{m}}$ represents the derivative with respect to time, and the time is measured in units γM_{s} with γ being the gyromagnetic ratio. The effective field $\boldsymbol{h}_{\text{eff}}$ includes both external and internal fields, and its form is given below. The parameter α is the Gilbert damping factor. The last term on the right side in Eq. (1) denotes the inertial effect, in which τ is the angular momentum relaxation time,[12,36] and we found that in Refs. [9,33] the approximate estimation result of the theoretically derived inertial parameter τ is about 10 fs–100 fs. In Ref. [1], the fundamental value of τ collected experimentally was extracted from the resonance linewidth by approximately 10 ps. In Ref. [37], the estimated time-resolved magneto-optical measurements of Co thin films τ value is approximately 1 ps. It is obvious that the first term on the right side of Eq. (1) denotes the magnetization precession and the second term represents the damping effect of precession. From Eq. (1) we have $\ddot{\boldsymbol{m}} = -(1/\alpha\tau)(\boldsymbol{m} \times \dot{\boldsymbol{m}} + \boldsymbol{m} \times (\boldsymbol{m} \times \boldsymbol{h}_{\text{eff}})) - \alpha \dot{\boldsymbol{m}}/\tau - |\dot{\boldsymbol{m}}|^2 \boldsymbol{m}$. It is to say that under the action of the inertial effect, i.e., the second derivative of magnetization $\ddot{\boldsymbol{m}}$ on the femtosecond time scale, this will include the trend toward the direction of $\boldsymbol{m}$. That is exactly what causes magnetization nutation. In this short time scale, the precession motion of magnetization is superimposed with the rotating circuit, and the inertial effect becomes very important. For ultrafast switching, the state space of the possible paths is different from the one in the absence of inertial effect — it will produce nutation. It is pointed out that the existence of inertia-driving magnetization dynamics opens up a way to surpass the limit of precession for ultrafast magnetic switch.[38,39] By iterating Eq. (1) once, we have

$$\left(1+\alpha^2\right) \dot{\boldsymbol{m}} = -\boldsymbol{m} \times \boldsymbol{h}'_{\text{eff}} - \alpha \boldsymbol{m} \times \left(\boldsymbol{m} \times \boldsymbol{h}'_{\text{eff}}\right), \tag{2}$$

where $\boldsymbol{h}'_{\text{eff}} = \boldsymbol{h}_{\text{eff}} - \alpha\tau\ddot{\boldsymbol{m}}$. Equation (2) clearly shows that the inertia effect not only affects the precession of magnetization, but also acts as an effective damping. It also implies that the motion of magnetization can be driven by the inertial effect in some special conditions.

3. Inertial effect on the magnetization reversal field

For simplicity, we consider only the uniaxial anisotropy model, in which the z axis is selected as the easy axis. Then, the effective field has the expression as follows, $\boldsymbol{h}_{\text{eff}} = \boldsymbol{h} + h_{\text{i}}\boldsymbol{e}_z$, where $\boldsymbol{h}$ is the applied external field, $h_{\text{i}} = km_z$ denotes the internal field due to the magnetic anisotropy. In the spherical coordinates $\{\boldsymbol{e}_r, \boldsymbol{e}_\theta, \boldsymbol{e}_\varphi\}$, i.e., $\boldsymbol{m} = (1,0,0)$, we can simplify Eq. (2) into the form

$$\begin{aligned} \left(1+\alpha^2\right)\dot{\theta} &= h'_\varphi + \alpha h'_\theta - \alpha k \cos\theta \sin\theta, \\ \left(1+\alpha^2\right)\sin\theta\,\dot{\varphi} &= \alpha h'_\varphi - h'_\theta + k\cos\theta\sin\theta, \end{aligned} \tag{3}$$

