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Floquet dynamical quantum phase transitions in transverse XY spin chains under periodic kickings

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Floquet dynamical quantum phase transitions (DQPTs), which are nonanalytic phenomena recuring periodically in time-periodic driven quantum many-body systems, have been widely studied in recent years. In this article, the Floquet DQPTs in transverse XY spin chains under the modulation of δ-function periodic kickings are investigated. We analytically solve the system, and by considering the eigenstate as well as the ground state as the initial state of the Floquet dynamics, we study the corresponding multiple Floquet DQPTs emerged in the micromotion with different kicking moments. The rate function of return amplitude, the Pancharatnam geometric phase and the dynamical topological order parameter are calculated, which consistently verify the emergence of Floquet DQPTs in the system.

Keywords: Floquet dynamical quantum phase transitions, periodic kick, XY spin chains

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1. Introduction

Quantum phase transitions (QPTs) have attracted a great deal of attention for decades. Different from traditional phase transitions, QPTs are solely driven by quantum fluctuations,[1] and in the context of quantum information theory,[2] such quantum phenomena have been deeply investigated and many essential properties have been revealed.[3–5] In recent years, dynamical quantum phase transitions (DQPTs), which have gained much attention as an extended study of QPTs to the non-equilibrium case, have been theoretically investigated in different physical dimensions, and huge progress has been provoked by the experimental observations,[6–21] for example, quenching dynamics between different Floquet topological phases is simulated by quantum walking dynamics, where DQPTs and underlying dynamical topological order parameters (DTOPs) are characterized by interference-based measurement experiments.[10] DQPTs concern the nonanalytic behaviors in many-body systems after a sudden change of the parameters, so-called quantum quench,[11] and appear whenever the time evolved state becomes orthogonal to the initial state, which are often measured by the Fisher zeros in the context of Loschmidt amplitudes.[12,22–24] Furthermore, DQPTs have also been extended to reveal the changes of topological invariants for quenched topological quantum systems.[25–27]

The Floquet theory serves as a highly controllable and versatile tool in quantum control, and has revealed many nontrivial effects in periodic driven quantum systems,[28–38] especially the topological quantum system.[29–31,35,38–45] Floquet DQPTs, which exhibit exotic new properties in many non-equilibrium quantum many-body systems beyond quenches, have been proposed in time periodically driven Floquet quantum systems.[18] Different from conventional DQPTs usually observable only in transient time scales, the non-analytic cusps in the return rate of Floquet DQPTs are found to recur periodically in time with a non-decaying global profile.[46–50] As an extended research area of DQPTs, richer non-equilibrium phases and intrinsic features have been revealed in time-periodic driven quantum systems and experimentally verified for Floquet DQPTs, and the related problems have become one of the most attractive areas of developing non-equilibrium research topics. Research of Floquet DQPTs has been extended to various quantum systems, such as the spin chain model,[47,49] synchronized periodic driving systems,[51] dissipative systems,[52] periodically driven topological systems,[53,54] and periodic quenching topological systems.[50] Many non-equilibrium phases have been revealed in time-periodic driven quantum systems and experimentally verified.[18,44–46,54]

In order to deeply understand the essential nature of Floquet DQPTs, some exactly solvable periodically time-dependent models have also been proposed, and it has been shown that, when a periodical driven system is initialized in a periodic quenched Floquet topological phase, the Floquet system can also show topologically nontrivial signatures.[50] As we know, δ-function periodic kicking is another special periodic driving and may provide an interesting analytical solvable model, which may also change the system and induce new phenomena, for instance, periodic behavior may result in a new phase diagram in a topological system, changes the critical point of the quantum phase transition, etc.[55–58] It is an interesting topic to consider effect of periodic kickings in Floquet systems with DQPTs.

The exactly solvable transverse XY spin chain[59] has

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been widely used to investigate quantum phase transitions by many kinds of perspectives from quantum information theory,[60–71] such as the Loschmidt echo,[72] ground state fidelity,[73] quantum entanglement,[74,75] and geometric phase.[76–80] The XY spin chain model has also been experimentally realized and studied, such as the relation between geometric phase and the ground state quantum phase transitions,[81] the geometric phase of the ground state of the three-self-spin XY model using nuclear magnetic resonance (NMR) technology.[82] A periodically driven XY spin chain has been proposed as an analytically solvable model to study changes of quantum phase transitions. Floquet DQPTs of pure and mixed states in an extended XY model driven periodically by a staggered magnetic field are also investigated.[47] Motivated by the above studies, in this paper we propose an analytically solvable periodic driven model, which consists of the usual transverse XY spin chain under periodic kicking, to study the Floquet DQPTs emerging in such a system. The critical properties and phase diagram of the model will be changed by the periodic kicking, by both the kicking strength and kicking period. For each micromotion of the periodic driven, we will show that the Floquet DQPTs will appear, which can be verified by the Loschmidt echo, the geometric phase, or even the dynamical topological order parameter. It is a simple and analytically solvable Floquet system and we believe that the Floquet DQPTs in the periodic kicking systems are also valuable used to reveal the essential mechanism of the intrinsic features of Floquet DQPTs.

The remainder of the paper is arranged as follows. In Section 2, we introduce the exact solution of the Floquet effective Hamiltonian of the periodically kicked XY spin chain. In Section 3, we study the Floquet DQPTs emerging in the micromotion of the Floquet dynamics, by considering the eigenstate and the ground state of the effective system as initial states respectively, and discuss the Floquet DQPTs of the system from different physical quantities characterization. Finally, we summarize our study in Section 4.

2. Transverse XY spin chains under periodic kickings

In this section, we briefly give the solution of our model. Consider the transverse XY spin chain under the periodic kicking, whose Hamiltonian can be expressed as

$$H(t) = H_0 + H_1(t), \tag{1}$$

where H_0 is the Hamiltonian for original XY spin chains, i.e.,

$$H_0 = -\sum_l \left(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z \right), \tag{2}$$

and $H_1(t)$ is the driving Hamiltonian, which consists of periodic kicking along z-direction,

$$H_1(t) = -\alpha\sum_l \sigma_l^z \sum_{m=-\infty}^{\infty} \delta(t - mT). \tag{3}$$

In the above expressions, σ_l^x, σ_l^y and σ_l^z are the usual Pauli operators at l-site in the chain; λ and γ are the strength of the transverse field and the anisotropy parameter of the XY coupling; α is the kicking strength; and T is the driving period. We will always consider the periodic boundary condition for our system.

