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Fixed-time group consensus of second-order multi-agent systems based on event-triggered control

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The problem of fixed-time group consensus for second-order multi-agent systems with disturbances is investigated. For cooperative-competitive network, two different control protocols, fixed-time group consensus and fixed-time event-triggered group consensus, are designed. It is demonstrated that there is no Zeno behavior under the designed event-triggered control. Meanwhile, it is proved that for an arbitrary initial state of the system, group consensus within the settling time could be obtained under the proposed control protocols by using matrix analysis and graph theory. Finally, a series of numerical examples are propounded to illustrate the performance of the proposed control protocol.

Keywords: event-triggered control, group consensus, multi-agent system, fixed-time

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1. Introduction

The cooperative control of multi-agent systems (MASs) has attracted many concerns in recent decades, and it has solved many important problems. The cooperative control of MASs has been widely used in traffic vehicle control,[1] formation,[2] sensor networks,[3] and other fields.[4] Among them, the consensus, which is the basic issue in the cooperative control of MASs, requires all agents in a system to finally reach the same state under a certain control protocol.

It is worth mentioning that one type of the consensus is the group consensus,[5] which means that various subgroup of a system converges to different states and the agents in the same subgroup achieve together. The group consensus theory could be applied to systems with multiple tasks.

In recent years, much attention was paid to the consensus problem for the first-order multi-agent system (MAS).[6–8] Compared with the first-order MAS, problems considered in second-order MASs[9–13] are more complex. In Ref. [6], the leader-following consensus for the first-order MAS was studied. A method of the dynamic hybrid quantizer was proposed, which overcomes the shortcomings of the static quantizer and improves the performance of the system. In Ref. [7], average consensus tracking for the first-order MAS was investigated. In Ref. [10], the second-order discrete-time MAS consensus with nonlinear terms was discussed via impulsive control. In Ref. [12], the leader-following consensus for nonlinear second-order MASs with intermittent control was studied.

It is worth noting that most of the above-mentioned works are concerned with asymptotic consensus, i.e., the consensus of MASs could be realized only in infinite time. In order to solve the convergence rate problem of a system, scholars proposed the finite-time consensus,[14,15] which has features such as stronger robustness against uncertain influences and faster convergence rate compared with asymptotic consensus. In Ref. [14], for second-order MASs, the finite-time consensus was investigated. A finite-time distributed observer was used to calculate each agent's mismatched disturbance. For finite-time output consensus and tracking consensus, two alternative control techniques have been developed. In Ref. [15], the finite-time consensus of MASs with input saturation and disturbances was investigated. A sliding mode control approach was introduced to deal with bounded disturbances. Different control protocols were designed for both leader-following and leaderless scenarios.

Noticeably, for the finite-time consensus, the convergence time is related to initial conditions of a system. However, the fixed-time consensus overcomes this constraint. For fixed-time consensus, a system could achieve consensus in a bounded time, and the settling time is independent of initial states.[16–20] In Refs. [17,18], the fixed-time consensus of nonlinear heterogeneous MASs was studied. In Ref. [19], for high-order non-global limited chain systems with unknown parameters and nonlinear uncertainties, the robust fixed-time tracking consensus was explored. An upgraded power integration technique was proposed to make the fixed-time consensus in MASs with uncertain parameters.

Compared with biological populations in nature, there are not only cooperative but also competitive relationships between species, for example, the relationship between pouncing and being pounced between sheep and wolves. Groups of the

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same species cooperate with each other, while the relationship between different species is competitive. Therefore, considering both the cooperative and competitive relationships in a system is very important for practical applications.[21–24] The consensus achieved under the cooperative-competitive system is called bipartite consensus, which is a unique case of group consensus. In Ref. [21], the group consensus of heterogeneous MASs in weakly connected topology with cooperative-competitive relationships was investigated. In the design of control protocols, adaptive and hybrid pinning control methods are adopted. In Ref. [24], the cooperative-competitive group consensus for discrete-time MASs without the velocity of second-order agents was investigated by considering the input delay, communication delay, and packet loss.

In addition, in order to reduce the energy consumption of a system in control process, the event-triggered control method was proposed.[25–28] In Ref. [25], the problem of event-triggered consensus for nonlinear MASs was investigated, in which the continuous communication between agents in the system was not required. In Ref. [27], the event-triggered consensus for the first-order MAS with input delay and uncertain disturbances was investigated.

Contributions Inspired by the above discussion, the fixed-time event-triggered group consensus for second-order MASs with disturbances is proposed in this work. The primary contributions of this work could be summarized as follows:

(i) Fixed-time group consensus of second-order MASs is investigated, which ensures that the settling time of the system is independent of initial conditions.

(ii) Unlike in traditional studies, cooperative-competitive relations between agents is taken into account. A signed structurally balanced communication topology is considered.

