

Dynamic decision and its complex dynamics analysis of low-carbon supply chain considering risk-aversion under carbon tax policy

Jin-Chai Lin(林金钗), Ru-Guo Fan(范如国), Yuan-Yuan Wang(王圆缘), and Kang Du(杜康)
Citation: Chin. Phys. B, 2023, 32 (**10**): 100502. DOI: 10.1088/1674-1056/acd2b5
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

A class of two-dimensional rational maps with self-excited and hidden attractors

Li-Ping Zhang(张丽萍), Yang Liu(刘洋), Zhou-Chao Wei(魏周超),Hai-Bo Jiang(姜海波), and Qin-Sheng Bi(毕勤胜)
Chin. Phys. B, 2022, 31 (**3**): 030503. DOI: 10.1088/1674-1056/ac4025

Transient transition behaviors of fractional-order simplest chaotic circuit with bi-stable locally-active memristor and its ARM-based implementation

Zong-Li Yang(杨宗立), Dong Liang(梁栋), Da-Wei Ding(丁大为), Yong-Bing Hu(胡永兵), and Hao Li(李浩)
Chin. Phys. B, 2021, 30 (**12**): 120515. DOI: 10.1088/1674-1056/ac1fdf

A novel class of two-dimensional chaotic maps with infinitely many coexisting attractors

Li-Ping Zhang(张丽萍), Yang Liu(刘洋), Zhou-Chao Wei(魏周超), Hai-Bo Jiang(姜海波), Qin-Sheng Bi(毕勤胜)
Chin. Phys. B, 2020, 29 (**6**): 060501. DOI: 10.1088/1674-1056/ab8626

Complex dynamics of an archetypal self-excited SD oscillator driven by moving belt friction

Zhi-Xin Li(李志新), Qing-Jie Cao(曹庆杰), Léger Alain
Chin. Phys. B, 2016, 25 (**1**): 010502. DOI: 10.1088/1674-1056/25/1/010502

Complex dynamics analysis of impulsively coupled Duffing oscillators with ring structure

Jiang Hai-Bo (姜海波), Zhang Li-Ping (张丽萍), Yu Jian-Jiang (于建江)
Chin. Phys. B, 2015, 24 (**2**): 020502. DOI: 10.1088/1674-1056/24/2/020502

Dynamic decision and its complex dynamics analysis of low-carbon supply chain considering risk-aversion under carbon tax policy

Jin-Chai Lin(林金钗)[1,†], Ru-Guo Fan(范如国)[2], Yuan-Yuan Wang(王圆缘)[2], and Kang Du(杜康)[2]

[1] *Management School, Chongqing University of Technology, Chongqing* 400054, *China*
[2] *Economics and Management School, Wuhan University, Wuhan* 430072, *China*

(Received 21 March 2023; revised manuscript received 23 April 2023; accepted manuscript online 5 May 2023)

This study establishes a low-carbon supply chain game model under the centralized decision situation and the decentralized decision situation considering the manufacturer risk-aversion behavior, and discusses the influence of the manufacturer risk-aversion behavior on the optimal decision, profit, coordination, and complex dynamics of the supply chain. We find that comparing with the risk-neutral decentralized decision, the increase of manufacturer's risk tolerance attitude can narrow the gap between the supply chain profit and the centralized decision, but it will further reduce the carbon emission reduction level. The increase of risk tolerance of the manufacturer and carbon tax will narrow the stable region of the system. Under this situation, the manufacturer should carefully adjust parameters to prevent the system from losing stability, especially the adjustment parameters for carbon emission reduction level. When the system is in a chaotic state, the increase of carbon tax rate makes the system show more complex dynamic characteristics. Under the chaotic state, it is difficult for the manufacturer to make correct price decision and carbon emission reduction strategy for the next period, which damages its profit, but increases the profit of the retailer and the supply chain. Finally, the carbon emission reduction cost-sharing contract is proposed to improve the carbon emission reduction level and the supply chain efficiency, achieving Pareto improvement. The stability region of the system is larger than that in the centralized decision situation, but the increase of the cost sharing coefficient will reduce the stability of the system in the decentralized decision-making situation.

Keywords: carbon tax, risk-avers, low-carbon supply chain, complex dynamics

PACS: 05.45.–a, 05.45.Pq **DOI:** 10.1088/1674-1056/acd2b5

1. Introduction

In recent years, a series of natural disasters caused by global warming has become increasingly prominent, seriously restricting the high-quality development of the global economy. To cope with the global climate change and improve the living environment of mankind, developing low-carbon economy has become the universal consensus of the international community. The carbon emissions of global brands and multinational companies are mainly concentrated in their supply chains, generally accounting for more than 70% of the total carbon emissions of brands, and some even reach 80% or 90%.[1] This poses a great challenge to the living environment, competition rules, cost composition and operation mode of the global supply chain.

Many countries have introduced policies such as carbon tax, carbon quota and carbon trading to regulate the carbon emissions of enterprises. Among them, carbon tax can not only increase government revenue for investment and development of new emission reduction technologies, but also have lower management and operation costs than carbon trading. Scholars have introduced carbon tax into the study of supply chain operation to explore its influence on decision and performance of supply chain.

It is worth noting that the carbon emission reduction decision and pricing decision are management activities mainly conducted by person. In an uncertain environment, the degree of cooperation and related decisions are affected by subjective factors such as people's psychology, emotions, and their cognitive capabilities.[2–4] With the increase of complexity and uncertainty of the market environment, enterprises often show risk aversion when making decisions. For example, after McKinsey surveyed 1500 executives in 90 countries, it is found that managers show a high level of risk aversion regardless of the size of the investment.[5] Therefore, it is necessary to consider the influence of enterprises' risk aversion on carbon emission reduction decision-making, pricing, and profit of the supply chain.

It is not difficult to find that in the current study, scholars have abstracted and simplified the social and economic systems such as the supply chain into deterministic and linear systems. In reality, the evolution process of supply chain system often has complex characteristics such as nonlinearity and uncertainty, owing to heterogeneity and autonomy attributes of social subjects, randomness of social environment and other factors.[6,7] Therefore, using static linear model to analyze supply chain system has certain limitations.

Above all, this study introduces the risk aversion behavior of the manufacturer and constructs the game models of low-carbon supply chain. We explore the influence of the manufacturer's risk aversion on the optimal decision, profit

[†]Corresponding author. E-mail: jc_lin@cqut.edu.cn

© 2023 Chinese Physical Society and IOP Publishing Ltd

and coordination of the supply chain from the static perspective and dynamic evolution perspective, respectively. Then, the operation of the supply chain system under the decentralized decision-making situation and the centralized decision-making situation are compared and analyzed, with the centralized decision-making situation taken as a benchmark. Finally, a carbon emission reduction cost-sharing contract is proposed to improve the carbon emission reduction level and supply chain efficiency, which realizes Pareto improvement.

The rest of this study is organized as follows. In Section 2, the existing reseraches are reviewed. In Section 3, model assumptions and notation are described. In Sections 4 and 5, model construction and analysis are shown in the frame of decentralized decision-making model and centralized decision-making, respectively. In Section 6, cost-sharing contract game model is designed. Conclusions and suggestions are presented in Section 7.

2. Review on existing researches

This paper mainly combs the existing researches from the following three aspects: (i) carbon emission reduction decision and pricing in the supply chain; (ii) risk avoidance in the supply chain; (iii) the application of nonlinear dynamical system theory to the supply chain.

2.1. Carbon emission reduction decision and pricing in supply chain

Considering the influence of consumers' low-carbon preferences on enterprise decision-making is a current research hotspot.[8,9] In addition, the influence of the government's carbon tax on the decision-making of carbon emission reduction in the supply chain is also the focus of attention of academia and enterprises. Wang *et al.*[10] found that the carbon tax policy affects the product pricing and emission reduction strategy of enterprises. Huang and Zhang[11] found that the increase of carbon tax rate can increase the emission reduction level of products in low emission industries, but it reduces the emission reduction level of products in high emission industries. Lu and Yi[12] found that with the increase of carbon tax rate, the phenomenon of "less new products and more remanufactured products" appears.

The above studies on carbon emission reduction in supply chain are all based on the risk neutral assumption, ignoring the risk aversion characteristics of decision makers, and the obtained research results can not accurately provide theoretical reference for real supply chain decision makers.

2.2. Risk avoidance in supply chain

In recent years, scholars have introduced risk aversion into the study of supply chain management to explore its influence on enterprise pricing,[13,14] order quantity,[15] supply chain profit and coordination.[16] Niederhoff and kouvelis[17] proved that the risk aversion behavior of suppliers will invalidate the revenue sharing contract by using the method of behavior laboratory. Chen *et al.*[18] extended to the study of decentralized supply chain coordination with multiple risk averse suppliers and one retailer. Li and Jiang[19] explored the influence of consumer sensitivity to revenue policies on corporate decision-making and revenue under different risk attitudes of the retailer. Bai *et al.*[20] found that the risk aversion behavior of supply chain members enables dominant enterprise to obtain more profit under a fixed carbon quota.

The above-mentioned researches show that risk aversion behavior of enterprises will change the optimal decisions and corporate profits. However, there have been many studies on the influence of the risk aversion behavior of decision-makers on supply chain financing,[21,22] but few researches involve with the influence of low-carbon supply chain decision-making and coordination, which mainly focus on the study of static models.

2.3. Applications of nonlinear dynamical system theory to supply chain

On the construction of low-carbon supply chain game model, the existing literature mainly focuses on static models. However, supply chain management system is a nonlinear system with uncertainty, variability, openness, and dynamics. Xi and Zhang[23] found that the dynamic subsidy of the government to the nitrogen emission reduction behavior of agricultural enterprises will affect the stability of the supply chain system and thus the performance of the supply chain. Tian *et al.*[24] showed that when consumers' channel preference is too strong, it will damage the profits of supply chain members, no matter how hard enterprises try to improve sales efforts. Ma *et al.*[25] studied dynamic complexity of a multi-channel supply chain considering cap-and-trade regulation. Li *et al.*[26] incorporated fairness concern into a dynamic model of the closed-loop supply chain. This literature only studied the pricing decision and took the carbon emission reduction strategy as an exogenous variable. Unlike this literature, the present study takes carbon emission reduction strategy as a decision variable of the manufacturer.

The contributions of this study are as follows. First, from the dynamic evolution perspective, the influence of the manufacturer's risk aversion behavior on the optimal carbon emission reduction decision, profit, and supply chain coordination are analyzed. Second, this study investigates the dynamic evolution characteristics of the system, and the influence of the manufacturer's risk aversion and carbon tax on the system. Finally, the cost-sharing contract of carbon emission reduction investment is proposed to optimize the supply chain system to improve the level of carbon emission reduction and economic

benefits of the supply chain. Unlike the conclusion of traditional supply chain, it is concluded in the present study that the increase of the cost-sharing coefficient will reduce the stability region of the supply chain system.

