Synchronization coexistence in a Rulkov neural network based on locally active discrete memristor

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At present, many neuron models have been proposed, which can be divided into discrete neuron models and continuous neuron models. Discrete neuron models have the advantage of faster simulation speed and the ease of understanding complex dynamic phenomena. Due to the properties of memorability, nonvolatility, and local activity, locally active discrete memristors (LADMs) are also suitable for simulating synapses. In this paper, we use an LADM to mimic synapses and establish a Rulkov neural network model. It is found that the change of coupling strength and the initial state of the LADM leads to multiple firing patterns of the neural network. In addition, considering the influence of neural network parameters and the initial state of the LADM, numerical analysis methods such as phase diagram and timing diagram are used to study the phase synchronization. As the system parameters and the initial states of the LADM change, the LADM coupled Rulkov neural network exhibits synchronization transition and synchronization coexistence.

Keywords: locally active discrete memristor (LADM), Rulkov, synchronization coexistence

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1. Introduction

The brain is one of the most remarkably complex systems in nature, and it has a strong ability of information processing, learning, and memory. It controls almost all complex life activities. Studying the brain is of great significance to understanding the structure and function of the brain and effective diagnosis and treatment of brain diseases. The brain relies on its complex neural network to transmit information. At present, neural network is a hot topic in artificial intelligence, neurophysiology, nonlinear dynamics and other related fields.[1–4] It is widely used and has a good development prospect in many fields such as data processing, signal detection, and feature recognition. Neurons are the basic units of neural networks.[5] Neuron models are divided into continuous neuron models and discrete neuron models. Up to now, many continuous neuron models have been constructed, such as Morris–Lecar neuron models,[6,7] Hindmarsh–Rose (HR) neuron models,[8–10] and Fitzhugh–Nagumo (FN) neuron models.[11–14] With the intense research on continuous neurons, some scholars break the routine and put forward the idea of discretizing continuous neuron models. Neuron models formed by nonlinear map are called discrete neuron models.[15–21] The discrete neuron model has the advantage of faster speed of simulations and the ease of understanding complex phenomena.[22–24] Map-based discrete neuron models have become most popular in recent years.[25–30]

Neurons are directly or indirectly related to each other, the connection and information transmission between neurons are mainly accomplished through synapses. Chaeles Sherrington proposed the concept of synapses for the first time in 1897. It is responsible for the tight functional connection and special structural differentiation between two neurons. According to the different ways of information transmission, synapses can be divided into electric synapses and chemical synapses. We usually refer to electric synapses and chemical synapses as traditional synapses. The dynamic behaviors of two electric synapse-coupled Sherman neural networks and a ring-connected electrical synapse-coupled Sherman neuron network were studied in Ref. [31], and the results show that they have similar synchronization transition phenomena. An Aihara neural network coupled by electric synapses and the conditions for its complete synchronization were analyzed in Ref. [32]. It was found that when the firing patterns of neurons are chaotic, it is impossible to have complete synchronization. The dynamic behavior of Rulkov neural networks coupled by excitatory and inhibitory chemical synapses were discussed in Ref. [33].

In 1971, Chua proposed the concept of memristor for the first time.[34] It has the dimension of resistance, indicating the relationship between magnetic flux and charge, and has become the fourth basic circuit element after the traditional resistance, inductance and capacitance. In 2008, HP laboratory successfully manufactured the solid memristor by nanotechnology,[35] which physically proved the existence of memristor. Memristor has the characteristics of nonlinearity and memory function, and it is widely used in nanotechnology,[36,37] neural network,[38–41] circuit design,[42–45] secure communication,[46,47] and other

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fields. Researchers applied memristors to simulate biological synapses based on the remarkable bionic characteristics of memristors, such as memorability and nonlinearity.[48–50] Neural networks with memory elements and their applications in image encryption were studied in Refs. [51,52]. A neural network with HR and FN neurons coupled by memristors was studied and its dynamic behaviors were analyzed in Ref. [53]. A Wilson neural model based on memristors was proposed in Ref. [54]. A Hopfield neural network model based on memristive synapses was constructed in Refs. [55,56], and its dynamic behaviors were discussed in detail.