where $h'_\theta = h_\theta - \alpha\tau\ddot{\theta} + \alpha\tau\dot{\varphi}^2\sin\theta\cos\theta$, $h'_\varphi = h_\varphi - 2\alpha\tau\dot{\theta}\dot{\varphi}\cos\theta - \alpha\tau\ddot{\varphi}\sin\theta$, with $\boldsymbol{h} \equiv \left(h_r, h_\theta, h_\varphi\right)$, in which h_r, h_θ and h_φ is the field components along $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$ and $\boldsymbol{e}_\varphi$, respectively. The above results show that h'_θ and h'_φ can be considered as an effective magnetic field, including the correction caused by the inertial effect. Using Eq. (3), it is obtained that $h'_\theta = \alpha\dot{\theta} - \sin\theta\dot{\varphi} + k\cos\theta\sin\theta$, $h'_\varphi = \dot{\theta} + \alpha\sin\theta\dot{\varphi}$. Let $g \equiv h'^2_\theta + h'^2_\varphi = \left(1+\alpha^2\right)\dot{\theta}^2 + 2\alpha k\cos\theta\sin\theta\dot{\theta} + (\alpha\sin\theta\dot{\varphi})^2 + \sin^2\theta\left(\dot{\varphi} - k\cos\theta\right)^2$, here g does not depend explicitly on φ for a uniaxial model. According to Ref. [24], we adopt mathematical methods to find extreme values $\partial g/\partial\dot{\varphi} = 0$, $\partial^2 g/\partial\dot{\varphi}^2 > 0$, where it can be obtained that $\dot{\varphi} = (k/\alpha_1)\cos\theta$ with $\alpha_1 \equiv 1+\alpha^2$. Under such a reversal trajectory we find that $h'_\theta = \alpha h'_\varphi$, with which we can simplify Eq. (3) as the form

$$h_\theta = \alpha\dot{\theta} + \frac{\alpha^2 k}{2\alpha_1}\sin 2\theta + \alpha\tau\ddot{\theta} - \frac{\alpha}{2}\tau\dot{\varphi}^2\sin 2\theta, \tag{4}$$

$$h_\varphi = \dot{\theta} + \frac{\alpha k}{2\alpha_1}\sin 2\theta + \alpha\tau\ddot{\varphi}\sin\theta + 2\alpha\tau\dot{\theta}\dot{\varphi}\cos\theta. \tag{5}$$

In the following, we need to consider what kind of time-dependent magnetic field is added, so that the magnetization can be easily reversed, and the smaller the applied magnetic field, the better. It is obvious that the h_r does not appear in Eq. (3), and one can reduce the switching field by always putting $h_r = 0$, and then $h^2 = h_\theta^2 + h_\varphi^2$. We also know that when the magnetic moment reverses, $0 \le \theta \le \pi$, i.e., $\dot{\theta} \ge 0$. In order to obtain a smaller magnetic moment reversal field, we put $\dot{\theta} = 0$, and from Eqs. (4) and (5) obtain $h = (\alpha k/(2\alpha_1))\sin 2\theta\sqrt{1 + (\alpha - (\tau k/\alpha_1)\cos^2\theta)^2}$.

By using $\partial h/\partial\theta = 0$, $\partial^2 h/\partial\theta^2 < 0$, we can obtain $\theta = \pi/4$, which implies the theoretical limit value of the minimum external magnetization reversal field (the maximum value of the minimum applied magnetization reversal field in the trajectory of magnetic moment changing from bit "1" to "0") under the ultrafast magnetization mechanism is determined by

$$h_{\text{c}} = \frac{k\alpha}{2\alpha_1}\sqrt{1 + \left[1 - \tau k/(2\alpha\alpha_1)\right]^2\alpha^2}. \tag{6}$$

The above equation clearly clarifies how the inertial parameter τ determines the theoretical limit value of the magnetization reversal field with the different damping α, and anisotropy k.