3. Model assumptions and notation

Under the carbon tax policy, this study takes a two-level supply chain consisting of a single manufacturer and a single retailer as the research object. We only consider the carbon emission of product production, *i.e.*, only the manufacturer is affected by the carbon tax. To improve competitiveness and meet consumers' demand for low-carbon products, the manufacturer needs to invest in carbon emission reduction and sells the products to consumers through the retailers. The manufacturer is the leader of the game and first determines optimal carbon emission reduction effort level e and wholesale price w. The retailer is the follower and makes optimal retail price p maximize its own profit according to the manufacturer's decision.

Assuming that there are uncertain factors in the market demand. According to Ref. [27], the market demand can be expressed by the following function:

$$D = \bar{a} - p + te, \tag{1}$$

where $\bar{a} = a + \varepsilon$, a is the factor determined by the potential demand of the market, ε is the random item, which obeys the normal distribution $\varepsilon \sim N(0, \sigma^2)$, and the distribution information is common knowledge of both sides; t is the low-carbon preference of consumers. Equation (1) indicates that the increasing of the carbon emission reduction level of the low-carbon product can increase the market demand.

Assume that the initial carbon emission of unit product is e_0, and unit carbon tax is set to f, so the manufacturer needs to pay carbon emission tax of unit product $f(e_0 - e)$. Assume that c is unit cost when carbon emission reduction investment is not considered, then $w - c - f(e_0 - e) > 0$ will need satisfying in order to ensure that the manufacturer is profitabl. The cost function of carbon emission reduction is $C = (1/2)ke^2$, which has been widely used,[11,28] where $C' > 0, C'' > 0$, and the parameter k is the cost coefficient related to emission reduction investment, parameters a and k are much larger than other parameters.

According to the above assumptions, the profit function of the retailer, the manufacturer and the supply chain are as follows:

$$\pi_{\mathrm{r}} = (p - w)(a - p + te), \tag{2}$$

$$\pi_{\mathrm{m}} = (w - c - f(e_0 - e))(a - p + te) - ke^2/2, \tag{3}$$

$$\pi_{\mathrm{sc}} = (p - c - f(e_0 - e))(a - p + te) - ke^2/2. \tag{4}$$

When the manufacturer shows risk aversion characteristics, owing to market uncertainty, emission reduction investment, and carbon tax, not only should attention be paid to its own profit, but also to how to avoid risks. As for the utility function of risk aversion, this study uses the mean variance method which is widely used in theory and practice to measure the utility function of decision-makers' risk aversion behavior.[29,30] The manufacturer's expected utility is expressed as $E(U_{\mathrm{m}}) = E(\pi_{\mathrm{m}}) - \mathrm{Var}(\pi_{\mathrm{m}})/2R_{\mathrm{m}}$, where R_{m} is the manufacturer's risk tolerance. According to the mean variance method, the variance of the manufacturer's expected return is $\mathrm{Var}(\pi) = (w - c - f(e_0 - e))^2\sigma^2$. Therefore, the manufacturer's utility function can be expressed as

$$\begin{aligned} E(U_{\mathrm{m}}) \;=\; & (w - c - f(e_0 - e))(a - p + te) \\ & - \frac{1}{2}ke^2 - \frac{(w - c - f(e_0 - e))^2\sigma^2}{2R_{\mathrm{m}}}. \end{aligned} \tag{5}$$

4. Decentralized decision game model of low-carbon supply chain

4.1. Static model

The superscript "R_{m}^*" represents the optimal solution of the manufacturer in the case of risk aversion. Reverse induction method is used to solve the Stackelberg game between the manufacturer and the retailer. Solving the second partial derivative of Eq. (5) with respect to p, we obtain $\partial E^2(\pi_{\mathrm{r}})/\partial p^2 = -2 < 0$. Therefore, π_{r} has a unique optimal solution with respect to p. Letting $\partial \pi_{\mathrm{r}}/\partial p = 0$, the response function of the retailer is calculated from

$$p(w, e) = \frac{a + w + te}{2}. \tag{6}$$

Substituting Eq. (6) into Eq. (5) and calculating the first-partial derivatives of the decision variables w and e, from the resulting equation, we obtain

$$\begin{cases} \dfrac{\partial E(U_{\mathrm{m}})}{\partial w} = \dfrac{(f(e_0 - e) + te + a + c - 2w)R_{\mathrm{m}} + 2(f(e_0 - e) + c - w)\sigma^2}{2R_{\mathrm{m}}}, \\ \dfrac{\partial E(U_{\mathrm{m}})}{\partial e} = \dfrac{-2\sigma^2(e - e_0)f^2 + (((2e - e_0)t + a - w)R_{\mathrm{m}} + 2\sigma^2(c - w))f - ((c - w)t + 2ke)R_{\mathrm{m}}}{2R_{\mathrm{m}}}. \end{cases} \tag{7}$$

When $4k(R_{\mathrm{m}} + \sigma^2) > R_{\mathrm{m}}(f + t)^2$, the Hessian matrix of Eq. (7) is negative-definite, and the utility function of the manufacturer has a unique maximum solution with respect to w and e. Under this constraint, the optimal decision of the manufacturer can be

obtained by solving Eq. (7):

$$e^{R^*} = \frac{R_{\rm m}(f+t)(a-c-fe_0)}{4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2}, \tag{8}$$

$$w^{R^*} = \frac{4k\sigma^2(c+fe_0)+R_{\rm m}(k(2a+2c)-ct^2-(t(a+c+e_0t)-2e_0k)f-(e_0t+a)f^2)}{4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2}. \tag{9}$$

Substituting the above two optimal solutions into Eq. (5), the optimal retail price is as follows:

$$p^{R^*} = \frac{2k\sigma^2(a+c+fe_0)+R_{\rm m}(k(3a+c)-ct^2-(t(a+c+e_0t)-e_0k)f-(e_0t+a)f^2)}{4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2}. \tag{10}$$

Substituting the optimal decisions obtained from Eqs. (8)–(10) into Eqs. (2)–(4), respectively, we obtain

$$\pi_{\rm m}^{R^*} = \frac{kR_{\rm m}(a-c-fe_0)^2(4k(R_{\rm m}+2\sigma^2)-R_{\rm m}(f+t)^2)}{(4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2)^2}, \tag{11}$$

$$\pi_{\rm r}^{R^*} = \frac{k^2(R_{\rm m}+2\sigma^2)^2(a-c-fe_0)^2}{(4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2)^2}, \tag{12}$$

$$\pi_{\rm sc}^{R^*} = \frac{((6k-(f+t)^2)R_{\rm m}^2+16R_{\rm m}k\sigma^2+8k\sigma^4)(a-c-fe_0)^2}{2(4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2)^2}. \tag{13}$$

To show more clearly the influence of risk aversion behavior on the optimal decisions, profits, and dynamic characteristics of the supply chain system, we solve the decentralized decision-making situation of the supply chain game model with the manufacturer's risk neutral. The optimal solution is represented by a superscript "$*$". When $4k > (f+t)^2$, the Hessian matrix of Eq. (5) with respect to the decision variables w and e is negative-definite and the manufacturer's utility function has a unique maximum. The optimal decisions and the profits can be obtained as follows:

$$e^* = \frac{(f+t)(a-c-fe_0)}{4k-(f+t)^2}, \quad w^* = \frac{2k(a+c)-((e_0t+a)f^2+(t^2e_0+(a+c)t-2e_0k)f-ct^2}{4k-(f+t)^2},$$

$$p^* = \frac{k(3a+c)-(e_0t+a)f^2-(t^2e_0+t(a+c)-e_0k)f-ct^2}{4k-(f+t)^2},$$

$$\pi_{\rm m}^* = \frac{k(a-c-fe_0)^2}{8k-2(f+t)^2}, \quad \pi_{\rm r}^* = \frac{k^2(a-c-fe_0)^2}{(4k-(f+t)^2)^2}, \quad \pi_{\rm sc}^* = \frac{k(6k-(f+t)^2)(a-c-fe_0)^2}{2(4k-(f+t)^2)^2}.$$

Proposition 1 $\partial e^{R^*}/\partial R_{\rm m} > 0$, $\partial w^{R^*}/\partial R_{\rm m} > 0$, $\partial p^{R^*}/\partial R_{\rm m} > 0$, $\partial \pi_{\rm m}^{R^*}/\partial R_{\rm m} > 0$, $\partial \pi_{\rm r}^{R^*}/\partial R_{\rm m} < 0$.

Proof The first derivative of Eq. (8) can be obtained to be

$$\frac{\partial e^{R^*}}{\partial R_{\rm m}} = \frac{4k\sigma^2(f+t)(a-c-fe_0)}{(4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2)^2},$$

and we can derive $\partial e^{R^*}/\partial R_{\rm m} > 0$. Similarly, it can be obtained that

$$\frac{\partial w^{R^*}}{\partial R_{\rm m}} = \frac{4k\sigma^2(2k+tf-f^2)(a-c-fe_0)}{(4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2)^2},$$

$$\frac{\partial p^{R^*}}{\partial R_{\rm m}} = \frac{2k\sigma^2(2k+t^2-f^2)(a-c-fe_0)}{(4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2)^2},$$

and according to Eqs. (9) and (10), we can derive $\partial w^{R^*}/\partial R_{\rm m} > 0$, and $\partial p^{R^*}/\partial R_{\rm m} > 0$. Then, the profits function of the manufacturer and the retailer are analyzed, according to Eqs. (11) and (12), we obtain

$$\frac{\partial \pi_{\rm m}^{R^*}}{\partial R_{\rm m}} = \frac{16k^3\sigma^4(a-c-fe_0)^2}{(4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2)^2} > 0,$$

$$\frac{\partial \pi_{\rm r}^{R^*}}{\partial R_{\rm m}} = \frac{-4k\sigma^2(R_{\rm m}+2\sigma^2)(2k-(f+t)^2)(a-c-fe_0)^2}{(4k(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2)^2} < 0.$$

Proposition 1 shows that with $R_{\rm m}$ increasing, e^{R^*}, w^{R^*}, p^{R^*}, and $\pi_{\rm m}^{R^*}$ increase, but $\pi_{\rm r}^{R^*}$ decreases. The manufacturer's risk aversion behavior affects not only its own decision-making, but also the decision-making of the retailer downstream the supply chain. When the manufacturer's risk tolerance is smaller, the e^{R^*} and w^{R^*} will decrease, then, the retailer will reduce p^{R^*} and adopt the strategy of small profit but quick turnover. However, this will reduce the profit of the manufacturer. Proposition 1 shows that when the manufacturer's risk aversion awareness is strong, it is beneficial to the retailer but reduces the manufacturer's own profit.

