Enhanced phase sensitive amplification towards improving noise immunity

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Quantum states are essential resource for quantum-enhanced applications. Loss incurred in the distribution channel, however, dissipates the high signal-to-noise ratio advantage enjoyed by the squeezed state. Here, we first demonstrate noise immunity enhancement by using phase-sensitive amplifier (PSA) with measurement-based noiseless linear amplifier (MB-NLA). We explore the signal transfer capability with the amplifier in a noisy channel. The MB-NLA enhanced PSA has obvious suppression effect on channel noises, especially it has improvement for the noise contaminated signal. Better performance can be achieved by flexibly adjusting amplifier parameters. With the amplifier, it is promising to overcome the entanglement-distribution loss and show its superiority in squeezing based quantum sensing.

Keywords: phase sensitive amplification, noiseless linear amplification, quantum sensing

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1. Introduction

Quantum information science enabled by quantum entanglement shows great superiority in implementing information processing tasks, such as quantum communication,[1] quantum key distribution,[2] quantum metrology,[3] quantum spectroscopy and sensing.[4] Optical amplifiers, as an essential tool, guarantee the quantum properties of fragile quantum entanglement in noisy environments and facilitate their development towards practical applications. Improvement of the signal-to-noise ratio (SNR) is the key requirement of optical amplifier research. There are various amplifier schemes for applicable scenarios, each with its advantages and disadvantages. Phase-insensitive amplifiers (PIA) amplify the two orthogonal signal quadratures equally, and have the well-known 3 dB quantum-limited noise figure. In contrast, a standard phase-sensitive amplifier (PSA) theoretically has a 0 dB noise figure for the in-phase quadrature, which means that excess noise only arises from the link loss. PSA has many potential advantages, such as providing the best possible sensitivity for long-haul free-space links,[5] advancing the rate and scalability of future quantum communications systems.[6]

PSAs can be realized by nonlinear parametric effects or using electro–optic methods. Although most PSA experiments to date have relied on the former way, such as in fiber,[7] in centimeter long χ^2[8] or χ^3 devices.[9] These experiments commonly require more auxiliary devices making PSA experiments complex and practically challenging to perform. One can benefit from the advantages of being compact and easy to implement with the electro–optic way. This scheme does not employ any nonlinear optical process and preserves optical coherence. The feedforward amplifier is initially proposed and implemented by Lam *et al.*[10] on amplitude quadrature amplification. It soon shows great superiority in optical power amplification,[11] quantum-limited signal transmission,[12] and the general quantum nondemolition measurement (QND) scheme.[13] However, restricted by quantum noise and transmission losses, the SNR of the feedforward amplifier output is always lower than the input SNR. Such deterministic linear amplifier (DLA) cannot achieve noise reduction in principle, thus limiting its overall utility.

Fortunately, there is a way to circumvent the excess noise by allowing a probabilistic operation. Fiurasek[14] proposed an intriguing idea that could subtract noise in the probabilistic amplifier. Ralph and Lund extended this idea and independently proposed a noiseless linear amplifier (NLA).[15] This amplifier outperforms the perfect linear amplifier by preserving the noise characteristic of the input state, so from a classical point of view, it is a noise-reduced amplifier. Its subsequent theoretical studies[16,17] and physical implementations[18–20] in principle provide a method to build high efficient quantum tasks. These schemes based on photon subtraction most require sophisticated operations to realize.[21] A more compatible and simplicity scheme of NLA is via a dual homodyne measurement directly and classical post-processing, so called measurement-based noiseless linear amplifier (MB-NLA).[22–24] Recently, a concatenation scheme of an MB-NLA and an electro–optic feedforward DLA was proposed and its fidelity was proved to surpass the deterministic no-cloning bound.[25] With this technology, Zhao *et al.* for the first time realized a quantum enhancement of SNR for arbitrary coherent states and amplification gains in a PIA.[26] This useful tool had also shown its promise in quantum information

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processing, such as high-fidelity gate operations.[27,28]

Additionally, a phase-sensitive parametric amplifier was experimentally implemented in optical sensing for the first time, showing strong tolerance of an interferometer to inefficiency detection.[29] However, the research on MB-NLA enhanced PSA has not yet been reported. Among them, the noise immunity performance of quantum states in quantum sensing is a significant topic.

