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Citation: Chin. Phys. B, 2023, 32 (**7**): 070504. DOI: 10.1088/1674-1056/acc058
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Symmetry phases of asymmetric simple exclusion processes on two lanes with an intersection

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(Received 2 November 2022; revised manuscript received 20 February 2023; accepted manuscript online 2 March 2023)

This paper studies two-lane asymmetric simple exclusion processes (ASEPs) with an intersection. In the upstream segments of the intersection, one particle can move to the next site with rate 1 if the site is empty, and the other particle can move forward with rate p in the sites of downstream segments. The parameter p can represent the rate of slowing of motion, and the parameter is introduced to investigate spontaneous symmetry breaking (SSB) phenomenon. Extensive Monte Carlo simulations are carried out. It is shown that three symmetric phases exist and the SSB does not exist in the system. Simple mean field approach in which correlation of sites is ignored is firstly adopted to analyze the system, and the system is divided into four independent segments. It is found that the analytical results deviate from the simulation ones, especially when p is small. In addition, the inexsitence of SSB can only be explained qualitatively. Motivated by this, we carry out the cluster mean field analysis in which correlation of five sites is considered. It is shown that densities of the two upstream segments are equal, which demonstrates that the SSB does not exist. It is also shown that, as expected, the cluster mean field analysis performs much better than the simple mean field analysis.

Keywords: asymmetric simple exclusion process, spontaneous symmetry breaking, simple mean-field analysis, cluster mean-field analysis

PACS: 05.70.Fh, 02.50.Ey, 05.60.Cd **DOI:** 10.1088/1674-1056/acc058

1. Introduction

Transport phenomena of nonequilibrium systems is very common in our world. Asymmetric simple exclusion process (ASEP) has become a prominent model in studying the motion of active species in nonequlibrium systems,[1] and the simplest model is the totally asymmetric simple exclusion process (TASEP). It was first introduced to study the synthesis of proteins by ribosomes moving on an mRNA strand in living organisms.[2] The model consists of some lattices, and each lattice site can be either empty or occupied by a single particle. Particles of the model interact only through hard-core exclusion principle. Despite its simplicity, the model has been used as a basis for many generalizations, such as dense media polymer dynamics,[3] dynamics of motor proteins moving along rigid filaments,[4,5] membrane channels diffusion,[6] gel electrophoresis,[7] traffic flow,[8–10] and information flow.[11]

In many realistic situations, transportation is complex. For example, transport along some parallel or intersect lanes is very common in vehicular traffic or intracellular transport. Then, studies of ASEP with single lane have been generalized to ASEP with multilane which exhibits a variety of cooperative phenomena, such as shock formation,[12,13] phase separation,[14,15] and spontaneous symmetry breaking (SSB).[16–20]

SSB is an interesting phenomenon in which symmetric conditions of the dynamic processes lead to existence of phases with broken symmetries. The phenomenon was first observed in the bridge model in which two species of particles move in the opposite directions in the single lane.[21] Later, the phenomenon was also observed in two-lane asymmetric simple exclusion processes with narrow entrances. In the model, two species of particles interact at entrances of lanes.[22] In addition, the original bridge model was also extended to a new class of bridge models fed by junctions.[23] It has been shown that local interaction can influence macroscopic particle dynamics of systems.

The SSB phenomenon attract many researchers' interests in resent decades. For example, in Ref. [24], the connection between fluctuation and SSB has been studied. In Ref. [25], how dissociations of localized particles can affect the SSB has been investigated. In Ref. [26], it has been found that SSB also occurs in a bidirectional asymmetric simple exclusion process coupled to a reservoir featuring crowding effect. In Ref. [27], the SSB in three-lane asymmetric simple exclusion process has been discussed.

In theoretical studies of SSB, simple mean field analysis which is an approximate approach is often adopted because of confirmation by comparison with computer simulations and available exact solutions.[28,29] However, correlation of sites is often neglected in the simple mean field analysis, which make the analytical results deviate from simulation ones. Motivated by this, cluster mean field analysis is often adopted, the correlation of sites is considered, and it performs much better than the simple mean field analysis.[17,18,20,24,25,27]

Yuan *et al.* have investigated totally asymmetric simple exclusion processes on two intersected lattices.[16] Three mod-

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els are studied. The first model is for molecular motor motion, and molecular motors do not have pre-defined destination. The second and third models are for vehicle traffic. In the second model, drivers know where the destination is and do not change the destination. In the third model, drivers can change the destination at the intersection. The SSB is observed in the second and third models, while the first model not. The occurrence of the SSB phenomenon has been qualitatively explained by domain-wall approach. However, understanding the essence of occurrence of the phenomenon is still an open question.

In the second and third models of Ref. [16], when the SSB occurs, it is asymmetric that the motion of one particle of the intersection to the nearest sites of downstream segments. It is unclear whether symmetric condition of the motion of particle of intersection leads to the occurrence of SSB. Motivated by this, we study the first model with revision in this paper. The particles of downstream segments of intersection will move to the next site with rate p if the site is empty. Here, the parameter p can represent the rate of slowing of particles when $p < 1$.

Extensive Monte Carlo simulations are carried out in the paper, and density profiles and phase diagram are also investigated. Simple mean field and cluster mean field analyses are adopted, in which correlation of sites is ignored and considered separately. It is found that only symmetric phases exist and the SSB does not exist in the system. The rest of this paper is organized as follows. In Section 2, the model is introduced. In Section 3, the simulation, simple mean field analysis, and cluster mean field analysis are discussed. In Section 4, conclusions are given.