4.2. Dynamic model

On the assumption that the supply chain operates for a long time and is discounted at the same discount rate ρ, given that the initial internal price of the product is known, the long-term profits of the retailer and the manufacturer can be expressed as $V_{\rm r} = \max\limits_{p(t)} \sum\limits_{t=1}^{\infty} \rho^{t-1}\pi_{\rm r}(p(t))$ and $V_{\rm m} = \max\limits_{w(t),e(t)} \sum\limits_{t=1}^{\infty} \rho^{t-1}u_{\rm m}(w(t),e(t))$. In order to solve the above-

mentioned optimal decision, the information about complete market is required, but in reality, enterprises have limited information, and they can only make decisions based on their limited information. In this study, we assume that the manufacturer adjusts decisions by limited rational expectations.[26,27] The discrete dynamic system of dynamic evolutionary game of the low-carbon supply chain considering the manufacturer's risk aversion behavior is as follows:

$$\begin{cases} w(t+1) = w(t) + d_1 w(t)((f(e_0-e)+te+a+c-2w)R_{\mathrm{m}} + 2(f(e_0-e)+c-w)\sigma^2)/(2R_{\mathrm{m}}), \\ e(t+1) = e(t) + d_2 e(t)(-2\sigma^2(e-e_0)f^2 + (((2e-e_0)t+a-w)R_{\mathrm{m}} + 2\sigma^2(c-w))f - ((c-w)t+2ke)R_{\mathrm{m}})/(2R_{\mathrm{m}}). \end{cases} \tag{14}$$

Equation (14) is the manufacturer's decision variable, where d_i, $i=1,2$ represent the adjustment parameter of wholesale price and carbon emission reduction level, respectively. According to Eq. (6), the dynamic retail price can be expressed as

$$p(t+1) = \frac{a + w(t+1) + te(t+1)}{2}. \tag{15}$$

According to the definition of fixed point, letting $w(t+1) = w(t)$, $e(t+1) = e(t)$, we obtain four equilibrium points of system (14)

$$E_1 = (0,0),$$
$$E_2 = \left(\frac{(a+c+e_0 f)R_{\mathrm{m}} + 2\sigma^2(c+fe_0)}{2(R_{\mathrm{m}}+\sigma^2)}, 0\right),$$
$$E_3 = \left(0, \frac{(2e_0\sigma^2 t_2 + f((a-fe_0)R_{\mathrm{m}} + 2c\sigma^2) - tcR_{\mathrm{m}}}{2R_{\mathrm{m}}(k-tf) + 2\sigma^2 f^2}\right),$$

and $E_4 = \left(w^{R^*}, e^{R^*}\right)$, where E_1, E_2, and E_3 are the unstable boundary equilibrium points, and E_4 is the Stackelberg equilibrium point of the system. To analyze the local stability of the equilibrium points of the system, the Jacobian matrix is obtained as follows:

$$J(E_4) = \begin{pmatrix} 1+d_1\rho_1 & d_1\dfrac{w(R_{\mathrm{m}}t - f(R_{\mathrm{m}}+2\sigma^2))}{2R_{\mathrm{m}}} \\ d_2\dfrac{e(R_{\mathrm{m}}t - f(R_{\mathrm{m}}+2\sigma^2))}{2R_{\mathrm{m}}} & 1+d_2\rho_2 \end{pmatrix}, \tag{16}$$

where

$$\rho_1 = \frac{2\sigma^2(f(e_0-e)+c-2w) + R_{\mathrm{m}}(f(e_0-e)+a+c+te-4w)}{2R_{\mathrm{m}}},$$
$$\rho_2 = \frac{2f\sigma^2((e_0-2e)f+c-w) + R_{\mathrm{m}}(f((4e-e_0)t+a-w)+t(w-c)-4ke)}{2R_{\mathrm{m}}}.$$

Then, the characteristic equation of $J(E_4)$ can be expressed as $\lambda^2 - \mathrm{Tr}(J(E_4))\lambda + \mathrm{Det}(J(E_4)) = 0$, where $\mathrm{Tr}(J(E_4))$ is the trace of $J(E_4)$, and $\mathrm{Det}(J(E_4))$ is the determinant of $J(E_4)$. The necessary and sufficient conditions for the local asymptotic stability of system (14) can be derived according to the July criterion.[24–26] The following conditions for the system stability at E_4 should be met: $1 + \mathrm{Tr}(J(E_4)) + \mathrm{Det}(J(E_4)) > 0$, $1 - \mathrm{Tr}(J(E_4)) + \mathrm{Det}(J(E_4)) > 0$, and $1 - \mathrm{Det}(J(E_4)) > 0$. Otherwise, the system will lose stability and become disordered.

4.3. Complex dynamics analysis of supply chain system

Then, the dynamic characteristics of the system are simulated to explore the influence of adjustment parameters, risk tolerance, demand uncertainty, carbon emission tax on the system stability domain, wholesale price, carbon emission reduction level, and the profits of the manufacturer, the retailer, and the supply chain. Assume that $a = 15$, $c = 0.5$, $k = 10$, $R_{\mathrm{m}} = 2$, $t = 0.3$, $f = 0.8$, and $\sigma^2 = 1$, the values of these parameters meet the assumptions and constraints of this study.

4.3.1. Influence of adjustment parameters on system

Firstly, the influence of the adjustment parameters on the dynamic properties of the system is analyzed. Figure 1 shows the bifurcation diagram and the corresponding maximum Lyapunov exponent diagram of the system (14) with the change of adjustment parameters d_1 and d_2, respectively. First, when $d_2 = 0.1$, we observed the change of the system with adjustment parameter d_1 as shown in Figs. 1(a) and 1(b). With the increase of d_1, the system will experience period-doubling bifurcation from the stable state to the chaotic state, and the corresponding maximum Lyapunov exponent will change from negative to positive. When $d_1 = 0.1$, figures 1(c) and 1(d) show that the system changes with d_2, which is similar to the change of the system with d_1, but the system enters into the chaotic state much later. Therefore, compared with the increase of d_2, the increase of price adjustment parameter is easy to make the system lose stability.

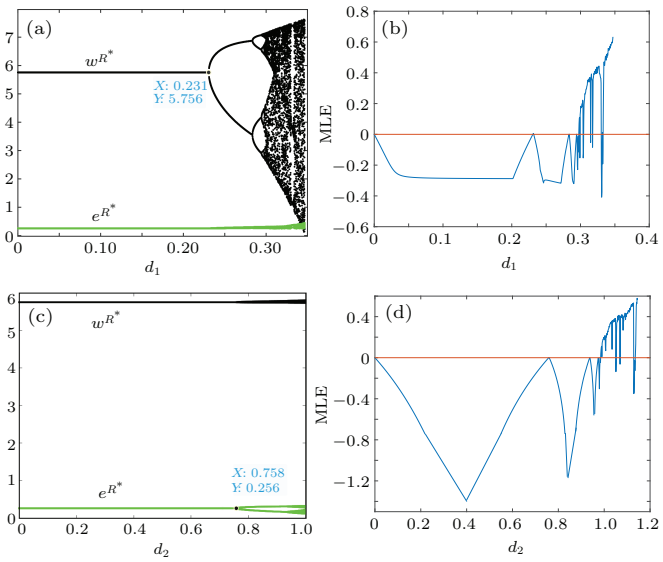


Fig. 1. Bifurcation diagram of system varying (a) with d_1 when $d_2 = 0.1$ and (c) with d_2 when $d_1 = 0.1$; maximum Lyapunov exponent diagrams with respect to (b) d_1 and (d) d_2.

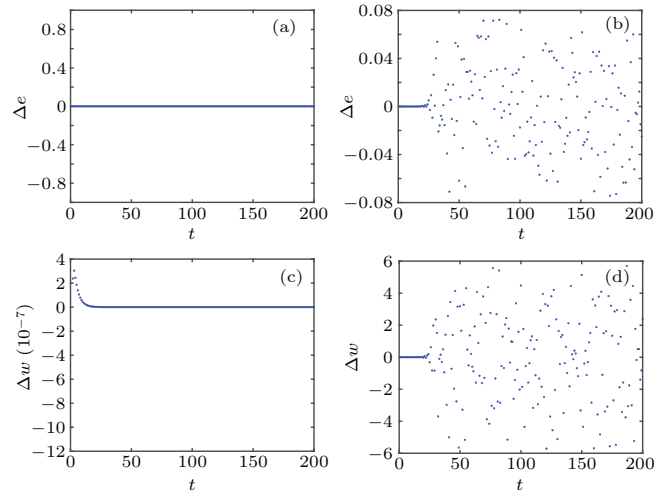


Fig. 2. Initial value sensitivity diagrams, showing change of (a) e and (b) w when the system is in stable state and change of (b) e and (d) w when the system is in chaotic state.

The sensitivity of initial value is another important feature of a chaotic system. When $(w, e) = (5.756, 0.256)$, the system is in a stable state as shown in Figs. 2(a) and 2(c). The equilibrium point has some fluctuations in the initial period time, but it will soon return to the stable state. However, when $(w', e') = (5.756001, 0.256001)$, the system is in a chaotic state. The system variables are completely different from the stable state over time as shown in Figs. 2(b) and 2(d), and w and e change only 10^{-6}. The final results are very different and unpredictable.

Next, we analyze $\pi_{\mathrm{m}}^{R^*}$, $\pi_{\mathrm{r}}^{R^*}$, and $\pi_{\mathrm{sc}}^{R^*}$ with the increase of price adjustment parameters d_1, as shown in Fig. 3. Comparing Fig. 3 with Fig. 1, it can be seen that the decision variables and profits have experienced the process from stable state to period-doubling bifurcation and finally enter into chaotic state with the increase of adjustment parameters. Figure 4 shows the analysis of the profit fluctuations of the system in different states.

When the system is in the stable state ($d_1 = 0.1$, $d_2 = 0.1$) as shown in Fig. 4(a), $\pi_{\mathrm{m}}^{R^*}$, $\pi_{\mathrm{r}}^{R^*}$, and $\pi_{\mathrm{sc}}^{R^*}$ are also stable, and their optimal profits can be obtained. When the system is in the period-doubling bifurcation state ($d_1 = 0.25$, $d_2 = 0.1$) as shown in Fig. 4(b), $\pi_{\mathrm{m}}^{R^*}$, $\pi_{\mathrm{r}}^{R^*}$, and $\pi_{\mathrm{sc}}^{R^*}$ begin to fluctuate up and down between two points and show stable periodic values. When the system is in the chaotic state ($d_1 = 0.32$, $d_2 = 0.1$) as shown in Fig. 4(c), the profits fluctuate disorderly. Because chaos means disorder, the manufacturer cannot make optimal decisions in a disordered market environment, so it cannot maximize its own profit.