(iii) There is no Zeno behavior under the designed event-triggered group consensus control protocol, and the system is resistant to interference.

Organization Some of the necessary concepts, lemmas, and problem formulation are given in Section 2. Sufficient conditions for the fixed-time group consensus and fixed-time event-triggered group consensus, and related proofs are presented in Section 3. Several numerical examples are presented to check the accuracy of the obtained results in Section 4. The conclusions and perspective on the future work are given in Section 5.

2. Preliminaries

2.1. Notations

Symbols and meanings used in this article are listed in Table 1.

The sign function is defined as follows:

$$\operatorname{sign}(x)=\begin{cases}1, & \text{if } x>0,\\ 0, & \text{if } x=0,\\ -1, & \text{if } x<0.\end{cases}$$

Let $\operatorname{softsign}(x)=\frac{x}{1+|x|}$. Note that $\operatorname{sign}(x)\approx\operatorname{softsign}(\sigma x)$ when $\sigma\gg 1$.

Table 1. Symbols and nomenclature.

Symbol	Meaning
$\mathcal{R}$	Set of all real numbers
$(\cdot)^{\mathrm{T}}$	Transpose of vector or matrice
$\lambda_i(\mathcal{A})$	The i_{th} eigenvalue of matrix $\mathcal{A}$
$\mathcal{R}^n$	Set of n-dimensional real column vectors
$\lvert\cdot\rvert$	Absolute value of a real number
$D^+(\cdot)$	Right upper derivative
$\mathbf{1}$	Column vector $[1,1,\ldots,1]^{\mathrm{T}}$ with appropriate dimension
$\mathcal{R}^{m\times n}$	Set of $m\times n$-dimensional real matrices
$\inf(S)$	Less than or equal to the maximum of all elements in S

2.2. Graph theory

An undirected signed graph $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$ represents the exchange of information among agents in a system, where $\mathcal{V}=\{v_1,v_2,\ldots,v_K\}$ is the vertex set, and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ the edge set. If $(v_j,v_k)\in\mathcal{E}$, it implies that agent j and agent k can communicate with each other directly. $\mathcal{A}=(a_{jk})\in\mathcal{R}^{K\times K}$ is the adjacency matrix, and $\mathcal{A}$ is symmetric. Define $\mathcal{I}=\{1,2,\ldots,K\}$. When $(v_j,v_k)\in\mathcal{E}$, $a_{jk}\neq 0$, otherwise, $a_{jk}=0$, $j,k\in\mathcal{I}$. Assuming that self-loops in the network $\mathcal{G}$ are ignored, then $a_{jj}=0$, $j\in\mathcal{I}$. The neighborhood node set of agent j is denoted as $\mathcal{N}_j=\{v_k\in\mathcal{V}|(v_k,v_j)\in\mathcal{E}\}$. Here $d_j=\sum_{k=1}^{K}|a_{jk}|$ is the degree of node j, and the degree matrix $\mathcal{D}=\operatorname{diag}\{d_1,d_2,\ldots,d_K\}$. $\mathcal{L}=(l_{jk})\in\mathcal{R}^{K\times K}$ is defined as Laplacian matrix, and

$$l_{jk}=\begin{cases}-a_{jk}, & \text{if } j\neq k,\\ d_j, & \text{if } j=k,\end{cases}\quad j,k\in\mathcal{I}.$$

2.3. Problem statement

Consider a second-order MAS consisting of K agents with disturbances. The dynamics of each agent could be described as

$$\begin{cases}\dot{x}_j(t)=v_j(t),\\ \dot{v}_j(t)=u_j(t)+\mu\left(x_j(t),v_j(t),t\right),\end{cases}\quad j\in\mathcal{I},\tag{1}$$

where $x_j(t),v_j(t),u_j(t)\in\mathcal{R}$ denote the position state, velocity state, and control input of the agent j, respectively, $\mu\left(x_j(t),v_j(t),t\right)\in\mathcal{R}$ is the bounded disturbance.

2.4. Some definitions, lemmas and assumptions

Definition 1 An undirected signed graph $\mathcal{G}$ is stated to connect if there exists a path between any two nodes v_j and

v_k. $\mathcal{G}$ is said to be structurally balanced. If the vertex set $\mathcal{V}$ could be divided into two parts $\mathcal{V}_1$ and $\mathcal{V}_2$, then we have the following conditions:

$$\begin{cases} a_{jk} > 0, \text{ if } \forall v_j, v_k \in \mathcal{V}_1, \text{ or } \forall v_j, v_k \in \mathcal{V}_2, \\ a_{jk} < 0, \text{ if } \forall v_j \in \mathcal{V}_1, \ \forall v_k \in \mathcal{V}_2, \text{ or } \forall v_j \in \mathcal{V}_2, \ \forall v_k \in \mathcal{V}_1, \end{cases}$$

where $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$.