2. Model

The model consists of two lanes with an intersection, see Fig. 1. Lane 1 is in horizontal direction, and lane 2 in vertical direction. This situation can be widely observed in vehicle traffic. In the model, the lattice sites of the two lanes are numbered form 1 to L, and $L+1$ to $2L$, respectively. Every site can be empty or occupied by one particle. The update rule is random update. At the entrance sites, one particle can enter the system with rate α. At the exit sites, one particle can leave the system with rate β. In the upstream section of intersection site (site C) of the two lanes, one particle can move forward with rate 1 if the next site is empty. In the downstream section, one particle will move forward with rate p if the next site is unoccupied. In the intersection site (site C), one particle can hop to site C_2 with rate p if site C_2 is empty and site C_4 is occupied. When site C_4 is empty and site C_2 is occupied, the particle of intersection hops to site C_4 with rate p. When sites C_2 and C_4 are both empty, the particle can hop to site C_2 or C_4 with rate $0.5p$. The parameter p represents the rate of slowing of moving particles when $p < 1$. The model can be used in simulation of vehicle motion when downstream of intersection are in traffic congestion, and conditions of the intersected roads can be interpreted qualitatively.

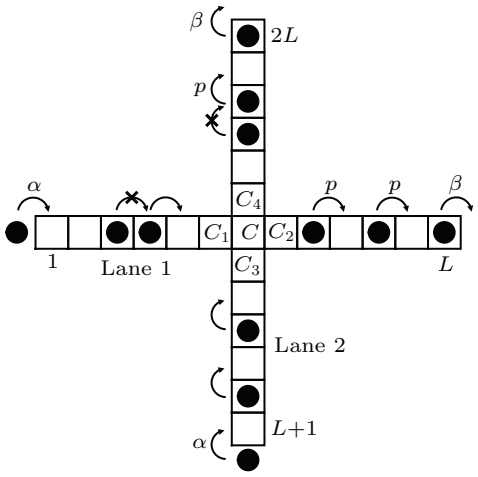


Fig. 1. Schematic view of the model. The arrows show allowed hopping and the crossed arrows show prohibited hopping. The entrance rates of two lanes are both α, exit rates are both β. The particles of upstream segments of intersection will move forward with rate 1 if next sites are empty, and particles of downstream segments will move to the next empty sites with rate p. Filled circles show that sites are occupied by particles.

3. Simulation and analytical results

3.1. Simulation results

Monte Carlo simulations show that there are three phases in the system, i.e., symmetric LL phase, symmetric HH phase and symmetric HL phase (LL: low density/low density, HH: high density/high density, LH: low density/high density), see Fig. 2.

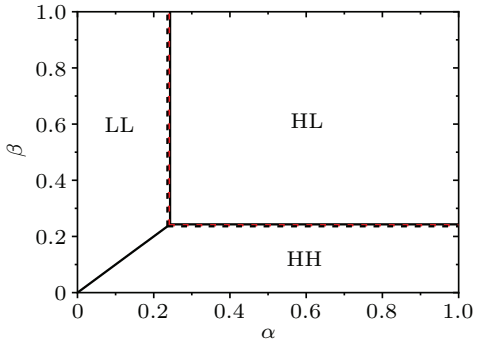


Fig. 2. Phase diagram of the model. Solid lines represent the results obtained from simulations. Black dash lines are obtained from simple mean field analysis. Red dash lines are from cluster mean field analysis. The parameter is taken as $p = 0.5$.

The three phases are all symmetric, which means that the conditions of the two lanes are identical, see Fig. 3. In symmetric LL phase, the two lanes are both in low density, and the density is α. In symmetric HH phase, the two lanes are in high density, and the density is $1-\beta$. In symmetric HL phase, the upstream of intersection of two lanes are in high density, the density is $1-\lambda$, downstream is in low density, the density is λ. The value of λ can vary with p.

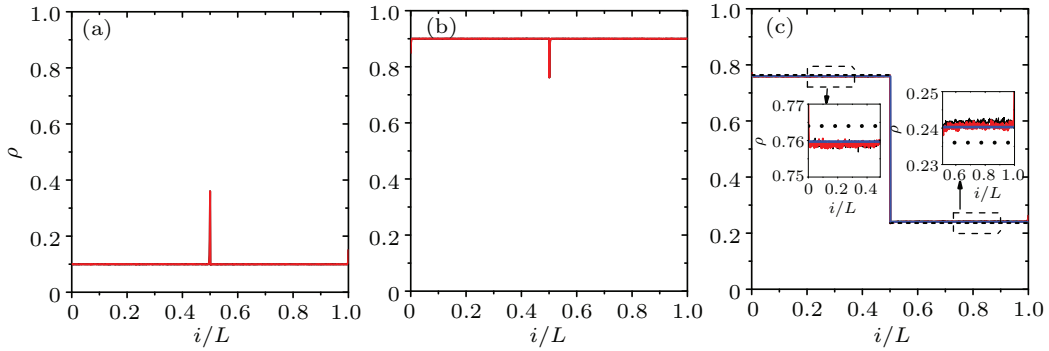


Fig. 3. Density profiles of (a) symmetric LL phase, (b) symmetric HH phase, and (c) symmetric HL phase for $p = 0.5$: (a) $\alpha = 0.1$, $\beta = 0.6$, (b) $\alpha = 0.6$, $\beta = 0.1$, (c) $\alpha = 0.8$, $\beta = 0.7$. Black and red solid lines represent the results obtained from simulations. In symmetric HL phase, black dash lines and blue solid lines are from simple mean field and cluster mean field analysis, respectively.

