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Theoretical research on the transverse spin of structured optical fields inside a waveguide

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Structured optical fields inside a waveguide possess the transverse spin, i.e., the spin angular momentum perpendicular to the direction of the waveguide. The physical origin of the transverse spin can be attributed to the presence of an effective rest mass of photons in guided waves, or equivalently, to the existence of a longitudinal field component, such that the transverse and longitudinal fields together form an elliptical polarization plane. In contrary to the traditional viewpoint, the transverse spin of photons in guided waves is also quantized, and its quantization form is related to the ellipticity of the polarization ellipse. The direction of the transverse spin depends on the propagation direction of electromagnetic waves along the waveguide, such a spin-momentum locking may have important applications in spin-dependent unidirectional optical interfaces. By means of a coupling between the transverse spin of guided waves and some physical degrees of freedom, one can develop an optical analogy of spintronics, i.e., spinoptics.

Keywords: transverse spin, topological photonics, the spin Hall effect of light, spinoptics

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1. Introduction

Light carries spin angular momentum (spin for short) associated with the polarization of light. For example, the right-hand and left-hand circular polarizations of a paraxial beam respectively correspond to the positive and negative helicities of photons.[1] Interestingly enough, in addition to the spin of photons in the conventional sense, there is an intriguing transverse spin that was first studied in 2012 by Bliokh and Nori,[2] where the term transverse means that the spin is orthogonal to the mean propagation direction of a structured optical field. Up to now, people have studied the transverse spin of evanescent waves, focused beams, and two-wave interference,[1–18] and have found important applications in spin-dependent unidirectional optical interfaces.[9,10,13,19–26] Similar counterparts of which have been found universally in other wave systems, such as acoustic and elastic waves.[27–32]

On the other hand, waveguides and cavities are two key device structures in many important engineering areas, people have presented many investigations on them.[33–38] Though the transverse spin and the related topics in two-dimensional (2D) optical waveguides have been studied,[27,39] it is not clear whether electromagnetic waves inside a 3D waveguide also possess the transverse spin. In particular, the physical origin of the transverse spin still remains obscure. In fact, the transverse spin is previously mysterious and lacks a unified physical picture. For example, according to Bliokh and Nori,[1] there are several kinds of transverse spin, which differ strongly in their origins and physical properties. In this paper, the transverse spin of electromagnetic waves inside a 3D waveguide is studied, based on which the physical origin and the quantization form of the transverse spin are revealed. Moreover, some potential applications of the transverse spin are tentatively explored.

We work in the international system of units (SI), the 4D metric tensor is taken as $\eta^{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ ($\mu,\nu = 0,1,2,3$). Let ε_0 and μ_0 denote the permittivity and permeability in vacuum, and then $c = 1/\sqrt{\varepsilon_0\mu_0}$ is the velocity of light in vacuum. Complex conjugation is denoted by $*$. Repeated indices must be summed according to the Einstein rule.

2. Transverse spin of electromagnetic waves in a waveguide

Let a Cartesian coordinate system be spanned by an orthonormal basis $\{\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z\}$ with $\boldsymbol{e}_z = \boldsymbol{e}_x \times \boldsymbol{e}_y$, where the unit vectors $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, and $\boldsymbol{e}_z$ are respectively along the x-, y-, and z-axes. A straight hollow-tube waveguide is placed along the z-axis, it has a rectangular cross section with perfectly conducting walls at $x = 0$, $x = a$, $y = 0$, and $y = b$ ($a > b$). Let us consider that the electromagnetic waves of frequency ω propagate along the waveguide, they consist of two independent fields,[40–42] called transverse magnetic (TM) and transverse electric (TE). Let $\boldsymbol{k} = (k_x, k_y, k_z)$ is the 3D wavenumber vector of photons inside the waveguide, where $\omega = c|\boldsymbol{k}|$, $k_x = m\pi/a$, and $k_y = n\pi/b$. The cutoff frequency of the waveguide is $\omega_c = c\sqrt{k_x^2 + k_y^2} = c\pi\sqrt{(m/a)^2 + (n/b)^2}$. The electric field intensity $\boldsymbol{E}$ and the magnetic induction density $\boldsymbol{B}$ inside the waveguide are

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$$\text{TM mode}: \begin{cases} E_x = \mathrm{i}\frac{m\pi}{a}\frac{k_z}{\omega_c^2}c^2E_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\exp[-\mathrm{i}\,(\omega t-k_z z)], \\ E_y = \mathrm{i}\frac{n\pi}{b}\frac{k_z}{\omega_c^2}c^2E_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\exp[-\mathrm{i}\,(\omega t-k_z z)], \\ E_z = E_0\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\exp[-\mathrm{i}\,(\omega t-k_z z)],\ \ B_z = 0, \\ B_x = -\mathrm{i}\frac{n\pi}{b}\frac{\omega}{\omega_c^2}E_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\exp[-\mathrm{i}\,(\omega t-k_z z)], \\ B_y = \mathrm{i}\frac{m\pi}{a}\frac{\omega}{\omega_c^2}E_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\exp[-\mathrm{i}\,(\omega t-k_z z)], \end{cases} \tag{1}$$