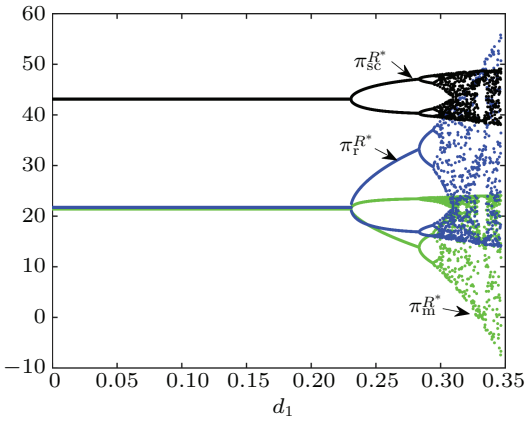


Fig. 3. Bifurcation diagram of expected profits with respect to d_1.

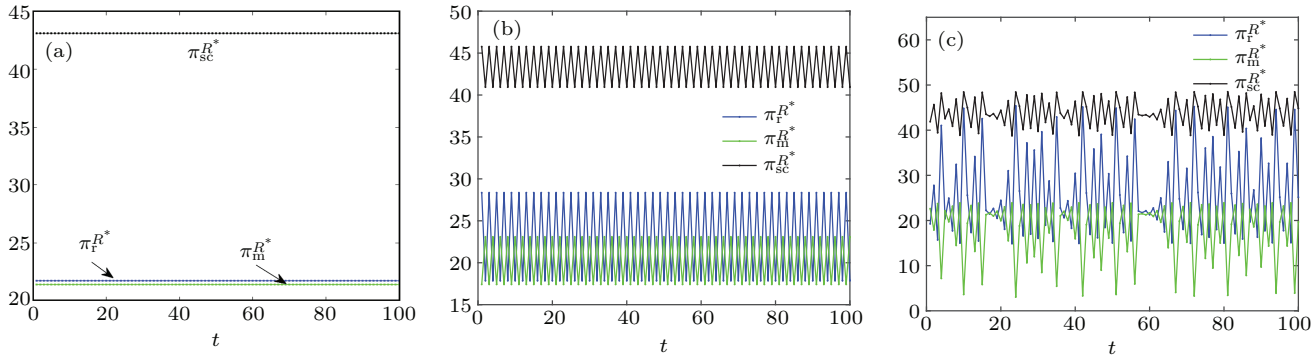


Fig. 4. Variation of profit with time when system is in (a) stable state, (b) period-doubling bifurcation state, and (c) chaos state.

Table 1 shows the corresponding average profits of the system in these three states. It can be seen that as the system goes through period-doubling bifurcation from the stable state to the chaotic state, the manufacturer's long-term average profit π_m continues to decline, but π_r and π_{sc} increase.

Table 1. Average profits of system in different states.

	$d_1 = 0.1, d_2 = 0.1$	$d_1 = 0.25, d_2 = 0.1$	$d_1 = 0.32, d_2 = 0.1$
π_m	21.393	20.872	19.077
π_r	21.722	22.340	24.534
π_{sc}	43.115	43.212	43.611

Conclusion 1 when the adjustment parameters are beyond the stability region of the system, the system enters into chaos through period-doubling bifurcation; when the system loses stability, the profits of the manufacturer, the retailer and the supply chain show fluctuation, which reduces the average manufacturer's profit, but increases the average profits of the retailer and the supply chain.

4.4. Influence of manufacturer's risk tolerance on system

In this subsubsection, we analyze the influence of the manufacturer's risk tolerance attitude on the dynamic characteristics of the system. Figure 5 shows the two-dimensional (2D) bifurcation diagram of the system with the increase of R_m. It can be seen that risk aversion behavior will not change the path to chaos of the system, but will affect thestable region size of the system. With the increase of R_m, the influence on the stability region in the direction d_1 is not obvious, but it can significantly reduce the stability region in the direction d_2, resulting in the system stability decreasing as a whole. This shows that with the increase of manufacturer's risk tolerance, the manufacturer needs to carefully increase d_2 to prevent the system from entering into a chaotic state.

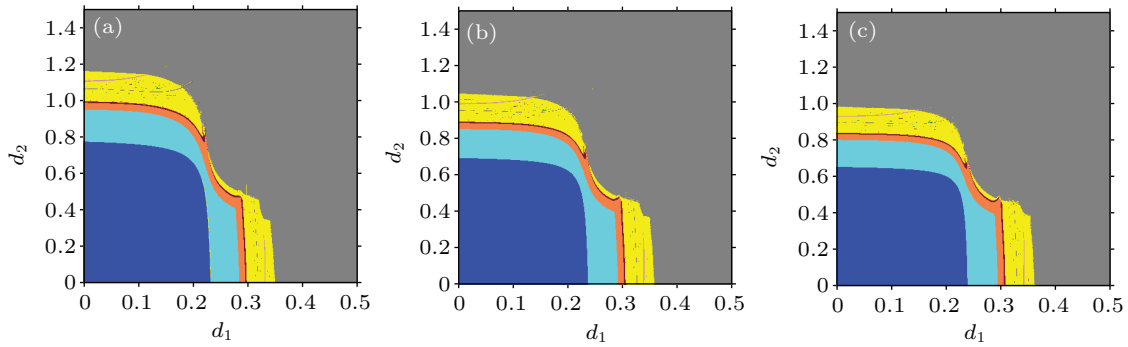


Fig. 5. Two-dimensional bifurcation diagram of the system with the increase when manufacturer's risk tolerance $R = 2$ (a), 3 (b), and 4 (c).

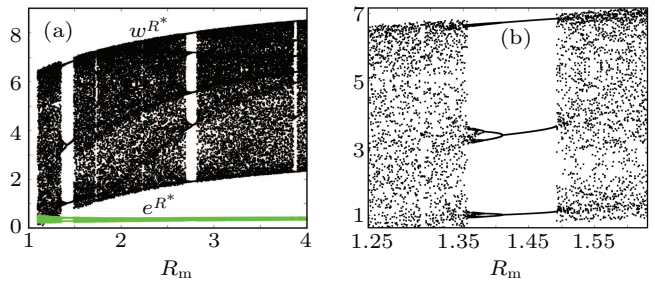


Fig. 6. Bifurcation diagram of system varying with R_m when $d_1 = 0.32, d_2 = 0.1$, showing (a) influence of R_m on system under chaotic state and (b) magnified part of panel (a) when $R_m \in (1.25, 1.6)$.

Figure 6(a) shows the system changes with the increase of R_m, when it is in a chaotic state ($d_1 = 0.32$, $d_2 = 0.1$). It can be seen from the figure that the system has entered into three cycles directly from chaos. From the magnified figure in Fig. 6(b), it can be clearly seen that the system experiences three cycles from chaos, and reenters into the chaotic state. Figure 6 shows that the system directly enters into periodic motion from chaos, which appears alternately, presenting a complex evolution process. This shows that when the system is disturbed, it can also be adjusted back to the ordered state without taking any control measure, but the system may jump back to the disordered state randomly again. This process occurs repeatedly.

Figure 7 shows the influence of manufacturer's risk tolerance on $\pi_m^{R^*}$, $\pi_r^{R^*}$, and $\pi_{sc}^{R^*}$ when the system is in a chaotic state (). We can know that although the system is in a chaotic state, the influences of manufacturer's risk tolerance on $\pi_m^{R^*}$, $\pi_r^{R^*}$, and $\pi_{sc}^{R^*}$ are similar to those in a stable state. This indicates that the increase of R_m will improve its own profit, but will reduce $\pi_r^{R^*}$ and $\pi_{sc}^{R^*}$.

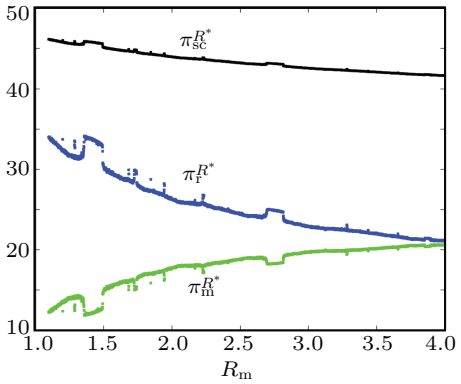


Fig. 7. Influence of manufacturer's risk tolerance on profit under chaos when $d_1 = 0.32$ and $d_2 = 0.1$.

Conclusion 2 As the risk tolerance increases, the stable

region of the system decreases, and the manufacturer needs to carefully increase the carbon emission reduction adjustment parameters to prevent the system from entering into a chaotic state. The increase of the manufacturer's risk tolerance will improve its own profit, but reduces the profits of the retailer and the supply chain.

4.4.1. Influence of carbon tax on system

Figure 8 shows the 2D bifurcation diagram of the system with the increase of carbon tax rate. We can see that with the increase of f, the stability region (dark blue) of the system decreases in the direction of the wholesale price, but the change in the direction of e^{R^*} is not obvious.

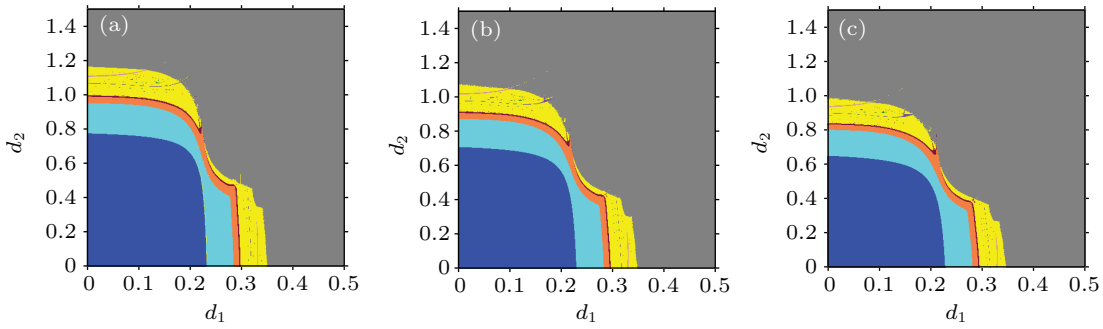


Fig. 8. Two-dimensional bifurcation diagram of system with carbon tax $f = 0.8$ (a), 0.9 (b), and 1 (c).