The Hamiltonian of the transverse XY spin chain $H(t)$ can be mapped to a quasi-Fermion system by the Jordan–Wigner transformation,

$$\begin{aligned}
&\sigma_l^z = a_l a_l^\dagger - a_l^\dagger a_l,\\
&\frac{1}{2}(\sigma_l^x + \mathrm{i}\sigma_l^y) = \prod_{j<l}(1-2a_j^\dagger a_j)a_l,\\
&\frac{1}{2}(\sigma_l^x - \mathrm{i}\sigma_l^y) = \prod_{j<l}(1-2a_j^\dagger a_j)a_l^\dagger,
\end{aligned} \tag{4}$$

and by the discrete Fourier transformation in the case of periodic boundary condition,

$$a_{k'} = \frac{1}{\sqrt{N}}\sum_l a_l \exp\left(\frac{-2\pi k' l}{N}\right), \tag{5}$$

one can obtain the quasi-Fermion form Hamiltonian

$$H(t) = \sum_k (a_k^\dagger \ \ a_{-k}) h_k(t) \begin{pmatrix} a_k \\ a_{-k}^\dagger \end{pmatrix}, \tag{6}$$

where the kernel Hamiltonian is $h_k(t) = h_{0,k} + h_{1,k}(t)$, which is defined as

$$\begin{aligned}
&h_{0,k} = (\lambda - \cos k)\sigma^z - \gamma \sin k \sigma^y,\\
&h_{1,k}(t) = \alpha\sigma^z \sum_{m=-\infty}^{\infty} \delta(t - mT).
\end{aligned} \tag{7}$$

In the above expression, $k = 2\pi k' l/N$ is used for simplicity; σ^x, σ^y, and σ^z are the Pauli operator for the two-level decoupled pseudo-spin subspaces. The above Hamiltonian H_0 can be diagonalized as $H_0 = \sum_k \varepsilon_k c_k^\dagger c_k$, by defining $\tan\theta_k = (\gamma \sin k)/(\lambda - \cos k)$, and the following Bogoliubov transformation

$$\begin{aligned}
&c_k^\dagger = \cos\frac{\theta_k}{2}a_k^\dagger + \mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}\sin\frac{\theta_k}{2}a_{-k},\\
&c_k = \cos\frac{\theta_k}{2}a_k + \mathrm{e}^{\mathrm{i}\frac{\pi}{2}}\sin\frac{\theta_k}{2}a_{-k}^\dagger.
\end{aligned} \tag{8}$$

Assume that the δ-pulses kick is added at moment T_1 for each period, then each period of time will be divided into two parts by the kick, $T = T_1 + T_2$. However, different evolutions of the state can always be obtained by a unitary transformation, i.e.,

$$U(T) = \prod_k \mathrm{e}^{-\mathrm{i}h_{0,k}T_2}\mathrm{e}^{-\mathrm{i}\alpha\sigma^z}\mathrm{e}^{-\mathrm{i}h_{0,k}T_1}$$

$$
\begin{aligned}
&= \prod_k \mathrm{e}^{-\mathrm{i}h_{0,k}T_2}(\mathrm{e}^{-\mathrm{i}\alpha\sigma^z}\mathrm{e}^{-\mathrm{i}h_{0,k}T})\mathrm{e}^{\mathrm{i}h_{0,k}T_2} \\
&= \prod_k \mathrm{e}^{-\mathrm{i}h_{0,k}T_2}(\mathrm{e}^{-\mathrm{i}\tilde{h}_{\mathrm{eff},k}T})\mathrm{e}^{\mathrm{i}h_{0,k}T_2} \\
&= \prod_k \mathrm{e}^{-\mathrm{i}h_{\mathrm{eff},k}T}, \qquad (9)
\end{aligned}
$$

where we have defined two forms of the unitary-transform-equivalent effective Hamiltonian, i.e., $\tilde{h}_{\mathrm{eff},k}$ is the effective Hamiltonian defined by assuming the δ-kick added at the end of each period, while the effective Hamiltonian $h_{\mathrm{eff},k}$ is defined for a general case of the kicking time. The above unitary operator determines the dynamics at time $t = nT$, $n = 0, 1, 2, \ldots$ of the system. For many more periods, the system will only obtain an extra phase factor $\exp(-\mathrm{i}h_{\mathrm{eff},k}nT)$.

Using the definition (9) for the effective Hamiltonian by one period of evolution, we can analytically obtain $\tilde{h}_{\mathrm{eff},k}$ as

$$
\tilde{h}_{\mathrm{eff},k} = \xi_k \begin{pmatrix} \tilde{n}_{k,z} & \tilde{n}_{k,x} - \mathrm{i}\tilde{n}_{k,y} \\ \tilde{n}_{k,x} - \mathrm{i}\tilde{n}_{k,y} & -\tilde{n}_{k,z} \end{pmatrix}, \qquad (10)
$$

where we have defined

$$
\begin{aligned}
&\varepsilon_k = \sqrt{(\lambda - \cos k)^2 + \gamma^2 \sin^2 k}, \\
&\xi_k = \frac{1}{T}\arccos\left[\cos\alpha\cos(\varepsilon_k T) - \frac{\lambda - \cos k}{\varepsilon_k}\sin\alpha\sin(\varepsilon_k T)\right], \qquad (11)
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{n}_{k,x} &= \frac{\frac{\gamma \sin k}{\varepsilon_k}\sin\alpha\sin(\varepsilon_k T)}{\sin(\xi_k T)}, \\
\tilde{n}_{k,y} &= \frac{-\frac{\gamma \sin k}{\varepsilon_k}\cos\alpha\sin(\varepsilon_k T)}{\sin(\xi_k T)}, \\
\tilde{n}_{k,z} &= \frac{\sin\alpha\cos(\varepsilon_k T) + \frac{\lambda - \cos k}{\varepsilon_k}\cos\alpha\sin(\varepsilon_k T)}{\sin(\xi_k T)}. \qquad (12)
\end{aligned}
$$