Inspired by the above investigations, in this paper, we construct a simple discrete neuron network model by coupling two Rulkov neurons together by an LADM, and analyze its dynamic behaviors. The remaining structure of this paper is arranged as follows: Section 2 shows the LADM model and its related characteristics in detail. Section 3 introduces the Rulkov neural network model coupled by an LADM. The multiple firing patterns of the neural network in different parameters and the coexistence phenomenon in different initial conditions are analyzed in Section 4. The synchronization transition with the change of coupling strength and the synchronization coexistence in different LADM initial conditions are discussed in Section 5. Finally, we conclude the results and prospects for future research in Section 6.

2. LADM model

The LADM proposed in this paper is based on the memristor proposed in Ref. [57] and described by the following equation:

$$\begin{cases} i(n) = M(q(n)) \cdot v(n) = q(n) \cdot v(n), \\ q(n+1) = F(q(n), v(n)) = q(n) + \alpha f(q(n)) + bv(n), \\ f(q(n)) = (\mathrm{sgn}(q(n)+1) + \mathrm{sgn}(q(n)-1) - q(n)), \end{cases} \tag{1}$$

where $i(n)$ is the output current of the memristor, $v(n)$ is the input voltage of the memristor, $M(q(n))$ is the memristor value, $q(n)$ is the resistance of the memristor in the current state and $q(n+1)$ is the resistance of the memristor in the next state. α and b are two parameters, and f is a function of $q(n)$.

In the study of the characteristics of the memristor, the pinched hysteresis loops can visually identify the characteristics of memristors.[58–62] According to its mathematical model, we built the Matlab simulation model of the LADM and verified it. We add a sine signal $v(n) = A\sin(2\pi\omega T(n))$ to it, and find that the area of the pinched hysteresis loops varies with amplitude A and angular frequency ω. The detailed memristive characteristics of this model have been verified in Ref. [15].

3. Rulkov neural network coupled by an LADM

3.1. Rulkov neuron model

According to biological neurons, scientists have established many corresponding neuron models. The Rulkov neuron model was proposed by Rulkov in 2001,[63] including supercritical Rulkov neuron model, chaotic Rulkov neuron model and non-chaotic Rulkov neuron model.[64] In this paper, we use the chaotic Rulkov neuron model, which has a fast variable and a slow variable to describe the properties of neurons. Its dynamic equation is

$$\begin{cases} x(n+1) = F(x(n), y(n) + \beta), \\ y(n+1) = y(n) - \mu(x(n) - \sigma), \end{cases} \tag{2}$$

where $x(n)$ is the fast variable of the system, $y(n)$ is the slow variable of the system, n is the discrete time, $x(n)$ means the membrane potential of neurons, $y(n)$ means restore variable. σ is a control parameter and the speed of change of the slow variable $y(n)$ is controlled by μ and σ. Generally, we choose μ as a very small value ($0 < \mu \ll 1$), β indicates the external influence on the system, so that the variable $y(n)$ will change slowly with time. The expression of the function $F(xy)$ is

$$F(xy) = \frac{\alpha}{1+x^2} + y, \tag{3}$$

where α is the system parameter which determines the firing mode of the system.

3.2. Proposed Rulkov neural network model

We connect two Rulkov neurons by an LADM to establish a new neural network, and this neural network is defined by the equations

$$\begin{cases} x_1(n+1) = \dfrac{a}{1+x_1(n)^2} + y_1(n) - kz(n)(x_1(n) - x_2(n)), \\ y_1(n+1) = y_1(n) - b(x_1(n) - \sigma), \\ x_2(n+1) = \dfrac{a}{1+x_2(n)^2} + y_2(n) + kz(n)(x_1(n) - x_2(n)), \\ y_2(n+1) = y_2(n) - b(x_2(n) - \sigma), \\ z(n+1) = z(n) + \alpha f(z(n)) + (x_1(n) - x_2(n)), \\ f(z(n)) = (\mathrm{sgn}(z(n)+1) + \mathrm{sgn}(z(n)-1) - z(n)), \end{cases} \tag{4}$$

where x_1 and x_2 denote the membrane potential of neurons, y_1 and y_2 are neuronal recovery variables, and k is the coupling strength, b and σ are system parameters. What needs to be noted is that the system parameters are fixed with $b = 0.001\sigma = -1$ in this paper. And z is the memristor used to couple the two neurons.

