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# Engineered photonic spin Hall effect of Gaussian beam in antisymmetric parity-time metamaterials

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A model of the photonic spin Hall effect (PSHE) in antisymmetric parity-time (APT) metamaterials with incidence of Gaussian beams is proposed here. We derive the displacement expression of the PSHE in APT metamaterials based on the transport properties of Gaussian beams in positive and negative refractive index materials. Furthermore, detailed discussions are provided on the APT scattering matrix, eigenstate ratio, and response near exceptional points in the case of loss or gain. In contrast to the unidirectional non-reflection in parity-time (PT) symmetric systems, the transverse shift that arises from both sides of the APT structure is consistent. By effectively adjusting the parameters of APT materials, we achieve giant displacements of the transverse shift. Finally, we present a multi-layer APT structure consisting of alternating left-handed and right-handed materials. By increasing the number of layers, Bragg oscillations can be generated, leading to an increase in resonant peaks in transverse shift. This study presents a new approach to achieving giant transverse shifts in the APT structure. This lays a theoretical foundation for the fabrication of related nano-optical devices.

**Keywords:** antisymmetric parity-time, photonic spin Hall effect, Gaussian beam

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## 1. Introduction

When a linearly polarized beam is reflected or transmitted at a non-uniform interface, its left circularly polarized (LCP) and right circularly polarized (RCP) components split, resulting in transverse shift (TS) in the direction perpendicular to the refractive index gradient. This phenomenon is known as the photonic spin Hall effect (PSHE), which originates from the conservation of total angular momentum of photons. It was first proposed by Onoda and other scientists in 2004,[1] and it was expected to have significant applications in nano-optics and quantum information. However, the observation of the PSHE was challenging until 2008 when Hosten and Kwiat indirectly detected the PSHE shift for photons passing through an air-glass interface. They used a weak measurement technique to amplify the shift by about $10^4$ times.[2] In the last decades, PSHE shifts have also been observed in various systems, such as left-handed materials (LHMs),[3] uniaxial crystals,[4] metals,[5] epsilon-near-zero slabs,[6,7] hyperbolic metamaterial waveguides,[8] graphene,[9,10] and chiral materials.[11–13] It was found that the spin Hall effect in LHMs is unreversed, although the sign of refractive index gradient is reversed.[3,14] The PSHE has a wide range of applications which are not limited to high-sensitivity optical sensors,[15] barcode encryption,[16] biosensors,[17] magneto-optical devices,[18,19] and optical edge detection.[20]

The concept of parity-time (PT) symmetry was first proposed by Bender *et al.* in 1998.[21,22] In 2007, El-Ganainy *et al.* first introduced the concept of PT symmetry into the optical system by founding a PT-symmetric optical system whose refractive index satisfies the condition $n(\boldsymbol{r}) = n^*(-\boldsymbol{r})$.[23] In 2013, Ge *et al.* designed an antisymmetric parity-time (APT) optical structure, whose refractive index satisfies the condition $n(\boldsymbol{r}) = -n^*(-\boldsymbol{r})$.[24] APT mematerials can control the PSHE by introducing only loss or gain, without the need for a complete balance of loss and gain. Therefore, APT mematerials can control more flexibly the transmission and distribution of photons, which is important for specific optical device designs.

Mathematically, a Hamiltonian satisfying PT symmetry is multiplied by i to satisfy the APT, meaning that the properties of an APT system are conjugated with the properties of a PT symmetric system. Thus, in the symmetry-unbroken regime, lossless propagation in a PT system corresponds to refractionless (or unit-refraction) propagation in an anti-PT system.[25,26] However, they also have similarities. It was found that the PSHE transverse shift at exceptional points (EPs) is zero, but largely enhanced in their vicinity. In addition, due to spontaneous PT symmetry breaking, the sign of the transverse shift switches across the exceptional point.[27] It was found that Bragg oscillation can be generated by increasing the period number of PT symmetric metamaterial layers.[28] These phenomena also exist in APT systems.

In this study, we present a theoretical analysis of impact of a Gaussian optical beam on APT metamaterials. We observe that the TSs generated by both incidences from left-handed materials (LHMs) and right-handed materials (RHMs) coincide, which is closely related to the elements of the scattering matrix and leads to a conservation relation in terms of the transmittance and (left and right) reflectances of APT

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metamaterials.[29,30] Furthermore, we discover that by increasing the number of stacked layers in the APT metamaterial layer, we can generate Bragg oscillations and increase the number of peaks in transverse displacement. These findings suggest a promising approach for modulating PSHE in APT metamaterials.