In Eq. (6), we can see that when $\tau = 0$, which is the influence of inertia effect is not considered, the results are consistent with that of the mechanism of fast magnetization dynamics in Ref. [24], $h_{1c} = k\alpha/(2\sqrt{\alpha_1})$, which has a linear relationship with anisotropy k and is directly proportional to α. Also, when α is very large, whether it is ultrafast magnetism or fast magnetism, h_c gradually tend to the same value and this value is close to the result of Stoner–Wohlfarth.[30] Theoretical analysis of Eq. (6), we can know that under the influence of the inertia effect, the relationship between h_c and the anisotropic parameter k is no longer a simple linear relationship, and the relationship with α becomes more complex. From the simple analysis, we can see that compared with the fast magnetization dynamics, the theoretical limit value of the magnetization reversal field in ultrafast magnetism has changed significantly. So does this change make sense, that is can we get a better magnetization reversal field in ultrafast magnetism than that of fast magnetism?

From the analytical results of Eq. (6), we can find that there is a critical value for the inertia parameter, i.e., when the value of the external magnetic field generated is equal to the theoretical limit value of the magnetization reversal magnetic field under the fast magnetic mechanism, the corresponding angular momentum relaxation time

$$\tau_c = 4\frac{\alpha\alpha_1}{k}, \tag{7}$$

which is affected by the damping α and anisotropy parameter k of the system. When the inertial parameter factor τ is greater than this critical value, i.e., $\tau > \tau_c$, the theoretical limit value of the magnetization reversal field under the ultrafast magnetic mechanism is greater than that of the fast magnetic mechanism, i.e., $h_c > h_{1c}$. When the inertial parameter factor is less than this critical value, i.e., $\tau < \tau_c$, we have $h_c < h_{1c}$. This result entirely confirms that, in ultrafast magnetism, to get a smaller magnetic moment switching field, it is not that greater τ is better. What is interesting is how the theoretical limit value of the magnetization reversal field changes with the increase of the inertial factor under the ultrafast magnetic mechanism. In order to reveal this phenomenon more accurately, we take the derivative of h_c against the inertia factor τ, i.e., $\partial h_c/\partial\tau = 0$. Through detailed calculation, we find that there is another critical value, i.e., $\tau = \tau_c/2$. The inertial effect on the theoretical limit value of the magnetization reversal field is illustrated in Fig. 1. From Fig. 1 we observe that the theoretical limit value of the magnetization reversal field under the ultrafast magnetic mechanism decreases with the increasing inertial factor, as the inertial factor is less than the critical value, i.e., $\tau < \tau_c/2$. However, when the inertial factor is greater than the critical value, i.e., $\tau > \tau_c/2$, the situation is just the opposite. Also, when $\tau = \tau_c/2$, we get the smallest limit value of magnetization reversal field under the ultrafast magnetic mechanism

$$h_{c\,\min} = \frac{k\alpha}{2\alpha_1}, \tag{8}$$

this equation indicates that when $\tau = \tau_c/2$, the limit value h_c of the magnetization inversion field is only related to Gilbert damping and anisotropy, which is illustrated in Fig. 2. From Fig. 2 we find that the smallest magnetization reversal field $h_{c\,\min}$ has a linear relationship with anisotropy k and $h_{c\,\min}$ is directly proportional to α firstly and then inversely proportional.

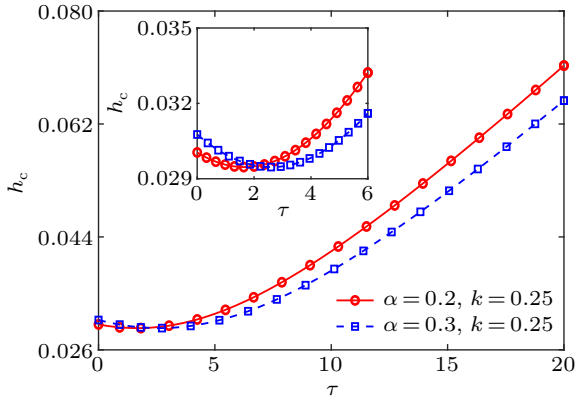


Fig. 1. The variation of the limit value of the magnetization reversal field h_c depends on the inertia parameter τ. The parameters are $\alpha = 0.2$, 0.3, and $k = 0.25$, respectively. Inset: h_c vs. τ in a small range.