Here, we demonstrate noise immunity enhancement of phase-sensitive amplifier by a concatenation of DLA and MB-NLA. We explore three kinds of amplification schemes against noisy channel from the signal transfer capability. We also investigate the transmission characteristics for both squeezed and coherent state. The MB-NLA enhanced PSA not only improves loss-tolerant against upcoming detection loss and channel loss, but also has obvious noise suppression effect on the noise contaminated signal. The results could provide useful guidance for the MB-NLA enhanced PSA implementation in practical applications. With this technique, it promises a full use of quantum resources in entanglement-distribution and squeezing based quantum sensing.[29–32]

2. The principle

In the following, we derive the input–output relations for our system. The theoretical model presented here considers three types of lossy channels. The channels A, B, and C respectively simulate the signal channel before the amplifier, the signal channels after the amplifier and the signal channel inside the amplifier with transmission efficiency $\eta_{\rm ch}^{\rm A}$, $\eta_{\rm ch}^{\rm B}$, and $\eta_{\rm ch}^{\rm C}$, respectively. As illustrated in Fig. 1, we firstly consider an input probe state $\hat{X}_{\rm in}$. Before being amplified by the PSA, the probe state suffers from channel noise

$$\hat{X}_{\rm a}=\sqrt{\eta_{\rm ch}^{\rm A}}\hat{X}_{\rm in}+\sqrt{1-\eta_{\rm ch}^{\rm A}}\hat{X}_{\rm v1}, \tag{1}$$

where $\hat{X}_{\rm v1}$ is the vacuum noise induced by the lossy channel. As the schematic inset in Fig. 1, the injection signal is divided into two beams by a beam splitter (BS) with a transmittance of T, where the vacuum field is coupled through the other port of BS. The transmitted signal and the reflected signal are written as

$$\hat{X}_{\rm t}=\sqrt{T}\hat{X}_{\rm a}+\sqrt{1-T}\hat{X}_{\rm v0},\ \ \hat{X}_{\rm r}=\sqrt{1-T}\hat{X}_{\rm a}-\sqrt{T}\hat{X}_{\rm v0}. \tag{2}$$

The reflected beam is received by the detector PD1 after passing through the lossy channel and its amplitude is measured by

$$\hat{X}_{\rm m}=\sqrt{\eta_{\rm ch}^{\rm C}}\hat{X}_{\rm r}+\sqrt{1-\eta_{\rm ch}^{\rm C}}\hat{X}_{\rm v2}. \tag{3}$$

Then, the measured statistical outcomes are post-selected by the filter function

$$P(X_{\rm m})=\begin{cases}\exp[(|X_{\rm m}|^2-|X_{\rm c}|^2)(1-\frac{1}{g_{\rm NLA}^2})], & |X_{\rm m}|\le|X_{\rm c}|,\\ 1, & \text{otherwise},\end{cases} \tag{4}$$

where all measurements $X_{\rm m}$ less than $X_{\rm c}$ are accepted or rejected according to the probability of Eq. (4), while those greater than $X_{\rm c}$ remain unchanged. The filter function predicts successful amplification and scales both the mean and variance of the statistic by $g_{\rm NLA}^2$. It transforms $\hat{X}_{\rm m}$ into $\hat{X}_{\rm m}^{\rm ps}$

$$\langle\hat{X}_{\rm m}^{\rm ps}\rangle=g_{\rm NLA}^2\langle\hat{X}_{\rm m}\rangle,\ \ \langle\Delta^2\hat{X}_{\rm m}^{\rm ps}\rangle=g_{\rm NLA}^2\langle\Delta^2\hat{X}_{\rm m}\rangle. \tag{5}$$