Then, the effective Hamiltonian $h_{\mathrm{eff},k} = \xi_k \boldsymbol{\sigma}\cdot\boldsymbol{n}_k$ with $\boldsymbol{n}_k = (n_{k,x}, n_{k,y}, n_{k,z})$ will have the same eigenvalues with $\tilde{h}_{\mathrm{eff},k}$, and $\boldsymbol{n}_k$ reads

$$
\begin{aligned}
n_{k,x} &= \cos(2\varepsilon_k T_2)\tilde{n}_{k,x} + \sin(2\varepsilon_k T_2)\left[\frac{-\gamma \sin k}{\varepsilon_k}\tilde{n}_{k,z}\right. \\
&\quad \left. - \frac{\lambda - \cos k}{\varepsilon_k}\tilde{n}_{k,y}\right], \\
n_{k,y} &= \cos(2\varepsilon_k T_2)\tilde{n}_{k,y} + \sin(2\varepsilon_k T_2)\left[\frac{(\lambda - \cos k)}{\varepsilon_k}\tilde{n}_{k,x}\right] \\
&\quad + 2\sin^2(\varepsilon_k T_2)\frac{-\gamma \sin k}{\varepsilon_k}\left[\frac{-\gamma \sin k}{\varepsilon_k}\tilde{n}_{k,y} + \frac{\lambda - \cos k}{\varepsilon_k}\tilde{n}_{k,z}\right], \\
n_{k,z} &= \cos(2\varepsilon_k T_2)\tilde{n}_{k,z} + \sin(2\varepsilon_k T_2)\left[\frac{\gamma \sin k}{\varepsilon_k}\tilde{n}_{k,x}\right] \\
&\quad + 2\sin^2(\varepsilon_k T_2)\frac{\lambda - \cos k}{\varepsilon_k}\left[\frac{-\gamma \sin k}{\varepsilon_k}\tilde{n}_{k,y}\right. \\
&\quad \left. + \frac{(\lambda - \cos k)}{\varepsilon_k}\tilde{n}_{k,z}\right]. \qquad (13)
\end{aligned}
$$

The effective Hamiltonian can also be diagonalized as $H_{\mathrm{eff}} = \sum_k \xi_k b_k^\dagger b_k$. Defining $\tan\Phi_k = n_y/n_x$ and $\tan\Theta_k = \sqrt{(n_{k,x}^2 + n_{k,y}^2)}/n_{k,z}$, we have the following Bogoliubov transformation:

$$
\begin{aligned}
b_k^\dagger &= \cos\frac{\Theta_k}{2}a_k^\dagger + \mathrm{e}^{\mathrm{i}\Phi_k}\sin\frac{\Theta_k}{2}a_{-k}, \\
b_k &= \cos\frac{\Theta_k}{2}a_k + \mathrm{e}^{-\mathrm{i}\Phi_k}\sin\frac{\Theta_k}{2}a_{-k}^\dagger. \qquad (14)
\end{aligned}
$$

One can easily verify that, in the case of $\alpha = 0$ and $T_2 = 0$, the above effective Hamiltonian can be simplified to the original XY spin-chain Hamiltonian H_0.

The kicking strength α and kicking period T will change the effective Hamiltonian of the system, and different time intervals of the periodic δ-kick added on the system will result in different micro-motions, giving possible chances for the Floquet DQPTs in each period of time evolution. In the present study, we only consider one periodic micromotion during the stationary evolution of the system, i.e, the initial state for each period keeps the same, except for a phase factor.

Assume that the initial state for each period is $|\psi_k(0)\rangle$ and the periodic δ-kicks are added during arbitrary time T_1 in every period, i.e., each period of evolution have been divided into two parts, $0 < t < T_1$ and $T_1 < t < T$, $(T = T_1 + T_2)$, then the evolution of the system in one period can be expressed as $|\psi_k(t)\rangle = U_k(t)|\psi_k(0)\rangle$, with the time-dependent evolution operator defined as

$$
U_k(t) = \begin{cases} \mathrm{e}^{-\mathrm{i}h_{0,k}t}, & 0 < t < T_1. \\ \mathrm{e}^{-\mathrm{i}h_{0,k}(t-T_1)}\mathrm{e}^{-\mathrm{i}\alpha\sigma^z}\mathrm{e}^{-\mathrm{i}h_{0,k}T_1}, & T_1 < t < T. \end{cases} \qquad (15)
$$

3. Floquet DQPTs in the periodically kicked *XY* spin chain

In this section, we consider the eigenstate as well as the ground state of the effective Hamiltonian as the initial state of the system, which ensures the same evolution for each period except a phase factor, and we show that multiple Floquet DQPTs can emerge in the micromotion for each period of the Floquet dynamics. Some physical quantities have been used to characterize the Floquet DQPTs in the periodically kicked XY-spin system, e.g., the rate function of return amplitude, the Pancharatnam geometric phase and the dynamical topological order parameter have been calculated, and these concepts are briefly introduced in Appendix.

3.1. Eigenstate as the initial state

Consider that the initial state for each period of the Floquet system is given by the eigenstate $|\chi_{\pm,k}\rangle$ of the effective Hamiltonian, i.e.,

$$
\begin{aligned}
|\chi_{+,k}\rangle &= \frac{1}{\sqrt{2(1 - n_{k,z})}}\begin{pmatrix} n_{k,x} - \mathrm{i}n_{k,y} \\ 1 - n_{k,z} \end{pmatrix}, \\
|\chi_{-,k}\rangle &= \frac{1}{\sqrt{2(1 + n_{k,z})}}\begin{pmatrix} n_{k,x} - \mathrm{i}n_{k,y} \\ -(1 + n_{k,z}) \end{pmatrix}, \qquad (16)
\end{aligned}
$$

which satisfies

$$U_k(T)|\chi_{\pm,k}\rangle = \mathrm{e}^{\mp \mathrm{i}(\xi_k T)}|\chi_{\pm,k}\rangle, \tag{17}$$

or equally $h_{\mathrm{eff},k}|\chi_{\pm,k}\rangle = \pm\xi_k|\chi_{\pm,k}\rangle$, then the state for any momentum k can also be obtained as

$$|\psi_{\pm,k}(t)\rangle = U_k(t)|\chi_{\pm,k}\rangle. \tag{18}$$

By Eq. (15), we can obtain the return amplitude (see Eq. (A1) in Appendix), i.e.,