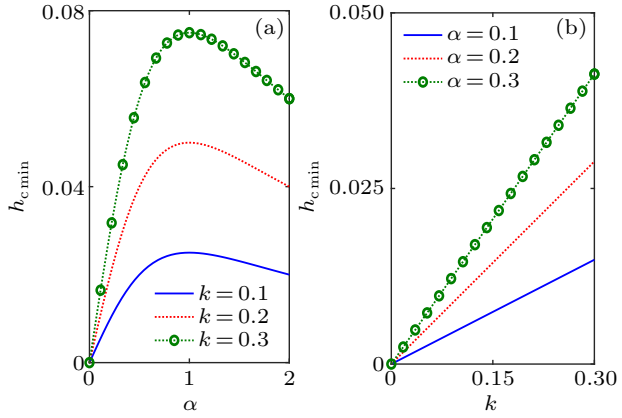


Fig. 2. The smallest limit value of the magnetization reversal field $h_{c\,\min}$ changes with damping α and the anisotropy k, in which (a) $k = 0.1$, 0.2, 0.3, and (b) $\alpha = 0.1$, 0.2, 0.3, respectively.

We also find that the limit value of the magnetization reversal field h_c and the inertia factor τ are in a quadratic parabolic relationship under the condition $|\tau - \tau_c/2| \ll 2\alpha_1/k$ with the help of Eq. (6). However, a larger reversal field will be generated and the limit value of the magnetization reversal field h_c increases linearly with the inertial factor τ under the the condition $\tau \gg \tau_c/2 + 2\alpha_1/k$. It is very similar to the case that the limit value of the magnetization reversal field h_c increases linearly with the anisotropy k with the absence of inertial effect $\tau = 0$. This result shows that under the ultrafast

magnetic mechanism, the inertial effect profoundly affects the limit value of the magnetization reversal field. As can be seen from Eq. (2), the inertial effect can be considered as part of the modified effective field, which will affect the magnitude of the critical magnetic field value. Therefore, the relationship between the reversal field and the inertial parameter is non-monotonic. In the experiment, the minimum magnetic field can be driven to flip the magnetization by adjusting the inertia effect under different materials and damping, which has great energy-saving significance in the field of information processing.

From Eq. (6) we see that the limit value of the magnetization reversal field increases linearly with the anisotropy k in the fast magnetic mechanism, i.e., $\tau = 0$. However, under the ultrafast magnetic mechanism, the dependence of limit value of the magnetization reversal field on the anisotropy of the system material has some characteristics, as shown in Fig. 3. The critical value τ_c changes with the anisotropy k is plotted in Fig. 3(a). Also, in this ultrafast magnetic mechanism it satisfies that $h_c > h_{1c}$, as $\tau > \tau_c$, and $h_c < h_{1c}$, as $\tau < \tau_c$, such as the four lines above and below the blue solid line in Fig. 3(b). With the help of Eq. (6), we find that when the inertial parameter factor $\tau \ll \tau_c$, the magnetization reversal field h_c increases almost linearly with the anisotropy k of the system. It is amazing that when the inertial parameter factor is far from this critical inertia value, i.e., $\tau \gg \tau_c$, the magnetization reversal field h_c almost increases quadratically with the anisotropy k.

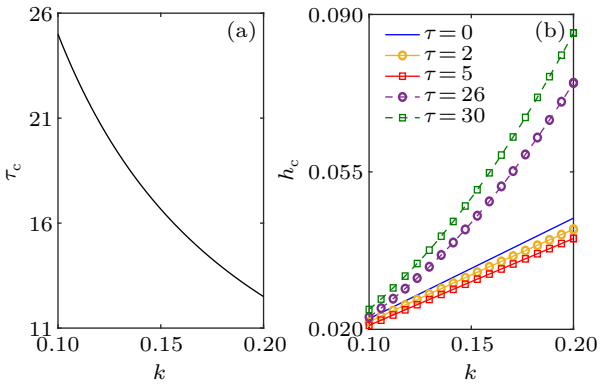


Fig. 3. The graphical representation of the limit value of the magnetization reversal field h_c with the anisotropy k. The parameters are $\alpha = 0.5$, (a) the variation of critical inertial value τ_c changes with the anisotropy k, (b) the inertial parameter factor is less, i.e., $\tau = 2$, 5, and is larger, i.e., $\tau = 26$, 30, respectively.