Finally, the signal is further amplified by the electronic gain $g_{\rm e}$ and coupled with the transmitted input light to complete the amplification operation

$$\hat{X}_{\rm c}=\hat{X}_{\rm t}+g_{\rm e}\hat{X}_{\rm m}^{\rm ps}. \tag{6}$$

Transmitted through the output lossy channel, the probe states are received by the detector PD2

$$\hat{X}_{\rm out}=\sqrt{\eta_{\rm ch}^{\rm B}}\hat{X}_{\rm c}+\sqrt{1-\eta_{\rm ch}^{\rm B}}\hat{X}_{\rm v3}. \tag{7}$$

Thus, the mean and variance of the quadrature amplitude at receiver are derived as

$$\begin{aligned}\langle\hat{X}\rangle_{\rm out}&=\sqrt{\eta_{\rm ch}^{\rm A}\eta_{\rm ch}^{\rm B}}(\sqrt{T}+g_{\rm NLA}^2g_{\rm e}\sqrt{(1-T)\eta_{\rm ch}^{\rm C}})\langle\hat{X}\rangle_{\rm in},\\ \langle\Delta^2\hat{X}\rangle_{\rm out}&=\eta_{\rm ch}^{\rm A}\eta_{\rm ch}^{\rm B}\left(\sqrt{T}+g_{\rm NLA}g_{\rm e}\sqrt{(1-T)\eta_{\rm ch}^{\rm C}}\right)^2\\ &\quad\times\left(\langle\Delta^2\hat{X}\rangle_{\rm in}-\langle\Delta^2\hat{X}\rangle_{\rm vac}\right)\\ &\quad+\left(1+\eta_{\rm ch}^{\rm B}g_{\rm NLA}^2g_{\rm e}^2\right)\langle\Delta^2\hat{X}\rangle_{\rm vac},\end{aligned} \tag{8}$$

where $\langle\Delta^2\hat{X}\rangle_{\rm vk}=\langle\Delta^2\hat{X}\rangle_{\rm vac}$ $(k=0,\ 1,\ 2,\ 3)$ is the vacuum fluctuation. The gain of the PSA is defined as the ratio of the output mean to the input mean $g=\langle\hat{X}\rangle_{\rm out}/\langle\hat{X}\rangle_{\rm in}$. In order to diminish the additional noise induced by the beam splitter, one can optimize the electronic gain when $g_{\rm e}=\sqrt{(1-T)/(T\eta_{\rm ch}^{\rm C})}/g_{\rm NLA}$.

To quantify the performance of the amplifier, we introduce the signal transfer coefficient

$$\tau_{\rm s}=\frac{\rm SNR_{out}}{\rm SNR_{in}}=\frac{\langle\hat{X}\rangle_{\rm out}^2\langle\Delta^2\hat{X}\rangle_{\rm in}}{\langle\Delta^2\hat{X}\rangle_{\rm out}\langle\hat{X}\rangle_{\rm in}^2}. \tag{9}$$

When $g_{\rm NLA}=1$, the PSA reduces to the conventional DLA. The overall effect of enhancement is described by the adjustable gain $g_{\rm NLA}$. Here, we consider both squeezed and coherent probe states with the same signal amplitude, $\langle\Delta^2\hat{X}\rangle_{\rm in}=1$ for coherent state and $\langle\Delta^2\hat{X}\rangle_{\rm in}={\rm e}^{-2r}$ for squeezed state when normalized to shot noise limit.