3.2. Simple mean field analytical results

In this section, the simple mean field approach is presented to investigate the system in which correlation of sites is ignored, see Fig. 4. The system is divided into four segments and each segment is regarded as single-lane TASEP. In segments I and Ⅲ, β_1 and β_2 are introduced to represent effective extraction rates of the two lanes, and effective insertion rates are denoted by α_1 and α_2, respectively. Because of symmetry of the system, we have $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$.

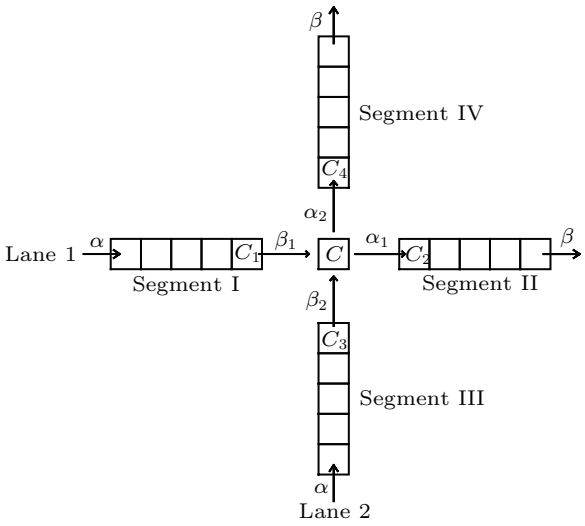


Fig. 4. The system is divided into four independent segments by site C in the simple mean field analysis.

We can obtain

$$\alpha_1 = \alpha_2 = \rho_c[(1-\rho_{c4})0.5p + \rho_{c4}p] = 0.5p\rho_c(1+\rho_{c4}), \quad (1)$$

and

$$\beta_1 = \beta_2 = 1 - \rho_c. \quad (2)$$

When the system is in the symmetric LL phase, the two lanes are both in low density phases. We have

$$\begin{cases} \alpha < \beta_1 & \text{and} \quad \alpha < 0.5, \\ \alpha_1 < \beta & \text{and} \quad \alpha_1 < 0.5, \\ \alpha < \beta_2 & \text{and} \quad \alpha < 0.5, \\ \alpha_2 < \beta & \text{and} \quad \alpha_2 < 0.5. \end{cases} \quad (3)$$

In addition, let J_1 and J_2 denote the currents of upstream and downstream segments of lane I, we can obtain

$$J_1 = \alpha(1-\alpha), \quad (4)$$

$$J_2 = \alpha_1(1-\alpha_1). \quad (5)$$

From Eqs. (4) and (5), we can get $\alpha = \alpha_1$. From Eq. (1), we can obtain $\rho_c = \frac{2\alpha}{p(1+\rho_{c4})}$. Due to the low density of segments Ⅳ, $\rho_{c4} = \alpha$. Then

$$\rho_c = \frac{2\alpha}{p(1+\alpha)}. \quad (6)$$

Combine with inequations (3), we can obtain condition of existence of the symmetric LL phase

$$\begin{cases} \alpha < \frac{-1+\sqrt{1+p^2}}{p}, \\ \alpha < \beta. \end{cases} \quad (7)$$

Similarly, when the system is in the symmetric HH phase, the two lanes are both in high density phases, and we can get

$$\begin{cases} \alpha > \beta_1 & \text{and} \quad \beta_1 < 0.5, \\ \alpha_1 > \beta & \text{and} \quad \beta < 0.5, \\ \alpha > \beta_2 & \text{and} \quad \beta_2 < 0.5, \\ \alpha_2 > \beta & \text{and} \quad \beta < 0.5. \end{cases} \quad (8)$$

From the equation $J_1 = J_2$, we can get $\beta_1 = \beta$. In combination with Eq. (2), $\rho_c = 1 - \beta$ is satisfied. Additionally, the current of segment Ⅳ can also be expressed as $\alpha_2(1-\rho_{c4})$, then $\alpha_2(1-\rho_{c4}) = \beta(1-\beta)$. We obtain

$$\rho_{c4} = \sqrt{1 - \frac{2\beta}{p}}. \quad (9)$$

In combination of Eq. (1) and inequation (8), we can get the condition of existence of symmetric HH phase as follows:

$$\begin{cases} \beta < \frac{-1+\sqrt{1+p^2}}{p}, \\ \beta < \alpha. \end{cases} \quad (10)$$

When the system is in symmetric HL phase, the two lanes are both in HL phase. Segments I and Ⅲ are in high density

phase, and segments Ⅱ and Ⅳ are in low density phase. We have

$$\begin{cases} \alpha > \beta_1 & \text{and} \quad \beta_1 < 0.5, \\ \alpha_1 < \beta & \text{and} \quad \alpha_1 < 0.5, \\ \alpha > \beta_2 & \text{and} \quad \beta_2 < 0.5, \\ \alpha_2 < \beta & \text{and} \quad \alpha_2 < 0.5. \end{cases} \tag{11}$$

The currents of upstream and downstream of lane 1 are equal, i.e., $J_1 = J_2$. Then

$$(1-\beta_1)\beta_1 = \alpha_1(1-\alpha_1). \tag{12}$$

From Eq. (12), we can obtain $\alpha_1 = \beta_1$ or $\alpha_1 = 1-\beta_1$. Because of low density of lane 2, $\rho_{c4} < 1$, and $\alpha_1 < p\rho_c$. Then $\alpha_1 = 1-\beta_1$ cannot be satisfied. Thus, we get $\alpha_1 = \beta_1$. Combining Eqs. (1) and (2), we have

$$0.5p\rho_c(1+\rho_{c4}) = 1-\rho_c. \tag{13}$$

Since segment Ⅳ is in low density,

$$\rho_{c4} = \alpha_2. \tag{14}$$

From Eqs. (1), (13), and (14), we can obtain

$$\rho_{c4} = -\frac{1}{p} + \sqrt{\frac{1}{p^2}+1}. \tag{15}$$

Combined with inequation (11), we get the condition of existence of symmetric HL phase as follows:

$$\begin{cases} \beta > \frac{-1+\sqrt{1+p^2}}{p}, \\ \alpha > \frac{-1+\sqrt{1+p^2}}{p}. \end{cases} \tag{16}$$