Under the influence of different inertial parameter factors τ, the effect of damping on the limit value of magnetization reversal field is different. In order to fully observe the relationship between the magnetic moment reversal field and damping, we have adopted a wide range of damping parameter values because the damping value is theoretically unlimited. For comparison with the results in Ref. [24], we chose the larger α, as shown in Fig. 4. The critical value τ_c changes with the damping factor α, as plotted in Fig. 4(a). It can still be proved that when the inertial parameter factor is τ less than critical value τ_c, the limit value of the magnetization reversal field under the ultrafast magnetic mechanism is less than that of the fast magnetic mechanism, i.e., $h_c < h_{1c}$, while when the inertial parameter factor is greater than the critical value τ_c, we get $h_c > h_{1c}$. These results are shown by the four lines above and below the blue solid line in Fig. 4(b), in which the blue solid line denotes the case of τ_c. We also found when the inertial parameter factor is smaller, the magnetization reversal field h_c increases almost proportional with the damping factor α. However, when the inertial parameter is larger, the h_c increases first, then decreases, and then increases finally. Both cases eventually increase until it is close to the result of the Stoner–Wohlfarth.[3]

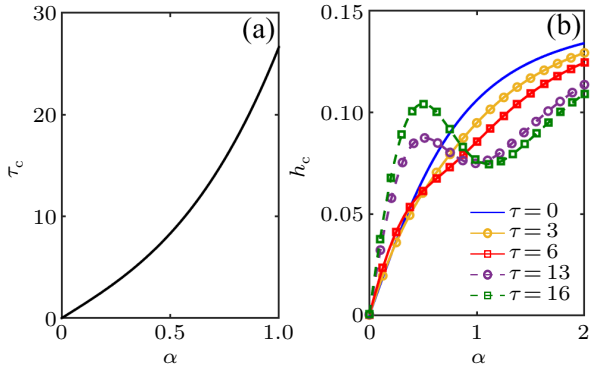


Fig. 4. The illustration of the limit value of the magnetization reversal field h_c with the damping α (theoretically, α can take a larger value). The parameters are $k = 0.3$. (a) The variation of critical inertial value τ_c changes with damping α. (b) The inertial factor τ is less, i.e., $\tau = 3$, 6, and is larger, i.e., $\tau = 13$, 16, respectively.

4. Conclusion

In summary, we investigate the magnetization reversal field for the ultrafast (femtosecond time) processes, which is described by the inertial LLG equation. The rich properties of the limit value of the magnetization reversal field are discussed in detail. The most important findings are that the inertial effect greatly affects the magnetization reversal field. In the ultrafast magnetism, there is a critical value τ_c for the inertia parameter, which determines that the limit value of the magnetization reversal field, when $\tau < \tau_c$, the minimum magnetic field theoretical limit value less than the fast magnetic mechanism will be generated. When $\tau = \tau_c/2$, we get the smallest limit value of the magnetization reversal field under the ultrafast magnetic mechanism. We expect that these results will have potential energy-saving significance in the field of information processing in the future.

Appendix A: Magnetization dynamics under inertial effect

By iterating Eq. (1) once, we have

$$\dot{\boldsymbol{m}} = -\boldsymbol{m} \times \boldsymbol{h}_{\text{eff}} + \alpha \boldsymbol{m} \times (\dot{\boldsymbol{m}} + \tau \ddot{\boldsymbol{m}})$$

$$= -\boldsymbol{m}\times\boldsymbol{h}_{\rm eff}+\alpha\tau\boldsymbol{m}\times\ddot{\boldsymbol{m}}$$
$$+\alpha\boldsymbol{m}\times\left(-\boldsymbol{m}\times\boldsymbol{h}_{\rm eff}+\alpha\boldsymbol{m}\times\left(\dot{\boldsymbol{m}}+\tau\ddot{\boldsymbol{m}}\right)\right). \quad \text{(A1)}$$