Table 2 specifically shows the influence of f on stability region of the system under the manufacturer's risk aversion decision situation. Let respectively. When $d_1 = 0.1$, with the increase of f, d_2 becomes smaller and the first bifurcation point occurs. When $d_2 = 0.1$, d_1 has the same varying tendency. This indicates that the increase of f will reduce the stability region of the system.

Table 2. Influence of f on stability region of system.

	$f = 0.8$	$f = 0.9$	$f = 1$
$d_1 = 0.1$	$d_2 = 0.757$	$d_2 = 0.688$	$d_2 = 0.628$
$d_2 = 0.1$	$d_1 = 0.231$	$d_1 = 0.229$	$d_1 = 0.227$
$e^{R^*}(t)$	0.256	0.279	0.30
$w^{R^*}(t)$	5.756	5.794	5.829
$p^{R^*}(t)$	10.416	10.439	10.460
$\pi_{\mathrm{r}}^{R^*}$	21.722	21.574	21.441
$\pi_{\mathrm{m}}^{R^*}$	21.393	21.186	20.988
$\pi_{\mathrm{sc}}^{R^*}$	43.115	42.760	20.988

Figure 9(a) shows the change of the system with the increase of f in a chaotic state at $d_1 = 0.32$ and $d_2 = 0.1$. It can be seen from the figure that when the carbon tax rate is about $f \approx 0.216$, the system bifurcates to six-cycle state from chaos. From the changes of the maximum Lyapunov exponent in Fig. 9(b), we can know that the system experiences a series of complex processes, during which the system enters into quasi periodic from chaos and then restores to a short-term stable state, and enters into the chaotic state again. Figure 9(c) shows the attractor of the system when $f = 0.1$, which shows that the trajectory of the system at the beginning is attracted into two chaotic attractors. Figure 9 shows that the system directly enters into a periodic motion from chaotic motion, which occurs alternately, showing a complex evolution process. Owing to the cataclysm, the attractor changes from the previous two pieces to one as shown in Fig. 9(d). This indicates that when the system is disturbed, it can be adjusted back to the ordered state without taking any external control measures, but the system may jump back to the disordered state randomly again, and this process occurs repeatedly. With the continuous increase of f, the dynamic characteristics of the system will become more and more complex. Therefore, the enterprise itself has a regulation mechanism when the system enters into chaotic state, but it still needs some artificial control measures to restore the system to the normal trajectory.

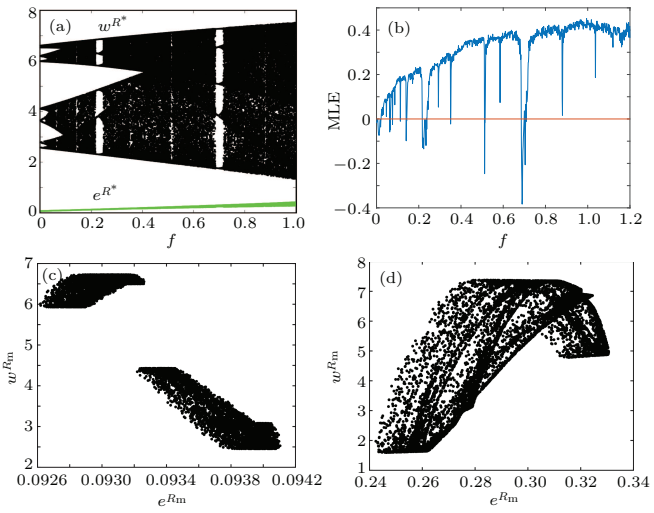


Fig. 9. (a) Bifurcation diagram when $d_1 = 0.32$ and $d_2 = 0.1$, (b) maximum Lyapunov exponent graph, (c) chaotic attractors when $f = 0.1$, and (d) chaotic attractors when $f = 0.8$.

Figure 10 shows the changes of $\pi_{\mathrm{m}}^{R^*}$, $\pi_{\mathrm{r}}^{R^*}$, and $\pi_{\mathrm{sc}}^{R^*}$ with the increase of f, when the system is in the chaotic state ($d_1 = 0.32$ and $d_2 = 0.1$). It can be seen from Fig. 10 that when the system loses stability, the influences of f on $\pi_{\mathrm{m}}^{R^*}$, $\pi_{\mathrm{r}}^{R^*}$, and $\pi_{\mathrm{sc}}^{R^*}$ are similar to those in the stable state. The increase of f will reduce $\pi_{\mathrm{m}}^{R^*}$, $\pi_{\mathrm{r}}^{R^*}$, and $\pi_{\mathrm{sc}}^{R^*}$. Compared with the

retailer, the manufacturer is more affected by carbon tax.

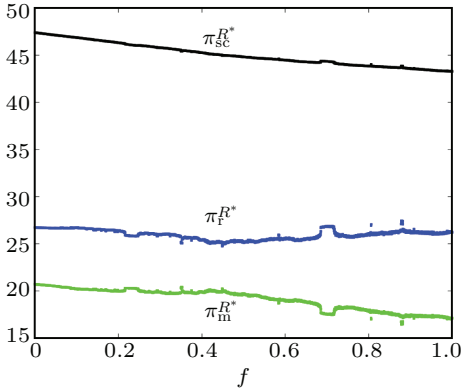


Fig. 10. Effects of f on profit of decision-maker and the supply chain under chaos.

Conclusion 3 under the carbon tax policy, with the increase of carbon tax rate, the carbon emission reduction level will increase, but the profits of the retailer, the manufacturer and the supply chain will decrease, and the stability region of the system will also decrease.

Conclusion 3 shows that the carbon tax policy can encourage the manufacturer to increase carbon emission reduction investment, but it will also damage the profits of the manufacturer and the retailer, resulting in the decrease of the supply chain profit. Therefore, when the government formulates the carbon tax rate, the goal of carbon emission reduction and the enterprises' profits need balancing, so as to avoid striking the manufacturers' enthusiasm for emission reduction.

5. Centralized decision model of low-carbon supply chain

5.1. Static model

The superscript "o" represents the optimal solution in the centralized decision situation. Under this decision situation, the goal is to maximize the overall profit of the supply chain by formulating the optimal p and e. We calculate the first-order partial derivative of the profit function of the centralized decision supply chain with respect to p and e from

$$\begin{cases} \dfrac{\partial \pi_{sc}^{o}}{\partial p} = f(e_0 - e) + te + a + c - 2p, \\ \dfrac{\partial \pi_{sc}^{o}}{\partial e} = f(t(2e - e_0) + a - p) + t(p - c) - ke. \end{cases} \tag{17}$$

When $2k > (f+t)^2$ is satisfied, the Hessian matrix of Eq. (4) with respect to the decision variables p and e is negative-definite, and the supply chain profit function has a unique maximum value. We calculate the optimal decisions and the supply chain profit under the centralized decision by solving $\partial \pi_{sc}^{o}/\partial p = 0$ and $\partial \pi_{sc}^{o}/\partial e = 0$ simultaneously and obtain

$$e^{o} = \frac{(f+t)(a-c-fe_0)}{2k-(f+t)^2}, \tag{18}$$

$$p^{o} = \frac{k(a+c) - (e_0 t + a)f^2 - (t^2 e_0 + t(a+c) - e_0 k)f - ct^2}{2k-(f+t)^2}, \tag{19}$$

$$\pi_{sc}^{o} = \frac{k(a-c-fe_0)^2}{4k-(f+t)^2}. \tag{20}$$

By comparing the centralized decision situation with the decentralized decision situation, Proposition 2 can be obtained.

Proposition 2 $e^{o} > e^{*} > e^{R^*}$, $\pi_{sc}^{o} > \pi_{sc}^{R^*} > \pi_{sc}^{*}$.

Proposition 2 states that under the carbon tax policy, whether the manufacturer has risk aversion behavior or not, e and π_{sc} under the decentralized decision are smaller than those under the centralized decision. This is because enterprises seek for maximizing their own benefits and ignore the overall performance of the supply chain, resulting in "double marginalization effect" under the decentralized decision. This leads to failing to achieve the optimal economic and environmental benefits. The supply chain profit considering the manufacturer's risk aversion behavior is greater than that under the risk neutrality situation of manufacturer, which slows down the "double marginalization effect". However, it is not conducive to the carbon emission reduction targets.

5.2. Dynamic model

As indicated in Subsection 4.2, under the centralized decision, the manufacturer and the retailer determine carbon emission reduction levels and retail price according to the bounded rational expectation.[26,27] The dynamic model under centralized decision can be expressed as

$$\begin{cases} p(t+1) = p(t) + d_3 p(t) \\ \qquad \times (f(e_0 - e) + te + a + c - 2p), \\ e(t+1) = e(t) + d_4 \\ \qquad \times (f(t(2e - e_0) + a - p) + t(p - c) - ke). \end{cases} \tag{21}$$

As indicated in Subsection 4.2, this subsection focuses on the analysis of the stability of $E = (w^{o}, e^{o})$. First, we solve the Jacobian matrix

$$J(E) = \begin{pmatrix} 1 + d_3\rho_3 & d_3 p(t-f) \\ d_4 e(t-f) & 1 + d_4\rho_4 \end{pmatrix}, \tag{22}$$

where $\rho_3 = a + c + te - 4p + f(e_0 - e)$ and $\rho_4 = (a - p + t(4e - e_0)f) + (p - c)t - 2ke$.

Then, the characteristic equation of $J(E)$ can be expressed as $\lambda^2 - \mathrm{Tr}(J(E))\lambda + \mathrm{Det}(J(E)) = 0$, where $\mathrm{Tr}(J(E))$ is the trace of $J(E)$, and $\mathrm{Det}(J(E))$ is the determinant of $J(E)$. According to the July criterion,[24–26] the necessary and sufficient conditions for the local stability of the system (21) can be given by $1 + \mathrm{Tr}(J(E)) + \mathrm{Det}(J(E)) > 0$, $1 - \mathrm{Tr}(J(E)) + \mathrm{Det}(J(E)) > 0$, and $1 - \mathrm{Det}(J(E)) > 0$. Otherwise, the system will lose stability and become disordered.

5.3. Complex dynamics analysis of supply chain system

Figure 11 shows the stable area for the adjustment parameters d_3 and d_4 of equation system (21). From Figs. 11(a) and 11(c), we know that when the adjustment parameters satisfy the inequalities $0 < d_3 < 0.1235$ and $0 < d_4 < 0.2018$, the system is stable. When the adjustment parameters exceed this range, the system begins to create period-doubling bifurcation and even leads to chaos. Figures 11(b) and 11(d) are the corresponding maximum Lyapunov exponent. Figure 12 is a 2D fork diagram about the adjustment of the parameters d_3 and d_4, which is similar to Fig. 5(a). Comparing with the decentralized decision, the sizes of the stability region, two-cycle and four-cycle of the system decrease under the centralized decision situation. This indicates that the centralized decision situation makes the system easier to lose stability.

