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A kinetic description of the impact of agent competence and psychological factors on investment decision-making

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The kinetic theory is employed to analyze influence of agent competence and psychological factors on investment decision-making. We assume that the wealth held by agents in the financial market is non-negative, and agents set their own investment strategies. The herding behavior is considered when analyzing the impact of an agent's psychological factors on investment decision-making. A nonlinear Boltzmann model containing herding behavior, agent competence and irrational behavior is employed to investigate investment decision-making. To characterize the agent's irrational behavior, we utilize a value function which includes current and ideal-investment decisions to describe the agent's irrational behavior. Employing the asymptotic procedure, we obtain the Fokker–Planck equation from the Boltzmann equation. Numerical results and the stationary solution of the obtained Fokker–Planck equation illustrate how herding behavior, agent competence, psychological factors, and irrational behavior affect investment decision-making, i.e., herding behavior has both advantages and disadvantages for investment decision-making, and the agent's competence to invest helps the agent to increase income and to reduce loss.

Keywords: kinetic theory, investment decisions, Fokker–Planck equation, value function

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1. Introduction

Investment decision-making is related to many factors, such as the agent's competence, herding behavior, and irrational behavior. Among these factors, herding behavior and irrational behavior are regarded as a manifestation of psychological factors in the investment market. Thus, these factors relating to the agent's behavior need to be considered in the model describing the evolution of investment decisions. Xuan and Dang[1] analyzed herding behavior patterns under different stock portfolios and revealed the existence of herding behavior in particular industry. Zhang and Wang[2] employed the complex network method to investigate the correlation between herding behavior and the overconfidence behavior of investors. Wu *et al.*[3] studied herding behavior in China's stock market during the COVID-19 epidemic. Because of the influence of agent's competence on investment decision-making, the factors affecting the agent's competence level are discussed. Weller *et al.*[4] explained the differences in decision-making competence between agents from the perspectives of their personality and gender. Liang and Zou[5] discussed the difference in agent's competence level and risk prediction under different educational backgrounds, and pointed out the important role that education plays in the process of competence formation. Bryan and Paul[6] utilized the numerical simulation method to analyze the factors affecting the committee's collective decision-making competence. To better study the socioeconomic problems, a suitable value function[7,8] is employed to describe the agent's irrational behavior. From the rules of immigration and emigration, Gualandi and Toscani[9] introduced a suitable value function into the interaction rule to depict the migration phenomenon, and analyzed the development process of urban agglomeration. Gualandi and Toscani[10] introduced a suitable value function as the main criterion for decision-making and obtained a linear Fokker–Planck equation with log-normally distributed density. Other similar studies were given in Refs. [11–19].

For the research on decision-making, Maldarella and Pareschi[20] considered the influence of herding behavior on investment decisions and investigated the asymptotic behavior of investment and the distribution of stock prices in financial markets. In Ref. [20], the influence of herding behavior on investment decision-making was considered. However, the impact of agents' irrational behaviors on investment decision-making is difficult to present. Pareschi *et al.*[21] utilized the kinetic theory to discuss the impact of competence on collective decision-making when there is an equality bias in financial markets. In Ref. [21], the process of competence formation and some factors affecting the evolution of agent's competence were considered. Although some behaviors of agents are difficult to describe, they all strive to improve their wealth status, that is, to maximize their utility. Generally, agents with higher competence can better predict market risks and potential returns and are more rational when facing the herding behavior. Agents with lower competence may be more affected by the herding behavior. When the agent's investment de-

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cision deviates from the ideal-investment decision, the agent will gradually adjust the investment decision to achieve the ideal-investment decision (the investment decision that maximizes the agent's utility).

Inspired by the works in Refs. [20,21], we assume that each agent is allowed to choose their own portfolio in the financial market. The agent adopts specific investment decisions to obtain higher returns. The choice of investment proportion (investment decision) is affected by many factors, such as herding behavior, agent competence, and irrational behavior. In the investment process, if most agents make the same investment decisions, this behavior causes other agents to make the same investment decisions. In other words, herding behavior affects the agent's investment decision-making. In addition, the agent's competence in financial investment makes the agent more rational when investing, which helps the agent to reduce losses and to increase income. To characterize the agent's competence, we analyze the process of competence evolution. Factors that affect the agent's competence in some aspects include not only heredity, living environment, and educational background but also the agent's subjective initiative. We utilize a value function to describe the agent's irrational behavior. A kinetic model containing herding behavior, agent's competence, irrational behavior is used to characterize the evolution of investment decisions.

The current work provides a kinetic explanation for the evolution of investment decisions. Compared to the works in Refs. [20,21], we use the kinetic theory to analyze the influence of herding behavior, agent's competence, and irrational behavior on investment decision-making, and introduce the agent's competence and value function describing agent's irrational behavior into the binary interaction rules. The value function includes the current investment decision and the ideal investment decision, in which the ideal investment decision is to maximize the agent's utility. The deviation between the current investment decisions and the ideal investment decision determines how agents adjust their investment decision to maximize the utility. These works are different from those in Ref. [20], where the investment strategy refers to the properties of the agents and does not consider the influence of the agent's irrational behavior on investment decision-making. Moreover, we consider the impacts of herding behavior and agents' competence on investment decision-making, which is different from those in Ref. [21], in which the herding behavior and agents' competence are not considered.

This paper is organized as follows. A kinetic model of competence evolution is given in Section 2. The binary interaction rules and investment decision models containing herding behavior, agent competence, and irrational behavior are presented in Sections 3 and 4. Section 5 uses the asymptotic procedure method to obtain the Fokker–Planck equations corresponding to the Boltzmann model. Some stationary solutions for the Fokker–Planck equations are given in Section 6. The stationary behavior of investment decisions and the numerical experimental results are analyzed in Section 7.

2. The basic model for competence evolution

Competence is interpreted as the comprehensive quality of an agent to complete a goal or task. Genetic factors, the agent's living environment, and education affect the formation of an agent's competence. In addition, interaction has the effect to improve the agent's competence. Especially, when agents interact with agents possessing high competence, they gain more competence.

In general, every small change in an agent's competence is accompanied by an increase or loss of part of the competence. We assume that $y \in Y$ is the quantities of the agent's competence, where $Y \subset \mathbb{R}_+$. Let $z \in \mathbb{R}_+$ be the competence that an agent obtains from the environment. Pareschi *et al.*[21] constructed an interaction model to depict the changes in the competence of two agents with competences y and y_* after a binary interaction,

$$\begin{cases} y' = (1-\lambda_1(y))y + \lambda_2(y)y_* + \lambda_3(y)z + \eta_1 y, \\ y'_* = (1-\lambda_1(y_*))y_* + \lambda_2(y_*)y + \lambda_3(y_*)z + \eta_2 y_*, \end{cases} \tag{1}$$

where y' and y'_* are the competences of two agents after a binary interaction, λ_1 represents the competence that an agent loses in the process of growth, λ_2 indicates the competence acquired by an agent in interaction with other individuals, λ_3 is the agent's competence to obtain from the environment, random variable z follows a uniform distribution on $(0, a_1)$, where a_1 is a constant. Random variables η_i $(i = 1, 2)$ (zero-mean and finite variance σ^2) describe the unpredictable factors of competence evolution.

System (1) describes the influence of interaction and environment on individual competence evolution, but does not consider the impact of an agent's subjective initiative on competence evolution. Subjective initiative means that agents actively understand the objective world, that is, individuals improve their competence through active participation in learning and social practice. Based on the work reported in Ref. [21], we consider the influence of subjective initiative on the evolution of the agent's competence. From Eq. (1), we consider the following interaction model:

$$\begin{cases} y' = (1-\lambda_1(y))y + \lambda_2(y)y_* + \lambda_3(y)z + \lambda_4(y)s + \eta_1 y, \\ y'_* = (1-\lambda_1(y_*))y_* + \lambda_2(y_*)y + \lambda_3(y_*)z + \lambda_4(y_*)s + \eta_2 y_*, \end{cases} \tag{2}$$

where λ_4 denotes the amount of competence acquired by the agent through active learning or social practice, random variable s obeys uniform distribution on interval $(0, b_1)$, where b_1 is the constant. In the process of studying competence evolution, we assume that $\lambda_- \le \lambda_1 \le \lambda_+$, where $\lambda_- > 0$ and $\lambda_+ < 1$.