Let $h_j \in \{-1, 1\}$, $j \in \mathcal{I}$. For any $j, k \in \mathcal{I}$, if v_j, $v_k \in \mathcal{V}_1$, or v_j, $v_k \in \mathcal{V}_2$, then $h_j h_k = 1$. If $v_j \in \mathcal{V}_1$, $v_k \in \mathcal{V}_2$, or $v_j \in \mathcal{V}_2$, $v_k \in \mathcal{V}_1$, then $h_j h_k = -1$.

Definition 2 For MAS (1), if for any initial state, there exists a settling time $0 < T \le T_{\max}$, and $T_{\max}$ is independent of the initial state, such that $\lim\limits_{t \to T} \left| x_j(t) - h_j h_k x_k(t) \right| = 0$, $j, k \in \mathcal{I}$, and $x_j(t) = h_j h_k x_k(t)$ when $t \ge T$, then MAS (1) is said to achieve group consensus within a fixed time T.

Lemma 1[29] For a structurally balanced graph, there exists a transformation $H = \text{diag}\{h_1, h_2, \ldots, h_K\}$, such that all elements of $H\mathcal{A}H$ are non-negative. Define a new Laplacian matrix $\mathcal{L}_g$ whose elements are expressed as

$$l_{g,jk} = \begin{cases} -h_j h_k a_{jk}, & \text{if } j \ne k, \\ \sum\limits_{k=1, k \ne j}^{K} \left| a_{jk} \right|, & \text{if } j = k, \end{cases} \quad j, k \in \mathcal{I}.$$

Then the eigenvalues of $\mathcal{L}$ are the same as those of $\mathcal{L}_g$.

Lemma 2[30] For an unsigned undirected connected graph $\mathcal{G}$, the Laplacian matrix $\mathcal{L}$ satisfies the following properties:

(i) $\mathcal{L}$ is semi-positive definite and the eigenvalues of $\mathcal{L}$ can be sequenced as $\lambda_K > \cdots > \lambda_2 > \lambda_1 = 0$.

(ii) $x^{\mathrm{T}} \mathcal{L} x = \frac{1}{2} \sum\limits_{j=1}^{K} \sum\limits_{k=1}^{K} a_{jk} (x_j - x_k)^2$,

where $x = [x_1, x_2, \ldots, x_K]^{\mathrm{T}}$.

(iii) If $\mathbf{1}^{\mathrm{T}} x = 0$, then $x^{\mathrm{T}} \mathcal{L} x \ge \lambda_2 x^{\mathrm{T}} x$.

Lemma 3[31] If $\ell_1, \ell_2, \ldots, \ell_K \ge 0$, then there is

(i) $\sum\limits_{j=1}^{K} \ell_j^m \ge \left(\sum\limits_{j=1}^{K} \ell_j \right)^m, \ 0 < m \le 1,$

(ii) $\left(\sum\limits_{j=1}^{K} \ell_j \right)^n \ge \sum\limits_{j=1}^{K} \ell_j^n \ge K^{1-n} \left(\sum\limits_{j=1}^{K} \ell_j \right)^n, \ 1 < n \le \infty.$

Lemma 4[25] If there exists a continuous radially unbounded function $V : \mathcal{R}^n \to \mathcal{R}^+ \cup \{0\}$ satisfying $V(x) = 0$ when and only when $x = 0$, and $\dot{V}(x(t)) \le -\rho_1 V^m(x(t)) - \rho_2 V^n(x(t))$, where $\rho_1, \rho_2 > 0$, $m \in (0,1)$, $n \in (1, \infty)$, then the globally fixed-time stable could be derived and the settling time T satisfies

$$T \le T_{\max} := \frac{1}{\rho_1 (1-m)} + \frac{1}{\rho_2 (n-1)}.$$

Assumption 1 The topology of system (1) is undirected signed, and structurally balanced.

Assumption 2 For the disturbance $\mu(x_j(t), v_j(t), t)$, there is $\left| \mu(x_j(t), v_j(t), t) \right| \le \delta$, $j \in \mathcal{I}$, where δ is a non-negative constant. That is, the disturbance is bounded.

3. Main results

The problem of group consensus for second-order MASs with disturbances is analyzed in this section. Fixed-time control protocols and fixed-time event-triggered control protocols are designed, respectively.

3.1. Fixed-time group consensus

In the following, $x_j(t) - x_k(t)$ and $x_j(t) + x_k(t)$ represent the relationship between agent j and agent k, which are cooperative and competitive, respectively.

Define

$$\begin{aligned} \chi_j(t) &= \sum_{k \in S_j} a_{jk} (x_j(t) - x_k(t)) \\ &\quad + \sum_{k \in D_j} a_{jk} (x_j(t) + x_k(t)), \\ \hbar_j(t) &= \sum_{k \in S_j} a_{jk} (v_j(t) - v_k(t)) \\ &\quad + \sum_{k \in D_j} a_{jk} (v_j(t) + v_k(t)), \end{aligned} \qquad j, k \in \mathcal{I},$$

where S_j and D_j represent the set of neighbors that are in the same subgroup and different subgroup of agent j, respectively.