With the help of the cross multiplication formula of vectors, we obtain

$$\boldsymbol{m}\times\left(\boldsymbol{m}\times\dot{\boldsymbol{m}}\right)=\left(\boldsymbol{m}\cdot\dot{\boldsymbol{m}}\right)\boldsymbol{m}-\left(\boldsymbol{m}\cdot\boldsymbol{m}\right)\dot{\boldsymbol{m}}=-\dot{\boldsymbol{m}}. \quad \text{(A2)}$$

Substituting the above result into Eq. (A1), we get

$$\left(1+\alpha^2\right)\dot{\boldsymbol{m}} = -\boldsymbol{m}\times\left(\boldsymbol{h}_{\rm eff}-\alpha\tau\ddot{\boldsymbol{m}}\right)$$
$$-\alpha\boldsymbol{m}\times\left[\boldsymbol{m}\times\left(\boldsymbol{h}_{\rm eff}-\alpha\tau\ddot{\boldsymbol{m}}\right)\right], \quad \text{(A3)}$$

in which we take a correction $\boldsymbol{h}'_{\rm eff}=\boldsymbol{h}_{\rm eff}-\alpha\tau\ddot{\boldsymbol{m}}$, and equation (2) can be obtained.

Appendix B: Inertial LLG equation in spherical coordinates

In spherical coordinates, the second derivative of the magnetization can be written as

$$\ddot{\boldsymbol{m}} = -\left(\dot{\theta}^2+\dot{\varphi}^2\sin^2\theta\right)\boldsymbol{e}_r+\left(\ddot{\theta}-\dot{\varphi}^2\sin\theta\cos\theta\right)\boldsymbol{e}_\theta$$
$$+\left(2\dot{\theta}\dot{\varphi}\cos\theta+\ddot{\varphi}\sin\theta\right)\boldsymbol{e}_\varphi. \quad \text{(B1)}$$

Substituting this relation into Eq. (1) we obtain

$$\alpha\tau\ddot{\theta}=h_\theta+\dot{\varphi}\sin\theta-\alpha\dot{\theta}+\frac{1}{2}\left(\alpha\tau\dot{\varphi}^2-k\right)\sin 2\theta, \quad \text{(B2)}$$

$$\alpha\tau\sin\theta\ddot{\varphi}=h_\varphi-\dot{\theta}-\alpha\dot{\varphi}\sin\theta-2\alpha\tau\dot{\theta}\dot{\varphi}\cos\theta. \quad \text{(B3)}$$

The equilibrium condition in our simulation is defined as $\theta=\pi/2$, $\varphi=\pi/2$, $\dot{\theta}=0$, $\dot{\varphi}=0$. Then, by using the fourth-order Runge–Kutta algorithm we numerically integrate Eqs. (B2) and (B3). The diagram of the numerical inertial dynamics has been shown in Fig. B1(a), and the schematic diagram in Fig. B1(b) can be used to better understand the inertial magnetization dynamics.

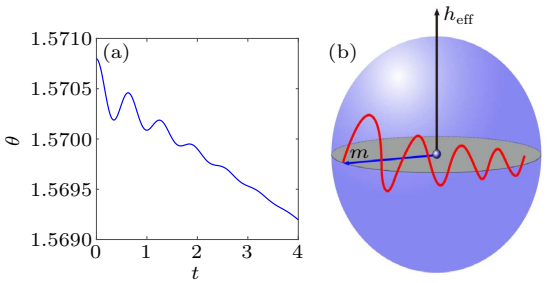


Fig. B1. The diagram of numerical inertial dynamics and schematic diagram. (a) Time dependence of θ for the inertial LLG equation. (b) The red solid dashed curve denotes schematically the magnetization nutation by the superposition effect of Gilbert damping and inertia.

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