Similarly, we set boundaries for $\lambda_2(y)$, $\lambda_3(y)$, and $\lambda_4(y)$, that is, $0 \le \lambda_2(y) \le \bar{\lambda}_2$, $0 \le \lambda_3(y) \le \bar{\lambda}_3$, and $0 \le \lambda_4(y) \le \bar{\lambda}_4$, where $\bar{\lambda}_2 < 1$, $\bar{\lambda}_3 < 1$, and $\bar{\lambda}_4 < 1$. The random variables η_i $(i = 1,2)$ are assumed to satisfy $\eta_i \ge -(1-\lambda_+)$. Under these assumptions, the value of y_* is non-negative.

We use $A(z)$ to denote the probability density function of competence achieved in each interaction from the environment and $B(s)$ to indicate the probability density function of competence acquired from active learning and social practice, where $A(z)$ and $B(s)$ satisfy

$$\int_{\mathbb{R}_+} A(z)\mathrm{d}z = 1, \qquad \int_{\mathbb{R}_+} zA(z)\mathrm{d}z = m_1, \tag{3}$$

$$\int_{\mathbb{R}_+} B(s)\mathrm{d}s = 1, \qquad \int_{\mathbb{R}_+} sB(s)\mathrm{d}s = m_2. \tag{4}$$

Let $f(y,t)$ be the probability density function of an agent with competence $y \in Y$ at time $t > 0$. For Eq. (2) and any smooth function $\varphi(y)$, we employ the kinetic theory to obtain the following model that characterizes the evolution of competence. Namely,

$$\begin{aligned}&\frac{\partial}{\partial t}\int_Y f(y,t)\varphi(y)\mathrm{d}y\\ &= \left\langle \int_{\mathbb{R}_+^2\times Y} (\varphi(y') - \varphi(y)) f(y,t)A(z)B(s)\mathrm{d}y\mathrm{d}z\mathrm{d}s \right\rangle,\end{aligned} \tag{5}$$

where y' is the agent's competence after interaction given by Eq. (2), $\langle\cdot\rangle$ denotes mathematical expectation, $\varphi(y)$ is a test function. At any time $t > 0$, the left side of Eq. (5) depicts the change in the agent competence, and the interaction operator on the right side of Eq. (5) measures this change. Selecting $\varphi(y) = y$, the average competence $m_y(t)$ satisfies

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}m_y(t) = &\int_{Y^2} (\lambda_2(y)y_* - \lambda_1(y)y) f(y,t)f(y_*,t)\mathrm{d}y\mathrm{d}y_*\\ &+ \int_{\mathbb{R}_+\times Y} \lambda_3(y)zA(z)f(y,t)\mathrm{d}y\mathrm{d}z\\ &+ \int_{\mathbb{R}_+\times Y} \lambda_4(y)sB(s)f(y,t)\mathrm{d}y\mathrm{d}s,\end{aligned} \tag{6}$$

where

$$m_y(t) = \int_Y yf(y,t)\mathrm{d}y.$$

When $\lambda_1(y) = \lambda_1$, $\lambda_2(y) = \lambda_2$, $\lambda_3(y) = \lambda_3$ and $\lambda_4(y) = \lambda_4$ are all constant, Eq. (6) becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}m_y(t) = (\lambda_2 - \lambda_1)m_y(t) + \lambda_3 m_1 + \lambda_4 m_2. \tag{7}$$

Equation (7) has the solution

$$m_y(t) = m_y(0)\mathrm{e}^{-(\lambda_1-\lambda_2)t} + (1 - \mathrm{e}^{-(\lambda_1-\lambda_2)t})\frac{m_2\lambda_4 + \lambda_3 m_1}{\lambda_1 - \lambda_2}, \tag{8}$$

where $\lambda_1 > \lambda_2$, $m_y(0)$ is the initial value.

When t tends to infinity, the average competence $m_y(t)$ converges to the limit value $\frac{m_2\lambda_4+\lambda_3 m_1}{\lambda_1-\lambda_2}$. From Eq. (8), we set $m_y(0) = 0.5$, $\lambda_1 = 0.6$, $\lambda_2 = 0.1$, $\lambda_3 = 0.2$, $m_1 = 0.5$ and $m_2 = 0.5$, and the average competence evolutions with different subjective initiatives are depicted in Fig. 1. As shown in Fig. 1, when $\lambda_4 = 0.2$, the average competence level of the agent decreases. Noticeably, when $\lambda_4 \ge 0.4$, the agent's competence level increases with time, and finally converges to its limit value. From Fig. 1, we know that agents with strong subjective initiative have a high competence level. In other words, the agent's subjective initiative improves average competence level.

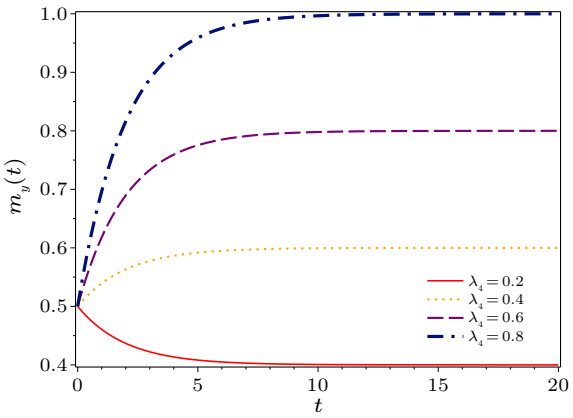


Fig. 1. The average competence with different subjective initiatives.

3. The binary interaction rules of investment decisions-making

In the investment process, investment decision-making is related to herding behavior, agent competence, and irrational behavior. To set an ideal-investment decision, the agent communicates with other agents and predicts the potential return of the investment. We utilize v and y to denote the investment decision and agents' competence, respectively. Assume that $I = [-1,1]$ is the set of possible investment decisions for each interaction, where ± 1 are two extreme investment decisions, indicating two opposite investment decisions. We use $v_d \in [-1,1]$ to represent the ideal-investment decision. Motivated by the works in Refs. [20,21], the binary interaction model of the investment decision-making is given as follows:

$$\begin{aligned}v' =& v - \alpha_1 H(v)(v - v_\mathrm{d}) - \alpha_2 v - \alpha_3 P(y,y_*,v,v_*)(v - v_*)\\ &+ \alpha_2 \Phi(\frac{v_\mathrm{d}}{v}) + D(y,v)\eta_3,\\ v'_* =& v_* - \alpha_1 H(v_*)(v_* - v_\mathrm{d}) - \alpha_2 v_*\\ &- \alpha_3 P(y_*,y,v_*,v)(v_* - v) + \alpha_2 \Phi(\frac{v_\mathrm{d}}{v}) + D(y_*,v_*)\eta_4,\end{aligned} \tag{9}$$

where the evolutions of competences y and y_* are given by Eq. (2), $v \in I$ and $v_* \in I$ represent the investment decisions of two agents before interaction, respectively; v' and v'_* denote the post-interaction investment decisions of two agents. $H(\cdot) \in [0,1]$ describes the influence of herding behavior on the agent's investment decision-making, $\alpha_1 \in [0,1]$ measures the

importance that agents attach to the opinions of other agents, $\alpha_2 \in [0,1]$ represents the agent's expectation of the potential return generated by the investment, $\alpha_3 \in [0,1]$ denotes the interaction rate. The interaction function $P(\cdot)$ is related to the competences of two agents and the investment viewpoint, η_j $(j=3,4)$ are random variables with zero mean and variance σ^2, $D(\cdot)$ is the local correlation between a given investment decision and the diffusion of agent competence. $\Phi(v_d/v)$ is the value function that depicts the agent's irrational behavior.

When $\eta_3, \eta_4 = 0$ and $H(v) = h \leq 1$ in system (9), we obtain

$$|v' - v'_*| = |1 - \alpha_1 h - \alpha_2 - \alpha_3 (P(y, y_*, v, v_*) + P(y_*, y, v_*, v))||v - v_*|. \tag{10}$$

From Eq. (10), we can get Proposition 1.