For simplicity, rewrite $\chi_j(t), \hbar_j(t)$ as

$$\begin{aligned} \chi_j(t) &= \sum_{k=1}^{K} a_{jk} \left(x_j(t) - \text{sign}(a_{jk}) x_k(t) \right), \\ \hbar_j(t) &= \sum_{k=1}^{K} a_{jk} \left(v_j(t) - \text{sign}(a_{jk}) v_k(t) \right), \end{aligned} \qquad j, k \in \mathcal{I}.$$

For MAS (1), the virtual velocity $\tilde{v}_j(t)$ is introduced,

$$\begin{aligned} \tilde{v}_j(t) \ = \ & -\sigma_1 \chi_j^m(t) - \sigma_2 \chi_j^n(t) \\ & - \sigma_3 \text{softsign}(\sigma \chi_j(t)) + d, \ \ j \in \mathcal{I}. \end{aligned} \tag{2}$$

Define the velocity error $z_j(t)$ between $v_j(t)$ and $\tilde{v}_j(t)$ as

$$z_j(t) = v_j(t) - \tilde{v}_j(t). \tag{3}$$

The control protocol $u_j(t)$ is designed to keep the actual velocity following the virtual velocity. That is,

$$\begin{aligned} u_j(t) \ = \ & -\sigma_1 m \chi_j^{m-1}(t) \hbar_j(t) - \sigma_2 n \chi_j^{n-1}(t) \hbar_j(t) \\ & - \sigma_3 \sigma \frac{1}{\left(1 + \left| \sigma \chi_j(t) \right| \right)^2} \hbar_j(t) - \sigma_4 z_j^r(t) \\ & - \sigma_5 \text{softsign}(\sigma z_j(t)), \ \ j \in \mathcal{I}, \end{aligned} \tag{4}$$

where $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5 > 0$ are control parameters, $m \in (0,1)$, $n, r \in (1, \infty)$ are the ratio of positive odd numbers, $d > 0$, $\sigma \gg 1$.

Then, for derivative of $z_j(t)$, there is

$$\begin{aligned} \dot{z}_j(t) \ &= \ \dot{v}_j(t) - \dot{\tilde{v}}_j(t) \\ &= \ u_j(t) + \mu_j(x_j(t), v_j(t), t) + \sigma_1 m \chi_j^{m-1}(t) \hbar_j(t) \end{aligned}$$

$$+\sigma_2 n\chi_j^{n-1}(t)\hbar_j(t)+\sigma_3\sigma\frac{1}{(1+|\sigma\chi_j(t)|)^2}\hbar_j(t)$$
$$=\mu_j(x_j(t),v_j(t),t)-\sigma_4 z_j^r(t)-\sigma_5\text{softsign}(\sigma z_j(t)). \quad (5)$$

Theorem 1 Under the control protocol (4) with σ_1, σ_2, σ_3, σ_4, $\sigma_5>0$, $0<d<\sigma_3$, $\sigma\gg 1$, and $m\in(0,1)$, $n,r\in(1,\infty)$ are the ratio of positive odd numbers, if $\delta<\sigma_5$, and assumptions 1 and 2 hold, then MAS (1) reaches group consensus within a fixed time T, and T satisfies

$$T\le T_{\max}=T_{1\max}+T_{2\max},$$

where λ_2 is the minimum nonzero eigenvalue of $\mathcal{L}_g$, and

$$T_{1\max}:=\frac{2^{\frac{1}{2}}}{\sigma_5}+\frac{1}{\sigma_4(r-1)K^{\frac{1-r}{2}}2^{\frac{r}{2}}},$$
$$T_{2\max}:=\frac{1}{\sigma_1(1-m)K^{\frac{1-m}{2}}2^{\frac{m}{2}}\lambda_2^{\frac{m+1}{2}}}+\frac{1}{\sigma_2(n-1)2^{\frac{n}{2}}\lambda_2^{\frac{n+1}{2}}}.$$