## 2. Theory and models

### 2.1. Spin Hall effect of reflected and transmitted light

Assuming a Gaussian-distributed optical beam transmitted and reflected at the air-APT metamaterials interface with an angle of incidence $\theta$, its angular spectrum can be expressed as $\tilde{E}_i = w_0 \exp[-w_0^2(k_{ix}^2+k_{iy}^2)/4]/\sqrt{2\pi}$, where $w_0$ is the beam waist, and $k_{ix}$ and $k_{iy}$ are the magnitude of the wave vector components in the direction of $x_i$ and $y_i$, respectively. In the spin group set, the incidence angle spectrum can be expressed as $\tilde{E}_i^{\mathrm{H}} = (\tilde{E}_{i+}+\tilde{E}_{i-})/\sqrt{2}$ and $\tilde{E}_i^{\mathrm{V}} = \mathrm{i}(\tilde{E}_{i-}-\tilde{E}_{i+})/\sqrt{2}$, where $\tilde{E}_i^{\mathrm{H}}$ and $\tilde{E}_i^{\mathrm{V}}$ represent the angular spectra of the horizontal and vertical polarizations of the incident wave, $\tilde{E}_{i+}$ and $\tilde{E}_{i-}$ represent LCP and RCP of light. By matching the boundary conditions, we can derive the relationship between the incident and reflected fields at the air-APT metamaterials interface, the angle spectrum between the reflected beam and the incident beam is written as[31]

$$\begin{pmatrix} \tilde{E}_r^{\mathrm{H}} \\ \tilde{E}_r^{\mathrm{V}} \end{pmatrix} = \begin{pmatrix} r_{\mathrm{p}} & \frac{k_{ry}(r_{\mathrm{p}}+r_{\mathrm{s}})\cot\theta}{k_0} \\ -\frac{k_{ry}(r_{\mathrm{p}}+r_{\mathrm{s}})\cot\theta}{k_0} & r_{\mathrm{s}} \end{pmatrix} \begin{pmatrix} \tilde{E}_i^{\mathrm{H}} \\ \tilde{E}_i^{\mathrm{V}} \end{pmatrix}, \quad (1)$$

where $\tilde{E}_{r(i)}^{\mathrm{H}}$ and $\tilde{E}_{r(i)}^{\mathrm{V}}$ represent the horizontal (H) and vertical (V) components of the angular spectrum of the reflected (incident) beam component, respectively. The terms $r_{\mathrm{p}}$ and $r_{\mathrm{s}}$ denote the Fresnel reflectivity coefficients for p- and s-polarizations, respectively, $k_0 = 2\pi/\lambda$ is the wave vector in air, $\theta$ represents the angle of incidence, $k_{ry} = k_{iy}$ and $k_{ry} = -k_{iy}$ satisfy the boundary conditions of RHMs and LHMs. In the given context, p refers to parallel polarized light and s refers to vertically polarized light. When the incident beam is p-polarized, its Jones vector can be written as $[1,0]^{\mathrm{T}}$, and when it is s-polarized, the Jones vector is represented as $[0,1]^{\mathrm{T}}$, where $[1,0]^{\mathrm{T}}$ and $[0,1]^{\mathrm{T}}$ are column vectors representing the polarization states. By obtaining the angular spectrum of the reflected light for both horizontal and vertical polarizations at the interface between air and APT metamaterials, we can use inverse Fourier transform to obtain the electric field expression of the reflected field as follows:

$$E(x_r,y_r,z_r) = \frac{1}{2\pi}\iint \tilde{E}(k_{rx},k_{ry},z)\exp\left[\mathrm{i}(k_{rx}x+k_{ry}y) - \frac{k_{rx}^2+k_{ry}^2}{2k_r}z_r\right]\mathrm{d}k_{rx}\mathrm{d}k_{ry}. \quad (2)$$

According to the definition of PSHE, the transverse shifts of the reflected light can be expressed as

$$\delta_{\pm} = \frac{\iint y_r|E_{\pm}(x_r,y_r,z_r)|^2\mathrm{d}x_r\mathrm{d}y_r}{\iint |E_{\pm}(x_r,y_r,z_r)|^2\mathrm{d}x_r\mathrm{d}y_r}. \quad (3)$$