$$\text{TE mode}: \begin{cases} E_x = -\mathrm{i}\frac{n\pi}{b}\frac{\omega}{\omega_c^2}c^2B_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\exp[-\mathrm{i}\,(\omega t-k_z z)], \\ E_y = \mathrm{i}\frac{m\pi}{a}\frac{\omega}{\omega_c^2}c^2B_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\exp[-\mathrm{i}\,(\omega t-k_z z)], \\ E_z = 0, B_z = B_0\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\exp[-\mathrm{i}\,(\omega t-k_z z)], \\ B_x = -\mathrm{i}\frac{m\pi}{a}\frac{k_z}{\omega_c^2}c^2B_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\exp[-\mathrm{i}\,(\omega t-k_z z)], \\ B_y = -\mathrm{i}\frac{n\pi}{b}\frac{k_z}{\omega_c^2}c^2B_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\exp[-\mathrm{i}\,(\omega t-k_z z)], \end{cases} \tag{2}$$

where E_0 and B_0 are two real constants, $m,n = 1,2,3,\ldots$ for the TM mode, and $m = 1,2,3,\ldots$, $n = 0,1,2,\ldots$ for the TE mode. For propagation modes k_z is a real number, for evanescent modes $k_z = \mathrm{i}\beta$ with β being a positive real number.

The field intensities $\boldsymbol{E}$ and $\boldsymbol{B}$ of the guided waves inside the hollow waveguide can be determined from the potentials $A^\mu = (\varphi/c, \boldsymbol{A})$ or $C^\mu = (c\phi, \boldsymbol{C})$ ($\mu = 0,1,2,3$, where the superscripts or subscripts of $i = 1,2,3$ respectively correspond to the x-, y- and z-components). Let us take the gauge as $\varphi = 0$ and $\nabla\cdot\boldsymbol{A} = 0$, or $\phi = 0$ and $\nabla\cdot\boldsymbol{C} = 0$, and then we have $\boldsymbol{E} = -\partial\boldsymbol{A}/\partial t$ and $\boldsymbol{B} = \nabla\times\boldsymbol{A}$, or $\boldsymbol{E} = -\nabla\times\boldsymbol{C}$ and $\boldsymbol{B} = -\partial\boldsymbol{C}/c^2\partial t$. In this way the field intensities are only determined by $\boldsymbol{A}$ or $\boldsymbol{C}$. When all field quantities are expressed as real functions, one can prove that the total angular momentum of the electromagnetic field is given by[43–46]

$$\begin{aligned} \boldsymbol{J} &= \int \varepsilon_0 \boldsymbol{r}\times(\boldsymbol{E}\times\boldsymbol{B})\mathrm{d}x\mathrm{d}y\mathrm{d}z \\ &= \varepsilon_0\int E_i(\boldsymbol{r}\times\nabla)A_i\mathrm{d}x\mathrm{d}y\mathrm{d}z + \varepsilon_0\int \boldsymbol{E}\times\boldsymbol{A}\mathrm{d}x\mathrm{d}y\mathrm{d}z \\ &= \varepsilon_0\int B_l(\boldsymbol{r}\times\nabla)C_l\mathrm{d}x\mathrm{d}y\mathrm{d}z + \varepsilon_0\int \boldsymbol{B}\times\boldsymbol{C}\mathrm{d}x\mathrm{d}y\mathrm{d}z, \end{aligned} \tag{3}$$

which provides the field falls off suitably as $|\boldsymbol{r}| \to +\infty$, where $\boldsymbol{r} = (x,y,z)$, $i,l = 1,2,3$ and repeated indices must be summed according to the Einstein rule. Then the total angular momentum can be decomposed into an orbital angular momentum depending on the reference frame and an angular momentum (spin) independent of the frame of reference. We refer to $\boldsymbol{s}^{\mathrm{e}} = \varepsilon_0\boldsymbol{E}\times\boldsymbol{A}$ and $\boldsymbol{s}^{\mathrm{m}} = \varepsilon_0\boldsymbol{B}\times\boldsymbol{C}$ as the electric-like and magnetic-like spin densities, respectively. For the electromagnetic waves of frequency ω propagating inside the waveguide, in our gauge one has $\boldsymbol{A} = -\mathrm{i}\boldsymbol{E}/\omega$ and $\boldsymbol{C} = -\mathrm{i}c^2\boldsymbol{B}/\omega$. When all field quantities are expressed as complex functions, the time-averaged electric-like and magnetic-like spin densities can be expresses as

$$\boldsymbol{s}^{\mathrm{e}} = \frac{\varepsilon_0}{2}\mathrm{Re}(\boldsymbol{E}\times\boldsymbol{A}^*) = \frac{\varepsilon_0}{2\omega}\mathrm{Re}(\mathrm{i}\boldsymbol{E}\times\boldsymbol{E}^*), \tag{4}$$

$$\boldsymbol{s}^{\mathrm{m}} = \frac{\varepsilon_0}{2}\mathrm{Re}(\boldsymbol{B}\times\boldsymbol{C}^*) = \frac{1}{2\mu_0\omega}\mathrm{Re}(\mathrm{i}\boldsymbol{B}\times\boldsymbol{B}^*). \tag{5}$$