$$G_k(t) = \cos(\varepsilon_k t) - \mathrm{i}\left(\frac{-\gamma\sin k}{\varepsilon_k} n_{k,y} + \frac{\lambda - \cos k}{\varepsilon_k} n_{k,z}\right)\sin(\varepsilon_k t) \tag{19}$$

for $0 < t < T_1$, and

$$G_k(t) = \mathrm{e}^{-\mathrm{i}(\xi_k T)}\Big\{\cos[\varepsilon_k(t-T)] - \mathrm{i}\left(\frac{-\gamma\sin k}{\varepsilon_k} n_{k,y} + \frac{\lambda - \cos k}{\varepsilon_k} n_{k,z}\right)\sin[\varepsilon_k(t-T)]\Big\} \tag{20}$$

for $T_1 < t < T$. We numerically simulate the quantum phase transition properties of the periodic kicking on the spin chain from the two aspects of the kicking strength and time. In the case of $G_k(t) = 0$, both the real part and imaginary part should be zero, which will result in the following condition:

$$\begin{cases} (\varepsilon_k t) = \frac{2n-1}{2}\pi, \ (n \in \mathbb{Z}), \\ \frac{-\gamma\sin k}{\varepsilon_k} n_{k,y} + \frac{\lambda - \cos k}{\varepsilon_k} n_{k,z} = 0, \end{cases} \tag{21}$$

for $0 < t < T_1$, and

$$\begin{cases} \varepsilon_k(t-T) = \frac{2n-1}{2}\pi, \ (n \in \mathbb{Z}), \\ \frac{-\gamma\sin k}{\varepsilon_k} n_{k,y} + \frac{\lambda - \cos k}{\varepsilon_k} n_{k,z} = 0, \end{cases} \tag{22}$$

for $T_1 < t < T$. The equations can be used to determine the critical momentum k_c and the corresponding critical time t_c.

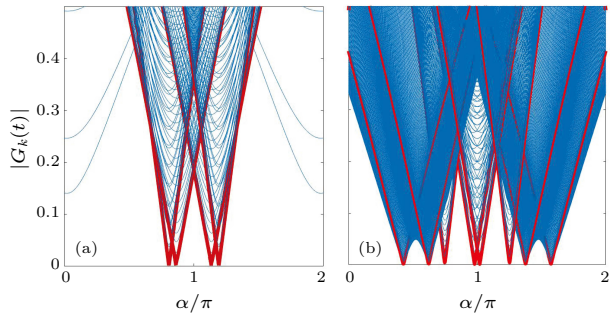


Fig. 1. The return amplitude $|G_k(t)|$ of the system for momenta $k \in (-\pi, \pi)$, $\lambda = 0.5$, $t \in (0, 2\pi)$, $T_1 = \pi$ and $T = 2\pi$: (a) $\gamma = 0.15$, (b) $\gamma = 1$.

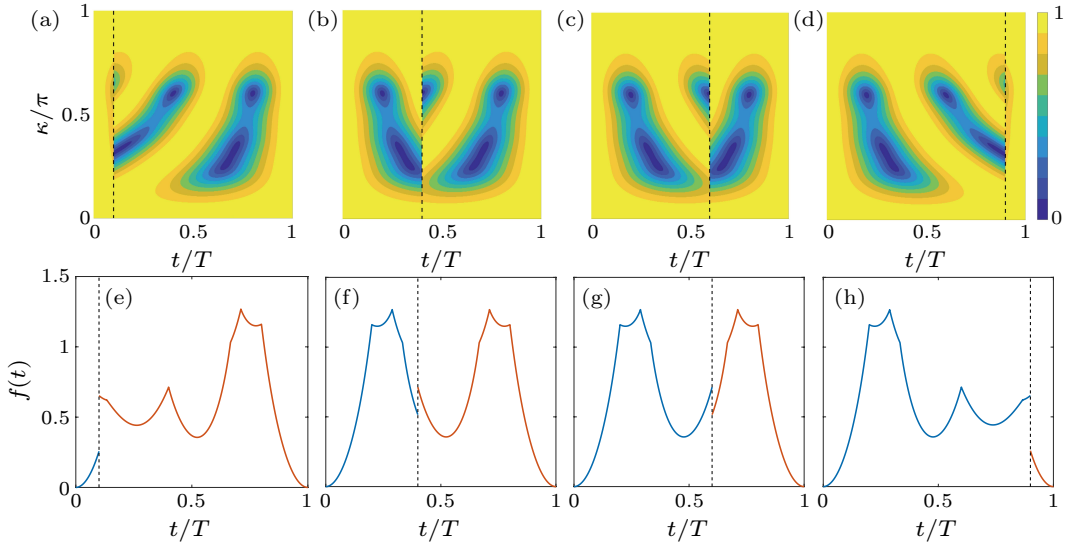


Fig. 2. [(a)–(d)] The return amplitude $|G_k(t)|$ of the system for momenta $k \in (-\pi, \pi)$, $\gamma = 1$, $\lambda = 0.5$, $\alpha = 1.5\pi$ and $T = 2\pi$: (a) $T_1/T = 0.1$, (b) $T_1/T = 0.4$, (c) $T_1/T = 0.6$, (d) $T_1/T = 0.9$. [(e)–(h)] The rate function of return amplitude $f(t)$ with the kicking time for one period $T \in (0, 2\pi)$: (e) $T_1/T = 0.1$, (f) $T_1/T = 0.4$, (g) $T_1/T = 0.6$, (h) $T_1/T = 0.9$. The black dotted line represents the boundary line of the periodic kicking time $t = T_1/T$.

The anisotropy parameter γ characterizes the transformation from the Ising chain to the XX spin chain of the system, and it may affect the properties of the Floquet DQPTs. Figure 1 shows the numerical results of the return amplitude versus the kicking strength, where we have set $\alpha \in (0, 2\pi)$ and $T_1/T = 0.5$. From the above analytical expressions and the numerical simulations, we can clearly find that with the increasing $\gamma \in (0, 1]$, more zero points appear, and the DQPTs may appear several times with the kicking strength varies. Thus, a feasible strategy for researching richer Floquet DQPTs can be provided by the anisotropy of the spin chain.