By substituting the electric field expression (2) into Eq. (3), we can derive the expressions for the transverse shifts of the reflected beam in terms of the PSHE as follows:

$$\delta_{\mathrm{R}\pm}^{\mathrm{H(V)}} = \pm C w_0 \frac{|r_{\mathrm{p(s)}}|^2 + \mathrm{Re}[r_{\mathrm{p(s)}} r_{\mathrm{s(p)}}^*]}{|r_{\mathrm{p(s)}}|^2 C^2 + |r_{\mathrm{p}}+r_{\mathrm{s}}|^2 + \left|\frac{\partial r_{\mathrm{p(s)}}}{\partial\theta}\right|^2 \tan^2\theta}, \quad (4)$$

where $C = k_0 w_0 \tan\theta$. By using the similar methods and boundary conditions, we can derive the displacement expression of the transmitted light at the air-APT metamaterials interface:

$$\delta_{\mathrm{T}\pm}^{\mathrm{H(V)}} = \pm C w_0 \frac{|t_{\mathrm{p(s)}}|^2 - \mathrm{Re}[t_{\mathrm{p(s)}} t_{\mathrm{s(p)}}^*]}{|t_{\mathrm{p(s)}}|^2 C^2 + |t_{\mathrm{p}}-t_{\mathrm{s}}|^2 + \left|\frac{\partial t_{\mathrm{p(s)}}}{\partial\theta}\right|^2 \tan^2\theta}. \quad (5)$$

### 2.2. Model of the APT metamaterials in PSHE

The structure shown in Fig. 1 is placed at the $z = 0$ plane, where its background medium is air. The relative permittivity and relative permeability values for the air, positive refractive index dielectric layer (RHM), and negative refractive index dielectric layer (LHM) are denoted as $\varepsilon_0$, $\varepsilon_1$, $\varepsilon_2$, respectively, and $\varepsilon_1 = \varepsilon' \pm \mathrm{i}\varepsilon''$, $\varepsilon_2 = -\varepsilon' \pm \mathrm{i}\varepsilon''$. Here, plus or minus of $\varepsilon''$ indicates that loss or gain is introduced into the APT metamaterials, and the relative permeability of the three dielectric layers is $\mu_0 = \mu_1 = 1$, $\mu_2 = -1$. The thicknesses of the LHM and RHM both are the same as $d$.

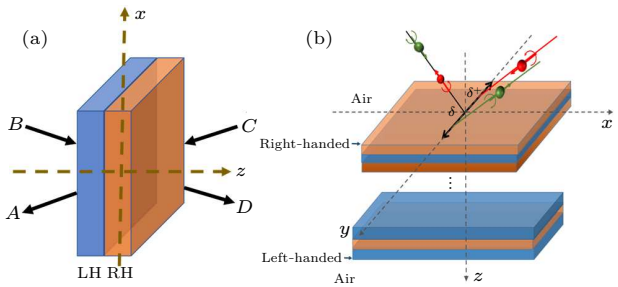



**Fig. 1.** Schematic illustration of the wave transmission in the APT structure. (a) Diagram of the scattering of circularly polarized plane waves by a double-layer APT optical system. (b) Schematic of the transverse displacement of the reflected PSHE in the multi-layer APT structure.

By the method of transmission matrix, we can derive the specific expressions for the reflectivity and transmission coefficient of an APT system as follows:[24]

$$\begin{pmatrix} A \\ B \end{pmatrix} = Q \begin{pmatrix} C \\ D \end{pmatrix}. \quad (6)$$

When the beam is transmitted from the air into the APT metamaterials and back to the air medium, the matrix is expressed in the specific form[32]

$$Q = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} = V_0^{-1} M V_0 = \begin{pmatrix} 1 & 1 \\ \eta_0 & -\eta_0 \end{pmatrix}^{-1} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ \eta_0 & -\eta_0 \end{pmatrix}, \quad (7)$$

with

$$M_j = \begin{pmatrix} \cos\delta_j & -\frac{\mathrm{i}\sin\delta_j}{\eta_j} \\ -\mathrm{i}\eta_j\sin\delta_j & \cos\delta_j \end{pmatrix}. \tag{8}$$