Firstly, we consider the perfect PSA placed at the output of the lossy quantum channel, $\eta_{\rm ch}^{\rm A}<1$ and $\eta_{\rm ch}^{\rm B}=\eta_{\rm ch}^{\rm C}=1$. We calculate the signal transfer coefficient as a function of the channel loss under three situations: without using a PSA, using a PSA without NLA and using a PSA with NLA. The gains of the PSA with and without NLA are respectively 2.5 and

2.16 according to the experimental parameters and the definition. The theoretical results are shown in Figs. 2(a1) and 2(a2), where the black horizontal line is marked as a comparison reference as experiencing 50% channel loss. We find that using a PSA without NLA has no SNR improvement for a noisy signal, where the transmission curves of the coherent state (blue solid line) and the squeezed state (red solid line) coincide with the curves without PSA respectively. In contrast, the PSA with NLA shows obvious noise reduction capability for both coherent and squeezed states, increasing the noise immunity by 12.7% (to 62.7%) and 12.4% (to 45.7%), respectively. Among them, the squeezed state is more sensitive to the channel losses.

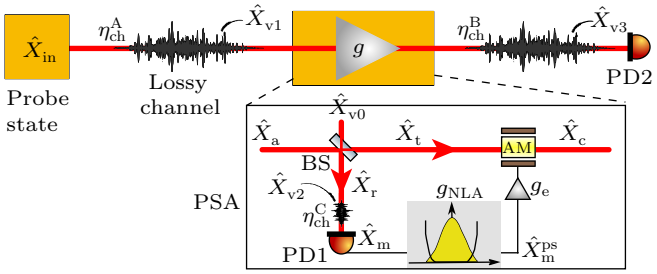


Fig. 1. Example of a protocol for noise immunity against channel noise in sensing link. The probe state is received through the lossy channel and amplification is attempted via a MB-NLA enhanced PSA. Inset: schematic diagram of the PSA.

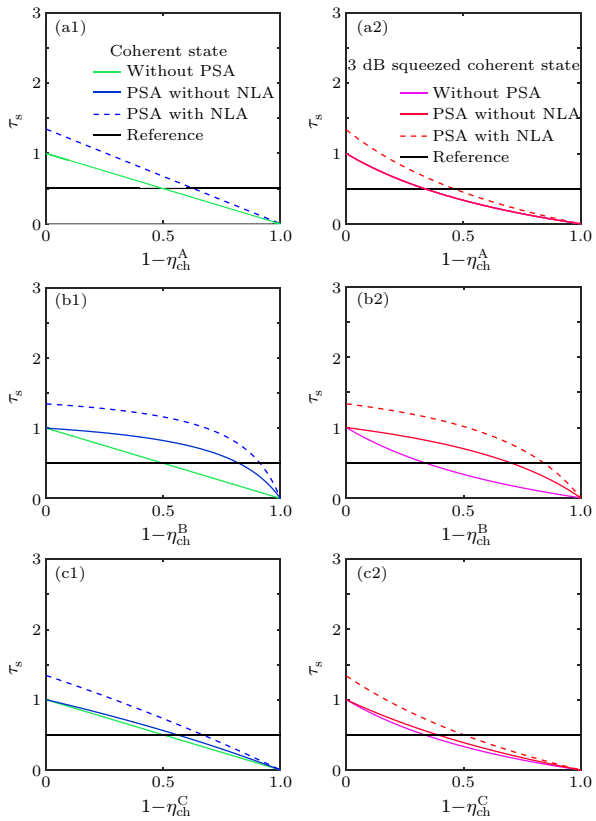


Fig. 2. The noise immunity comparison of the PSA with NLA (dotted curve) and without NLA (solid curve). The left column corresponds to the coherent state and the right column corresponds to the 3 dB squeezed coherent state with the same input amplitude when the PSA operating (a) after the lossy channel, (b) before the lossy channel and (c) inside the lossy channel, $T = 0.215$, $g_{NLA} = 1.2$. The black horizontal line is marked as a comparison reference.