Let $h = E_0$ for the TM mode and $h = cB_0$ for the TE mode (the same below). Applying Eqs. (1), (2), (4), and (5), one can obtain $\boldsymbol{s}^{\mathrm{e}} = (s_x^{\mathrm{e}}, s_y^{\mathrm{e}}, 0)$ and $\boldsymbol{s}^{\mathrm{m}} = (0,0,0)$ for the TM mode, while $\boldsymbol{s}^{\mathrm{m}} = (s_x^{\mathrm{m}}, s_y^{\mathrm{m}}, 0)$ and $\boldsymbol{s}^{\mathrm{e}} = (0,0,0)$ for the TE mode, where

$$\text{TM}: \begin{cases} s_x^{\mathrm{e}} = -\frac{n\pi}{b}\frac{k_z}{2\mu_0\omega_c^2\omega}h^2\sin^2\left(\frac{m\pi}{a}x\right)\sin\left(\frac{2n\pi}{b}y\right), \\ s_y^{\mathrm{e}} = \frac{m\pi}{a}\frac{k_z}{2\mu_0\omega_c^2\omega}h^2\sin\left(\frac{2m\pi}{a}x\right)\sin^2\left(\frac{n\pi}{b}y\right), \end{cases} \tag{6}$$

$$\text{TE}: \begin{cases} s_x^{\mathrm{m}} = \frac{n\pi}{b}\frac{k_z}{2\mu_0\omega_c^2\omega}h^2\cos^2\left(\frac{m\pi}{a}x\right)\sin\left(\frac{2n\pi}{b}y\right), \\ s_y^{\mathrm{m}} = -\frac{m\pi}{a}\frac{k_z}{2\mu_0\omega_c^2\omega}h^2\sin\left(\frac{2m\pi}{a}x\right)\cos^2\left(\frac{n\pi}{b}y\right). \end{cases} \tag{7}$$

Obviously, both Eqs. (6) and (7) show that the spin densities are transverse with respect to the longitudinal direction of the waveguide, and then they are the so-called transverse spin. It is worth noting that Eqs. (6) and (7) show that the transverse spin is proportional to the wave number component of k_z, such a spin-momentum locking may have important applications in spin-dependent unidirectional optical interfaces. In fact, the transverse spin of structured optical fields has been associated with the spin Hall effect of light and topological photonics,[39,47,48] etc.

To exhibit the density distribution of these transverse-spin vectors within a cross section of the waveguide (i.e., at an arbitrary fixed z), we take the TM_{11}, TM_{21}, TE_{10}, and TE_{11} modes as examples, and show them by Figs. 1(a), 1(b), 2(a), and 2(b), respectively, where the diagrammatic arrows represent local spin vectors, with the thickness of arrows being directly proportional to the spin density. For convenience let $a = b = 1$ and $\pi k_z h^2/(2\mu_0\omega_c^2\omega) = 1$ (it is just a matter of a relative density distribution).

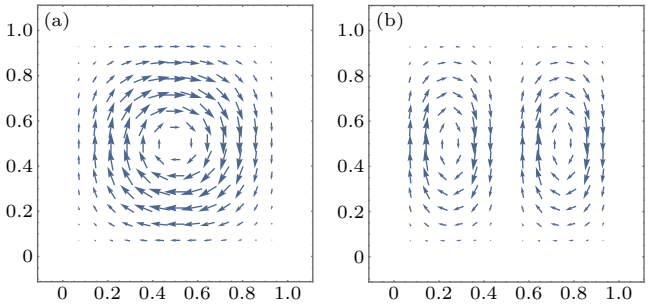


Fig. 1. (a) The transverse-spin density distribution of TM_{11}. (b) The transverse-spin density distribution of TM_{21}.

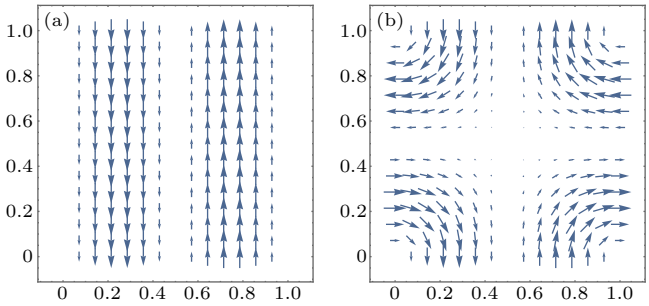


Fig. 2. (a) The transverse-spin density distribution of TE_{10}. (b) The transverse-spin density distribution of TE_{11}.

3. Quantization form of the transverse spin of guided waves

The quantization of a spin angular momentum means that the projection of the spin along a given direction is quantized. In contrast with the previous studies,[1,2] here we will show that the transverse spin of photons in guided waves is also quantized. As for the nonvanishing spin densities, let $s_i = s_i^{\mathrm{e}}$ for the TM mode and $s_i = s_i^{\mathrm{m}}$ for the TE mode, $i = x, y$. Considering that they fluctuate with spatial positions, conventionally, let us define the average of $(s_x^2 + s_y^2)$ over the cross section of the waveguide as follows:

$$\langle s_\perp^2 \rangle = \frac{1}{ab}\int_0^b \int_0^a (s_x^2 + s_y^2)\,\mathrm{d}x\mathrm{d}y. \tag{8}$$

Substituting Eq. (6) or Eq. (7) into Eq. (8), let $m, n = 1, 2, \ldots$, using $c^2 = 1/(\varepsilon_0\mu_0)$ and $\omega_{\mathrm{c}} = c\sqrt{(m\pi/a)^2 + (n\pi/b)^2}$, one has (for both the TM and TE modes)

$$\langle s_\perp^2 \rangle = \frac{3}{64}\left(\frac{\varepsilon_0 c k_z h^2}{\omega_{\mathrm{c}}\omega}\right)^2. \tag{9}$$