Here, $\delta_j = k_{zj}d$, $k_{zj} = k_0\sqrt{n_j^2 - n_0^2\sin^2\theta}$ when p-polarized light is incident $\eta_j = k_{jz}/\omega\varepsilon_j$. When s-polarized light is incident, we have $\eta_j = -k_{jz}/\omega\mu_j$, $j = 0,1,2$, where $\omega$ denotes the angular frequency of incident wave. While the material is a complex multilayer structure, the form of the transfer matrix $M$ can be written as

$$M = \prod_{j=1}^{N} M_j. \tag{9}$$

The scattering matrix of the APT metamaterials is defined as

$$\begin{pmatrix} A \\ D \end{pmatrix} = S_{\mathrm{APT}} \begin{pmatrix} B \\ C \end{pmatrix} = \begin{pmatrix} r_{\mathrm{L}} & t_{\mathrm{R}} \\ t_{\mathrm{L}} & r_{\mathrm{R}} \end{pmatrix} \begin{pmatrix} B \\ C \end{pmatrix}. \tag{10}$$

According to the strict formula derivation, we can obtain that the reflection coefficient and transmission coefficient of light in the APT metamaterials satisfy the following relations: $r_{\mathrm{L}} = \frac{Q_{12}}{Q_{22}}$, $r_{\mathrm{R}} = -\frac{Q_{21}}{Q_{22}}$, $r_{\mathrm{L}} = r_{\mathrm{R}}^*$, $t_{\mathrm{L}} = t_{\mathrm{R}} = \frac{1}{Q_{22}}$. Let $a = \mathrm{Re}[r_{\mathrm{L}}]$, $b = \mathrm{Im}[r_{\mathrm{L}}]$, then the scattering matrix can be written as

$$S = \begin{pmatrix} a+b\mathrm{i} & t \\ t & a-b\mathrm{i} \end{pmatrix}, \tag{11}$$

and its eigenvalues are $s_\pm = a \pm \sqrt{t^2 - b^2}$. It has two phases, and the phase transition occurs at $t = \pm b$. Let the scattering eigenstates be $\Psi_\pm = \begin{pmatrix} B_\pm \\ C_\pm \end{pmatrix}$, then the ratio of the scattered eigenstates is $p_\pm = B_\pm/C_\pm$.[24] In the APT phase ($|t| > |b|$), the eigenvalue of the scattering matrix is real, which means that the symmetric input is amplified or damped during scattering without adding phase. When the system is in the APT breaking phase ($|t| < |b|$), the eigenvalue of the system is complex, and the two scattering eigenstates have the same scattering intensity. The scattering eigenvalues are related to the APT properties: the system is in the APT phase at $s_\pm \neq s_\mp$.

## 3. Results and discussion

### 3.1. Double-layer APT with loss in PSHE

Firstly, we verify the relationship between the reflection coefficient and the transmission coefficient in the APT metamaterial through Mathematica software simulation, and the simulation results are consistent with the results derived from the previous theory, that is, $r_{\mathrm{L}} = r_{\mathrm{R}}^*$, $t_{\mathrm{L}} = t_{\mathrm{R}}$. Under the premise of ensuring that the real and imaginary parts of the refractive index meet the APT material, we first consider some new effects of double-layer APT with loss in PSHE. We then extend our discussion to multi-layer APT systems, where we expect to find similar results.

The proposed structure is shown in Fig. 1, and the APT structure composed of a layer of RHM and a layer of LHM with equal thickness is analyzed first, and the non-magnetic medium is selected from the APT metamaterials, so the specific value is $\varepsilon_1 = 0.1 + \mathrm{i}0.01$, $\varepsilon_2 = -0.1 + \mathrm{i}0.01$, $\mu_1 = 1$, $\mu_2 = -1$, $d_1 = d_2 = 0.785\lambda$, $w_0 = 15\lambda$. The LHM and RHM layers have the same thickness $d$. For ease of studying, the scattering matrix eigenvalues, reflections, and transmission coefficients are logarithms of the modulus, and the displacement is normalized to the incident wavelength, and the result is expressed as a multiple of the wavelength.

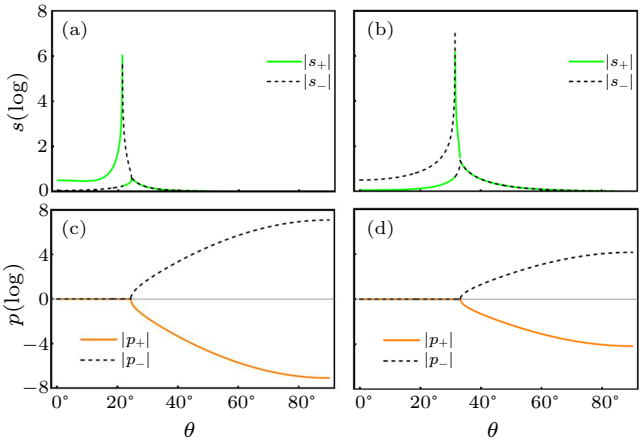



**Fig. 2.** [(a), (b)] Dependences of the eigenvalue of the scattering matrix. [(c), (d)] Display of ratio of the scattering eigenstates on $\theta$ for the cases of H polarization (the first column), and V polarization (the second column), respectively.