The inexistence of asymmetric phase of the system can be explained qualitatively. Following Ref. [16], when the system is in asymmetric phase, one lane is in HD phase, the bulk density is $1-\beta$. The other lane is in HL phase, the density of upstream segment is larger than $1-\beta$, and that of downstream segment is lower than β. Therefrom, we firstly suppose that lane 1 is in HD phase. When one particle reaches the exit site of segment Ⅱ, domain wall will form on the segment. The removal rate β is smaller than the effective entrance rate α_1, then the domain wall will propagate upstream. When the domain wall reaches site C, the stable HD phase will form in lane 1. Due to the update rules of the system, the particle of site C can hop to segment Ⅳ when site C_2 is occupied, and barrier cannot be formed. The effective entrance rate of segment Ⅳ does not decrease, and the LD phase does not exist in segment Ⅳ, which indicates that asymmetric phase does not exist in the system.

The results of simple mean field analysis are shown in Fig. 2. Both the simulation and analysis indicate that only the symmetric phases exist in the system. However, with decrease of p, deviation between the simulation and simple mean field analytical results increases, see Fig. 5. This is because the correlation between upstream and downstream segments increases when p decreases. In addition, inexistence of the asymmetric phase cannot be investigated numerically by the simple mean field analysis. Motivated by this, cluster mean field analysis in which correlation of sites is considered is also carried out.

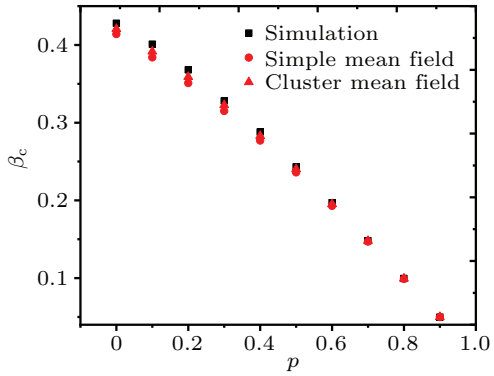


Fig. 5. The results of boundary between symmetric HH phase and symmetric HL phase obtained from Monte Carlo simulation, simple mean field analysis, and cluster mean field analysis.

3.3. Cluster mean field analytical results

In cluster mean field analysis, correlation of five sites (i.e. C, C_1, C_2, C_3, and C_4) is considered, see Fig. 6. Here, α_1 and α_2 denote effective entrance rates into site C_1 and C_3, and β_1 and β_2 denote effective existing rates of particles from site C_2 and C_4. In the analysis, $P_{\tau_1\tau_2\tau_3\tau_4\tau_5}$ is defined as the probability that sites C_1, C, C_2, C_3, and C_4 are in states τ_1, τ_2, τ_3, τ_4, and τ_5, respectively. Due to the update rules of model, sites C_1, C_2, C_3, and C_4 can be in two states, site C can be in three states. Then, τ_1, τ_3, τ_4, and τ_5 have two values (0 means empty, 1 means occupied), and τ_2 have three values (0 means empty, 1 means occupied by one particle of lane 1, 2 means occupied by one particle of lane 2). We have 48 possible states of the system. Due to the conservation of these probabilities, we can obtain $\Sigma P_{\tau_1\tau_2\tau_3\tau_4\tau_5} = 1$, i.e.,

$$\begin{aligned} &P_{00000}+P_{00001}+P_{00010}+P_{00011}+P_{00100} \\ &+P_{00101}+P_{00110}+P_{00111}+P_{01000}+P_{01001} \\ &+P_{01010}+P_{01011}+P_{01100}+P_{01101}+P_{01110} \\ &+P_{01111}+P_{02000}+P_{02001}+P_{02010}+P_{02011} \\ &+P_{02100}+P_{02101}+P_{02110}+P_{02111}+P_{10000} \\ &+P_{10001}+P_{10010}+P_{10011}+P_{10100}+P_{10101} \\ &+P_{10110}+P_{10111}+P_{11000}+P_{11001}+P_{11010} \\ &+P_{11011}+P_{11100}+P_{11101}+P_{11110}+P_{11111} \\ &+P_{12000}+P_{12001}+P_{12010}+P_{12011}+P_{12100} \\ &+P_{12101}+P_{12110}+P_{12111}=1. \end{aligned} \tag{17}$$

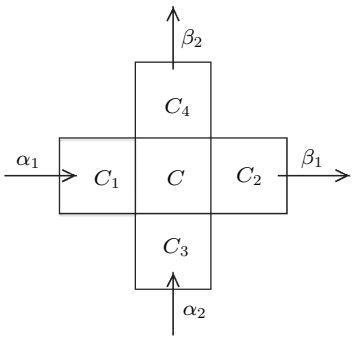


Fig. 6. The illustration of cluster mean field analysis.