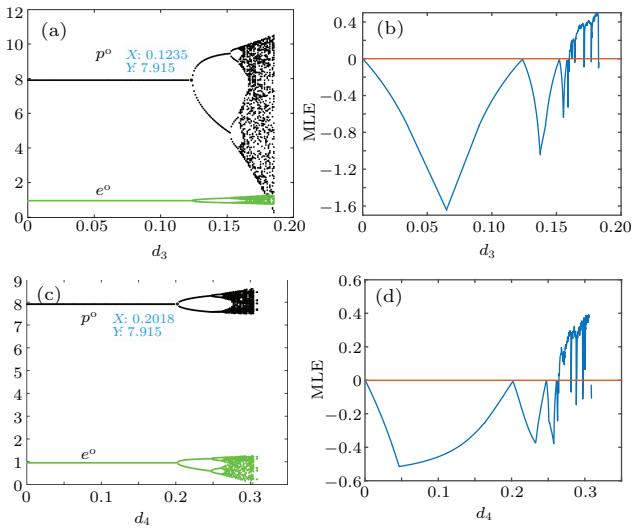


Fig. 11. Bifurcation diagram of the system varying with (a) d_3 and (c) d_4 and maximum Lyapunov exponent diagram of the system with respect to (b) d_3 and (d) d_4.

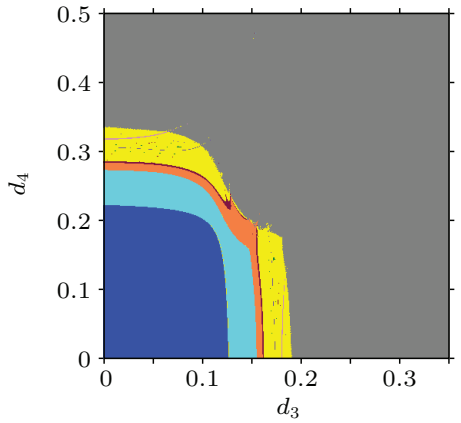


Fig. 12. The 2D bifurcation diagram of the system with d_3 and d_4 increasing.

Figure 13 shows the change of π_{sc}^{o} with d_3 and d_4 in the centralized decision. When the system is stable, π_{sc}^{o} is equivalent to the value of the equilibrium solution (blue line), which is greater than $\pi_{sc}^{R^*}$ in Fig. 3. However, when the system is not stable, π_{sc}^{o} begins to fluctuate (black), which leads π_{sc}^{o} to decrease and turns less than the value of the stable state. From Fig. 13, it can be seen that excessive pricing or the carbon emission reduction level adjustment parameters are disadvantageous to the supply chain.

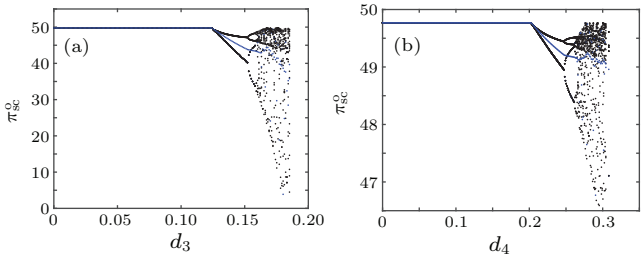


Fig. 13. Bifurcation diagram of supply chain profits varying with (a) d_3 and (b) d_4.

Conclusion 4 Compared with the decentralized decision model, the system under the centralized decision is more likely to lose stability with the increase of adjustment parameters; when the system enters into chaotic state, the long-term profit of the supply chain begins to fluctuate and is smaller than that in the stable situation.

5.4. Sensitivity analysis of exogenous variables

In this subsection, sensitivity analysis is carried out on the influence of exogenous variables, including f, t, R, *etc.*, on the optimal decision-making and profits. It can be seen from Fig. 14 that the increase of t and f both have positive influence on e, w, and p of three different decision situations. As can be seen from Fig. 14(a), e under the centralized decision is significantly higher than that in the decentralized decision. When the manufacturer has risk aversion, e is the smallest. In addition, we find that the values of e under the three decision situations all increase with f increasing, and the increased speed in the centralized decision situation is greater than that under the decentralized decision situation. Therefore, the carbon tax policy can encourage the manufacturer to invest in carbon emission reduction; however, it will increase w and p simultaneously as shown in Figs. 14(b) and 14(c). In addition, when the manufacturer has risk aversion, w is less than that of the risk neutral decentralized decision situation. The wholesale prices of the two situations both increase with f increasing, but the increase of w in the manufacturer's risk aversion situation is greater than that in the manufacturer's risk neutral situation.

The value of p in the manufacturer's risk aversion situation is less than that in the risk neutral decentralized decision situation, but it is greater than that of the centralized decision-making situation.

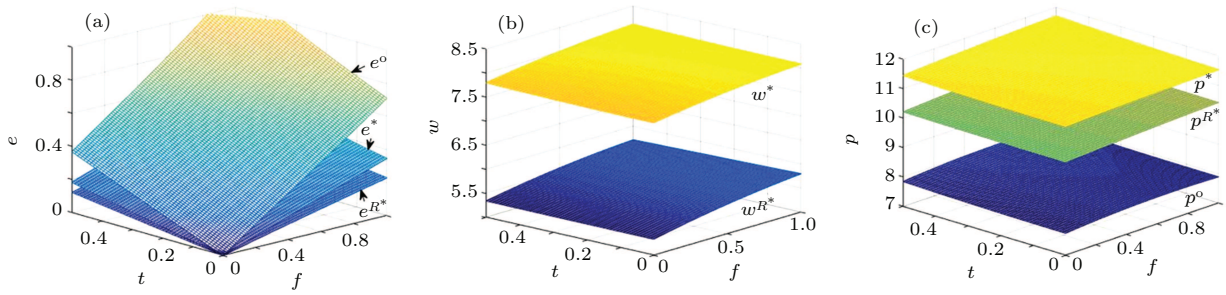


Fig. 14. Influence of f and t on (a) e, (b) w, and (c) p.

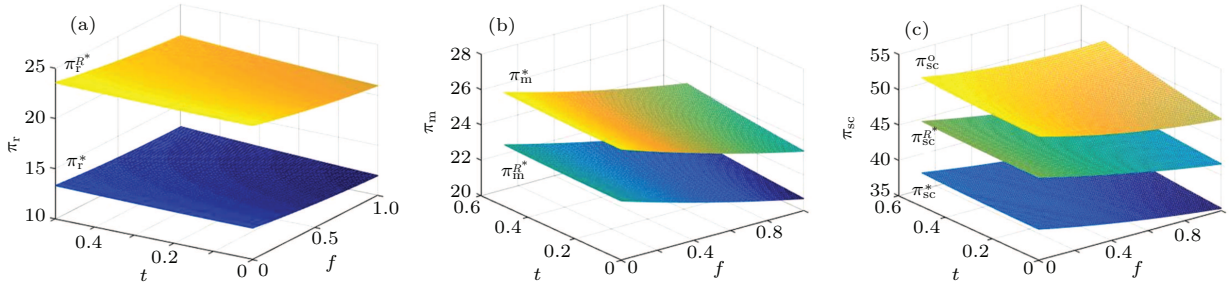


Fig. 15. Influence of carbon tax rate t and low-carbon preference coefficient f on profit (a) π_r, (b) π_m, and (c) π_{sc}.

Figure 15 shows the influence of f and t on the profits of the retailer, the manufacturer and the supply chain, respectively. With the increase of t, the profits of the retailer, the manufacturer and the overall supply chain increase. The increase of f will reduce the profits of the retailer, the manufacturer and the overall supply chain. In addition, when the manufacturer has risk aversion, it can slow down the "double marginalization effect" of the supply chain, which makes the profit of the supply chain greater than that under the risk neutral situation.

Conclusion 5 Whether the risk aversion behavior of the manufacturer is considered or not, the carbon emission reduction level and the supply chain profit under the decentralized decision are smaller than those under the centralized decision. Compared with risk neutral situation, the manufacturer's risk aversion behavior can narrow the gap of the supply chain profit under decentralized decision and centralized decision, but it will frustrate the manufacturer's enthusiasm for emission reduction.

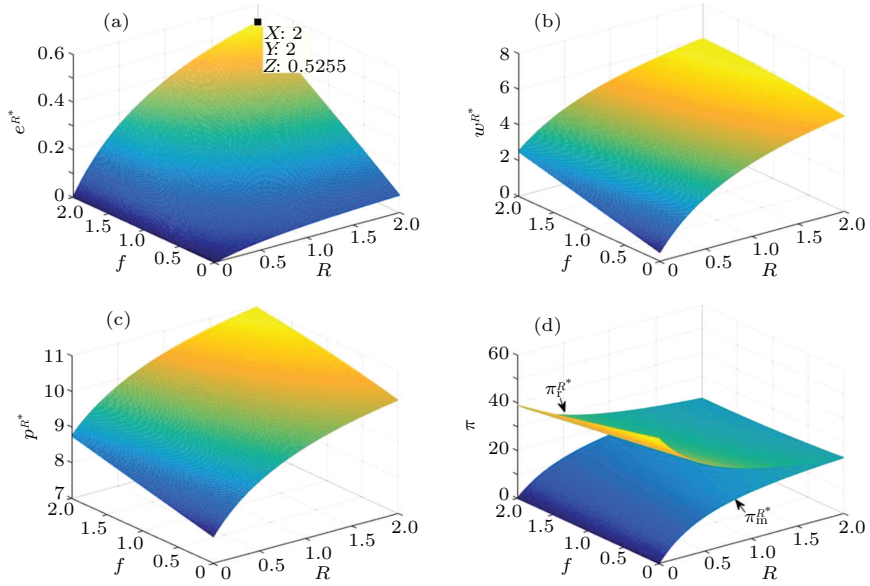


Fig. 16. Influence of carbon tax rate R and manufacturer's risk-tolerance f on (a) e, (b) w, (c) p, and (d) π_r and π_m

Figure 16 shows the influence of f and R on the optimal decision-making and the profit of the retailer and the manufacturer. It can be seen from Fig. 16 that both f and R have a positive influence on e, w and p, but the influence on the profit of the retailer and the manufacturer are different. From Fig. 16(d), we know that as the manufacturer's risk

tolerance increases, the manufacturer's own profit increases, but the profit of the retailer decreases. The increase of f will reduce the profit of the retailer and the manufacturer.