The δ-kicks may be added at arbitrary moment in each period, which will result in different regular patterns of Floquet DQPTs. Figures 2(a)–2(d) shows the return amplitude $|G_k(t)|$ with $\gamma = 1$, $\lambda = 0.5$ and different ratios of T_1 to T_2. We can find that the δ-function kick added in each period may hinder or postpone the emergence of the Floquet DQPTs points. Figures 2(a)–2(d) also show a clear regularity of symmetry

with time distribution of the two stages T_1 and T_2 before or after the kick.

The explicit expression of the return rate function $f(t)$ (see Appendix) of the periodical kicking system can be obtained by definition

$$f(t) = -\int_{-\pi}^{\pi} \frac{\mathrm{d}k}{2\pi} \ln\Big\{\cos^2(\varepsilon_k t) + \Big(\frac{-\gamma \sin k}{\varepsilon_k} n_{k,y} + \frac{\lambda - \cos k}{\varepsilon_k} n_{k,z}\Big)^2 \sin^2(\varepsilon_k t)\Big\}, \tag{23}$$

in the case of $0 < t < T_1$, and

$$f(t) = -\int_{-\pi}^{\pi} \frac{\mathrm{d}k}{2\pi} \ln\Big\{\cos^2[\varepsilon_k (t-T)] + \Big(\frac{-\gamma \sin k}{\varepsilon_k} n_{k,y} + \frac{\lambda - \cos k}{\varepsilon_k} n_{k,z}\Big)^2 \sin^2[\varepsilon_k (t-T)]\Big\}, \tag{24}$$

in the case of $T_1 < t < T$.

Figures 2(e)–2(h) show the rate function $f(t)$ with different kicking times for $T_1/T = 0.1, 0.4, 0.6, 0.9$, where we have set $t \in (0, 2\pi)$, $\gamma = 1$, $\lambda = 0.5$, and $T = 2\pi$ in the calculation. The rate function $f(t)$ consistently shows the mutation inflection points with the return amplitude $|G_k(t)|$. In each period, mutation inflection points will emerge if the time distribution is long enough before or after the kick, which indicates the same principle for the emergence of Floquet DQPTs. In the thermodynamic limit, the rate function of return amplitude (A1), and its time derivative becomes a non-analytic time function at the critical time, and the rate function graphs are consistent with the return amplitude shown in Figs. 2(a)–2(d), respectively. The mutation inflection points of $f(t)$ more intuitively reflect the critical time of Floquet DQPTs, which are consistent with the former results. Figures 2(e)–2(h) can also capture the regularity of symmetry with time distribution of the two stages T_1 and T_2 before or after the kick.

The Floquet DQPTs of the system can also be characterized by the Pancharatnam geometric phase (see Eq. (A3) in Appendix), and the geometric phase of the system can be expressed in detail as

$$\phi_k^{\mathrm{geo}}(t) = -\mathrm{i} \ln\Big[\frac{G_k(t)}{|G_k(t)|}\Big] + \Big(\frac{-\gamma \sin k}{\varepsilon_k} n_{k,y} + \frac{\lambda - \cos k}{\varepsilon_k} n_{k,z}\Big) t, \tag{25}$$

in the case of $0 < t < T_1$, the first stage before kicking, and

$$\phi_k^{\mathrm{geo}}(t) = -\mathrm{i} \ln\Big[\frac{G_k(t)}{|G_k(t)|}\Big] + \Big(\frac{-\gamma \sin k}{\varepsilon_k} n_{k,y} + \frac{\lambda - \cos k}{\varepsilon_k} n_{k,z}\Big) t + \alpha n_{k,z}, \tag{26}$$

in the case of $T_1 < t < T$, the second stage after kicking.

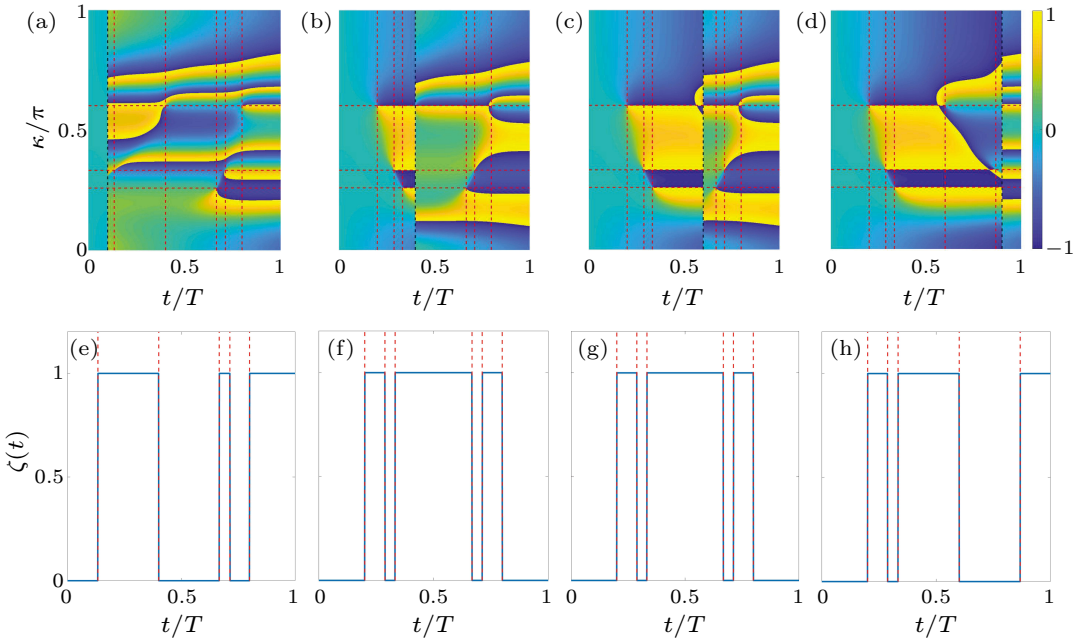


Fig. 3. [(a)—(d)] The Pancharatnam geometric phases $\phi_k^{\mathrm{geo}}(t)$ with different kicking time distributions for fixed period $T \in (0, 2\pi)$: (a) $T_1/T = 0.1$, (b) $T_1/T = 0.4$, (c) $T_1/T = 0.6$, (d) $T_1/T = 0.9$. [(e)–(h)] The order parameter $\zeta(t)$ with different kicking time distributions for fixed period $T \in (0, 2\pi)$: (e) $T_1/T = 0.1$, (f) $T_1/T = 0.4$, (g) $T_1/T = 0.6$, (h) $T_1/T = 0.9$. Other parameters of all pictures are set as $\gamma = 1$, $\lambda = 0.5$, $\alpha = 1.5\pi$, and $T = 2\pi$. The black dotted line represents the boundary line $t = T_1/T$ that separates the two kicking periods, and the red dotted line represents the the critical time t_c of Floquet DQPTs.