Proposition 1 Assume $0 \leq P(\cdot) \leq 1$, $v, v_* \in [-1,1]$, $H(\cdot) = h \leq 1$, and

$$|\eta_i| \leq C(1 - \alpha_1 - \alpha_2 - \alpha_3),$$

where

$$C = \min_{(y,v)\in Y\times I} \left\{ \frac{1-v}{D(y,v)}, D(y,v) \neq 0 \right\},$$

then the binary interaction rules in Eq. (9) can ensure that the post-interaction investment decision variables still belong to the interval $[-1,1]$.

Proof This proof is similar to that in Ref. [22], here we omit this proof.

From system (9), we know that the main factors that cause changes in investment decisions are herding behavior, agents' competence, irrational behavior and unpredictable factors, in which the agent's irrational behavior are described by value function $\Phi(\cdot)$. Inspired by the ideas in Ref. [10], we consider the following value function:

$$\Phi\left(\frac{v_d}{v}\right) = u \frac{(v_d/v)^\delta - 1}{w(v_d/v)^\delta + 1}, \tag{11}$$

where the constant values u $(0 < u < 1)$, δ $(0 < \delta < 1)$ and w $(w > 1)$ depict the rate of change of investment decisions around the ideal-investment decision v_d. In Eq. (11), v_d is the ideal investment decision to maximize the agent's utility. The value functions with different δ are given in Fig. 2.

As shown in Eq. (11) and Fig. 2, when $v_d > v$ $(v_d/v > 1)$, $\Phi(v_d/v)$ is positive and concave; when $v_d < v$ $(v_d/v < 1)$, $\Phi(v_d/v)$ is negative and convex. Thus, when $v_d \neq v$, the value function $\Phi(v_d/v)$ describes the evolution process from investment decision to ideal-investment decision v_d. In addition, the value function satisfies the following bound:

$$-u < \Phi\left(\frac{v_d}{v}\right) \leq \frac{u}{w}. \tag{12}$$

For a given value v, the value uv/w characterizes the maximal amount of investment decision change. In Eq. (9), when $v < v_d$, the plus sign in front of $\Phi(v_d/v)$ indicates that the investment decision tends to increase, while investment decision tends to decrease if $v > v_d$.

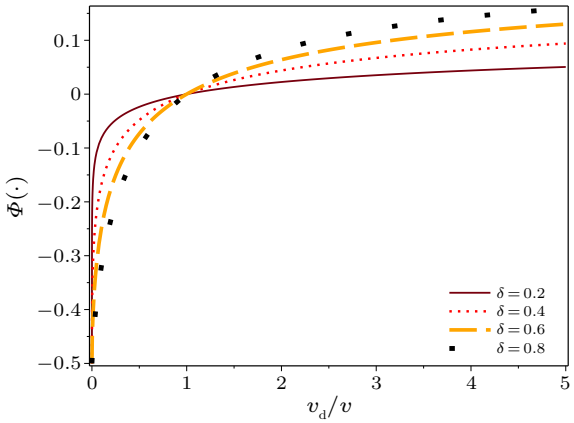


Fig. 2. The value function with different σ.

For the value function $\Phi(v_d/v)$, given a certain value $0 < \iota < 1$, it has the following properties:

$$-\Phi(1-\iota) > \Phi(1+\iota). \tag{13}$$

Assume that each agent has different investment decisions in the financial market. When most agents' investment decisions are consistent with the ideal-investment decision, the remaining agents are willing to adjust their investment decisions to coincide with the ideal investment decision. However, when most agents' investment decisions deviate from the ideal investment decision, the remaining agents are not willing to adjust their investment decisions to the ideal investment decision.

4. A kinetic model of Boltzmann type for competence and investment decision-making

In this section, we derive the investment decisions model that is influenced by agent competence, irrational behavior, and herding behavior. We use $g = g(y,v,t)$ to represent the density function of agents with investment decision $v \in I = [-1,1]$ and competence $y \in Y \subset \mathbb{R}_+$ at time $t > 0$. Let $h(y,t)$ be the marginal density function of the variable $y \in Y$, namely,

$$h(y,t) = \int_I g(y,v,t)\,\mathrm{d}v, \tag{14}$$

where $h(y,t)$ satisfies

$$\int_{Y\times\mathbb{R}_+} h(y,t)\mathrm{d}y\mathrm{d}t = 1.$$

For any smooth function $\psi(y,v)$, the density function $g(y,v,t)$ meets

$$\frac{\partial}{\partial t}\int_{Y\times I} \psi(y,v) g(y,v,t)\mathrm{d}v\mathrm{d}y = \frac{\beta}{2}\left\langle \int_{\mathbb{R}_+}\int_{Y^2\times I^2} (\psi(y',v') + \psi(y'_*,v'_*)) g(y,v,t) g(y_*,v_*,t)\right.$$

$$\times A(z)B(s)\mathrm{d}v\mathrm{d}v_*\mathrm{d}y\mathrm{d}y_*\mathrm{d}z\mathrm{d}s\Big\rangle$$
$$-\frac{\beta}{2}\Big\langle\int_{\mathbb{R}_+}\int_{Y^2\times I^2}(\psi(y,v)+\psi(y_*,v_*))g(y,v,t)g(y_*,v_*,t)$$
$$\times A(z)B(s)\mathrm{d}v\mathrm{d}v_*\mathrm{d}y\mathrm{d}y_*\mathrm{d}z\mathrm{d}s\Big\rangle, \tag{15}$$

where (y',v') and (y'_*,v'_*) are given by (y,v) and (y_*,v_*) which are defined in systems (1) and (9). It is worth noting that when $\psi(y,v)=1$ in Eq. (15), the total number of agents pre-interaction and post-interaction satisfies the conservation law. The average investment decision of the overall agent is defined as

$$G(t)=\int_{Y\times I}vg(y,v,t)\mathrm{d}v\mathrm{d}y, \tag{16}$$

which indicates the investment decisions of the system at time t. The evolution of investment decisions is expressed by

$$\begin{aligned}&\frac{\partial}{\partial t}\int_{X\times I}vg(y,v,t)\mathrm{d}v\mathrm{d}y\\&=\alpha_1\beta\int_{Y\times I}H(v)(v_\mathrm{d}-v)g(y,v,t)\mathrm{d}v\mathrm{d}y\\&\quad+\frac{\alpha_3\beta}{2}\int_{Y^2\times I^2}[P(y,y_*,v,v_*)-P(y_*,y,v_*,v)](v-v_*)\\&\quad\times g(y,v,t)g(y_*,v_*,t)\mathrm{d}v\mathrm{d}v_*\mathrm{d}y\mathrm{d}y_*\\&\quad+\frac{\beta}{2}\int_{Y^2\times I^2}\Big[2\alpha_2\Phi\Big(\frac{v_\mathrm{d}}{v}\Big)-\alpha_2(v+v_*)\Big]\\&\quad\times g(y,v,t)g(y_*,v_*,t)\mathrm{d}v\mathrm{d}v_*\mathrm{d}y\mathrm{d}y_*.\end{aligned} \tag{17}$$

When $P(\cdot)$ is a symmetric function, $\Phi(\frac{v_\mathrm{d}}{v})=\gamma\in[-1,1]$ and $H(v)=h$, Eq. (17) reduces to

$$\frac{\mathrm{d}}{\mathrm{d}t}G(t)=\alpha_1\beta hv_\mathrm{d}+\alpha_2\beta\gamma-(\alpha_1h+\alpha_2)\beta G(t), \tag{18}$$

which has the solution

$$G(t)=G_0\mathrm{e}^{-(\alpha_1h+\alpha_2)\beta t}+\frac{\alpha_1\beta hv_\mathrm{d}+\alpha_2\beta\gamma}{(\alpha_1h+\alpha_2)\beta}\big(1-\mathrm{e}^{-(\alpha_1h+\alpha_2)\beta t}\big) \tag{19}$$

for any $t>0$, where $(\alpha_1h+\alpha_2)\beta>0$, and G_0 is the initial value of the average investment decision. When $t\to\infty$, the average investment decision $G(t)$ converges exponentially to the limit value $(h\alpha_1v_\mathrm{d}+\alpha_2\gamma)/\alpha_1h+\alpha_2)$. If we assume that $\Phi(\frac{v_\mathrm{d}}{v})=\gamma=v_\mathrm{d}$, then when t tends to infinity, $G(t)$ converges to v_d.