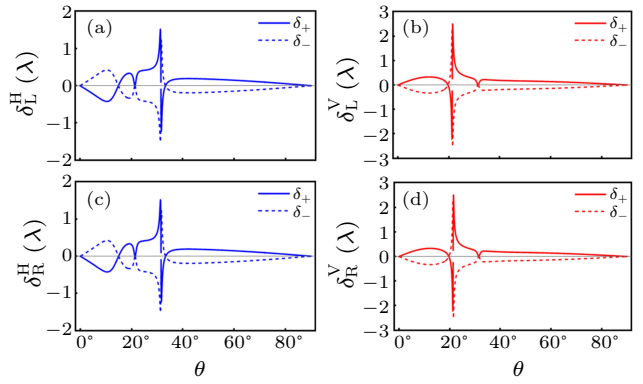



**Fig. 3.** Transverse shift of reflected light in the double-layer APT structure with loss. Blue (red) line represents p(s)-polarization incident from the LH or RH layer, and the solid (dashed) line represents the TS of LCP (RCP).

Figure 3 presents the simulation results for the spin splitting displacement in APT metamaterials with loss, using the same parameters as those in Fig. 2. The figure includes two rows showing the TS of reflected light when incident from the RHM and LHM sides, respectively. The TS produced by both of them were found to be identical when incident at the same angle, differing from PT symmetric metamaterials. The PT symmetric systems exhibit the non-reflection phenomenon due to balanced gain and loss, which is dependent strongly on the incidence direction of the optical beam. While PT materials can achieve unidirectional and non-reflection behavior, their control over the PSHE may be limited.[33–35] Both PT

and APT metamaterials are non-Hermitian, but they have different advantages: PT metamaterials have symmetry breaking, enabling unidirectional light propagation and nonlinear effects. APT materials have amplifier and loss compensation capabilities, enhancing and maintaining signal strength in signal transmission, and can be designed as single photon emitters, optical amplifiers, and high-efficiency optical energy saver devices.[36]

We proceed to simulate the TS of the transmitted light in the APT metamaterials that introduce loss, as shown in Fig. 4. A p-polarized optical beam exhibits a spin-splitting phenomenon when incident at an angle of 29.7° and again at 34.5°, resulting in two peaks in the transmitted beam. However, the spin-splitting remains unchanged in the incidence interval of 40°–60° as the incidence angle varies. The insensitivity of the spin-splitting phenomenon to changes in the angle of incidence within the 40°–60° interval is a desirable feature for developing optoelectronic devices with anti-interference properties. At the incidence of s-polarization, only one traverse peak appears in the figure, that is, around the angle of incidence of 21.2°, at which point the traverse peak is $7.3\lambda$. At that point, the peak will mutate, that is, from $\pm 7.3\lambda$ mutation to $\mp 7.3\lambda$. The results show that in the APT metamaterials which introduce loss, the enhanced PSHE can be achieved by properly adjusting the parameters of the system so that the APT systems spontaneous breaking phase occurs in the system.[37]

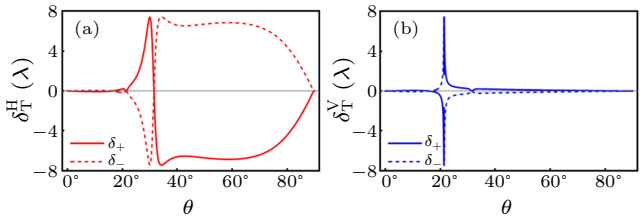



**Fig. 4.** Transmission shift in the double layers of the APT structure with loss.

### 3.2. Double-layer APT with gain in PSHE

In this section, we analyze the effect of a Gaussian beam on an APT symmetric metamaterial structure with gain only, where the imaginary part of the relative permittivity is negative. The scattering characteristics of this type of metamaterial, which consists of a double layer of alternating RHM and LHM with thickness of $d$, are shown in Fig. 5. Among them, the relative permittivities of the two layers are $\varepsilon_1 = 0.1 - \mathrm{i}0.1$, $\varepsilon_2 = -0.1 - \mathrm{i}0.1$. The other parameters are consistent with the above description. A similar result to the previous section can be observed, namely, spontaneous breaking can also occur in APT symmetric metamaterials with gain introduced. When the beam is p-polarized, the system exhibits two EP points, located at the incidence angles of 10.3° and 21.2°, and the eigenstates degenerate of the scattering matrix in the range of 10.3°–21.2°.