6. Cost-sharing contract game model of low-carbon supply chain

6.1. Model construction

To mitigate the adverse effect of the manufacturer's risk aversion behavior and reduce the "double-marginal effect" of the supply chain, in this subsection we introduce the carbon emission reduction cost-sharing contract (CSC) to improve the carbon emission reduction level and optimize the performance of the supply chain. Assuming that the proportion shared by the retailer is ϕ, after implementing the cost-sharing contract, the utility function of the manufacturer and the retailer respectively are as follows:

$$\begin{aligned} E(U_{\rm m}) &= (w-c-f(e_0-e))(a-p+te) \\ &\quad -\frac{1}{2}ke^2(1-\phi)-\frac{(w-c-f(e_0-e))^2\sigma^2}{2R_{\rm m}}, \quad (23) \\ E(U_{\rm r}) &= \pi_{\rm r}=(p-w)(a-p+te)-\frac{1}{2}ke^2\phi. \quad (24) \end{aligned}$$

The superscript c^* represents the optimal solution in the case of the CSC. After the CSC is implemented, the order of the game will not be changed. The optimal decisions are obtained by backward induction:

$$\begin{aligned} e^{c^*} &= \frac{R_{\rm m}(f+t)(a-c-fe_0)}{4k(1-\phi)(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2}, \quad (25) \\ w^{c^*} &= R_{\rm m}(2k(1-\phi)(a+c+e_0f) \\ &\quad -(f+t)(ct+f(e_0t+a)))+4k\sigma^2(c+fe_0) \\ &\quad \times(1-\phi)/(4k(R_{\rm m}+\sigma^2)(1-\phi)-R_{\rm m}(f+t)^2), \quad (26) \end{aligned}$$

$$\begin{aligned} p^{c^*} &= R_{\rm m}(2k(1-\phi)(2a+c+e_0f+et) \\ &\quad -(f+t)(t(a+c+(e_0+e)f+et^2+2af)) \\ &\quad +4k\sigma^2(1-\phi)(c+fe_0+a+te)/2(4k(R_{\rm m}+\sigma^2) \\ &\quad \times(1-\phi)-R_{\rm m}(f+t)^2). \quad (27) \end{aligned}$$

The profit function of the manufacturer, the retailer and the supply chain respectively are

$$\begin{aligned} \pi_{\rm m}^{c^*} &= (k(((4k(1-\phi)-(f+t)^2)R_{\rm m}^2+8k\sigma^2(1-\phi)R_{\rm m}) \\ &\quad \times(1-\phi)(a-c-fe_0)^2)/2(4k(R_{\rm m}+\sigma^2) \\ &\quad \times(1-\phi)-R_{\rm m}(f+t)^2)^2, \quad (28) \\ \pi_{\rm r}^{c^*} &= k((2k(1-\phi)-\phi(f+t)^2)R_{\rm m}^2+8k\sigma^2(1-\phi)^2R_{\rm m} \\ &\quad +8k\sigma^4(1-\phi)^2)(a-c-fe_0)^2/2(4k(R_{\rm m}+\sigma^2) \\ &\quad \times(1-\phi)-R_{\rm m}(f+t)^2)^2, \quad (29) \\ \pi_{\rm sc}^{c^*} &= k((6k(1-\phi)^2-(f+t)^2)R_{\rm m}^2+16k\sigma^2(1-\phi)^2R_{\rm m} \\ &\quad +8k\sigma^4(1-\phi)^2)(a-c-fe_0)^2/2(4k(R_{\rm m}+\sigma^2) \\ &\quad \times(1-\phi)-R_{\rm m}(f+t)^2)^2. \quad (30) \end{aligned}$$

Proposition 3 When the cost-sharing contract is implemented, the carbon emission reduction level will beimproved, and it increases with the cost-sharing proportion increasing.

Proof By comparing Eq. (7) with Eq. (25), we can have $e^{c^*}>e^{R^*}$. Then we derive the first derivative of Eq. (27) and obtain

$$\frac{\partial e^{c^*}}{\partial\phi}=\frac{4kR_{\rm m}(f+t)(a-c-fe_0)(R_{\rm m}+\sigma^2)}{4k(1-\phi)(R_{\rm m}+\sigma^2)-R_{\rm m}(f+t)^2}>0.$$

Proposition 4 When the cost-sharing contract coefficient satisfies $0<\phi<\bar{\phi}$, the retailer's profit and the manufacturer's utility are greater than those in the case of decentralized decision, and Pareto improvement is achieved, where

$$\bar{\phi}=\frac{((f+t)^2R_{\rm m}^2+12\sigma^2kR_{\rm m}+16k\sigma^4)(((f+t)^2-4k)R_{\rm m}-4k\sigma^2)}{2k(R_{\rm m}+2\sigma^2)^2(((f+t)^2-8k)R_{\rm m}-8k\sigma^2)}.$$

Proof The effective implementation of the CSC needs to meet the participation constraints, that is, and $E\left(u_{\rm m}^{c^*}\right)\geq E\left(u_{\rm m}^{R^*}\right)$, Under these conditions, the manufacturer and retailer will accept the CSC. To meet the condition $\pi_{\rm r}^{c^*}-\pi_{\rm m}^{R^*}\geq 0$, the cost-sharing coefficient needs to satisfy $\phi<\bar{\phi}$. To meet $E\left(u_{\rm m}^{c^*}\right)-E\left(u_{\rm m}^{R^*}\right)\geq 0$, we calculate the case $0<\phi$. Therefore, when the cost-sharing coefficient satisfies $0<\phi<\bar{\phi}$, Pareto improvement can be achieved. Then, we discuss whether the CSC can achieve Pareto optimality. According to Eqs. (18) and (25), when $e^{c^*}=e^{\rm o}$, we derive the cost-sharing coefficient $\phi=\frac{2\sigma^2+R_{\rm m}}{2(R_{\rm m}+\sigma^2)}$. By bringing the obtained cost-sharing proportion into p^{c^*}, and comparing p^{c^*} with the retail price $p^{\rm o}$ in the case of centralized decision-making, we can obtain

$$p^{c^*}-p^{\rm o}=\frac{kR_{\rm m}(a-c-fe_0)}{2(R_{\rm m}+\sigma^2)(2k-(f+t)^2)}>0.$$

Therefore, the Pareto optimal equilibrium solution cannot be obtained through the cost-sharing contract.

Under the CSC, the discrete dynamic system of dynamic evolutionary game of the low-carbon supply chain considering the manufacturer's risk aversion behavior is as follows:

$$\begin{aligned} w^T(t+1) &= w^T(t)+d_5w^T(t)(f(e_0-e^T) \\ &\quad +te^T+a+c-2w^T)R_{\rm m} \\ &\quad +2(f(e_0-e^T)+c-w^T)\sigma^2/2R_{\rm m}, \end{aligned}$$

$$
\begin{aligned}
e^T(t+1) &= e^T(t)+d_6e^T(t)(((2e^T-e_0)t+a-w^T)f \\
&\quad +2ke^T(\phi-1)-t(c-w))R_{\rm m} \\
&\quad +2f\sigma^2((e_0-e^T)f+c-w^T)/2R_{\rm m}, \\
p^T(t+1) &= \frac{a+w^T(t+1)+te^T(t+1)}{2}.
\end{aligned} \tag{31}
$$

6.2. Numerical simulation

In this subsection, we analyze the influence of CSC on the supply chain system. Figure 17(a) shows the influence of the CSC on the carbon emission reduction level. With the increase of ϕ, e^{c^*} increases, and it is greater than that in the decentralized decision, which verifies Proposition 3. Figure 17(b) shows the influence of ϕ on the expected profit of the manufacturer and the retailer. After implementing the cost-sharing contract, the manufacturer's profit increases with the cost-sharing coefficient increasing, and it is greater than that in the decentralized decision. For the retailer, with the increase of ϕ, the profit first increases and then decreases. Correspondingly, after implementing the CSC, the retailer's profit is first greater than that in the decentralized decision. When ϕ reaches the critical point, the retailer's profit is equal to that in decentralized decision, which is also the threshold for the retailer to be willing to accept the CSC. Figure 17(c) shows the influence of ϕ on the profit of the supply chain. After the implementation of the CSC, the profit of the supply chain is greater than that under the decentralized decision, and Pareto improvement is achieved, which verifies Proposition 4.

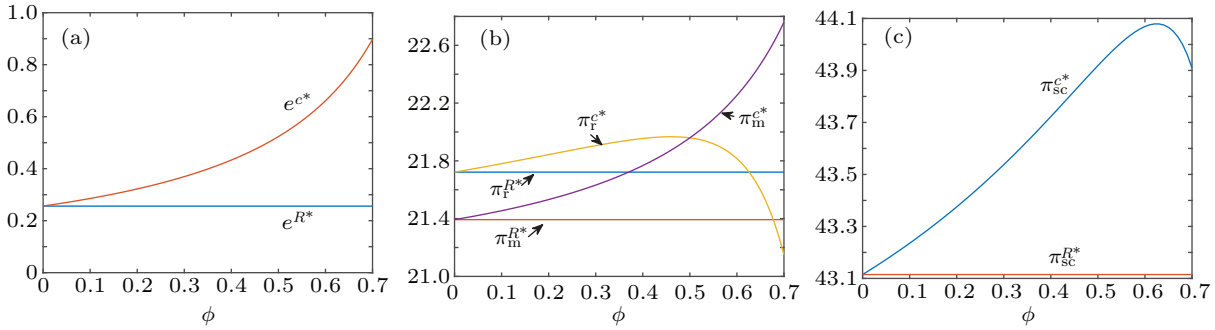


Fig. 17. Influence of ϕ on (a) carbon reduction level, (b) profits of manufacturer and retailer, and (c) profit of supply chain.

In this subsection, we analyze the influence of ϕ on the stability region of the system. Figure18 shows the 2D bifurcation diagram of the system when the manufacturer's risk tolerance coefficient is $R_{\rm m}=2,3,4$, respectively, in which the dark blue area represents the stable area of the system. Comparing the 2D bifurcation diagram of the system with that in Fig. 5, the CSC does not change the way the system enters into chaotic state, but affects the size of the system stability domain. When $R_{\rm m}$ has the same value, the stability region of the supply chain system under the CSC is smaller than that in Section 4. Therefore, although the CSC improves carbon reduction level and the supply chain performance, the stability of the supply chain system decreases.

Conclusion 6 Under the cost-sharing contract mechanism, the carbon emission reduction level and economic benefit of the supply chain can be improved simultaneously, and the stability region of the system is larger than that in the centralized decision-making situation, but the increase of the cost-sharing coefficient will reduce the stability of the system in the decentralized decision-making situation.