In addition, we can also get master equations of evolution of probabilities, e.g., take P_{00000} as an example,

$$\frac{\mathrm{d}P_{00000}}{\mathrm{d}t} = -(\alpha_1+\alpha_2)P_{00000} + \beta_1 P_{00100} + \beta_2 P_{00001}. \quad (18)$$

Here, $\mathrm{d}P_{00000}/\mathrm{d}t$ represents evolution with time of the state such that the five states are all empty. When one particle is injected into cite C_1, the state evolves to another state that sites C, C_2, C_3, C_4 are empty and C_1 is occupied, the probability is represented by $-\alpha_1 P_{00000}$. Similarly, when one particle enters the site C_3, the evolution probability is $-\alpha_2 P_{00000}$. In addition, when the particle of the state that sites C, C_1, C_3, C_4 are empty and site C_2 is occupied, the state evolves to the state such that five sites are all empty, the evolution probability is $\beta_1 P_{00100}$. Likewise, $\beta_2 P_{00001}$ represents the probability that the state in which C, C_1, C_2, C_3 are empty and site C_4 is occupied evolves to the state such that all sites are empty.

In the stationary state, $\mathrm{d}P_{00000}/\mathrm{d}t = 0$, we obtain

$$-(\alpha_1+\alpha_2)P_{00000} + \beta_1 P_{00100} + \beta_2 P_{00001} = 0. \quad (19)$$

and the rest 47 equations are shown in Appendix A. However, in all the 48 equations, only 47 of them are independent.

Let ρ_1 and ρ_2 denote densities of site C_1 and C_3. They can be expressed as

$$\begin{aligned}\rho_1 =\ & P_{10000}+P_{10001}+P_{10010}+P_{10011}\\ &+P_{10100}+P_{10101}+P_{10110}+P_{10111}\\ &+P_{11000}+P_{11001}+P_{11010}+P_{11011}\\ &+P_{11100}+P_{11101}+P_{11110}+P_{11111}\\ &+P_{12000}+P_{12001}+P_{12010}+P_{12011}\\ &+P_{12100}+P_{12101}+P_{12110}+P_{12111}, \quad (20)\\ \rho_2 =\ & P_{00010}+P_{00011}+P_{00110}+P_{00111}\\ &+P_{01010}+P_{01011}+P_{01110}+P_{01111}\\ &+P_{02010}+P_{02011}+P_{02110}+P_{02111}\\ &+P_{10010}+P_{10011}+P_{10110}+P_{10111}\\ &+P_{11010}+P_{11011}+P_{11110}+P_{11111}\\ &+P_{12010}+P_{12011}+P_{12110}+P_{12111}. \quad (21)\end{aligned}$$

Furthermore, the currents of upstream segments of intersection of lanes 1 and 2, which are denoted by J_1 and J_2, can be expressed as

$$J_1 = \rho_1(1-\rho_1), \quad (22)$$
$$J_2 = \rho_2(1-\rho_2), \quad (23)$$

J_1 and J_2 can also be obtained by

$$J_1 = \alpha_1(1-\rho_1), \quad (24)$$
$$J_2 = \alpha_2(1-\rho_2). \quad (25)$$

Let J_3 and J_4 denote the currents of downstream segments of lanes 1 and 2. They can be calculated by

$$\begin{aligned}J_3 =\ & \rho_1(1-\rho_1) - 0.5p(P_{01000}+P_{01010}+P_{11000}+P_{11010})\\ &+0.5p(P_{02000}+P_{02010}+P_{12000}+P_{12010})\\ &-0.5(P_{01100}+P_{01110}+P_{11100}+P_{11110})\\ &+0.5(P_{02001}+P_{02011}+P_{12001}+P_{12011}), \quad (26)\\ J_4 =\ & \rho_2(1-\rho_2) - 0.5p(P_{02000}+P_{02010}+P_{12000}+P_{12010})\\ &+0.5p(P_{01000}+P_{01010}+P_{11000}+P_{11010})\\ &-0.5(P_{02001}+P_{02011}+P_{12001}+P_{12011})\\ &+0.5(P_{01100}+P_{01110}+P_{11100}+P_{11110}). \quad (27)\end{aligned}$$

In addition, J_4 can also be expressed as

$$\begin{aligned}J_4 =\ & \beta_2(P_{00001}+P_{00011}+P_{00101}+P_{00111}\\ &+P_{01001}+P_{01011}+P_{01101}+P_{01111}\\ &+P_{02001}+P_{02011}+P_{02101}+P_{02111}\\ &+P_{10001}+P_{10011}+P_{10101}+P_{10111}\\ &+P_{11001}+P_{11011}+P_{11101}+P_{11111}\\ &+P_{12001}+P_{12011}+P_{12101}+P_{12111}). \quad (28)\end{aligned}$$

Due to the symmetry indicated by simulations, the properties are identical in the cluster mean field analysis, i.e., $\alpha_1=\alpha_2$, $\beta_1=\beta_2$, $\rho_1=\rho_2$, $J_1=J_2$, $J_3=J_4$. Thus, we have 53 unknowns, including 48 probabilities $P_{\tau_1\tau_2\tau_3\tau_4\tau_5}$, α_1 (α_2), β_1 (β_2), ρ_1 (ρ_2), J_1 (J_2), J_3 (J_4). We also have 53 equations, i.e., Eqs. (17), (19), (21), (22), (24), (26), (28), (A1)–(A46). We can obtain these unknown quantities by solving these equations. The boundary between symmetric HH phase and symmetric HL phase can be obtained by

$$\beta_c(1-\beta_c) = J_3. \quad (29)$$

The boundary between symmetric LL phase and symmetric HL phase is determined by $\alpha=\beta_c$ because of symmetry of phase diagram.

The results of boundaries between phases can also be obtained with change of p, see Fig. 5. We can see that the symmetric HL phase expands with increase of p. When $p=1$, the system is in symmetric HL phase independent of α and β, this is because particles cannot be injected into downstream

segments of the two lanes. It is also shown that, as expected, the cluster mean field analysis performs much better than the simple mean field analysis with decrease of p. We also investigate the density profiles when the system is in symmetric HL phase, and the analytical results are shown in Fig. 3(c). We can see that the analytical results and simulation ones are in excellent agreement.