We set $\beta=1$, $\alpha_1=\alpha_2=0.5$, $v_\mathrm{d}=0.6$, $\gamma=0.8$ in Eq. (19). When $G_0=0.3$ and $G_0=0.8$, the average investment decision of the overall agent with different h are displayed in Figs. 3(a) and 3(b), respectively.

From Fig. 3(a), when $G_0=0.3$, the average investment decision of the overall agent increases with time, and finally converges to its limit value. The greater the influence of herding behavior, the lower the final average investment decision value of the overall agent. In Fig. 3(b), the average investment decision of the overall agent tends to decrease and finally converges to its limit value. From Fig. 3 we can see that when G_0 takes different values, herding behavior has different effects on $G(t)$, that is, the dynamic evolution of the average investment decision is different.

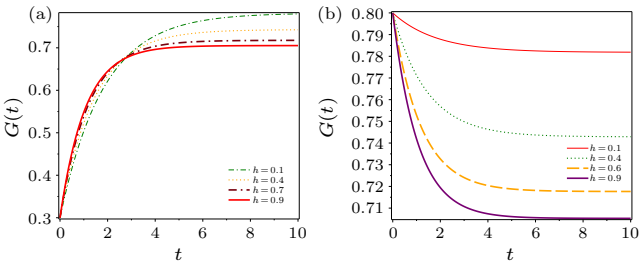


Fig. 3. The average investment decision of the overall agent with different herding behaviors: (a) $G_0=0.3$, (b) $G_0=0.8$.

For Eq. (15), if we choose $\psi(y,v)=v^2$, and the second order moment for the investment decision is obtained

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}R(t)&=\alpha_1^2\beta\int_{Y\times I}H^2(v)(v-v_\mathrm{d})^2g(y,v,t)\mathrm{d}v\mathrm{d}y\\&\quad+\frac{\alpha_3^2\beta}{2}\int_{Y^2\times I^2}(P^2(y,y_*,v,v_*)+P^2(y_*,y,v_*,v))(v-v_*)^2\\&\quad\times g(y,v,t)g(y_*,v_*,t)\mathrm{d}v\mathrm{d}v_*\mathrm{d}y\mathrm{d}y_*\\&\quad-\alpha_3\beta\int_{Y^2\times I^2}(v-v_*)(vP(y,y_*,v,v_*)-v_*P(y_*,y,v_*,v))\\&\quad\times g(y,v,t)g(y_*,v_*,t)\mathrm{d}v\mathrm{d}v_*\mathrm{d}y\mathrm{d}y_*\\&\quad+2\alpha_1\beta\int_{Y\times I}H(v)v(v-v_\mathrm{d})\\&\quad\times g(y,v,t)\mathrm{d}v\mathrm{d}y+\alpha_1\alpha_3\beta\int_{Y^2\times I^2}(v-v_*)\\&\quad\times(H(v)P(y,y_*,v,v_*)(v-v_\mathrm{d})-H(v_*)P(y_*,y,v_*v)\\&\quad\times(v_*-v_\mathrm{d}))g(y,v,t)g(y_*,v_*,t)\mathrm{d}v\mathrm{d}v_*\mathrm{d}y\mathrm{d}y_*\\&\quad+\sigma^2\beta\int_{Y\times I}D^2(y,v)g(y,v,t)\mathrm{d}v\mathrm{d}y\\&\quad+2\alpha_1\alpha_2\beta\int_{Y\times I}H(v)(v-v_\mathrm{d})\times vg(y,v,t)\mathrm{d}v\mathrm{d}y\\&\quad+\alpha_2^2\beta\int_{Y\times I}v^2g(y,v,t)\mathrm{d}v\mathrm{d}y+2\alpha_2^2\Phi^2\Big(\frac{v_\mathrm{d}}{v}\Big)\\&\quad-2\alpha_2\beta\int_{Y\times I}v^2g(y,v,t)\mathrm{d}v\mathrm{d}y\\&\quad-2\alpha_1\alpha_2\beta\int_{Y\times I}H(v)\Phi\Big(\frac{v_\mathrm{d}}{v}\Big)(v-v_\mathrm{d})g(y,v,t)\mathrm{d}v\mathrm{d}y\\&\quad+\alpha_2\alpha_3\beta\int_{Y^2\times I^2}(vP(y,y_*,v,v_*)\\&\quad-v_*P(y_*,y,v_*,v))(v-v_*)\\&\quad\times g(y,v,t)g(y_*,v_*,t)\mathrm{d}v\mathrm{d}v_*\mathrm{d}y\mathrm{d}y_*\\&\quad-2\alpha_2\alpha_3\beta\int_{Y^2\times I^2}\Phi(\frac{v_\mathrm{d}}{v})(P(y,y_*,v,v_*)\\&\quad-P(y_*,y,v_*,v))(v-v_*)g(y,v,t)g(y_*,v_*,t)\mathrm{d}v\mathrm{d}v_*\mathrm{d}y\mathrm{d}y_*.\end{aligned} \tag{20}$$

If we do not consider the diffusion term and set $P(y,y_*,v,v_*)=P\in[0,1]$, $H(v)=h\in[0,1]$, $\Phi(v_\mathrm{d}/v)=v_\mathrm{d}$, $\alpha_1=\alpha_2=\alpha_3=\alpha$,

then Eq. (20) becomes

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}R(t)=&\alpha^2\beta h^2\int_{Y\times I}(v-v_{\mathrm{d}})^2 g(y,v,t)\mathrm{d}v\mathrm{d}y\\&+2\alpha\beta h(1+\alpha)\int_{Y\times I}(v^2-vv_{\mathrm{d}})g(y,v,t)\mathrm{d}v\mathrm{d}y\\&+\alpha\beta(\alpha-2)\int_{Y\times I}v^2 g(y,v,t)\mathrm{d}v\mathrm{d}y+2\alpha^2 v_{\mathrm{d}}\\&+\alpha\beta P(\alpha P-1+\alpha h+\alpha)\int_{Y^2\times I^2}(v-v_*)^2\\&\times g(y,v,t)g(y_*,v_*,t)\mathrm{d}v_*\mathrm{d}v\mathrm{d}y_*\mathrm{d}y. \end{aligned}\tag{21}$$

From the solution of Eq. (18), we know that when $t\to\infty$, $G(t)\to v_{\mathrm{d}}$. If

$$\begin{aligned}&\beta=\frac{2\alpha}{(2-\alpha)v_{\mathrm{d}}},\\&\alpha\le\min\left[1,\frac{2(P-h+1)}{(h+P+1)+P^2}\right],\end{aligned}$$

then

$$\int_{Y\times I}(v-v_{\mathrm{d}})^2 g(y,v,t)\,\mathrm{d}v\mathrm{d}y=R(t)-v_{\mathrm{d}}^2.\tag{22}$$

We mention that under the above assumptions, when $t\to\infty$, $R(t)\to v_{\mathrm{d}}^2$, and the stationary solution of the interacting system is the Dirac delta $\delta(v-v_{\mathrm{d}})$ centered on the ideal-investment decision $v_{\mathrm{d}}\in[-1,1]$.

5. Derivation of the Fokker–Planck equations for investment decision-making

To analyze the long-time behavior of the Boltzmann model similarly to Refs. [23,24], we derive the Fokker–Planck equation from the Boltzmann equation (15) using the asymptotic procedure method. As inspired in Ref. [21], we assume that $\lambda_i(y)=\lambda_i$ $(i=1,2,3,4)$ in system (2) and consider the scaling

$$\begin{aligned}&\lambda_i\to\varepsilon\lambda_i\ \ (i=1,2,3,4),\quad \alpha_i\to\varepsilon\alpha_j\ \ (j=1,2,3),\\&\sigma\to\sigma\sqrt{\varepsilon},\quad \eta_i\to\varepsilon^{1/2}\eta_i,\end{aligned}\tag{23}$$

where α_i $(i=1,2,3,4)$ is a positive constant and $0<\varepsilon\ll 1$. For value function $\Phi(\cdot)$ which depicts irrational behavior, we consider the following scaling

$$\Phi\left(\frac{v_{\mathrm{d}}}{v}\right)\to\Phi_\varepsilon\left(\frac{v_{\mathrm{d}}}{v}\right)=u\frac{(v_{\mathrm{d}}/v)^{\varepsilon\delta}-1}{w(v_{\mathrm{d}}/v)^{\varepsilon\delta}+1}.\tag{24}$$