Proof Choose the Lyapunov function as

$$V_1(t)=\frac{1}{2}\sum_{j=1}^{K}z_j^2(t).$$

The derivative of $V_1(t)$ along Eq. (5) is

$$\begin{aligned}\dot{V}_1 &= \sum_{j=1}^{K}z_j(t)\dot{z}_j(t)\\ &\le -\sigma_4\sum_{j=1}^{K}z_j^{r+1}(t)-(\sigma_5-\delta)\sum_{j=1}^{K}|z_j(t)|\\ &\le -\sigma_4K^{\frac{1-r}{2}}\left(\sum_{j=1}^{K}z_j^2(t)\right)^{\frac{r+1}{2}}-(\sigma_5-\delta)\left(\sum_{j=1}^{K}z_j^2(t)\right)^{\frac{1}{2}}\\ &\le -\sigma_4K^{\frac{1-r}{2}}(2V_1)^{\frac{r+1}{2}}-(\sigma_5-\delta)(2V_1)^{\frac{1}{2}}\\ &\le -\sigma_4K^{\frac{1-r}{2}}2^{\frac{r+1}{2}}V_1^{\frac{r+1}{2}}-(\sigma_5-\delta)2^{\frac{1}{2}}V_1^{\frac{1}{2}}. \quad (6)\end{aligned}$$

Due to $\frac{r+1}{2}\in(1,\infty)$, according to Lemma 4, there exists a settling time T_1 satisfying

$$T_1\le T_{1\max}:=\frac{2^{\frac{1}{2}}}{\sigma_5}+\frac{1}{\sigma_4(r-1)K^{\frac{1-r}{2}}2^{\frac{r}{2}}}.$$

After the settling time T_1, the real velocity $v_j(t)$ could follow the virtual velocity $\tilde{v}_j(t)$, thus the velocity error $z_j(t)=0$ for $t>T_1$. From Eqs. (2) and (3), the real velocity $v_j(t)$ is

$$v_j(t)=-\sigma_1\chi_j^m(t)-\sigma_2\chi_j^n(t)-\sigma_3\text{softsign}(\sigma\chi_j(t))+d. \quad (7)$$

Define $x(t)=[x_1(t),x_2(t),\ldots,x_K(t)]^{\mathrm{T}}$. Select the Lyapunov function as

$$V_2(t)=\frac{1}{2}x^{\mathrm{T}}(t)\mathcal{L}_g x(t).$$

The derivative of $V_2(t)$ along Eqs. (1) and (7) is

$$\begin{aligned}\dot{V}_2 &= \sum_{j=1}^{K}\chi_j(t)v_j(t)\\ &\le -\sigma_1\sum_{j=1}^{K}\chi_j^{m+1}(t)-\sigma_2\sum_{j=1}^{K}\chi_j^{n+1}(t)+(d-\sigma_3)\sum_{j=1}^{K}|\chi_j(t)|\\ &\le -\sigma_1\sum_{j=1}^{K}\left(\chi_j^2(t)\right)^{\frac{m+1}{2}}-\sigma_2\sum_{j=1}^{K}\left(\chi_j^2(t)\right)^{\frac{n+1}{2}}\\ &\le -\sigma_1K^{\frac{1-m}{2}}\left(\sum_{j=1}^{K}\chi_j^2(t)\right)^{\frac{m+1}{2}}-\sigma_2\left(\sum_{j=1}^{K}\chi_j^2(t)\right)^{\frac{n+1}{2}}. \quad (8)\end{aligned}$$

From Lemma 2, one gets

$$\lambda_2 x^{\mathrm{T}}(t)\mathcal{L}_g x(t)\le\sum_{j=1}^{K}\chi_j^2(t)\le\lambda_K x^{\mathrm{T}}(t)\mathcal{L}_g x(t), \quad (9)$$

where λ_2 and λ_K are the minimum nonzero eigenvalue and the maximum eigenvalue of $\mathcal{L}_g$, respectively.

Combining Eq. (8) with Eq. (9) yields

$$\dot{V}_2\le -\sigma_1K^{\frac{1-m}{2}}(2\lambda_2)^{\frac{m+1}{2}}V_2^{\frac{m+1}{2}}-\sigma_2(2\lambda_2)^{\frac{n+1}{2}}V_2^{\frac{n+1}{2}}.$$

Due to $\frac{m+1}{2}\in(0,1)$, $\frac{n+1}{2}\in(1,\infty)$, based on Lemma 4, after $z_i(t)$ converges to 0, there exists a $T_2>0$, such that the states of all agents achieve consensus in T_2, and T_2 satisfies

$$T_2\le T_{2\max}:=\frac{1}{\sigma_1(1-m)K^{\frac{1-m}{2}}2^{\frac{m}{2}}\lambda_2^{\frac{m+1}{2}}}+\frac{1}{\sigma_2(n-1)2^{\frac{n}{2}}\lambda_2^{\frac{n+1}{2}}}.$$

By now, when $t\in[0,T_1]$, from Eq. (6), it follows that the errors $z_j(t)$ and $\tilde{v}_j(t)$ are bounded. Combining Eqs. (1) and (3), it yields $\dot{x}_j(t)=z_j(t)+\tilde{v}_j(t)$, hence both x and v are bounded. When $t\in[T_1,T_2]$, from Eq. (8), MAS (1) could achieve fixed-time group consensus, and the settling time T satisfies

$$T=T_1+T_2\le T_{\max}=T_{1\max}+T_{2\max}. \quad (10)$$

Remark 1 MAS (1) could achieve fixed-time group consensus under the control protocol (4). From Theorem 1, it could be shown that the settling time T of the system (1) is independent of the initial conditions.