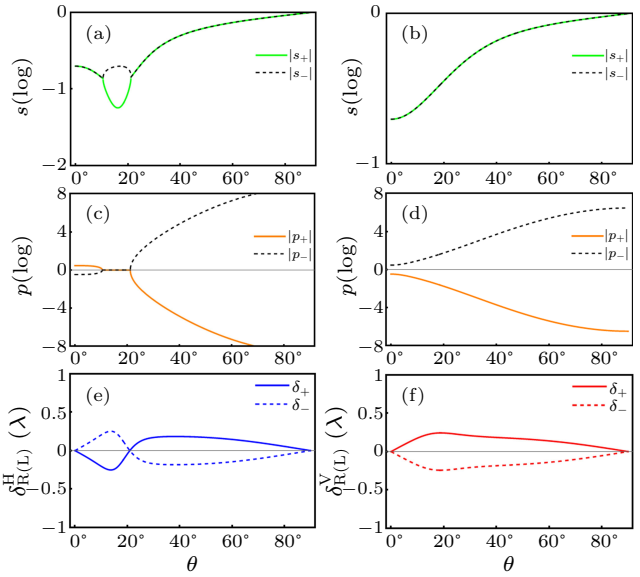



**Fig. 5.** [(a), (b)] Dependences of the eigenvalue of the scattering matrix. [(c), (d)] Display of the ratio of the scattering eigenstates on $\theta$ for the cases of H polarization (the first column), and V polarization (the second column), respectively. [(e), (f)] Transverse shift of reflected light in the double-layer APT structure with gain. Blue (red) line represents p(s)-polarization incident from the LH or RH layer, and the solid (dashed) line represents the TS of LCP (RCP).

We further discuss the spin splitting displacement in the APT symmetric metamaterial with gain introduced. Figures 5(e) and 5(f) show the TS of the parallel and vertically polarized reflected light, respectively. It is found that the TSs produced from RHM and LHM are exactly consistent at the same angular incidence, similar to the case in APT symmetric metamaterials that only introduce loss. Near the EP point, an enhanced spin-splitting traverse is still produced. In addition, in the second symmetry breaking point, that is, the angle of incidence is around 21.2°, the traverse generated by the LCP and RCP components has a positive and negative transition.

### 3.3. Controlling the thickness and $q$ of APT metamaterials

The difference in Fig. 6 is that the thickness of the material layer is changed, and the three sets of data $d = 0.785\lambda$, $d = 1.0\lambda$, $d = 1.5\lambda$ are taken for comparison. When a p-polarized beam is incident, the TS of LCP (RCP) light transits from positive to negative (from negative to positive) with increasing thickness at an incidence angle of 31.5°, 20.5°, and 12.3°, respectively, and reaches the peak displacement around this angle, and the displacement peak changes from $\pm 1.4\lambda$, $\pm 2.1\lambda$ to about $\pm 7.1\lambda$, gradually becoming larger. Moreover, we conducted simulations with varying thicknesses of the APT metamaterials and observed that as the thickness increases, the peak point of the transition displacement for the PSHE shifts towards the left. This shift is most significant for p-polarized incident beams and approaches 0° incidence angle under ideal conditions. When the s-polarization is incident, the displacement peak occurs at about 20°, as the material thickness increases, which shows that the photonic spin hall effect

generated by s-polarized beam is insensitive to small material thickness changes. In terms of craftsmanship, this characteristic allows the material to have certain production errors.

The system parameters will be adjusted appropriately as $\varepsilon_1 = 0.1 - \mathrm{i}0.1q$, $\varepsilon_2 = -0.1 - \mathrm{i}0.1q$; $\mu_1 = 1$, $\mu_2 = -1$. Keep the thickness and radius of the girdle unchanged, i.e., $d_1 = d_2 = 0.785\lambda$, $w_0 = 15\lambda$. The magnitude of the gain factor $q$ will be varied to observe its effect on the TS.