When $\varepsilon\to 0$, we get

$$\Phi_\varepsilon\left(\frac{v_{\mathrm{d}}}{v}\right)\to\frac{u}{w+1}\log\frac{v_{\mathrm{d}}}{v}.\tag{25}$$

From scaling (23), Eq. (15) becomes

$$\begin{aligned}&\frac{\partial}{\partial t}\int_{Y\times I}\psi(y,v)g(y,v,t)\mathrm{d}v\mathrm{d}y\\&=\frac{1}{\varepsilon}\Big\langle\int_{\mathbb{R}_+}\int_{Y^2\times I^2}(\psi(y',v')-\psi(y,v))g(y,v,t)g(y_*,v_*,t)\\&\times A(z)B(s)\mathrm{d}y\mathrm{d}y_*\mathrm{d}v\mathrm{d}v_*\mathrm{d}z\mathrm{d}s\Big\rangle.\end{aligned}\tag{26}$$

Using the properties of the random variables η_i $(i=1,2,3,4)$, we calculate the mathematical expectation of variables in systems (2) and (9), and obtain

$$\begin{aligned}\langle y'-y\rangle=&-\lambda_1 y+\lambda_2 y_*+\lambda_3 z+\lambda_4 s=:A_1(y,y_*,z,s),\\\langle v'-v\rangle=&-\alpha_1 H(v)(v-v_{\mathrm{d}})-\alpha_2 v\\&-\alpha_3 P(y,y_*,v,v_*)(v-v_*)+\alpha_2\Phi\left(\frac{v_{\mathrm{d}}}{v}\right)\\=&:H_1(v)+P_1(y,y_*,v,v_*)+\alpha_2\Phi\left(\frac{v_{\mathrm{d}}}{v}\right),\\\langle(y'-y)^2\rangle=&A_1^2(y,y_*,z,s)+\sigma^2y^2,\\\langle(v'-v)^2\rangle=&\left[H_1(v)+P_1(y,y_*,v,v_*)+\alpha_2\Phi\left(\frac{v_{\mathrm{d}}}{v}\right)\right]^2\\&+\sigma^2D(y,v),\\\langle(y'-y)(v'-v)\rangle=&A_1(y,y_*z,s)\\&\times\left[H_1(v)+P_1(y,y_*v,v_*)+\alpha_2\Phi(\frac{v_{\mathrm{d}}}{v})\right].\end{aligned}\tag{27}$$

From Eq. (27), we apply Taylor series expansion for the function $\psi(y',v')$ concerning (y,v) and obtain

$$\begin{aligned}&\psi(y',v')-\psi(y,v)\\=&\frac{\partial\psi}{\partial y}(y'-y)+\frac{\partial\psi}{\partial v}(v'-v)+\frac{1}{2}\frac{\partial^2\psi}{\partial y^2}(y'-y)^2\\&+\frac{1}{2}\frac{\partial^2\psi}{\partial v^2}(v'-v)^2+\frac{1}{2}\frac{\partial^2\psi}{\partial y\partial v}(y'-y)(v'-v)+R(y,y_*,v,v_*),\end{aligned}\tag{28}$$

or

$$\begin{aligned}&\langle\psi(y',v')-\psi(y,v)\rangle\\=&\frac{\partial\psi}{\partial y}A_1(y,y_*,z,s)+\frac{\partial\psi}{\partial v}\left[H_1(v)+P_1(y,y_*,v,v_*)+\alpha_2\Phi\left(\frac{v_{\mathrm{d}}}{v}\right)\right]\\&+\frac{1}{2}\frac{\partial^2\psi}{\partial v^2}\left[\left(H_1(v)+P_1(y,y_*v,v_*)+\alpha_2\Phi\left(\frac{v_{\mathrm{d}}}{v}\right)\right)^2+\sigma^2D^2(y,v)\right]\\&+\frac{1}{2}\frac{\partial^2\psi}{\partial y\partial v}\left[A_1(y,y_*,z,s)\left(H_1(v)+P_1(y,y_*,v,v_*)+\alpha_2\Phi\left(\frac{v_{\mathrm{d}}}{v}\right)\right)\right]\\&+\frac{1}{2}\left[\frac{\partial^2\psi}{\partial y^2}(A_1^2(y,y_*,z,s)+\sigma^2y^2\right]+R(y,y_*,v,v_*),\end{aligned}\tag{29}$$

where $R(y,y_*,v,v_*)$ is the remainder of the Taylor series expansion. From Eq. (23), we have

$$\begin{aligned}&A_1(y,y_*,z,s)\to\varepsilon A_1(y,y_*,z,s);\quad H_1(v)\to\varepsilon H_1(v),\\&P_1(y,y_*,v,v_*)\to\varepsilon P_1(y,y_*,v,v_*).\end{aligned}\tag{30}$$

Equation (26) becomes

$$\begin{aligned}&\frac{\partial}{\partial t}\int_{Y\times I}\psi(y,v)g(y,v,t)\mathrm{d}v\mathrm{d}y\\=&\int_{\mathbb{R}_+^2}\int_{Y^2\times I^2}\left[A_1(y,y_*,z,s)\frac{\partial\psi}{\partial y}(y,v)+H_1(v)\frac{\partial\psi}{\partial v}(y,v)\right.\\&\left.+P_1(y,y_*v,v_*)\frac{\partial\psi}{\partial v}(y,v)+\alpha_2\Phi\left(\frac{v_{\mathrm{d}}}{v}\right)\frac{\partial\psi}{\partial v}(y,v)\right.\end{aligned}$$

$$+\frac{\sigma^2}{2}y^2\frac{\partial^2\psi}{\partial y^2}(y,v)+\frac{\sigma^2}{2}D^2(y,v)\frac{\partial^2\psi}{\partial v^2}(y,v)\Big]$$
$$\times g(y,v,t)g(y_*,v_*t)A(z)B(s)\mathrm{d}v\mathrm{d}v_*\mathrm{d}y\mathrm{d}y_*\mathrm{d}z\mathrm{d}s$$
$$+\bar{R}(\varepsilon)+O(\varepsilon), \tag{31}$$

where

$$\bar{R}(\varepsilon)=\frac{1}{2\varepsilon}\int_{\mathbb{R}^2_+}\int_{Y^2\times I^2}\Big[\varepsilon^2\Big(A_1^2(y,y_*,z,s)\frac{\partial^2\psi}{\partial y^2}(y,v)$$
$$+\Big(H_1(v)+P_1(y,y_*,v,v_*)+\alpha_2\Phi\Big(\frac{v_\mathrm{d}}{v}\Big)\Big)^2\frac{\partial^2\psi}{\partial v^2}(y,v)$$
$$+A_1(y,y_*,z,s)\Big(H_1(v)+P_1(y,y_*,v,v_*)$$
$$+\alpha_2\Phi\Big(\frac{v_\mathrm{d}}{v}\Big)\Big)\frac{\partial^2\psi}{\partial y\partial v}(y,v)\Big)+R(y,y_*,v,v_*)\Big]$$
$$\times g(y,v,t)g(y_*,v_*,t)A(z)B(s)\mathrm{d}y\mathrm{d}y_*\mathrm{d}v\mathrm{d}v_*\mathrm{d}z\mathrm{d}s. \tag{32}$$

When $\varepsilon\to 0$, $\bar{R}(\varepsilon)$ tends to zero. Consequently, Eq. (31) becomes

$$\frac{\partial}{\partial t}\int_{Y\times I}\psi(y,v)g(y,v,t)\mathrm{d}v\mathrm{d}y$$
$$=\int_{\mathbb{R}^2_+}\int_{Y^2\times I^2}\Big[A_1(y,y_*,z,s)\frac{\partial\psi}{\partial y}(y,v)+H_1(v)\frac{\partial\psi}{\partial v}(y,v)$$
$$+P_1(y,y_*,v,v_*)\frac{\partial\psi}{\partial v}(y,v)+\alpha_2\Phi\Big(\frac{v_\mathrm{d}}{v}\Big)\frac{\partial\psi}{\partial v}(y,v)\Big]$$
$$\times g(y,v,t)g(y_*,v_*,t)A(z)B(s)\mathrm{d}y\mathrm{d}y_*\mathrm{d}v\mathrm{d}v_*\mathrm{d}z\mathrm{d}s$$
$$+\frac{\sigma^2}{2}\int_{Y^2\times I^2}y^2\frac{\partial^2\psi}{\partial y^2}(y,v)g(y,v,t)\mathrm{d}y\mathrm{d}v$$
$$+\frac{\sigma^2}{2}\int_{Y^2\times I^2}D^2(y,v)\frac{\partial^2\psi}{\partial v^2}(y,v)g(y,v,t)\mathrm{d}v\mathrm{d}y. \tag{33}$$