As an exception, for the TE mode with $m = 1, 2, \ldots$ and $n = 0$, one has

$$\langle s_\perp^2 \rangle = \frac{1}{8}\left(\frac{\varepsilon_0 c k_z h^2}{\omega_{\mathrm{c}}\omega}\right)^2. \tag{10}$$

Our discussion first bases on Eq. (9). Let L denotes the length of the waveguide, and then $V = abL$ is the volume of the waveguide. The total transverse spin $S_\perp$ can be expressed as

$$S_\perp = \int_0^L \int_0^b \int_0^a \sqrt{\langle s_\perp^2 \rangle}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z = \frac{\sqrt{3}}{8}\frac{\varepsilon_0 c k_z h^2 V}{\omega_{\mathrm{c}}\omega}, \tag{11}$$

where $h = E_0$ for the TM mode and $h = cB_0$ for the TE mode. Equation (11) shows that the total transverse spin $S_\perp$ also depends on the wave number component of k_z. On the other hand, in the time-averaged sense, the total momentum of the guided waves is

$$\begin{aligned} \boldsymbol{P} &= \int_0^L \int_0^a \int_0^b \frac{1}{2}\varepsilon_0 \mathrm{Re}(\boldsymbol{E}\times\boldsymbol{B}^*)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z = (0, 0, P_z), \\ P_z &= \frac{\varepsilon_0 \omega k_z h^2 V}{8\omega_{\mathrm{c}}^2}. \end{aligned} \tag{12}$$

Likewise, using $\omega^2 = \omega_{\mathrm{c}}^2 + c^2 k_z^2$ and $\omega_{\mathrm{c}} = c\pi\sqrt{(m/a)^2 + (n/b)^2}$ one can obtain the total energy of the guided waves

$$\begin{aligned} W &= \int_0^L \int_0^a \int_0^b \frac{1}{4}\varepsilon_0 \mathrm{Re}(\boldsymbol{E}\cdot\boldsymbol{E}^* + c^2\boldsymbol{B}\cdot\boldsymbol{B}^*)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z \\ &= \frac{\varepsilon_0 \omega^2 h^2 V}{8\omega_{\mathrm{c}}^2}. \end{aligned} \tag{13}$$

Then the energy velocity along the waveguide is $\boldsymbol{v}_E = c^2\boldsymbol{P}/W = (0, 0, v)$, where

$$v = P_z c^2/W = k_z c^2/\omega = \pm c\sqrt{1 - \omega_{\mathrm{c}}^2/\omega^2}. \tag{14}$$

That is, the energy velocity equals to the group velocity of the guided waves. The electromagnetic waves inside the waveguide consist of photons, which means that the vacuum energy can be discarded by taking the normal product of field operators

$$\begin{aligned} &W = \frac{\varepsilon_0 \omega^2 h^2 V}{8\omega_{\mathrm{c}}^2} = l\hbar\omega, \quad P_z = \frac{\varepsilon_0 \omega k_z h^2 V}{8\omega_{\mathrm{c}}^2} = l\hbar k_z, \\ &l = 1, 2, \ldots. \end{aligned} \tag{15}$$

Comparing with Eq. (11), using Eq. (14) one can obtain the total transverse spin of l photons

$$S_\perp = \sqrt{3}\frac{c k_z \omega_{\mathrm{c}}}{\omega^2} l\hbar = \sqrt{3} l\hbar \frac{v}{c}\sqrt{1 - \frac{v^2}{c^2}}, \quad l = 1, 2, \ldots. \tag{16}$$

Now, let us provide a physical interpretation for Eq. (16). It should be emphasized that, though the guided waves with the transverse spin are linearly polarized within the polarization plane perpendicular to the mean propagation axis, they are elliptically polarized in a polarization plane containing the mean propagation axis, and the ellipticity of the polarization ellipse that makes contribution to the transverse spin can be described by

$$\begin{aligned} \mathrm{TM}: \ \tan\theta &= \frac{\sqrt{\langle E_x E_x^* + E_y E_y^* \rangle}}{\sqrt{\langle E_z E_z^* \rangle}}, \\ \mathrm{TE}: \ \tan\theta &= \frac{\sqrt{\langle B_x B_x^* + B_y B_y^* \rangle}}{\sqrt{\langle B_z B_z^* \rangle}}, \end{aligned} \tag{17}$$

where $\langle F\rangle \equiv (ab)^{-1}\int_0^b\int_0^a F\,\mathrm{d}x\mathrm{d}y$ denotes the average of F over the cross section of the waveguide. Using Eqs. (1) and (2), one can show that the two modes give the same result

$$e=\tan\theta=\frac{k_z c}{\omega_\mathrm{c}}=\frac{v/c}{\sqrt{1-v^2/c^2}}. \tag{18}$$

Using Eq. (18) one can rewrite Eq. (16) as

$$S_\perp=\frac{\sqrt{3}}{2}l\hbar\sin 2\theta,\ \ l=1,2,\ldots. \tag{19}$$

Therefore, a single photon inside the waveguide possesses the total transverse spin of $\hbar\sqrt{3}\sin 2\theta/2$. Obviously, as the energy velocity $v=0$, one has $\theta=0$ and $e=0$, the elliptical polarization becomes a linear polarization, and then $S_\perp=0$. While, as $v=\sqrt{2}c/2$, the ellipticity $e=\tan\theta=1$ (i.e., the circular polarization), and the total transverse spin of a single photon inside the waveguide becomes $S_\perp=\sqrt{3}\hbar/2$.