3.3. Equilibrium point analysis

By calculating the equilibrium point of Eq. (4), we can get E_1 $(\sigma,\ \sigma - \alpha/(1+\sigma^2),\ \sigma,\ \sigma - \alpha/(1+\sigma^2),\ -2)$, E_2 $(\sigma,\ \sigma - \alpha/(1+\sigma^2),\ \sigma,\ \sigma - \alpha/(1+\sigma^2),\ 0)$, and E_3 $(\sigma, \sigma - \alpha/(1+\sigma^2), \sigma, \sigma - \alpha/(1+\sigma^2), 2)$. For Eq. (4) the Jacobi

matrix is calculated as

$$J=\begin{bmatrix} \frac{-2\alpha\sigma}{(1+\sigma^2)^2}-kz^{(e)} & 1 & 0 & 0 & k(x_1^{(e)}-x_2^{(e)}) \\ -b & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{-2\alpha\sigma}{(1+\sigma^2)^2}-kz^{(e)} & 1 & 0 \\ 0 & 0 & -b & 1 & 0 \\ 1 & 0 & -1 & 0 & f \end{bmatrix}, \quad (5)$$

with

$$f = 2\mathrm{dirac}\,(z^{(e)}-1)+2\mathrm{dirac}\,(z^{(e)}+1)+1. \quad (6)$$

The eigenvalues can be obtained from $P(\lambda)=\lambda I-J$, where I is the unit matrix. Considering $\alpha=4$, the eigenvalues of E_1 and E_3 are (0.9990, 1.0008, 2.1992, 1.0008, 2.1992) and (0.9990, 1.0013, 1.7987, 1.0013, 1.7987). It is obvious that these two eigenvalues are located outside the unit circle, so they are unstable. Similarly, we can also get the eigenvalue of E_2 by considering the vaule of α and the equilibrium points E_2 is stable.

4. Firing patterns of the proposed Rulkov neural network

In this section, the coexisting firing patterns of the proposed Rulkov neural network are investigated, focusing on the coexisting firing patterns induced by different initial states of the LADM and different coupling strengths.

4.1. Coexisting firing patterns

The system parameters are fixed as $\alpha=2$, $b=0.001$, $\sigma=-1$, and the coupling strength k is 0.1. The neural network shows a variety of coexisting firing patterns in different initial values. To make it easier to distinguish, we describe them with different colored lines. Figure 1 shows the time-series and phase diagrams with different initial values. The blue line corresponds to the initial value of $(x_1(0), y_1(0), x_2(0), y_2(0), z(0)) = (0, 0.1, 0.1, 0, 2)$. The green line corresponds to the initial value of (0, 0.1, 0.1, 0, 1). The red line corresponds to the initial value of (0, 0.1, 0.1, 0, −2).

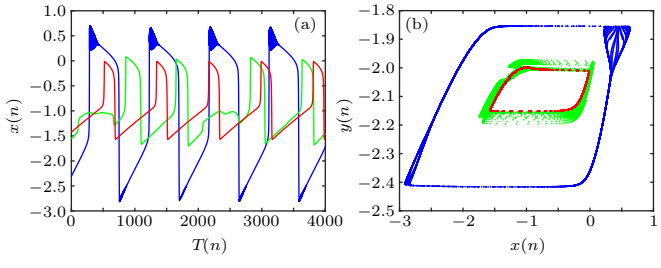


Fig. 1. Coexisting firing patterns: (a) the time-series with $\alpha=2$, $b=0.001$, $\sigma=-1$, and $z=2$ (blue), $z=1$ (green), $z=-2$ (red); (b) the phase diagrams with $\alpha=2$, $b=0.001$, $\sigma=-1$, and $z=2$ (blue), $z=1$ (green), $z=-2$ (red).