Secondly, we consider the perfect PSA placed at the input of the lossy quantum channel, $\eta_{ch}^{B} < 1$ and $\eta_{ch}^{A} = \eta_{ch}^{C} = 1$. We also calculate the signal transfer coefficient under the same three situations. These results are shown in Figs. 2(b1) and 2(b2). We observe that signal amplification before lossy channels can effectively improve the noise immunity. Compared with the case suffering the loss without amplifier, DLA increases the coherent state's immunity by 32.3% and MB-NLA enhanced PSA by another 9% (to 91.4%). For the squeezed state, DLA increases the performance by 36.6%, and MB-NLA increases by another 14.1% (to 84.0%).

Additionally, we consider a case that there is a lossy channel inside the amplifier, $\eta_{ch}^{C} < 1$ and $\eta_{ch}^{A} = \eta_{ch}^{B} = 1$, where η_{ch}^{C} is the feedforward efficiency of the amplifier relating to the imperfect feedforward channel and detection inefficiency. Similarly, we plot Figs. 2(c1) and 2(c2) according to Eqs. (8) and (9). It is shown that the scheme with the lossy channel inside the amplifier has better noise immunity than that of direct transmission scheme. The DLA improves the performance by 5.5% for both input states. In comparison, the MB-NLA enhanced PSA improves the capability by 16.7% (to 66.7%) and 11.4% (to 50.2%) for coherent and squeezed states, respectively.

3. Experiment results

Figure 3 depicts our linear optical scheme based on the MB-NLA enhanced PSA for transmitting the probe state. In the experiment, the probe states are prepared using a continuous-wave single-mode 1080 nm laser generated by an Nd: YAG solid-state laser, and modulated at 10 MHz by electro–optic modulators. At this frequency, the laser noise has reached the shot noise limit. The lossy channels are simulated by a half-wavelength plate (HWP) and a polarization beam splitter (PBS), and are placed in three positions marked by dotted box. To amplify the amplitude quadrature, the input state injects onto a beam splitter with a transmittance T, splitting into transmission and reflection modes. The detector (PD1) then measures the reflection modes, and the measurements are used in two ways. First, it is used to extract the 10 MHz information, which restores the $|X|_m$ term, and this information provides the heralding signal for post-selection according to Eq. (4). After 50 kHz low-pass filtering, the output results are collected by a 12-bit analog-to-digital acquisition card at a sampling rate of 625 kSa per second. Second, it is used to complete the feedforward operation. The measurement signal is electronically amplified by g_e and fed into an amplitude modulator (AM) in phase to complete the coupling of the transmitted beam. At last, the combined beam is detected by another detector (PD2).

Here, we demonstrate the post-amplification and pre-amplification schemes for a coherent probe state respectively,

where channels B and C contain fixed measurement losses. The feedback loop match efficiency is 96%, and the photodiode quantum inefficiency of detectors PD1 and PD2 is 75% and 78%, respectively. Due to the system inherent loss, there are some differences between the experimental results and the theoretical as shown in Fig. 2. We eliminate the impact of PD2 inefficiency in the data processing to better demonstrate the transmission characteristics. When there is no channel loss $\eta_{\rm ch}^{\rm A}=1$ or $\eta_{\rm ch}^{\rm B}=1$, the signal transfer coefficient begins with $\tau_{\rm s}<1$ because of the inherent losses of PSA. Nevertheless, all results satisfy the theoretical model as shown in Eqs. (1)–(9).

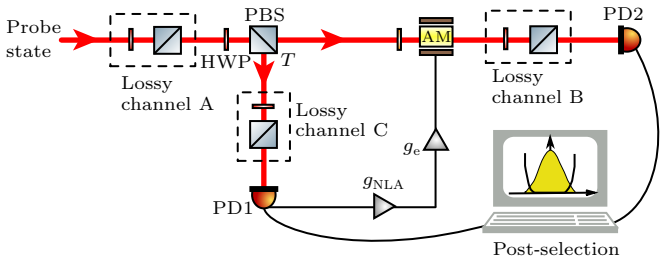


Fig. 3. Experimental setup for the NLA-enhanced PSA in the noisy channel. PBS: polarization beam splitter; HWP: half-wavelength plate; AM: amplitude modulator; PD: photodiode detector.