Likewise, as for the TE mode with $m=1,2,\ldots$ and $n=0$, substituting Eq. (10) into Eq. (11), and using Eq. (18), one can obtain

$$S_\perp=\sqrt{2}l\hbar\sin 2\theta,\ \ l=1,2,\ldots. \tag{20}$$

In the case of the circular polarization, i.e., $e=\tan\theta=1$ (and then $\sin 2\theta=1$ and $v=\sqrt{2}c/2$), the total transverse spin of a single photon inside the waveguide becomes the maximum value of $(S_\perp)_\mathrm{max}=\sqrt{2}\hbar$. Because of $(S_\perp^2)_\mathrm{max}=1(1+1)\hbar^2$, it is the usual total spin of a vector particle in free space.

4. Physical origin of the transverse spin of guided photons

Equations (1) and (2) show that, along the cross section of the waveguide, the electromagnetic waves are linearly polarized. However, along the longitudinal axis of the waveguide, the electric-field components (for the TM mode) or the magnetic-field components (for the TE mode) forms an elliptical polarization plane, such that the guided waves have the electric-like or magnetic-like transverse spin. This can be regarded as a direct physical explanation for the transverse spin of guided photons. Nonetheless, we will discuss the physical origin of the transverse spin from a more profound physical point of view, so as to obtain a unified physical picture.

The terminology of transverse spin was first introduced in 2012 by Bliokh and Nori,[2] it means that spin is perpendicular to the propagation direction, and then corresponds to a spin state with the helicity of $\lambda=0$. As we know, a massive vector field (the Proca field) has a longitudinal field component that can contribute to the spin state with the helicity of $\lambda=0$, and then has a transverse spin which is perpendicular to its propagation direction. In contrast to the Proca field, the photon field in free space acting as a massless vector field, its field intensities only contain two transverse field components describing two kinds of transverse photons with the helicity of $\lambda=\pm1$, without the longitudinal field component describing the admixture of the longitudinal and scalar photons with the helicity of $\lambda=0$ (the longitudinal and scalar photons can exist just in the form of virtual photons).

As mentioned above, the Proca field's transverse spin relates to its longitudinal field component, which is due to the existence of a nonvanishing rest mass. Likewise, we will show that the transverse spin of guided photons can be attributed to an effective rest mass of the guided photons, and it is associated with the presence of a longitudinal field component. However, here the transverse direction is defined as perpendicular to the average propagation direction of the guided photons, which is why the rest mass of the guided photons is just an effective one.

Let $\boldsymbol{k}=\boldsymbol{k}_\perp+\boldsymbol{k}_\mathrm{L}$, where $\boldsymbol{k}_\perp=k_x\boldsymbol{e}_x+k_y\boldsymbol{e}_y$ and $\boldsymbol{k}_\mathrm{L}=k_z\boldsymbol{e}_z$ stand for $\boldsymbol{k}$'s components perpendicular and parallel to the waveguide, respectively. According to the waveguide theory, the group and phase velocities of the guided waves are $\boldsymbol{v}_\mathrm{g}=\boldsymbol{e}_z c\sqrt{1-(\omega_\mathrm{c}/\omega)^2}$ and $\boldsymbol{v}_\mathrm{p}=\boldsymbol{e}_z c/\sqrt{1-(\omega_\mathrm{c}/\omega)^2}$, respectively, such that single photons inside the waveguide satisfy the de Broglie's relations of matter waves $\boldsymbol{v}_\mathrm{g}\cdot\boldsymbol{v}_\mathrm{p}=c^2$ and $w^2=\boldsymbol{p}^2c^2+m_0^2c^4$, where $w=\hbar\omega$ and $\boldsymbol{p}=\hbar\boldsymbol{k}_\mathrm{L}$ are the energy and momentum of a single photon along the waveguide, and $m_0=\hbar\omega_\mathrm{c}/c^2$ is called as the effective rest mass of single photons inside the waveguide. It is easy to prove that photons inside the waveguide satisfy

$$w=\hbar\omega=\frac{m_0c^2}{\sqrt{1-(\boldsymbol{v}_\mathrm{g}^2/c^2)}},\ \ \boldsymbol{p}=\hbar\boldsymbol{k}_\mathrm{L}=\frac{m_0\boldsymbol{v}_\mathrm{g}}{\sqrt{1-(\boldsymbol{v}_\mathrm{g}^2/c^2)}}. \tag{21}$$