The deviation between the cluster mean field analysis and simulation still exists even if correlation of the five sites is considered. However, we think that the deviation can decrease when correlation of more sites is considered, and this will be our future work.

In order to investigate asymmetric phase, we suppose that the phase exists in the system. Without loss of generality, we assume that lane 1 is in HD phase and lane 2 is in HL phase. We have

$$J_3 = \beta(1-\beta). \tag{30}$$

We have 58 unknowns because of asymmetry, including 48 probabilities $P_{\tau_1\tau_2\tau_3\tau_4\tau_5}$, α_1, α_2, β_1, β_2, ρ_1, ρ_2, J_1, J_2, J_3, and J_4. We also have 58 equations, including Eqs. (17), (19)–(28), (30), and (A1)–(A46). The ρ_1 and ρ_2 can be obtained, see Fig. 7. We can see that $\rho_1 = \rho_2$, which violates the assumption. In addition, the relationship can still be obtained even if p changes. Thus, we can conclude that the asymmetric phase does not exist and the spontaneous symmetry breaking does not exist in the system.

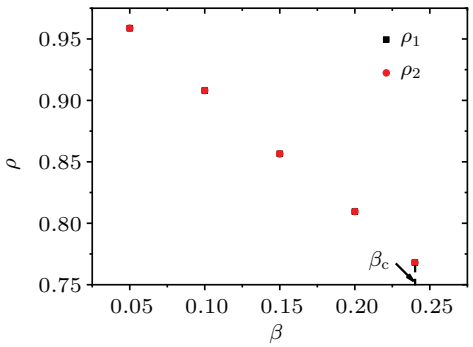


Fig. 7. The relationship between ρ_1 and ρ_2 with change of β. The black and red scattered pints are obtained from the cluster mean field analysis. The β_c corresponds to the value of the boundary between symmetric HH phase and symmetric HL phase. The parameter is $p = 0.5$.

4. Conclusion

In summary, we have investigated asymmetric simple exclusion processes on two lanes with an intersection under open boundaries. The update rule is random update. In the upstream segments of intersection, one particle can move forward with rate 1 if the next site is empty. In order to investigate the occurrence of spontaneous symmetry breaking phenomenon, one particle hops to the next site with rate p in the downstream segments, which can represent the rate of slowing of moving particles. At the intersection site, particles can change moving direction with rate p or $0.5p$. Extensive Monte Carlo simulations are carried out. It is shown that three symmetric phases (i.e., symmetric HH phase, symmetric LL phase, and symmetric HL phase) exist in the system.

The simple mean field analysis and cluster mean field analysis are adopted to investigate the system. In the simple mean field analysis, correlations of sites are ignored. The parameters α_1, α_2, β_1, and β_2 are introduced, and the system is divided into four independent segments. Due to the ignorance of correlation, the analytical results deviate from the simulation ones more and more with decrease of p. The inexistence of asymmetric phase can only be explained qualitatively. Motivated by this, the cluster mean field analysis is also carried out. In the cluster mean field analysis, the correlation of five sites including the intersection site is considered. The analytical boundaries are obtained. It is shown that, as expected, the cluster mean field analysis performs much better than the simple mean field analysis, especially when p is small. Furthermore, we also investigate the existence of asymmetric phase. We firstly assume that the asymmetric phase exists in the system. However, it is found that the densities of two upstream segments are always symmetric, which violates the assumption. Then, we can conclude that the asymmetric phase does not exist, and the spontaneous symmetry breaking does not exist either in the system. We have also investigated the density profiles of the symmetric HL phase, it is shown that the cluster mean field analytical result and simulation result are in excellent agreement.

Our findings can deepen understanding of occurrence of the spontaneous symmetry breaking phenomenon. In addition, the cluster mean field analysis can help to obtain more accurate analytical results of other systems with totally asymmetric simple exclusion processes.

Appendix A

$$\frac{\mathrm{d}P_{00001}}{\mathrm{d}t} = -(\alpha_1+\alpha_2+\beta_2)P_{00001} + \beta_1 P_{00101} + 0.5p(P_{01000}+P_{02000}) = 0, \tag{A1}$$

$$\frac{\mathrm{d}P_{00010}}{\mathrm{d}t} = -(\alpha_1+1)P_{00010} + \alpha_2 P_{00000} + \beta_1 P_{00110} + \beta_2 P_{00011} = 0, \tag{A2}$$

$$\frac{\mathrm{d}P_{00011}}{\mathrm{d}t} = -(\alpha_1+\beta_2+1)P_{00011} + \alpha_2 P_{00001} + \beta_1 P_{00111} + 0.5p(P_{01010}+P_{02010}) = 0, \tag{A3}$$

$$\frac{\mathrm{d}P_{00100}}{\mathrm{d}t} = -(\alpha_1+\alpha_2+\beta_1)P_{00100} + \beta_2 P_{00101} + 0.5p(P_{01000}+P_{02000}) = 0, \tag{A4}$$

$$\frac{\mathrm{d}P_{00101}}{\mathrm{d}t} = -(\alpha_1+\alpha_2+\beta_1++\beta_2)P_{00101}$$

$$+p(P_{01001}+P_{01100}+P_{02001}+P_{02100})=0, \quad \text{(A5)}$$

$$\frac{\mathrm{d}P_{00110}}{\mathrm{d}t}=-(\alpha_1+\beta_1+1)P_{00110}+\alpha_2P_{00100}$$
$$+\beta_2P_{00111}+0.5p(P_{01010}+P_{02010})=0, \quad \text{(A6)}$$