For incident p-polarized beams, the PSHE exhibits a displacement peak at an incidence angle of approximately 20°, with a gain factor of around 0.1. On the other hand, for s-polarization incidence, the reflected light displacement peak appears within the incidence angle range of 10°–20° and decreases gradually as the angle increases. However, increasing the gain factor has a minimal effect on the displacement of the PSHE. Therefore, it is concluded that the reflection displacement of s-polarized beam incidence is relatively tolerant to changes in the gain factor. Additionally, we conducted an investigation into the lateral displacement of the transmitted beam in APT metamaterials. As depicted in Fig. 7, the peak displacement for p-polarized incidence occurs at a large angle of 70°, while for s-polarization, it occurs at a smaller angle of around 30°. Similar to the reflection displacement of s-polarized light, the displacement of transmitted s-polarized light is less sensitive to changes in the gain factor when the incidence angle exceeds 20°.

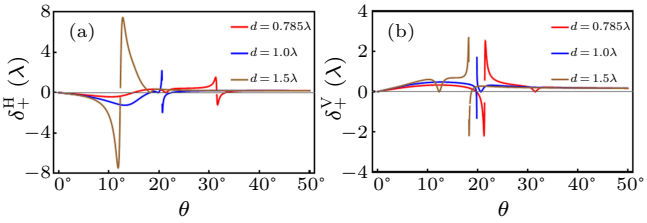



**Fig. 6.** Relationship of TS changing with the incident angle ($\theta$) under different metamaterials thicknesses.

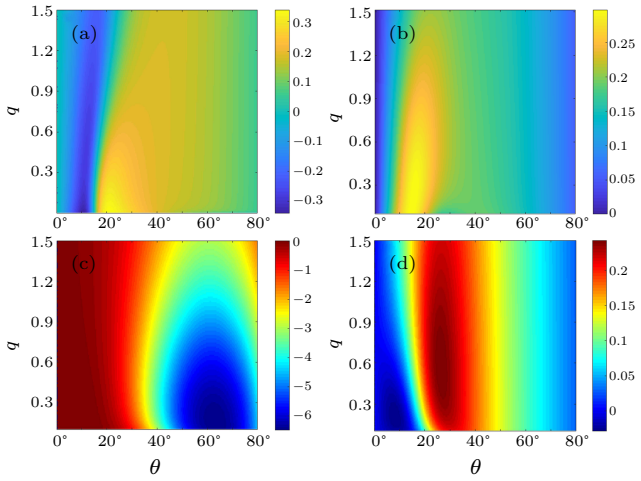



**Fig. 7.** Transverse shift contour (integer multiples of wavelength) of LCP reflected light [(a), (b)] and transmitted light [(c), (d)] for different Im[$\varepsilon$] with H [(a), (c)] and V [(b), (d)] inputs.

### 3.4. Different refraction index gradient in PSHE

The refractive index gradient plays an important role in the PSHE,[38] so the next research revolves around changing the effect of the real and imaginary parts of the refractive index of the APT material on PSHE, unlike the above section, the relative permittivity of the right-handed material layer and the left-handed material layer are, respectively, $\varepsilon_1 = 1 + \mathrm{i}0.1$, $\varepsilon_2 = -1 + \mathrm{i}0.1$, $\mu_1 = 1$, $\mu_2 = -1$, $d_1 = d_2 = 0.785\lambda$, $w_0 = 15\lambda$.

In Fig. 8, as a p-polarized beam is incident, the PSHE shift undergoes a transition from positive to negative (or negative to positive) at approximately 44.5°, accompanied by the appearance of the first displacement peak. Reflectivity is minimal during this time. Subsequently, the displacement curve becomes flattened, but a sudden change in sign occurs at 70.6°, where the second peak is observed. At this point, the maximum displacement of $\pm 6.9\lambda$ is reached and accompanied by strong reflection of the incident light field. When an s-polarized beam is incident, only one displacement peak is observed, which reaches $\pm 6.9\lambda$ at an angle of incidence around 72.5°. The reflection at this point is also strong. Compared with the PSHE using small refractive index APT materials, the one with large refractive index can achieve giant PSHE displacement at a large angle. Next, the transmission PSHE of the APT material is investigated. It is observed that the displacement of transmitted light under p/s-polarized incidence has only one peak at 70.6°/72.5°, which is consistent with the second peak in the reflection of p/s-polarized beam.