4.2. Multiple firing patterns in different coupling strengths

The system parameters are fixed as $\alpha=2$, $b=0.001$, $\sigma=-1$, and the system initial value is (0, 0.1, 0.1, 0, 2). When the coupling strength k is 0.1, the timing diagram of the neural network is presented in Fig. 2(a), and its phase diagram is presented in Fig. 2(b). The neural network shows a firing pattern of spiking. When the coupling strength k is −0.1, the timing diagram and the phase diagram of the neural network are shown in Figs. 2(c) and 2(d). And the neural network exhibits triangular firing pattern at this time. When the coupling strength k is 0.2, the timing diagram and the phase diagram are presented in Figs. 2(e) and 2(f) respectively. It can be seen that there are many different firing patterns in this neural network, such as spiking and triangular firing, with different coupling strength k.

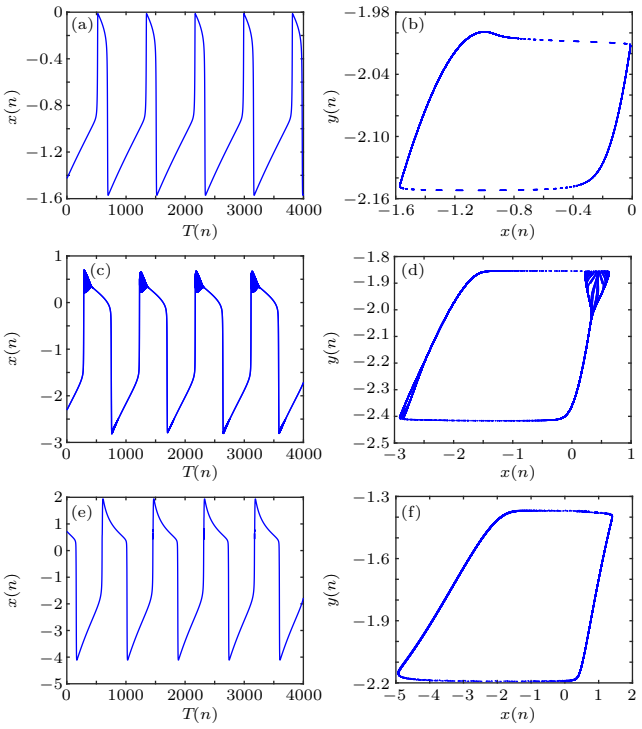


Fig. 2. Firing patterns in different coupling strengths: [(a), (b)] $k=0.1$, [(c), (d)] $k=0.1$, [(e), (f)] $k=0.2$.

5. Synchronization transition and synchronization coexistence

The study of synchronization phenomenon is an important part of the neural network research. In general, when information is transmitted between two neurons, the action potentials of the two neurons tend to be consistent. This kind of phenomenon is called synchronization phenomenon, which is the basis of information interaction in neuron networks. In this paper, we verify the synchronous firing behavior of the LADM coupled Rulkov neural network, which exhibits two kinds of synchronous firing behaviors: in-phase synchronization and anti-phase synchronization.

We keep the initial value of the system and the coupling strength k constant. When the system parameter α is changed, the neural network exhibits synchronization transition. With $\alpha = 2$, $k = 0.1$, $z = -2$, neuron x_1 and neuron x_2 exhibit in-phase spike synchronization as shown in Figs. 3(a) and 3(b). When the system parameter $\alpha = 3$, the synchronization between neuron x_1 and neuron x_2 is in-phase double triangular synchronization as shown in Figs. 3(c) and 3(d). When $\alpha = 4$, the synchronization phenomenon is in-phase chaotic bursting synchronization as shown in Figs. 3(e) and 3(f).

In addition, we find that the LADM coupled Rulkov neural network exhibits anti-phase synchronization phenomenon when the initial value of the LADM is changed. For the convenience of observation, the time-series and phase diagrams of the two coupled neurons are all shown in the same figure. When the initial value of the LADM $z_0 = 2$, the LADM coupled Rulkov neural network exhibits anti-phase triangle synchronization, anti-phase double triangle synchronization, and anti-phase chaotic bursting synchronization as shown in Figs. 4(a)–4(c). Coexistence is a special phenomenon in nonlinear systems.[65,66] The phenomenon that the synchronization mode of the neural network changes as the initial value of the LADM changes is shown in Figs. 3(a)–3(f) and 4(a)–4(c), which is called synchronization coexistence.