$$\frac{\mathrm{d}P_{00111}}{\mathrm{d}t}=-(\alpha_1+\beta_1+\beta_2+1)P_{00111}+\alpha_2P_{00101}$$
$$+p(P_{01011}+P_{02011}+P_{01110}+P_{02110})=0, \quad \text{(A7)}$$

$$\frac{\mathrm{d}P_{01000}}{\mathrm{d}t}=-(\alpha_1+\alpha_2+p)P_{01000}+P_{10000}$$
$$+\beta_1P_{01100}+\beta_2P_{01001}=0, \quad \text{(A8)}$$

$$\frac{\mathrm{d}P_{01001}}{\mathrm{d}t}=-(\alpha_1+\alpha_2+\beta_2+p)P_{01001}$$
$$+P_{10001}+\beta_1P_{01101}=0, \quad \text{(A9)}$$

$$\frac{\mathrm{d}P_{01010}}{\mathrm{d}t}=-(\alpha_1+p)P_{01010}+\alpha_2P_{01000}+P_{10010}$$
$$+\beta_1P_{01110}+\beta_2P_{01011}=0, \quad \text{(A10)}$$

$$\frac{\mathrm{d}P_{01011}}{\mathrm{d}t}=-(\alpha_1+\beta_2+p)P_{01011}+\alpha_2P_{01001}$$
$$+P_{10011}+\beta_1P_{01111}=0, \quad \text{(A11)}$$

$$\frac{\mathrm{d}P_{01100}}{\mathrm{d}t}=-(\alpha_1+\alpha_2+\beta_1+p)P_{01100}$$
$$+P_{10100}+\beta_2P_{01101}=0, \quad \text{(A12)}$$

$$\frac{\mathrm{d}P_{01101}}{\mathrm{d}t}=-(\alpha_1+\alpha_2+\beta_1+\beta_2)P_{01101}+P_{10101}=0, \quad \text{(A13)}$$

$$\frac{\mathrm{d}P_{01110}}{\mathrm{d}t}=-(\alpha_1+\beta_1+p)P_{01110}+P_{10110}$$
$$+\alpha_2P_{01100}+\beta_2P_{01111}=0, \quad \text{(A14)}$$

$$\frac{\mathrm{d}P_{01111}}{\mathrm{d}t}=-(\alpha_1+\beta_1+\beta_2)P_{01111}+P_{10111}+\alpha_2P_{01101}=0, \quad \text{(A15)}$$

$$\frac{\mathrm{d}P_{02000}}{\mathrm{d}t}=-(\alpha_1+\alpha_2+p)P_{02000}+P_{00010}$$
$$+\beta_1P_{02100}+\beta_2P_{02001}=0, \quad \text{(A16)}$$

$$\frac{\mathrm{d}P_{02001}}{\mathrm{d}t}=-(\alpha_1+\alpha_2+\beta_2+p)P_{02001}$$
$$+P_{00011}+\beta_1P_{02101}=0, \quad \text{(A17)}$$

$$\frac{\mathrm{d}P_{02010}}{\mathrm{d}t}=-(\alpha_1+p)P_{02010}+\alpha_2P_{02000}$$
$$+\beta_1P_{02110}+\beta_2P_{02011}=0, \quad \text{(A18)}$$

$$\frac{\mathrm{d}P_{02011}}{\mathrm{d}t}=-(\alpha_1+\beta_2+p)P_{02011}+\alpha_2P_{02001}+\beta_1P_{02111}=0, \quad \text{(A19)}$$

$$\frac{\mathrm{d}P_{02100}}{\mathrm{d}t}=-(\alpha_1+\alpha_2+\beta_1+p)P_{02100}+\beta_2P_{02101}+P_{00110}=0, \quad \text{(A20)}$$

$$\frac{\mathrm{d}P_{02101}}{\mathrm{d}t}=-(\alpha_1+\alpha_2+\beta_1+\beta_2)P_{02101}+P_{00111}=0, \quad \text{(A21)}$$

$$\frac{\mathrm{d}P_{02110}}{\mathrm{d}t}=-(\alpha_1+\beta_1+p)P_{02110}+\alpha_2P_{02100}+\beta_2P_{02111}=0, \quad \text{(A22)}$$

$$\frac{\mathrm{d}P_{02111}}{\mathrm{d}t}=-(\alpha_1+\beta_1+\beta_2)P_{02111}+\alpha_2P_{02101}=0, \quad \text{(A23)}$$

$$\frac{\mathrm{d}P_{10000}}{\mathrm{d}t}=-(\alpha_2+1)P_{10000}+\alpha_1P_{00000}$$
$$+\beta_1P_{10100}+\beta_2P_{10001}=0, \quad \text{(A24)}$$

$$\frac{\mathrm{d}P_{10001}}{\mathrm{d}t}=-(\alpha_2+\beta_2+1)P_{10001}+\alpha_1P_{00001}$$
$$+\beta_1P_{10101}+0.5p(P_{11000}+P_{12000})=0, \quad \text{(A25)}$$

$$\frac{\mathrm{d}P_{10010}}{\mathrm{d}t}=-2P_{10010}+\alpha_1P_{00010}+\alpha_2P_{10000}$$
$$+\beta_1P_{10110}+\beta_2P_{10011}=0, \quad \text{(A26)}$$

$$\frac{\mathrm{d}P_{10011}}{\mathrm{d}t}=-(\beta_2+2)P_{10011}+\alpha_1P_{00011}+\alpha_2P_{10001}$$
$$+\beta_1P_{10111}+0.5p(P_{11010}+P_{12010})=0, \quad \text{(A27)}$$