Figure 3 shows the geometric phase for one period of the periodically kicked system, $\gamma = 1$, $\lambda = 0.5$. We have also set the parameters of all figures as $T = 2\pi$, and set $T_1/T = 0.1$, 0.4, 0.6, 0.9 for Figs. 3(a)–3(d), respectively. For each figure, the intersection of the horizontal red dashed line and the vertical axis corresponds to the critical momentum k_c, and the intersection of the vertical red dashed line and the horizontal axis represents the critical time t_c. We have set $\phi_k^{\mathrm{geo}}(t)$ in the

range from $-\pi$ to π, neglecting the difference of 2π for the geometric phase. The Floquet DQPTs have been captured by the geometric phase, showing a jump from $-\pi$ to π for a period $T \in (0, 2\pi)$. It clearly shows that the critical time for the system to present Floquet DQPTs is consistent with the previously studied $G_k(t)$ and $f(t)$.

The dynamical topological order parameter $\zeta(t)$ with different kicking time distributions for fixed period $T \in (0, 2\pi)$, $\gamma = 1$, $\lambda = 0.5$ have also been shown in Fig. 3. The parameters of these figures have been set as $T = 2\pi$, and $T_1/T = 0.1$, 0.4, 0.6, 0.9 for Figs. 3(e)–3(h), respectively. The dynamical topological order parameter $\zeta(t)$ exhibits a perfect quantization as a function of time between changing times of two DQPTs. The change times are exactly corresponding to the critical points shown in Fig. 2, which means that the abrupt jumps in the dynamic topological order parameter $\zeta(t)$ consistently reflect the onset of Floquet DQPTs. In the absence of the Floquet topology, there is no direct correlation between Floquet DQPTs, fixed points, and dynamic topological order parameters, which are closely connected when $\zeta(t) = 0$, which indicate that the emergence of dynamic topological phenomena is protected by the Floquet topology.

3.2. Ground state as the initial state

Now, we consider that the initial state of the system is prepared as the ground state of the effective Hamiltonian, $|\psi_k(0)\rangle = |G_{\rm eff}\rangle$, which satisfies

$$b_k|G_{\rm eff}\rangle = 0. \tag{27}$$

and for each period, the analysis result of the return amplitude can be expressed as

$$\begin{aligned} G(t) &= \langle G_{\rm eff}|{\rm e}^{-{\rm i}H_0 t}|G_{\rm eff}\rangle, \quad (0 < t < T_1), \\ G(t) &= \langle G_{\rm eff}|{\rm e}^{-{\rm i}H_0(t-T)}{\rm e}^{-{\rm i}H_{\rm eff}T}|G_{\rm eff}\rangle \\ &= {\rm e}^{-{\rm i}\Sigma_k(\xi_k T)}\langle G_{\rm eff}|{\rm e}^{-{\rm i}H_0(t-T)}|G_{\rm eff}\rangle, \quad (T_1 < t < T). \end{aligned} \tag{28}$$

According to Eqs. (8) and (14), one can obtain the analysis result of $G(t)$,

$$\begin{aligned} G(t) &= \prod_k \Big[1 + ({\rm e}^{-{\rm i}(\varepsilon_k t)} - 1)\Big(\cos^2\frac{\theta_k}{2}\sin^2\frac{\Theta_k}{2} \\ &\quad + \sin^2\frac{\theta_k}{2}\cos^2\frac{\Theta_k}{2} + \frac{1}{2}\sin\theta_k\sin\Theta_k\sin\Phi_k\Big)\Big], \\ &\quad (0 < t < T_1), \\ G(t) &= {\rm e}^{-{\rm i}\Sigma_k(\xi_k T)}\prod_k \Big[1 + ({\rm e}^{-{\rm i}\varepsilon_k(t-T)} - 1)\Big(\cos^2\frac{\theta_k}{2}\sin^2\frac{\Theta_k}{2} \\ &\quad + \sin^2\frac{\theta_k}{2}\cos^2\frac{\Theta_k}{2} + \frac{1}{2}\sin\theta_k\sin\Theta_k\sin\Phi_k\Big)\Big], \\ &\quad (T_1 < t < T). \end{aligned} \tag{29}$$

and for the condition $G(t) = 0$, i.e., the DQPT appears at the time instances at which at least one factor becomes zero, whenever there is a mode k satisfying

$$\begin{aligned} &1 + ({\rm e}^{-{\rm i}(\varepsilon_k t)} - 1)\Big[\Big(\cos\frac{\theta_k}{2}\sin\frac{\Theta_k}{2}\Big)^2 + \Big(\sin\frac{\theta_k}{2}\cos\frac{\Theta_k}{2}\Big)^2 \\ &+ \frac{1}{2}\sin\theta_k\sin\Theta_k\sin\Phi_k\Big] = 0, \quad (0 < t < T_1), \\ &1 + ({\rm e}^{-{\rm i}\varepsilon_k(t-T)} - 1)\Big[\Big(\cos\frac{\theta_k}{2}\sin\frac{\Theta_k}{2}\Big)^2 + \Big(\sin\frac{\theta_k}{2}\cos\frac{\Theta_k}{2}\Big)^2 \\ &+ \frac{1}{2}\sin\theta_k\sin\Theta_k\sin\Phi_k\Big] = 0, \quad (T_1 < t < T). \end{aligned} \tag{30}$$