3.2. Fixed-time event-triggered consensus

The event-triggered control strategy is introduced on the basis of fixed-time group consensus. In comparison with general fixed-time group consensus methods, fixed-time event-triggered control could better reduce the communication burden between agents. The study of an event-triggered technique is increasingly important when dealing with resource-limited microprocessors and bandwidth-limited networks.

Therefore, design the control inputs for system (1) as

$$\begin{aligned}u_j(t) &= -\sigma_1 m\chi_j^{m-1}(t_k^j)\hbar_j(t_k^j)-\sigma_2 n\chi_j^{n-1}(t_k^j)\hbar_j(t_k^j)\\ &\quad -\sigma_3\sigma\frac{1}{\left(1+|\sigma\chi_j(t_k^j)|\right)^2}\hbar_j(t_k^j)-\sigma_4 z_j^r(t_k^j)\end{aligned}$$

$$-\sigma_5 \text{softsign}\left(\sigma z_j(t_k^j)\right),\ t\in\left[t_k^j, t_{k+1}^j\right),\quad j\in\mathcal{I}, \tag{11}$$

and $t_{k+1}^j = \inf\left\{t > t_k^j \,|\, \varphi_j(t) > 0\right\}$ is the next trigger instant of agent j, where $\varphi_j(t)$ represents the event trigger function to be designed, t_k^j is the k_{th} trigger instant of agent j.

Firstly, the measurement error is defined as

$$\begin{aligned} e_j(t) \;=\; & \sigma_1 m\chi_j^{m-1}(t_k^j)\hbar_j(t_k^j) + \sigma_2 n\chi_j^{n-1}(t_k^j)\hbar_j(t_k^j) \\ & + \sigma_3\sigma\frac{1}{\left(1+|\sigma\chi_j(t_k^j)|\right)^2}\hbar_j(t_k^j) + \sigma_4 z_j^r(t_k^j) \\ & + \sigma_5 \text{softsign}\left(\sigma z_i(t_k^j)\right) - \sigma_1 m\chi_j^{m-1}(t)\hbar_j(t) \\ & - \sigma_2 n\chi_j^{n-1}(t)\hbar_j(t) - \sigma_3\sigma\frac{1}{(1+|\sigma\chi_j(t)|)^2}\hbar_j(t) \\ & - \sigma_4 z_j^r(t) - \sigma_5 \text{softsign}\left(\sigma z_j(t)\right),\quad j\in\mathcal{I}. \end{aligned} \tag{12}$$

Then, define the trigger function as

$$\varphi_j(t) = \left|e_j(t)\right| - \sigma_4\iota z_j^r(t)\text{softsign}\left(\sigma z_j^r(t)\right) - \sigma_5\iota, \tag{13}$$

where $\iota\in(0,1)$. The performance of the system (1) could be adjusted by selecting the value of ι. When $\varphi_j(t) > 0$, the controller is triggered and the state of agent i is updated.

The derivative of $z_j(t)$ could be rewritten as

$$\begin{aligned} \dot{z}_j(t) \;=\; & -e_j(t) - \sigma_4 z_j^r(t) - \sigma_5 \text{softsign}\left(\sigma z_j(t)\right) \\ & + \mu_j\left(x_j(t), v_j(t), t\right). \end{aligned} \tag{14}$$

Theorem 2 Under the control protocol (11) with σ_1, σ_2, σ_3, σ_4, $\sigma_5 > 0$, $0 < d < \sigma_3$, $\sigma \gg 1$, and $m\in(0,1)$, $n, r\in(1,\infty)$ are the ratio of positive odd numbers, if $0 < \delta < \sigma_5(1-\iota)$, where $\iota\in(0,1)$, and assumptions 1 and 2 hold, then MAS (1) achieves fixed-time event-triggered group consensus, and the settling time T satisfies

$$T \le T_{\max} = T_{3\max} + T_{4\max},$$

where λ_2 is the minimum nonzero eigenvalue of $\mathcal{L}_g$, and

$$\begin{aligned} T_{3\max} &:= \frac{1}{\sigma_4(r-1)(1-\iota)K^{\frac{1-r}{2}}2^{\frac{r}{2}}} + \frac{2^{\frac{1}{2}}}{\sigma_5(1-\iota)-\delta}, \\ T_{4\max} &:= \frac{1}{\sigma_1(1-m)K^{\frac{1-m}{2}}2^{\frac{m}{2}}\lambda_2^{\frac{m+1}{2}}} + \frac{1}{\sigma_2(n-1)2^{\frac{n}{2}}\lambda_2^{\frac{n+1}{2}}}. \end{aligned}$$