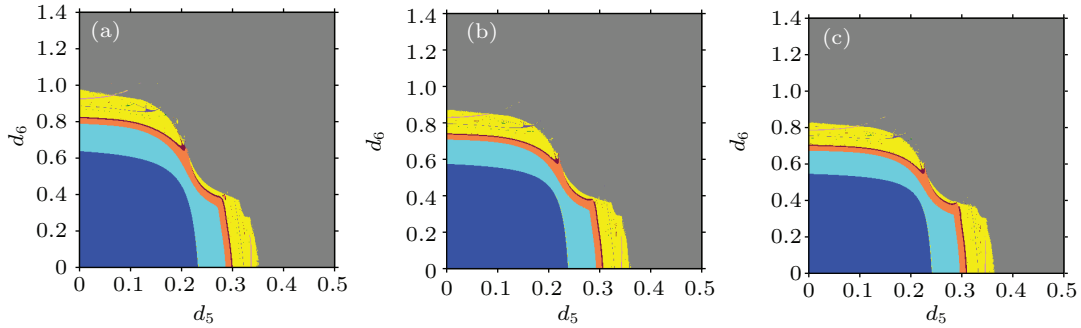


Fig. 18. The 2D bifurcation diagram of the system with $R_{\rm m}=2$ (a), 3 (b), and 4 (c) under the cost-sharing contract.

Figure 19 shows that the system changes with the increase of ϕ, when the system is in a chaotic state ($d_1=0.32$, $d_2=0.1$). It can be seen from Fig. 19(a) that although the system is in a chaotic state, the carbon emission reduction level shows an upward trend with the increase of ϕ. In addition, when ϕ is about 0.4, the system experiences the process from chaos to quasi period and then to chaos. It can be seen from Fig. 19(b) that the maximum Lyapunov exponent is less

than zero, when $\phi = 0.4$. It shows that the system enters into chaotic state again after a short period of stability.

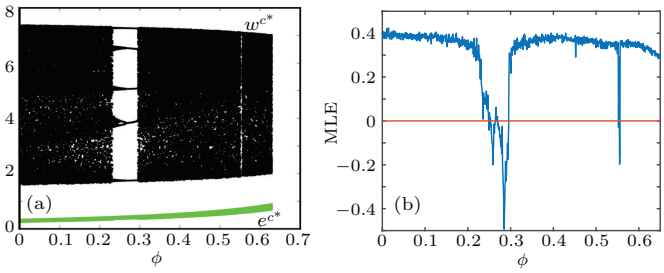


Fig. 19. (a) Bifurcation diagram varying with ϕ when $d_1 = 0.32$, $d_2 = 0.1$ and (b) maximum Lyapunov exponent (MLE) diagram.

7. Conclusions and suggestions

7.1. Conclusions

Under the carbon tax policy, this study establishes dynamic evolution game models of a low-carbon supply chain, respectively, under decentralized decision and centralized decision. We explore the influence of the manufacturer's risk-aversion behavior on the optimal decision-making, profits of enterprises and the supply chain, and also on complex dynamic characteristics of the supply chain system. In addition, we propose a carbon emission reduction cost-sharing contract to achieve Pareto improvement. The main conclusions are drawn as follows.

(i) Whether the risk aversion behavior of the manufacturer is considered or not, the carbon emission reduction level and the supply chain profit under decentralized decision are smaller than those under centralized decision. Comparing with the risk neutral model, the manufacturer's risk aversion behavior can narrow the gap between the supply chain profit under decentralized decision and that under centralized decision, but it will frustrate the manufacturer's enthusiasm for emission reduction. Under the carbon tax policy, the increase of carbon tax rate can encourage the manufacturer to improve the carbon emission reduction levels, and the increase is the biggest under the centralized decision-making situation. However, the increase of carbon tax rate will reduce the profits of the retailer, the manufacturer and the supply chain.

(ii) The increase of the manufacturer's risk tolerance and carbon tax will narrow the stable region of the system. Under this situation, the manufacturer should carefully adjust parameters to prevent the system from losing stability, especially the adjustment parameter for carbon emission reduction level. When the system is in a chaotic state, the increase of carbon tax rate makes the system show more complex dynamic characteristics. Under chaotic state, it is difficult for the manufacturer to make correct price decision and carbon emission reduction strategy for the next period. This damages the manufacturer profit, but increases the profits of the retailer and the supply chain.

(iii) The implementation of the cost-sharing contract can promote the manufacturer to improve the carbon emission reduction level, and increases the supply chain profit. With the increase of cost-sharing coefficient, the manufacturer's profit increases, and the retailer's profit first increases and then decreases. When certain conditions are met, the cost-sharing contract can improve the efficiency of the supply chain, and increase the system stability region compared with the scenarios in the centralized decision situation. However, the increase of the cost-sharing coefficient will reduce the system stability under the decentralized decision-making situation.

7.2. Suggestions

First, to achieve the goal of "carbon neutrality", both the government and enterprises should strive to achieve low-carbon supply chain. Although the carbon tax policy can encourage manufacturers to make efforts to increase carbon emission reduction level, it will lead their profits to decline and the stability of the supply chain system to decrease. Therefore, when formulating carbon tax rates, the government needs to weigh the relationship among carbon emission reduction target, economic benefit and supply chain stability.

Second, the manufacturer's risk aversion behavior and the increasing of carbon tax rate will reduce the stability of the supply chain system. The system in a chaotic state has anti-interference capability, which can adjust itself to an orderly state without external forces, and then jump back to disorder randomly. This process occurs repeatedly, and the dynamic characteristics of the system will become more and more complex. Therefore, enterprises still need to actively take control measures to make the chaotic system return to the normal trajectory. In addition, enterprises should strengthen the ability of them to cope with dynamic environmental and risk.

Third, retailers and manufacturers seek for maximizing their own benefits, which will lead to "double-marginal effect". Although manufacturers' risk aversion behavior can slow down this double-marginal effect, it will frustrate manufacturers' enthusiasm for emission reduction. By providing carbon emission reduction cost-sharing contracts, retailers can encourage manufacturers to improve carbon emission reduction level, increase sales, and ultimately improve the economic and environmental benefits of the supply chain. The emission reduction cost-sharing contract improves the system stability compared with the centralized decision. In addition, with the increase of the cost-sharing proportion, for retailers, the profits first increase and then decrease. For manufacturers, the profits continuously increase. Therefore, the cost-sharing proportion for the retailers to maximize profit is different from that for the manufacturers to maximize profit. In this situation, both parties need negotiating to determine the final cost-sharing proportion.

7.3. Future prospects

First, we study the case of information symmetry. Various cases of information asymmetry will be explored in the future, such as demand, cost, and risk avoidance information asymmetry. Second, this study takes a two-level supply chain composed of a single manufacturer and a single retailer as the research object. More complex supply chain structures such as three-level supply chain and competitive supply chain can be considered in the future. Third, this study mainly focuses on the same low-carbon product for research. In the future, research can be conducted on different product types or homogeneous products with different levels of carbon reduction. Finally, owing to the difficulty in collecting data of a specific industry, we do not use real data for simulation analysis. In the future, we will analyze a specific industry.

Acknowledgements

Project supported by the Social Science Planning Project of Chongqing, China (Grant No. 2022BS069), the Science and Technology Research Project of Chongqing Education Committee, China (Grant No. KJQN202201140), the National Social Science Foundation of China (Grant No. 20&ZD155), and the National Natural Science Foundation of China (Grant No. 72061003).

References

[1] Zhang J W 2021 *China Today* **70** 57
[2] Ho T H, Su X M and Wu Y Z 2014 *Prod. Oper. Manag.* **23** 161
[3] Qi Q, Zhang R Q and Bai Q G 2021 *Comput. Ind. Eng.* **162** 107783
[4] Sarkar S and Bhala S 2021 *Eur. J. Oper. Res.* **295** 140
[5] Wu M, Zhu S X and Teunter R H 2014 *Int. J. Prod. Econ.* **156** 13
[6] Zhu Y Y, Xia C Y, Wang Z and Chen Z Q 2022 *IEEE. Trans. Netw. Sci. Eng.* **9** 2450
[7] Li X P, Han W W, Yang W J, Wang J, Xia C Y, Li H J and Shi Y 2022 *Physica A* **594** 127055
[8] Liu M W, Liu Y Q and Fu Q L 2022 *Chin. J. Manag. Sci.* **30** 131
[9] Ji J N, Zhang Z Y and Yang L 2017 *J. Clean Prod.* **141** 852
[10] Wang J, Cheng X X, Jiang Y S and Dong Z M 2021 *Chin. J. Manag. Sci* **29** 128
[11] Huang D and Zhang J L 2021 *Chin. J. Manag. Sci.* **29** 57
[12] Lu Z N and Yi S S 2019 *Eco. Econ.* **35** 75
[13] Shi S Y, Sun J C and Deng F 2021 *Oper. Res. Manag. Sci.* **30** 1
[14] Wang H P, Yan X C, Zhao D and Li Y 2022 *Oper. Res. Manag. Sci. Forthcoming*
[15] Shang C, Guan Z and Mi L 2022 *Comput. Eng. Appl.* **58** 309
[16] Liu L and Li F T 2022 *J. Ind. Eng. Eng. Manag.* **36** 159
[17] Niederhoff J A and Kouvelis P 2019 *Eur. J. Oper. Res.* **277** 1060
[18] Chen X, Shum S and Simchi-Levi D 2014 *Prod. Oper. Manag.* **23** 379
[19] Li B and Jiang Y S 2019 *J. Retail. Consum. Serv.* **47** 104
[20] Bai Q G, Shi B Z and Xu J T 2019 *Syst. Eng.* **37** 86
[21] Yan N N, He X L and Liu Y 2019 *Omega* **88** 162
[22] Yan N N, Jin X Y, Zhong H C and Xu X 2020 *Int. J. Prod. Econ.* **227** 107665
[23] Xi X and Zhang J X 2020 *Chaos, Solitons and Fractals* **140** 110220
[24] Tian Y, Ma J H, Xie L, *et al.* 2020 *Chaos, Solitons and Fractals* **132** 109576
[25] Ma J H, Tian Y, Xu T T, Koivumaki T and Xu Y Q 2022 *Chaos, Solitons and Fractals* **160** 112131
[26] Li Q X, Shi M N and Huang Y M 2019 *Int J. Environ. Res. Publ. Health* **16** 1978
[27] Xie L, Ma J H and Han H S 2017 *Appl. Math. Model.* **55** 484
[28] Yang H X and Chen W B 2018 *Omega* **78** 179
[29] Choi T M, Ma C, Shen B and Sun Q 2019 *Omage* **88** 150
[30] Ding J F, Chen W D and Fu S S 2022 *Syst. Eng. Theory Pract.* **42** 637