In the case of $0 < t < T_1$, in order to find the Fisher zeros on the complex plane, one can extend ${\rm i}t$ to $z = \tau + {\rm i}t$, which means

$${\rm e}^{(\tau+{\rm i}t)\varepsilon_k} = -\Big[\Big(\Big(\cos\frac{\theta_k}{2}\sin\frac{\Theta_k}{2}\Big)^2 + \Big(\sin\frac{\theta_k}{2}\cos\frac{\Theta_k}{2}\Big)^2 + \frac{1}{2}\sin\theta_k\sin\Theta_k\sin\Phi_k\Big)\Big]^{-1} + 1, \tag{31}$$

and thus we obtain

$$\tau + {\rm i}t = \frac{1}{\varepsilon_k}\ln\Big[\Big(\cos\frac{\theta_k}{2}\sin\frac{\Theta_k}{2}\Big)^2 + \Big(\sin\frac{\theta_k}{2}\cos\frac{\Theta_k}{2}\Big)^2 + \frac{1}{2}\sin\theta_k\sin\Theta_k\sin\Phi_k\Big] + {\rm i}\frac{(2n-1)\pi}{\varepsilon_{\rm k}}, \tag{32}$$

similarly, in the case of $T_1 < t < T$, one can obtain

$$\tau + {\rm i}(t-T) = \frac{1}{\varepsilon_k}\ln\Big[\Big(\cos\frac{\theta_k}{2}\sin\frac{\Theta_k}{2}\Big)^2 + \Big(\sin\frac{\theta_k}{2}\cos\frac{\Theta_k}{2}\Big)^2 + \frac{1}{2}\sin\theta_k\sin\Theta_k\sin\Phi_k\Big] + {\rm i}\frac{(2n-1)\pi}{\varepsilon_k}. \tag{33}$$

Then, we will simulate the quantum phase transition properties of the periodic kick on the spin chain from the two aspects of the kicking strength and time.

Firstly, we consider the effect of the kicking strength to analyze the return amplitude of Floquet DQPTs. The returned amplitude of the system changes with α for the driving period $T_1/T = 0.5$, as shown in Fig. 4, with the value range of α being $(0, 2\pi)$. From the above exact calculations and the numerical simulations, we can conclude that the DQPTs appear as the kicking strength varies. This provides a feasible strategy for studying the DQPTs.

Figures 5(a)–5(d) show the numerical rate function, i.e., Eqs. (32) and (33), where we have set $\gamma = 1$, $\lambda = 0.5$, and $T_1/T = 0.1$, 0.4, 0.6, 0.9. The kick in each period can also hinder or postpone the emergence of the Floquet DQPTs points. The regularity of symmetry with time distribution of the two stages T_1 and T_2 before or after the kick have also been shown in these figures. Several phase transition points may emerge in the rate function $f(t)$ for sufficiently long time of the stage separated by the kick.

The dynamical topological order parameter $\zeta(t)$ can also display the Floquet DQPTs of the system, as shown in Fig. 5, where we have set $\gamma = 1$, $\lambda = 0.5$ in the calculation. We have considered different kicking time distributions for the fixed period $T \in (0, 2\pi)$, and set $T_1/T = 0.1$, 0.4, 0.6, 0.9 for Figs. 5(e)–5(h), respectively. The rate function of return

amplitude $f(t)$, as well as the dynamic topological order parameter $\zeta(t)$, consistently indicates the Floquet DQPTs points. The abrupt jumps in the dynamic topological order parameter $\zeta(t)$ also correctly reflect the onset of Floquet DQPTs, indicating that the emergence of dynamic topological phenomena is also protected by the Floquet topology.

From the results of the former two examples, we can find that periodic kickings added on the spin chain system can also induce and detune the Floquet DQPTs, and different physical quantities can consistently characterize the emergence of Floquet DQPTs in the system. Since both the eigenstate and the ground state of the effective Hamiltonian as the initial state of evolution can ensure the same evolution for each period evolution of the system, our model may be used to predict the Floquet DQPTs of the system for some long time stable states of the periodically kicked systems. The XY spin chain can be experimentally realized in many physics systems or simulated by simply systems, such as the nuclear magnetic resonance (NMR) technology,[81,82] the negatively charged nitrogen-vacancy center in diamond.[18] Our system may also be experimentally realized in such systems with extra periodic changed δ-pulse sequence kicking fields, the physical quantities such as geometric phase can be similarly measured.[18,81,82]

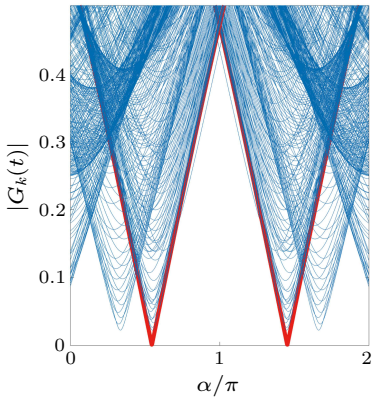


Fig. 4. The returned amplitude $|G_k(t)|$ of the system for momenta $k \in (-\pi, \pi)$, $t \in (0, 2\pi)$. We have set $\gamma = 1$, $\lambda = 0.5$, $T_1 = \pi$, and $T = 2\pi$ in the calculation.