For Eq. (33), we obtain the Fokker–Planck equation

$$\frac{\partial}{\partial t}g(y,v,t)=\frac{\partial}{\partial y}\mathcal{H}[g](y,t)g(y,v,t)+\frac{\partial}{\partial v}\mathcal{G}[g](y,t)g(y,v,t)$$
$$+\frac{\partial}{\partial v}\mathcal{K}[g]g(y,v,t)+\frac{\sigma^2}{2}\frac{\partial^2}{\partial y^2}(y^2g(y,v,t))$$
$$+\frac{\sigma^2}{2}\frac{\partial^2}{\partial v^2}(D^2(y,v)g(y,v,t))$$
$$+\frac{\partial}{\partial v}\Big(\alpha_2\Phi\Big(\frac{v_\mathrm{d}}{v}\Big)\Big)g(y,v,t), \tag{34}$$

where

$$\mathcal{G}[g](y,v,t)=\int_{Y\times I}P_1(y,y_*,v,v_*)g(y_*,v_*,t)\mathrm{d}v_*\mathrm{d}y_*, \tag{35}$$
$$\mathcal{K}[g](v_\mathrm{d})=\int_{Y\times I}H_1(v)g(y_*,v_*,t)\mathrm{d}y_*\mathrm{d}v_*$$
$$=-\alpha_1H(v)(v-v_\mathrm{d})-\alpha_2v \tag{36}$$

and

$$\mathcal{H}[g](y,t)=\int_{\mathbb{R}^2_+}\int_{Y\times I}A_1(y,y_*,z,s)g(y_*,v_*,t)$$
$$\times A(z)B(s)\mathrm{d}v_*\mathrm{d}y_*\mathrm{d}z\mathrm{d}s. \tag{37}$$

Since

$$m_y(t)=\int_{Y\times I}yg(y,v,t)\mathrm{d}y\mathrm{d}v,$$

Eq. (37) reads

$$\mathcal{H}[g](y,t)=\lambda_1y+\lambda_2m_y(t)-\lambda_3m_1-\lambda_4m_2. \tag{38}$$

Inspired by the works in Ref. [21], the boundary condition of the Fokker–Planck equation (34) is

$$\mathcal{G}[g](y,t)g(y,v,t)\Big|_{-1}^{1}=0. \tag{39}$$

6. The stationary distribution of investment decision-making

To get the stationary solution of the Fokker–Planck equation (34), we integrate the Fokker–Planck equation (34) for variable v to obtain

$$\frac{\partial}{\partial t}\int_I g(y,v,t)\mathrm{d}v$$
$$=\frac{\partial}{\partial y}\int_I\mathcal{H}[g]g(y,t)g(y,v,t)\,\mathrm{d}v+\mathcal{G}[g]g(y,v,t)\Big|_{-1}^{1}$$
$$+\frac{\partial}{\partial v}\int_I\mathcal{K}[g]g(y,v,t)\,\mathrm{d}v+\frac{\sigma^2}{2}\frac{\partial^2}{\partial y^2}(y^2\int_I g(y,v,t)\mathrm{d}v)$$
$$+\frac{\sigma^2}{2}\frac{\partial^2}{\partial v^2}\int_I D^2(y,v)g(y,v,t)\mathrm{d}v$$
$$+\frac{\partial}{\partial v}\int_I\alpha_2\Phi_\varepsilon(\frac{v_\mathrm{d}}{v})g(y,v,t)\mathrm{d}v. \tag{40}$$

From the boundary condition (39), Eq. (40) is simplified to

$$\frac{\partial}{\partial t}h(y,t)=\frac{\partial}{\partial y}(\lambda_1y-\lambda_2m_y-\lambda_3m_1-\lambda_4m_2)h(y,t)$$
$$+\frac{\partial}{\partial v}H_1(v)h(y,t)+\frac{\sigma^2}{2}\frac{\partial^2}{\partial y^2}(y^2h(y,t))$$
$$+\frac{\sigma^2}{2}\frac{\partial^2}{\partial v^2}D^2(y,v)h(y,t)$$
$$+\alpha_2\frac{\partial}{\partial v}\Big(\Phi_\varepsilon\Big(\frac{v_\mathrm{d}}{v}\Big)h(y,t)\Big). \tag{41}$$

When the diffusion term, herding behavior and the agent's irrational behavior are not considered, Eq. (41) becomes

$$\frac{\partial}{\partial t}h(y,t)=\frac{\partial}{\partial y}(\lambda_1y-\lambda_2m_y-\lambda_3m_1-\lambda_4m_2)h(y,t)$$
$$+\frac{\sigma^2}{2}\frac{\partial^2}{\partial y^2}(y^2h(y,t)). \tag{42}$$

Stationary solution of Eq. (42) is obtained by solving the following differential equation

$$\frac{\sigma^2}{2}\frac{\mathrm{d}}{\mathrm{d}y}(y^2h(y,t))+(\lambda_1y-\lambda_2m_y-\lambda_3m_1-\lambda_4m_2)h(y,t)=0. \tag{43}$$

The solution to Eq. (43) is

$$h_\infty(y)=\frac{C_{\lambda_1,\lambda_2,\lambda_3,\lambda_4,\sigma^2}}{y^{2(\sigma^2+\lambda_1)/\sigma^2}}\exp\Big\{-\frac{2}{\sigma^2y}\cdot\frac{\lambda_1(\lambda_3m_1+\lambda_4m_2)}{\lambda_1-\lambda_2}\Big\}, \tag{44}$$

where $C_{\lambda_1,\lambda_2,\lambda_3,\lambda_4,\sigma^2}$ is a constant and it satisfies $\int_{\mathbb{R}^+} h_\infty(y)\mathrm{d}y = 1$.

Considering the influence of agents' irrational behaviors on investment decision-making, we directly integrate Eq. (34) for variable y utilizing boundary condition (39), and obtain

$$\begin{aligned}
&\frac{\partial}{\partial t}\int_I g(y,v,t)\mathrm{d}y \\
&= \frac{\partial}{\partial y}\int_I \mathcal{H}[g]g(y,t)g(y,v,t)\mathrm{d}y + \mathcal{G}[g]g(y,v,t)\Big|_{-1}^{1} \\
&\quad + \frac{\partial}{\partial v}\int_I \mathcal{K}[g]g(y,v,t)\mathrm{d}y + \frac{\sigma^2}{2}\frac{\partial^2}{\partial y^2}\left(y^2\int_I g(y,v,t)\mathrm{d}y\right) \\
&\quad + \frac{\sigma^2}{2}\frac{\partial^2}{\partial v^2}\int_I D^2(y,v)g(y,v,t)\mathrm{d}y \\
&\quad + \frac{\partial}{\partial v}\int_I \alpha_2\Phi_\varepsilon\left(\frac{v_\mathrm{d}}{v}\right)g(y,v,t)\mathrm{d}y. \qquad (45)
\end{aligned}$$

We set $h_1(v,t)$ to denote the marginal density function of investment decision variable v, that is,

$$h_1(v,t) = \int_I g(y,v,t)\mathrm{d}y, \qquad (46)$$

where $h_1(v,t)$ meets

$$\int_{I\times\mathbb{R}_+} h_1(v,t)\mathrm{d}v\mathrm{d}t = 1.$$

From Eqs. (39) and (46), Eq. (45) becomes

$$\begin{aligned}
\frac{\partial}{\partial t}h_1(v,t) &= \frac{\partial}{\partial y}(\lambda_1 y - \lambda_2 m_y - \lambda_3 m_1 - \lambda_4 m_2)h_1(v,t) \\
&\quad + \frac{\partial}{\partial v}H_1(v)h_1(v,t) + \frac{\sigma^2}{2}\frac{\partial^2}{\partial y^2}(y^2 h_1(v,t)) \\
&\quad + \frac{\sigma^2}{2}\frac{\partial^2}{\partial v^2}D^2(y,v)h_1(v,t) \\
&\quad + \alpha_2\frac{\partial}{\partial v}\left(\Phi_\varepsilon\left(\frac{v_\mathrm{d}}{v}\right)h_1(v,t)\right). \qquad (47)
\end{aligned}$$