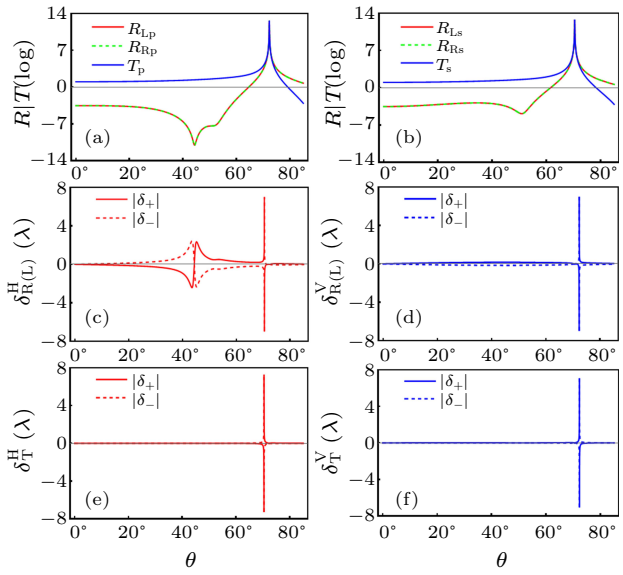



**Fig. 8.** The reflectivity and transmissivity of the Gaussian beam propagation (the first row), and the transverse shift of reflected light (the second row) on $\theta$ for the cases of H polarization (the first column), and V polarization (the second column), transverse shift of transmitted light (the third row).

### 3.5. Multi-layer APT system with gain and loss alternating

The number of cycles in the APT metamaterial, $N$, is a significant parameter that affects PSHE.[39] In order to further

investigate the influence of $N$ on lateral displacement, we consider an APT structure with 8 layers of LHM and RHM, which are center-symmetric and satisfy the properties of APT materials. We increase the refractive index difference between the material and air, as well as between neighboring layers of materials, in order to optimize the data and to observe the PSHE. The specific values are as follows: $\mathrm{Re}[\varepsilon_{13}] = 1$, $\mathrm{Re}[\varepsilon_{24}] = 8$, $\mathrm{Re}[\varepsilon_{57}] = -8$, $\mathrm{Re}[\varepsilon_{68}] = -1$. $\mathrm{Im}[\varepsilon_{1368}] = 0.1$, $\mathrm{Im}[\varepsilon_{2457}] = -0.1$, $d = 1.5\lambda$, $\mu_{1234} = 1$, $\mu_{5678} = -1$, $w_0 = 15\lambda$. As the number of layers in the APT material increases, more positive and negative TS peaks are obtained due to Bragg oscillation in the multi-layer structure. This process involves alternating gain and loss, which causes violent modulation of light during transmission, leading to the appearance of multiple exceptional points (EPs).

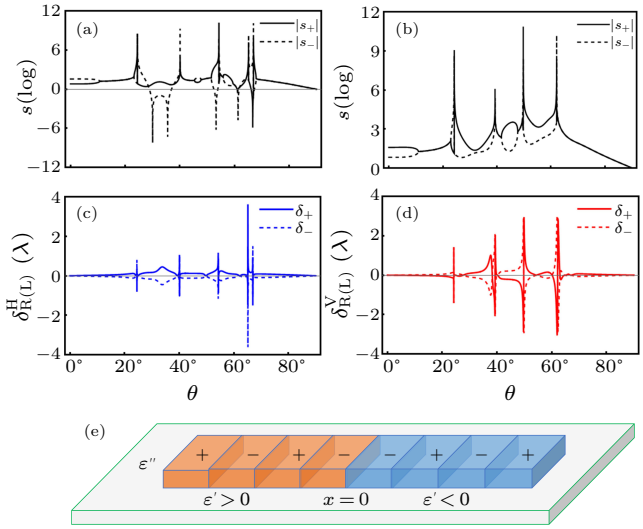



**Fig. 9.** [(a), (b)] Eigenvalues of the scattering matrix (first row), [(c), (d)] transverse displacement of reflected light (second row) on $\theta$ for the cases of H polarization (the first column), and V polarization (second column). (e) Eight-layer APT structure, with plus and minus signs denoting gain and loss.

## 4. Conclusion

In summary, we have investigated the interface of APT metamaterials for PSHE and found that when a Gaussian optical beam is incident on the material, the spin-dependent transverse shift (TS) of beam remains the same regardless of whether it is from the left-handed material (LHM) or right-handed material (RHM) side. Our findings suggest that it is possible to manipulate PSHE at multiple angles and over a wide range using this approach. By adjusting the structural parameters of the material and the angle of incidence of light, we can obtain giant displacements at $7.1\lambda$, whose upper limitation is the half of the beam waist. Furthermore, the TS shows little change in its trend with respect to the gain factor. However, adding more periodic layers amplifies the transverse displacement peaks caused by Bragg resonance. Our results provide a feasible pathway for the modulation of PSHE and the development of novel nanophotonic devices in some related fields.

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