Proof The Lyapunov function is chosen as

$$V_3(t) = \frac{1}{2}\sum_{j=1}^K z_j^2(t).$$

The derivative of $V_3(t)$ along Eq. (14) is

$$\begin{aligned} \dot{V}_3 \;=\; & \sum_{j=1}^K z_j(t)\dot{z}_j(t) \le \sum_{j=1}^K \left|z_j(t)\right|\left|e_j(t)\right| - \sigma_4\sum_{j=1}^K \left|z_j(t)\right|^{r+1} \\ & - \sigma_5\sum_{j=1}^K \left|z_j(t)\right| + \delta\sum_{j=1}^K \left|z_j(t)\right| \\ \le\; & -\sigma_4(1-\iota)\sum_{j=1}^K \left(z_j^2(t)\right)^{\frac{r+1}{2}} \\ & - (\sigma_5(1-\iota)-\delta)\sum_{j=1}^K \left(z_j^2(t)\right)^{\frac{1}{2}} \\ \le\; & -\sigma_4(1-\iota)K^{\frac{1-r}{2}}\left(\sum_{j=1}^K z_j^2(t)\right)^{\frac{r+1}{2}} \\ & - (\sigma_5(1-\iota)-\delta)\left(\sum_{j=1}^K z_j^2(t)\right)^{\frac{1}{2}} \\ \le\; & -\sigma_4(1-\iota)K^{\frac{1-r}{2}}2^{\frac{r+1}{2}}V_3^{\frac{r+1}{2}} \\ & - (\sigma_5(1-\iota)-\delta)2^{\frac{1}{2}}V_3^{\frac{1}{2}}. \end{aligned} \tag{15}$$

Due to $\frac{r+1}{2}\in(1,\infty)$, according to Lemma 4, there exists a settling time T_3 satisfying

$$T_3 \le T_{3\max} := \frac{1}{\sigma_4(r-1)(1-\iota)K^{\frac{1-r}{2}}2^{\frac{r}{2}}} + \frac{2^{\frac{1}{2}}}{\sigma_5(1-\iota)-\delta}.$$

After the settling time T_3, the real velocity $v_j(t)$ could follow the virtual velocity $\tilde{v}_j(t)$, thus the velocity error $z_j(t) = 0$ for $t > T_3$. From Eqs. (2) and (3), the real velocity of agent j, $j\in\mathcal{I}$, is

$$v_j(t) = -\sigma_1\chi_j^m(t) - \sigma_2\chi_j^n(t) - \sigma_3\text{softsign}\left(\sigma\chi_j(t)\right) + d.$$

Select the Lyapunov function as

$$V_4(t) = \frac{1}{2}x^{\mathrm{T}}(t)\mathcal{L}_g x(t),$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_K(t)]^{\mathrm{T}}$.

Along Eqs. (1) and (7), similar to the derivative of $V_2(t)$ in Eq. (8), one can get the following derivative of $V_4(t)$:

$$\begin{aligned} \dot{V}_4 \;=\; & x^{\mathrm{T}}(t)\mathcal{L}_g\dot{x}(t) \\ \le\; & -\sigma_1 K^{\frac{1-m}{2}}(2\lambda_2)^{\frac{m+1}{2}}V_4^{\frac{m+1}{2}} - \sigma_2(2\lambda_2)^{\frac{n+1}{2}}V_4^{\frac{n+1}{2}}. \end{aligned} \tag{16}$$

Due to $\frac{m+1}{2}\in(0,1)$, $\frac{n+1}{2}\in(1,\infty)$, based on Lemma 4, there exists a time T_4, such that the states of all agents achieve consensus and T_4 satisfies

$$\begin{aligned} T_4 \le T_{4\max} :\; = & \frac{1}{\sigma_1(1-m)K^{\frac{1-m}{2}}2^{\frac{m}{2}}\lambda_2^{\frac{m+1}{2}}} \\ & + \frac{1}{\sigma_2(n-1)2^{\frac{n}{2}}\lambda_2^{\frac{n+1}{2}}}. \end{aligned}$$

Then, one can get that MAS (1) achieves fixed-time event-triggered control group consensus, and the settling time T satisfies

$$T = T_3 + T_4 \le T_{\max} = T_{3\max} + T_{4\max}. \tag{17}$$

Theorem 3 For MAS (1), if the condition in Theorem 2 holds, then no Zeno behavior occurs under the event-triggered controller (11) with the event-triggered function (13).