$$\frac{\mathrm{d}P_{10100}}{\mathrm{d}t}=-(\alpha_2+\beta_1+1)P_{10100}+\alpha_1P_{00100}$$
$$+\beta_2P_{10101}+0.5p(P_{11000}+P_{12000})=0, \quad \text{(A28)}$$

$$\frac{\mathrm{d}P_{10101}}{\mathrm{d}t}=-(\alpha_2+\beta_1+\beta_2+1)P_{10101}+\alpha_1P_{00101}$$
$$+p(P_{11001}+P_{12001}+P_{11100}+P_{12100})=0, \quad \text{(A29)}$$

$$\frac{\mathrm{d}P_{10110}}{\mathrm{d}t}=-(\beta_1+2)P_{10110}+\alpha_1P_{00110}++\alpha_2P_{10100}$$
$$+\beta_2P_{10111}+0.5p(P_{11010}+P_{12010})=0, \quad \text{(A30)}$$

$$\frac{\mathrm{d}P_{10111}}{\mathrm{d}t}=-(\beta_1+\beta_2+2)P_{10111}+\alpha_1P_{00111}+\alpha_2P_{10101}$$
$$+p(P_{11011}+P_{12011}+P_{11110}+P_{12110})=0, \quad \text{(A31)}$$

$$\frac{\mathrm{d}P_{11000}}{\mathrm{d}t}=-(\alpha_2+p)P_{11000}+\alpha_1P_{01000}$$
$$+\beta_1P_{11100}+\beta_2P_{11001}=0, \quad \text{(A32)}$$

$$\frac{\mathrm{d}P_{11001}}{\mathrm{d}t}=-(\alpha_2+\beta_2+p)P_{11001}+\alpha_1P_{01001}+\beta_1P_{11101}=0, \quad \text{(A33)}$$

$$\frac{\mathrm{d}P_{11010}}{\mathrm{d}t}=-pP_{11010}+\alpha_1P_{01010}+\alpha_2P_{11000}$$
$$+\beta_1P_{11110}+\beta_2P_{11011}=0, \quad \text{(A34)}$$

$$\frac{\mathrm{d}P_{11011}}{\mathrm{d}t}=-(\beta_2+p)P_{11011}+\alpha_1P_{01011}$$
$$+\alpha_2P_{11001}+\beta_1P_{11111}=0, \quad \text{(A35)}$$

$$\frac{\mathrm{d}P_{11100}}{\mathrm{d}t}=-(\alpha_2+\beta_1+p)P_{11100}$$
$$+\alpha_1P_{01100}+\beta_2P_{11101}=0, \quad \text{(A36)}$$

$$\frac{\mathrm{d}P_{11101}}{\mathrm{d}t}=-(\alpha_2+\beta_1+\beta_2)P_{11101}+\alpha_1P_{01101}=0, \quad \text{(A37)}$$

$$\frac{\mathrm{d}P_{11110}}{\mathrm{d}t}=-(\beta_1+p)P_{11110}+\alpha_1P_{01110}$$
$$+\alpha_2P_{11100}+\beta_2P_{11111}=0, \quad \text{(A38)}$$

$$\frac{\mathrm{d}P_{11111}}{\mathrm{d}t} = -(\beta_1+\beta_2)P_{11111} + \alpha_1 P_{01111} + \alpha_2 P_{11101} = 0, \tag{A39}$$

$$\frac{\mathrm{d}P_{12000}}{\mathrm{d}t} = -(\alpha_2+p)P_{12000} + \alpha_1 P_{02000} + \beta_1 P_{12100} + \beta_2 P_{12001} + P_{10010} = 0, \tag{A40}$$

$$\frac{\mathrm{d}P_{12001}}{\mathrm{d}t} = -(\alpha_2+\beta_2+p)P_{12001} + \alpha_1 P_{02001} + \beta_1 P_{12101} + P_{10011} = 0, \tag{A41}$$

$$\frac{\mathrm{d}P_{12010}}{\mathrm{d}t} = -pP_{12010} + \alpha_1 P_{02010} + \alpha_2 P_{12000} + \beta_1 P_{12110} + +\beta_2 P_{12011} = 0, \tag{A42}$$

$$\frac{\mathrm{d}P_{12011}}{\mathrm{d}t} = -(\beta_2+p)P_{12011} + \alpha_1 P_{02011} + \alpha_2 P_{12001} + \beta_1 P_{12111} = 0, \tag{A43}$$

$$\frac{\mathrm{d}P_{12100}}{\mathrm{d}t} = -(\alpha_2+\beta_1+p)P_{12100} + \alpha_1 P_{02100} + \beta_2 P_{12101} + P_{10110} = 0, \tag{A44}$$

$$\frac{\mathrm{d}P_{12101}}{\mathrm{d}t} = -(\alpha_2+\beta_1+\beta_2)P_{12101} + \alpha_1 P_{02101} + P_{10111} = 0, \tag{A45}$$

$$\frac{\mathrm{d}P_{12110}}{\mathrm{d}t} = -(\beta_1+p)P_{12110} + \alpha_1 P_{02110} + \alpha_2 P_{12100} + \beta_2 P_{12111} = 0, \tag{A46}$$

$$\frac{\mathrm{d}P_{12111}}{\mathrm{d}t} = -(\beta_1+\beta_2)P_{12111} + \alpha_1 P_{02111} + \alpha_2 P_{12101} = 0. \tag{A47}$$

Acknowledgement

Project supported by the National Natural Science Foundation of China (Grant No. 11802003).

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