They are exactly the relativistic energy and moment of a massive particle. In fact, the velocity $\boldsymbol{v}_\mathrm{g}$ can be viewed as a relative velocity between an inertial observer and guided photons along the waveguide. Moreover, let us make an orthogonal decomposition for $p^\mu=(\hbar\omega/c,\hbar\boldsymbol{k})$ as follows: $p^\mu=p_\mathrm{T}^\mu+p_\mathrm{L}^\mu$, $p_\mathrm{T}^\mu\equiv(0,\hbar\boldsymbol{k}_\perp)$, $p_\mathrm{L}^\mu=(\hbar\omega/c,\hbar\boldsymbol{k}_\mathrm{L})=(w/c,\boldsymbol{p})$. This orthogonal decomposition is Lorentz invariant because of $p_{\mathrm{L}\mu}\,p_\mathrm{T}^\mu=0$. Likewise, let $\boldsymbol{r}_\mathrm{L}=z\boldsymbol{e}_z$ and $\boldsymbol{r}_\perp\equiv x\boldsymbol{e}_x+y\boldsymbol{e}_y$, one can make an orthogonal decomposition of $x^\mu=(ct,\boldsymbol{r})=x_\mathrm{T}^\mu+x_\mathrm{L}^\mu$, where $x_\mathrm{T}^\mu\equiv(0,\boldsymbol{r}_\perp)$ and $x_\mathrm{L}^\mu=(ct,\boldsymbol{r}_\mathrm{L})$. It is easy to show that $p_\mu x^\mu=p_{\mathrm{T}\mu}x_\mathrm{T}^\mu+p_{\mathrm{L}\mu}x_\mathrm{L}^\mu$. The operator $\hat{p}_\mu=\mathrm{i}\hbar\partial_\mu=\mathrm{i}\hbar\partial/\partial x^\mu$ represents the totally 4D momentum operator of photons inside the waveguide, while $\hat{p}_\mathrm{L}^\mu=\mathrm{i}\hbar\partial_{\mathrm{L}\mu}=\mathrm{i}\hbar\partial/\partial x_\mathrm{L}^\mu$ represents the 4D momentum operator of photons along the waveguide.

Now, let us associate the effective rest mass of guided photons with the longitudinal component of guided waves. In terms of $A^\mu=(\varphi/c,\boldsymbol{A})$ and $C^\mu=(c\phi,\boldsymbol{C})$ one can define an electromagnetic field tensor $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and a dual electromagnetic field tensor $G_{\mu\nu}=\partial_\mu C_\nu-\partial_\nu C_\mu$, respectively, and then $F_{ij}=\varepsilon_{ijk}B_k$, $F_{i0}=-F_{0i}=E_i/c$, $G_{ij}=-\varepsilon_{ijk}E_k$, and

$G_{i0} = -G_{0i} = B_i c$, where $\varepsilon_{ijk} = \varepsilon^{ijk}$ denotes the totally antisymmetric tensor with $\varepsilon_{123} = 1$ $(i, j, k = 1, 2, 3)$. One can prove that the time-averaged Lagrangian density satisfies

$$L = -\frac{1}{8}c^2 F_{\mu\nu}F^{\mu\nu *} = \frac{1}{8}G_{\mu\nu}G^{\mu\nu *} = \frac{1}{4}(\boldsymbol{E}\cdot\boldsymbol{E}^* - c^2\boldsymbol{B}\cdot\boldsymbol{B}^*). \tag{22}$$

It contains the contributions from the longitudinal fields (i.e., E_z or B_z). Let $F_{\perp\mu\nu}$ and $G_{\perp\mu\nu}$ represent the transverse parts of $F_{\mu\nu}$ and $G_{\mu\nu}$, respectively, that is, $F_{\perp\mu\nu} = F_{\mu\nu}$ and $G_{\perp\mu\nu} = G_{\mu\nu}$ for $\mu\nu \neq 12,\ 21,\ 03,\ 30$, $F_{\perp\mu\nu} = 0$ and $G_{\perp\mu\nu} = 0$ for $\mu\nu = 12,\ 21,\ 03,\ 30$. The time-averaged Lagrangian density of the TM mode and the one of the TE mode respectively satisfy

$$L_{\rm TM} = \frac{1}{8}G_{\perp\mu\nu}G_{\perp}^{\mu\nu *} + \frac{1}{4}E_z E_z^*,$$
$$L_{\rm TE} = -\frac{1}{8}c^2 F_{\perp\mu\nu}F_{\perp}^{\mu\nu *} - \frac{1}{4}c^2 B_z B_z^*. \tag{23}$$

For $L = L_{\rm TM},\ L_{\rm TE}$, the time-averaged Lagrangian per unit length along the waveguide can be written as $L = \int_0^a \int_0^b L\,{\rm d}x{\rm d}y$, where $L = L_{\rm TM},\ L_{\rm TE}$. In our case, $A_\mu A^{\mu *} = \boldsymbol{A}\cdot\boldsymbol{A}^*$ and $C_\mu C^{\mu *} = \boldsymbol{C}\cdot\boldsymbol{C}^*$, then one can prove that

$$L_{\rm TM} = \int_0^a \int_0^b \left(\frac{1}{8}G_{\perp\mu\nu}G_{\perp}^{\mu\nu *} + \frac{c^2}{4\hbar^2}m_0^2 C_\mu C^{\mu *}\right){\rm d}x{\rm d}y, \tag{24}$$

$$L_{\rm TE} = \int_0^a \int_0^b \left(-\frac{1}{8}c^2 F_{\perp\mu\nu}F_{\perp}^{\mu\nu *} - \frac{c^4}{4\hbar^2}m_0^2 A_\mu A^{\mu *}\right){\rm d}x{\rm d}y. \tag{25}$$

Equations (24) and (25) imply that the effective rest mass is carried by the longitudinal component of guided waves. As an example, applying $A^\mu = (0, \boldsymbol{A})$ and $\boldsymbol{A} = (A_1, A_2, 0)$, and substituting the second formula of Eq. (23) into the Euler–Lagrange equation

$$\frac{\partial L_{\rm TE}}{\partial A_\nu} - \frac{\partial}{\partial x^\mu}\frac{\partial L_{\rm TE}}{\partial(\partial_\mu A_\nu)} = 0, \tag{26}$$

one can obtain $(\partial_{{\rm L}\mu}\partial_{\rm L}^\mu - m_0^2 c^2/\hbar^2)A^\nu = 0$, which is equivalent to the Klein–Gordon equation of a massive particle moving along the waveguide.