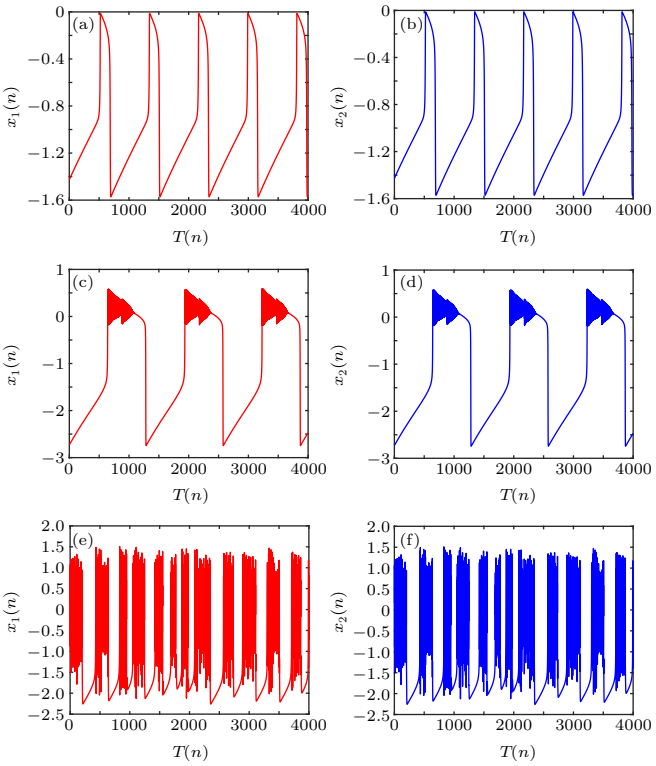


Fig. 3. In-phase synchronization with $z = -2$: [(a) and (b)] $\alpha = 2$, [(c) and (d)] $\alpha = 3$, [(e) and (f)] $\alpha = 4$.

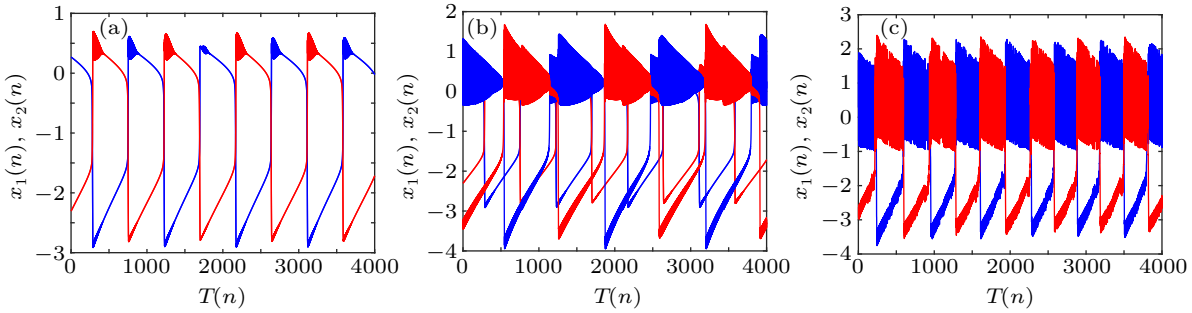


Fig. 4. Anti-phase synchronization with $k = 0.1$, $z = 2$, (a) anti-phase triangle synchronization with $\alpha = 2$; (b) anti-phase double triangle synchronization with $\alpha = 3$; (c) anti-phase chaotic bursting synchronization with $\alpha = 4$.

6. Conclusion

In this paper, the application of the LADM in discrete neural network is studied. Coupling two Rulkov neurons by an LADM, we got a new Rulkov neural network model and analyzed the dynamic behavior of the network in detail. By numerical simulation, we found several kinds of firing patterns, such as spike firing, triangle firing, and chaotic bursting firing, which are induced by different coupling strengthes and initial states of the LADM. The research shows that the LADM coupled neural network exhibits synchronization transition and synchronization coexistence. Synchronization coexistence has not been found in previous literature, and it can characterize different working modes of neural networks in different situations. Our next work should be to analyze the mechanism of the dynamic behavior shown in this paper and to study LADM-coupled complex neural networks.

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