Considering only agent irrational behavior and the influence of the diffusion term on the investment decisions, Eq. (47) reads

$$\begin{aligned}
\frac{\partial}{\partial t}h_1(v,t) &= \frac{\sigma^2}{2}\frac{\partial^2}{\partial v^2}D^2(y,v)h_1(v,t) \\
&\quad + \alpha_2\frac{\partial}{\partial v}\left(\Phi_\varepsilon\left(\frac{v_\mathrm{d}}{v}\right)h_1(v,t)\right), \qquad (48)
\end{aligned}$$

where

$$\Phi_\varepsilon\left(\frac{v_\mathrm{d}}{v}\right) = \frac{u}{w+1}\log\frac{v_\mathrm{d}}{v}.$$

When $D(y,v) = v$, the stationary solution to Eq. (48) is

$$h_{1,\infty}(v) = \frac{C_1}{v^2}\left(\frac{v_\mathrm{d}}{v}\right)^{\frac{2\alpha_2 u}{(1+w)\sigma^2 v}}\exp\left\{-\frac{2u\alpha_2}{(1+w)\sigma^2 v}\right\}, \qquad (49)$$

where C_1 is a constant and meets $\int_I h_{1,\infty}(v)\mathrm{d}v = 1$.

If we consider the impact of herding behavior, irrational behavior, and diffusion terms on investment decisions, Eq. (47) turns into

$$\begin{aligned}
\frac{\partial}{\partial t}h_1(v,t) &= \frac{\sigma^2}{2}\frac{\partial^2}{\partial v^2}D^2(y,v)h_1(v,t) \\
&\quad + \alpha_2\frac{\partial}{\partial v}\left(\Phi_\varepsilon\left(\frac{v_\mathrm{d}}{v}\right)h_1(v,t)\right) + \frac{\partial}{\partial v}H_1(v)h_1(v,t), \qquad (50)
\end{aligned}$$

where

$$H_1(v) = -\alpha_1 H(v)(v - v_\mathrm{d}) - \alpha_2 v.$$

When $D(y,v) = H(v) = v$, the stationary of Eq. (50) is obtained as follows:

$$\begin{aligned}
h_{1,\infty}(v) &= C_2 v_\mathrm{d}^{\frac{2u\alpha_2}{(1+w)\sigma^2 v}} v^{-\left(\frac{2(v_\mathrm{d}\alpha_1+\sigma^2-\alpha_2)}{\sigma^2}+\frac{2u\alpha_2}{\sigma^2 v(w+1)}\right)} \\
&\quad \times \exp\left\{-\frac{2(-v^2 w\alpha_1 - v^2\alpha_1 + u\alpha_2)}{\sigma^2 v(w+1)}\right\}, \qquad (51)
\end{aligned}$$

where constant C_2 satisfies $\int_I h_{1,\infty}(v)\mathrm{d}v = 1$.

From Eq. (44), we set $\lambda_1 = 0.6$, $\lambda_2 = 0.1$, $\lambda_3 = 0.2$, $\lambda_4 = 0.2$, $\sigma = 0.2$, $m_1 = 0.5$, and $m_2 = 0.5$, the competence evolution of agents with different λ_4 is presented in Fig. 4.

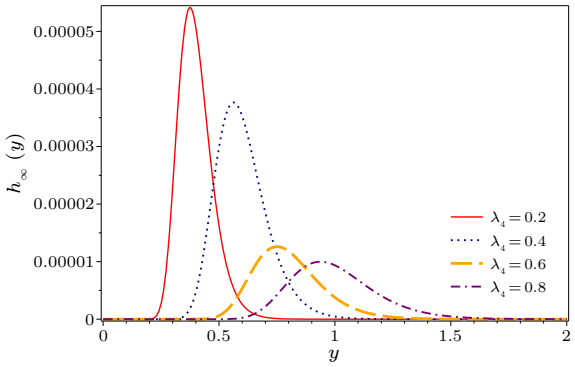


Fig. 4. Competence evolution of agents with different subjective initiatives.

As shown in Fig. 4, the competence level of the agent is closely related to the subjective initiative. Agents who actively learn or actively participate in social practice acquire more competence. For a given value of subjective initiative, the density of agents with competence y gradually increases first and then decreases. From Fig. 4, we know that the competence level of agents with the same subjective initiative fluctuates around a certain value. For example, when $\lambda_4 = 0.8$, the marginal density function under the steady state is similar to the density function of the normal distribution with a mean value of 1. In other words, when $\lambda_4 = 0.8$, the competence level of most agents fluctuates around $y = 1$.

We set $\alpha_2 = 0.5$, $\sigma = 0.5$, $u = 0.2$ and $w = 2$ in Eq. (49), the evolution of investment decisions with different v_d is displayed in Fig. 5. From Fig. 5, no matter what the agent's initial investment decision is, the final investment decision will achieve the ideal-investment decision. Some agents do not have strong investment intentions at the initial moment, but when most agents invest, their investment intentions will change. In the process of making investment decisions, agents are easily affected by herding behavior. Driven by herding behavior, agents constantly adjust their investment decisions until the investment decisions reach an ideal state.

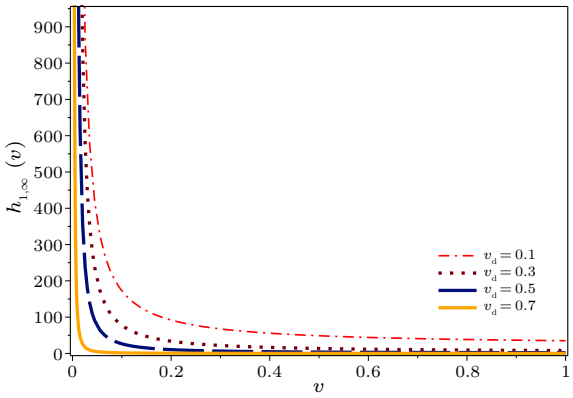


Fig. 5. Evolution of investment decisions with different ideal-investment decisions.

We choose $\alpha_2 = 0.5$, $\sigma = 0.5$, $u = 0.2$, $w = 2$ and $v_d = 0.05$ in Eq. (51), the influence of herding behavior on investment decisions with different α_1 is displayed in Fig. 6. Figure 6 shows that the higher the weightage given by agents to the opinions of others, the greater their susceptibility to herding behavior. When agents are influenced by herding behavior, it is easy to blindly follow others' investment strategies, resulting in shorter investment decision-making time. From Fig. 6, when $\alpha_1 = 0.8$, meaning that agents are highly susceptible to herding behavior, most agents make investment decisions quickly. When $\alpha_1 = 0.2$, indicating that agents are less influenced by herding behavior, they need more time to think, so making investment decisions is slower.

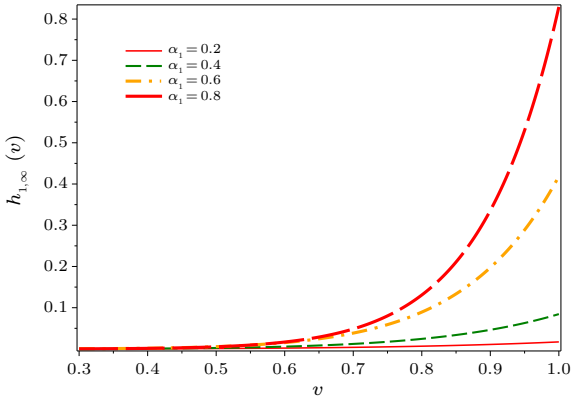


Fig. 6. Influence of herding behavior on investment decisions with different α_1.

Figure 7 shows the evolution of investment decisions with different w when $\alpha_1 = 0.4$ and other values are the same as those set in Fig. 6. As shown in Fig. 7, when w takes different values, that is, when the frequency of adjusting investment decisions by agents is different, the speed at which investment decisions converge to ideal investment decisions is also different. Agents exhibit a stronger investment inclination when they frequently adjust their investment decisions. During this process, irrational behaviors may occur to affect the evolution of investment decisions. For instance, when $w = 5$, agents frequently adjust their investment decisions, making investment decisions quickly reach an ideal state.