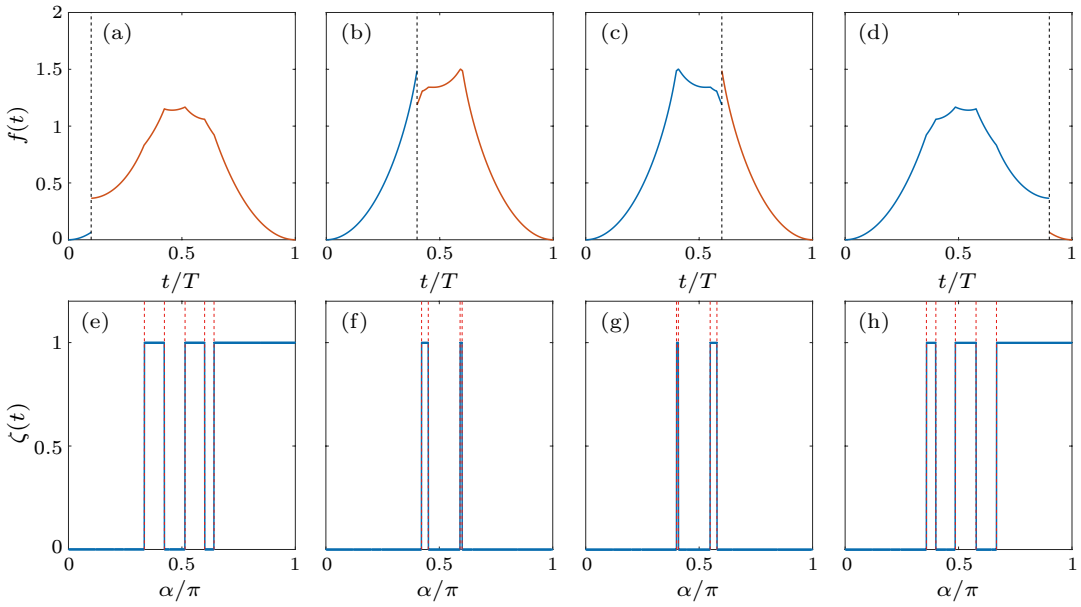


Fig. 5. [(a)–(d)] The rate function $f(t)$ of the system with momenta $k \in (-\pi, \pi)$ and period $T = 2\pi$: (a) $T_1/T = 0.1$, (b) $T_1/T = 0.4$, (c) $T_1/T = 0.6$, (d) $T_1/T = 0.9$ in the calculation. (e)–(h) The order parameter $\zeta(t)$ with different kicking time distributions for fixed period $T \in (0, 2\pi)$: (e) $T_1/T = 0.1$, (f) $T_1/T = 0.4$, (g) $T_1/T = 0.6$, (h) $T_1/T = 0.9$ in the calculation. Other parameters of all pictures are set as $\gamma = 1$, $\lambda = 0.5$, $\alpha = 1.5\pi$ and $T = 2\pi$. The black dotted line represents the boundary line of the periodic kicking time $t = T_1/T$.

4. Conclusion

In summary, we have proposed and investigated a new analytically solvable model for Floquet DQPTs, i.e., the Floquet DQPTs of the XY spin chain under the periodic kicking drive. We obtain the exact expression of the Floquet effective Hamiltonian, and investigate the Floquet DQPTs emerging in the micromotion of the Floquet dynamics. By considering the eigenstate and the ground state of the effective Hamiltonian, we discuss the return amplitude, the rate function of the return amplitude, the geometric phase, and the dynamical topological order parameter of the system, and show that all these physical quantities consistently display critical points for each period of the Floquet DQPTs. The periodic kicking can induce a new phase diagram for the XY spin chain, and also induce the Floquet DQPTs of the system, where multiple FDQPTs within each driving period will be introduced, which are affected by tuning both the anisotropy of the spin chain and the kicking parameters.

To effectively control closed non-equilibrium quantum many-body systems may pave the way to the development

of quantum technologies, while periodic kicking, as a special time-periodic driven system, provides a new kind of controllable system, which may also suggest new methods in controlling on the other quantum many-body systems.

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Appendix A: Some physical quantities used to characterize the Floquet DQPTs

In the main text, we have used the following physical quantities to characterize the emergence of Floquet DQPTs. Here, we give a brief introduction about these physical quantities.

(1) **Return amplitude of Floquet DQPTs** As we know, Loschmidt echo, or the return amplitude, has been widely used to identify DQPTs,[12,16] which is defined by the overlap between the initial state and its time-evolved state out of equilibrium. For the initial state $|\psi(k,0)\rangle$, the return amplitude at a later time t can be obtained by

$$G_k(t) = \langle\psi_k(0)|U_k(t)|\psi_k(0)\rangle, \tag{A1}$$

which tell us that, in the case that the finial state is orthogonal to the initial state, $G_k(t) = 0$, the corresponding values in parameter space will result in a non-analyticity in the associated dynamical free energy, and form a dense set of thermodynamic limits. It has been studied that, when this set crosses the imaginary time axis at a critical time, a real-time Fisher zero of $G_k(t)$ will appear.[18] This is analogue to the Lee–Yang theory of the phase transitions. In the following study, the return amplitude will also be used to show the non-analyticity in time-domain for the Floquet DQPTs.

(2) **Rate function of return amplitude** For the N lattice sites in the spin chain, the rate function of return amplitude $|G_k(t)|$ for a many-particle Floquet state is defined as

$$f(t) = -\lim_{N\to+\infty}\frac{1}{N}\sum_k \ln|G_k(t)|^2. \tag{A2}$$

In the case of $\ln|G_k(t)|^2$ diverging, $f(t)$ will disappear at a certain critical time and momentum, thus the rate function of the return amplitude can be used to characterize the Floquet DQPTs by revealing the existence of the non-analytical integral value in the range of $T \in (0, 2\pi)$.

(3) **Pancharatnam geometric phase** The Pancharatnam geometric phase is given by the difference between the total phase of the return amplitude $G_k(t)$ and the dynamic phase, under non-adiabatic conditions, i.e.,

$$\phi_k^{\rm geo}(t) = \phi_k^{\rm tot}(t) - \phi_k^{\rm dyn}(t), \tag{A3}$$

where the total phase can be derived from the returned amplitude, $\phi_k^{\rm tot}(t) = -\mathrm{i}\ln[\frac{G_k(t)}{|G_k(t)|}]$, and $\phi_k^{\rm dyn}(t) = -\int_0^t \mathrm{d}t\langle\psi_k(t')|h_k(t')|\psi_k(t')\rangle$ is the dynamic phase. This geometric phase can also be extend to characterize the emergence of Floquet DQPTs during each period.

(4) **Dynamical topological order parameter** Dynamical topological order parameter (DTOP) is proposed to characterize the emergence of DQPTs,[26] which is dynamically defined in the real-time evolution of a quantum many-body system, represented by a momentum space winding number of the Pancharatnam geometric phase of the Brillouin zone, i.e.,

$$\zeta(t) = \frac{1}{2\pi}\oint\frac{\partial\phi_k^{\rm geo}(t)}{\partial k}\mathrm{d}k. \tag{A4}$$

Because $\phi_k^{\rm geo}(t)$ shows a π phase flip at the critical momentum in the momentum-time plane, the DTOP sticks to a quantized value generally and shows a jump of unit magnitude at every DQPT, i.e., DTOP can change its integer value at discrete times where the so-called dynamical quantum phase transitions occur. Since $\zeta(t)$ makes a quantization jump when time passes the critical time of the Floquet DQPT, DTOP serves as a dynamical analog of an order parameter, and is capable of resolving if the topology of the system Hamiltonian has changed over the kicking.

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