Figure 4(a) illustrates the post-amplification performances against the channel losses. According to Eq. (8), the transmission efficiency of channels C is fixed $\eta_{\rm ch}^{\rm C}=0.72$. We plot the curve of the signal transfer coefficient with the loss of channel A when using a PSA with NLA (blue dash line) and without NLA (blue solid line). The green line is the signal transmission curve where the probe light through the lossy channel ($\eta_{\rm ch}=\eta_{\rm ch}^{\rm A}\eta_{\rm ch}^{\rm C}$) without using an amplifier. Our experimental data are marked with red dots with error bar, each of which is obtained from the post-processing of 1.6×10^8 measurements. Although the amplifier operates under certain conditions, a successful event produces a valid signal with a probability of success ranging from 0.1% to 30%. The measured $\tau_{\rm s}$ is in good agreement with the theory, indicating that the truncation factor ($X_{\rm c}=6.0$) used is sufficient. It is shown that DLA increases the noise immunity by 2.9%, which results from its contribution to the amplifier inherent loss and photodiode inefficiency. In contrast to the shortcoming of DLA, MB-NLA is effective to the noise contaminated signal. The MB-NLA enhanced PSA improves the noise immunity by 14.5% as a result.

We also demonstrate the amplifier performances against the upcoming channel losses. The theoretical predictions are drawn according to Eqs. (8) and (9), $\eta_{\rm ch}^{\rm A}=1$ and $\eta_{\rm ch}^{\rm C}=0.72$. We plot the curve of signal transmission capacity with the transmission loss of channel B, as shown in Fig. 4(b). We observe that the signal transfer coefficient slow decreases as the channel transmission noise increases, indicating that this amplification scheme is robust against the upcoming noise. The noise immunity is improved by 37.2% with DLA, and by 43.8% with MB-NLA enhanced PSA. Better performance can be achieved by increasing the gain of MB-NLA and the quantum efficiency of the photodiode.

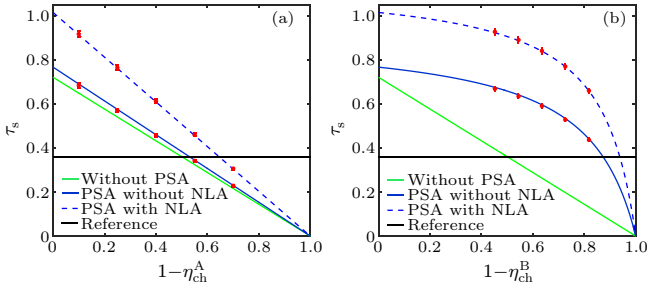


Fig. 4. Measured signal transfer coefficient for the PSA placed (a) after the lossy channel and (b) before the lossy channel. The parameters of the amplifier are $T=0.215$, $g_{\rm NLA}=1.2$. Red symbols represent experimental results for two amplification schemes with coherent state. Blue solid (dash) lines indicate corresponding theoretical predictions.

4. Conclusion

The channel losses can affect the performance of the continuous-variable quantum entanglement distribution and squeezing based quantum sensing protocols and are hard to circumvent experimentally. In this paper, we propose a method to improve the noise immunity in sensing link by using a MB-NLA enhanced PSA. Here, we present three kinds of amplification schemes and experimental to demonstrate the performance of the post-amplification and pre-amplification scheme for a coherent probe state. The pre-amplification scheme is robust against the upcoming noise compared to the post-amplification and in-line amplification scheme. In contrast to the shortcoming of DLA, the MB-NLA enhanced PSA not only shows obvious noise reduction capability against upcoming detection loss and channel loss, but also has obvious noise suppression effect on the noise contaminated signal. With the boost of higher NLA gain and squeezed probe state, the noise immunity capacity of the amplifier can be further improved. As such, the MB-NLA enhanced PSA is promising to improve the rate and scalability of future squeezing based quantum sensing systems.

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