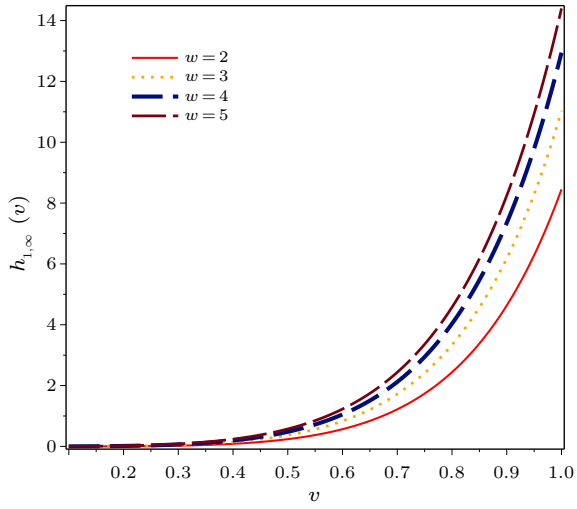


Fig. 7. Evolution of investment decisions with different w.

7. Numerical simulations

In this section, we apply the Monte Carlo method[25–28] to simulate the Boltzmann equation (26) and the equilibrium distribution (41). We set $D(y, v) = 0$, $w = 3$, $u = 0.4$, $\lambda_1 = 0.6$, $\sigma = 0.2$, $\lambda_2 = 0.1$, $\lambda_3 = 0.4$, $\lambda_4 = 0.6$, $m_1 = 0.5$ and $m_2 = 0.5$, the marginal densities of competence in the Boltzmann model (26) and the Fokker–Planck equation (41) with different values of ε and α_2 are displayed in Fig. 8.

Figure 8 shows that the relationship between the stationary solution of the Fokker–Planck equation (41) and the Boltzmann model (26) when ε and α_2 take different values. As shown in Fig. 8, the simulation results of the Boltzmann model (26) gradually approach the stationary solution of the Fokker–Planck equation (41). From Fig. 8(a), when $v = 0.8$ and $\varepsilon \to 1$, the density of agents with competence y agrees well with the stationary solution of the Fokker–Planck equation (41). Figure 8(b) shows that when $v = 0.8$ and $\alpha_2 \to 1$, the competence level of most agents fluctuates around $y = 1$. In other words, if the agent has a strong investment intention and high expectation for potential return on a project, it shows that the agent has a lot of information about the project and has a strong competence in financial investment. From Fig. 8, we know that the competence level of agents is not only related to the agent's subjective initiative but also related to the investment intention and the expectation of potential return. Agents with strong competence in financial investment can accurately predict market risks and potential returns and are more likely to make an ideal investment decision.

We set $\sigma = 0.2$, $v = 0.8$ and keep other parameter values as the same as those in Fig. 8, the equilibrium distributions (41) with different values v and σ are displayed in Fig. 9. From Fig. 9(a), when v is large, the density of agents with competence y is low, which indicates that some agents have low competence in financial investment, whereas their willingness to invest will increase due to the herding behavior. The

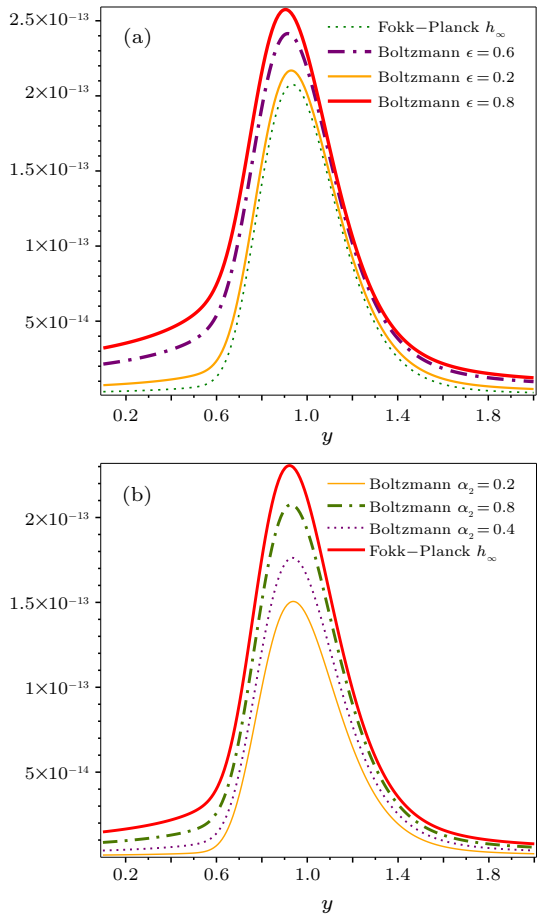


Fig. 8. The marginal densities of competence in the Boltzmann model (26) with the Fokker–Planck equation (41).

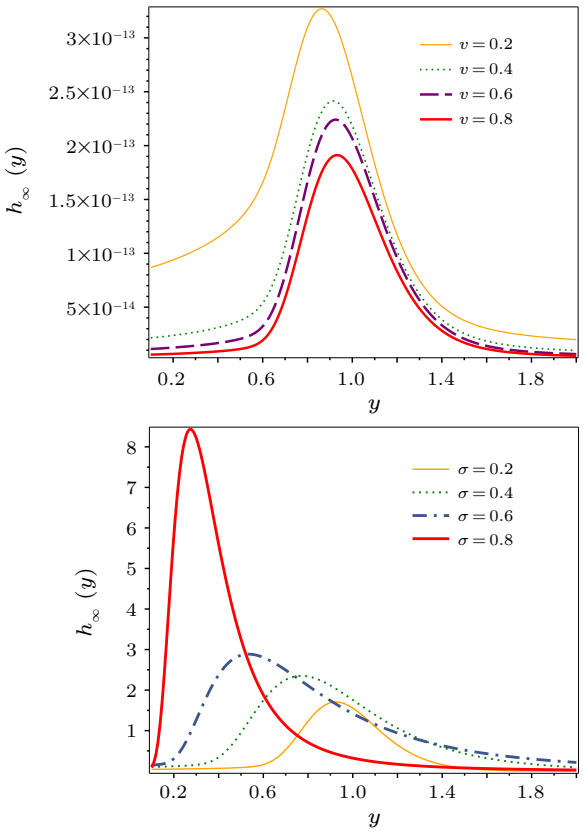


Fig. 9. The stationary behavior of the Fokker–Planck equation (41) with different values v and σ.

density function image of agents with the same investment intention and competence y is similar to the normal distribution image. In other words, there is little difference in the competence level of agents with the same investment intention in financial investment, that is, only a few agents have very high or low competence. Figure 9(b) illustrates the evolution of the density of agents with competence y when σ takes different values. When $v = 0.8$ and $\sigma = 0.8$, the density of agents whose competence level fluctuates around $y = 0.3$ is the largest. Contrarily, when $\sigma \to 0$, the density of agents whose competence level fluctuates around $y = 1$ gradually decreases. As shown in Fig. 9(b), when $\sigma \to 0$, the average competence of the agent gradually increases. Namely, when the agent actively learns or actively participates in social practice and is less interfered by unpredictable factors, the competence level of the agent will increase.

8. Conclusions

This paper uses the kinetic theory to analyze the evolution of investment decisions in the financial markets. The main feature of our model is to analyze the influence of herding behavior, agent competence, and irrational behavior on investment decision-making. We apply a value function which includes current and ideal-investment decisions to describe the agent's irrational behavior. The binary interaction rules and a Boltzmann model are given to describe the evolutions of investment decisions. Employing the asymptotic procedure, we obtain the Fokker–Planck equation from the Boltzmann model. The numerical simulation results and the stationary solution of the Fokker–Planck equation demonstrate that agents' living environment, education level, and subjective initiative affect agents' competence in financial investment. To some extent, subjective initiative increases the agent's competence level. In addition, the competence of agents and herding behavior affect investment decision-making. When the agent makes investment decisions, it will be interfered with the investment decisions made by other agents. An agent with strong competence in financial investment will be more rational in investing and more accurate in risk prediction, so the investment decision-making is closer to the ideal-investment decision. In short, herding behavior has both advantages and disadvantages for investment decision-making, and the agent's competence to invest helps the agent to increase income and reduce losses.

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