It is worth pointing out that, under the Lorentz transformations consisting of Lorentz boosts and spatial rotations with respect to the axial direction of the waveguide (they form a subgroup SO (1, 1) of the Lorentz group SO (1, 3)), Eq. (25) is covariant and the effective rest mass $m_0 = \hbar\omega_{\rm c}/c^2$ keeps invariant ($F_{\perp\mu\nu}F_{\perp}^{\mu\nu}$, $G_{\perp\mu\nu}G_{\perp}^{\mu\nu}$, $C_\mu C^\mu$, $A_\mu A^\mu$ are also invariant). The emergence of a rest mass is always accompanied with some symmetry breaking. Likewise, the appearance of the effective rest mass of guided photons relates to the symmetry breaking from the Lorentz group SO (1, 3) to the subgroup SO (1, 1). In fact, the effective rest mass is the rest energy of photons inside the waveguide (i.e., the energy as $v_{\rm g} = 0$), and arises by forming standing-waves along the cross-section of the waveguide. In other words, it arises by localizing and confining the electromagnetic energy along the cross-section of the waveguide.

5. Some potential applications of the transverse spin: an initial discussion

The propagation direction of a photon in the free vacuum is the special direction, and only along this direction the spin projection of the photon can be observed. In contrary to this, because of the nonvanishing rest mass, an electron has a measurable spin projection along the direction of an applied magnetic field, such that spintronics has been developed during the last twenty years.

However, a photon inside a waveguide has an effective rest mass, its spin projection along the cross section of the waveguide can also be observed, this is where the significance of the transverse spin lies. As a result, based on the transverse spin of guided waves, one can develop a new discipline that can be called as spinphotonics or spinoptics, as an optical analogy of spintronics. In fact, based on the effects of spin-symmetry breaking in nanoscale structures caused by spin–orbit interaction, a new branch in optics, i.e., spinoptics in plasmonics, has recently been developed.[49] Moreover, by means of the transverse spin of a surface plasmon wave, the authors of Ref. [10] have demonstrated a reciprocal effect of spin–orbit coupling when the direction of propagation of the surface plasmon wave determines a scattering direction of spin-carrying photons, where the spin–orbit coupling effect is an optical analogue of the spin injection in solid-state spintronic devices (inverse spin Hall effect), and may be important for optical information processing, quantum optical technology and topological surface metrology. Similarly, here we will initially discuss some potential applications of the transverse spin of guided waves.

Along the cross section of the waveguide, the electromagnetic waves are linearly polarized. While, along the longitudinal axis of the waveguide, the electric-field components (for the TM mode) or the magnetic-field components (for the TE mode) form an elliptical polarization. As a result, for the TM mode, $\boldsymbol{s}^{\rm e} \neq 0$ and $\boldsymbol{s}^{\rm m} = 0$; for the TE mode, $\boldsymbol{s}^{\rm e} = 0$ and $\boldsymbol{s}^{\rm m} \neq 0$. To illustrate the possibility of developing a new branch in optics based on the transverse spin of guided waves, let us first mention two key points as follows:

(i) Let us assume that a dielectric (or ferromagnetic) microsphere is placed near a microhole in the wall of the waveguide, where the effect of the microhole on the field distribution inside the waveguide can be ignored, and the microsphere has an isotropic polarizability α (or an isotropic susceptibility ζ). When the structured optical fields (SOF) inside the

waveguide impinge on the dielectric or ferromagnetic microsphere through the microhole, the microsphere becomes polarized or magnetized, the SOF-induced electric dipole moment $\boldsymbol{p}$ (or magnetic dipole moment $\boldsymbol{m}$) can be written as $\boldsymbol{p}=\alpha\boldsymbol{E}$ (or $\boldsymbol{m}=\zeta\boldsymbol{B}$, up to a proportional constant), their components satisfy (η_1 and η_2 are two real functions)

$$\begin{aligned} &p_z/p_x = E_z/E_x \propto \eta_1 \exp(\pm \mathrm{i}\pi), \\ &\text{or}\ \ m_z/m_y = B_z/B_y \propto \eta_2 \exp(\pm \mathrm{i}\pi). \end{aligned} \tag{27}$$

In return, the SOF-induced electric dipole moment $\boldsymbol{p}$ (or magnetic dipole moment $\boldsymbol{m}$) oscillates with SOF and can radiate an elliptically polarized wave. Starting from Eq. (27), one can study the transfer between the transverse spin of the structured optical fields and the spin angular momentum of the microsphere. In fact, Eq. (27) is also valid for a surface plasmon wave. Starting from Eq. (27) associated with the transverse spin of the surface plasmon wave, the authors in Ref. [10] have demonstrated the inverse spin Hall effect of photons.

(ii) Starting from the Dirac Hamiltonian for an electron (mass m, charge e) in an electromagnetic field with the gauge potential $A^{\mu}=(\Phi/c,\boldsymbol{A})$, one can obtain an angular magneto-electric (AME) coupling Hamiltonian[50,51]

$$H_{\mathrm{AME}}=\frac{e^2\hbar}{4m^2c^2}\boldsymbol{\sigma}\cdot(\boldsymbol{E}\times\boldsymbol{A}), \tag{28}$$

where $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ is the vector of Pauli matrices, $\boldsymbol{E}=\boldsymbol{E}_{\mathrm{ext}}-\nabla V/e$ is the total electric field (V is the crystal potential), $\boldsymbol{E}_{\mathrm{ext}}=-\nabla\Phi-\partial\boldsymbol{A}/\partial t$ is the external electric field (as an applied electric field). Correspondingly, $\boldsymbol{B}=\nabla\times\boldsymbol{A}$ is the external magnetic field. In general, the total electric field in a material can be split in terms of an internal electric field $\boldsymbol{E}_{\mathrm{int}}$, generated by all charges in the material, and an applied electric field $\boldsymbol{E}_{\mathrm{ext}}$ which would exist even in vacuum. The internal electric field depends on $\boldsymbol{E}_{\mathrm{ext}}$, such that $\boldsymbol{E}_{\mathrm{int}}=\boldsymbol{E}^0_{\mathrm{int}}+\gamma\boldsymbol{E}_{\mathrm{ext}}$, where $\boldsymbol{E}^0_{\mathrm{int}}=-\nabla V/e$ is the internal electric field when no external field is applied and, for a cubic or isotropic material, γ is a scalar.

In view of the fact that $\hbar\boldsymbol{\sigma}/2$ is the spin operator of the electron and $\boldsymbol{s}^{\mathrm{e}}=\varepsilon_0\boldsymbol{E}\times\boldsymbol{A}$ is the spin angular momentum density of the total electromagnetic field, Eq. (28) describes a coupling between the spin of electrons and the one of photons. In particular, the formula of $\boldsymbol{s}^{\mathrm{e}}=\varepsilon_0\boldsymbol{E}\times\boldsymbol{A}$ implies that Eq. (28) can also describe a coupling between the spin of electrons and the transverse spin of guided waves. Based on Eq. (28) one can investigate all potential applications of the transverse spin. For example, the coupling of Eq. (28) can produce an energy level splitting between the spin-up and spin-down states of an electron (or a photon), which can serve as a two-level electronic (or optical) system. As a result, one can study the transmission and transfer of quantum information between electronic and optical system, in terms of the spin-encoded electronic and photonic qubits.

The main purpose of the present paper is to provide a theoretical framework for the transverse spin of guided waves, so as to pave the way for our next work. Here we just mention a preliminary idea about the potential applications of the transverse spin. In fact, by means of photonic spin Hall effect and acoustic spin transport in waveguides,[30,31,39] one can hopefully find more potential applications of the transverse spin in waveguides.

6. Conclusions and prospect

As we know, a massive vector field has a longitudinal field component and possesses a state with transverse spin, i.e., the state with zero helicity. Similarly, photons inside a waveguide possess an effective rest mass which is related to the existence of a longitudinal field component along the waveguide, such that they also possess a transverse spin whose projection along the mean propagation direction of the guided waves vanishes. The appearance of the effective rest mass of guided photons can be attributed to the symmetry breaking from the Lorentz group SO (1, 3) to its subgroup SO (1, 1).

From a unified point of view, when an electromagnetic wave has a longitudinal (or transverse) spin, it should be an elliptically polarized wave whose polarization plane is orthogonal (or parallel) to the mean propagation direction of the wave. For example, though the guided waves with the transverse spin are linearly polarized within the polarization plane perpendicular to the mean propagation axis, they are elliptically polarized within a polarization plane containing the mean propagation axis. Some previous researches on the transverse spin imply that the transverse spin is not quantized. In this paper we show that the transverse spin of guided waves is also quantized. Compared with the quantization form of the usual spin, the quantization expression of the transverse spin contains an extra factor related to the ellipticity of the polarization ellipse. In particular, in the case of circular polarization, the quantization expression of the transverse spin becomes the usual form.

The research on the transverse spin of light has both theoretical and practical interests. For example, because photons inside the waveguide have an effective rest mass, their spin projection along a transverse direction can also be observed. Moreover, the sign of the transverse spin depends on the propagation direction of the guided waves. As a result, the transverse spin of the guided waves may have important applications in spin-dependent unidirectional optical interfaces, and the quantum spin Hall effect of light can be implemented via guided waves. Based on which, one can develop a new branch in optics that can be called as spinoptics, which can be regarded as the optical analogy of spintronics.

In our next work, we will make a detailed research on the potential applications of the transverse spin of guided waves, starting from the two key points (1) and (2) mentioned in Section 5, and with the aid of the $(1,0)\oplus(0,1)$ representation of SL(2, C).[52] Moreover, laser range-finding techniques are of great interests for many industrial applications, where a laser rangefinder determines the range to a target object based on the round-trip time-of-flight of a laser pulse to and from the object.[53] Though considerable progress has been achieved in the last few decades, to meet more and more special requirements, people have to seek for more laser range-finding techniques based on new physical principles. For example, when a target object is a black body (but not an ideal black body), to guarantee that the return pulse can hit the detector and trigger a range measurement, a new kind of receiver with high sensitivity is needed. We will design such a receiver based on a coupling related to the transverse spin of guided waves.

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