Proof From Eq. (9), one obtains $\sum_{j=1}^K z_j^2(t) \le 2\lambda_K V_3(t) \le 2\lambda_K V_3(0)$, and $\sum_{j=1}^K \chi_j^2(t) \le 2\lambda_K V_4(t) \le 2\lambda_K V_4(0)$. Taking the upper right differential of $e_j(t)$ in Eq. (12) along t yields

$$
\begin{aligned}
D^+\left|e_j(t)\right| \le \left|\dot{e}_j(t)\right| &\le \left|\sigma_4 \iota r z_j^{r-1}(t)\right| \left|\dot{z}_j(t)\right| \\
&\le \left(\sigma_4 \iota r (2\lambda_K V_3(0))^{\frac{r-1}{2}}\right) \left|u_j(t)+\mu_j\left(x_j(t),v_j(t),t\right)-\dot{\tilde{v}}(t)\right| \\
&\le \omega_1 \left|u_j(t)+\delta\right| \left|\left(\sigma_1 m \chi_j^{m-1}(t)+\sigma_2 n \chi_j^{n-1}(t)+\sigma_3\sigma\right)\hbar_j(t)\right| \\
&\le \omega_1 \left|u_j(t)+\delta\right| \left|\left(\sigma_1 m (2\lambda_K V_4(0))^{\frac{m-1}{2}}+\sigma_2 n (2\lambda_K V_4(0))^{\frac{n-1}{2}}+\sigma_3\sigma\right)\hbar_j(t)\right| \\
&\le \omega_1 \left|u_j(t_k^j)+\delta\right| \left|\left(\sigma_1 m (2\lambda_K V_4(0))^{\frac{m-1}{2}}+\sigma_2 n (2\lambda_K V_4(0))^{\frac{n-1}{2}}+\sigma_3\sigma\right)\hbar_j(t_k^j)\right| \le \omega_1\omega_2(t_k^j)\omega_3(t_k^j), \qquad (18)
\end{aligned}
$$

where

$$
\omega_1 = \sigma_4 \iota r (2\lambda_K V_3(0))^{\frac{r-1}{2}}, \quad \omega_2(t_k^j) = \left|u_i(t_k^j)+\delta\right|, \quad \omega_3(t_k^j) = \left|\left(\sigma_1 m (2\lambda_K V_4(0))^{\frac{m-1}{2}}+\sigma_2 n (2\lambda_K V_4(0))^{\frac{n-1}{2}}+\sigma_3\sigma\right)\hbar_i(t_k^j)\right|.
$$

Suppose that ω is the maximum upper bound of $\omega_1\omega_2(t_k^j)\omega_3(t_k^j)$. From Eq. (18), one gets

$$
\left|e_j(t)\right| \le \int_{t_k^j}^{t} \left|\dot{e}_j(s)\right| \mathrm{d}s \le \int_{t_k^j}^{t} \omega_1\omega_2(t_k^j)\omega_3(t_k^j)\mathrm{d}s. \qquad (19)
$$

Combined with the event-triggered condition $\varphi_j(t) \ge 0$, one obtains

$$
\begin{aligned}
\left|e_j(t_{k+1}^j)\right| &= \sigma_4 \iota \left|z_j^r(t_{k+1}^j)\right| + \sigma_5 \iota \\
&\le \int_{t_k^j}^{t_{k+1}^j} \omega_1\omega_2(t_k^j)\omega_3(t_k^j)\mathrm{d}s \le \int_{t_k^j}^{t_{k+1}^j} \omega \mathrm{d}s. \qquad (20)
\end{aligned}
$$

There is $t_{k+1}^j - t_k^j \ge \frac{\sigma_5\iota}{\omega} > 0$. Hence, there is no Zeno behavior under the event-triggered controller (11) with the event-triggered function (13).

Remark 2 If design control protocols based on sign function as usual, when the state error of agents in the system tends to 0, it will cause the control input to oscillate. Therefore, softsign function is introduced to replace the sign function in the control protocol, and it turns out that no Zeno behavior exists under the designed event-triggered controller (11) with the event-triggered function (13), which will be further illustrated in simulation examples.

4. Examples

Suppose that the second-order MAS (1) contains seven agents, the communication topology is depicted in Fig. 1, where the black solid lines represent a cooperative relationship between agents, and $a_{jk} = 1$, the red solid lines represent a competitive relationship between agents, and $a_{jk} = -1$.

Example 1 The initial positions of agents are selected as $x(0) = (1.3, -2.2, 0.3, -3.5, -1.8, 2.6, 1.0)^{\mathrm{T}}$ and the initial velocity as $v(0) = (1, -3, 5, -2, -6, 2, -1)^{\mathrm{T}}$.

Under control protocol (4), set $\sigma_1 = 1.5$, $\sigma_2 = 2.0$, $\sigma_3 = 5.5$, $\sigma_4 = 3.0$, $\sigma_5 = 3.5$, $\sigma = 20$, $m = 5/9$, $n = 23/9$, $r = 7/5$, $d = 1.6$, $\delta = 1.3$, and $\mu\left(x_j(t), v_j(t), t\right) = 0.8\sin(x_j(t)) + 0.5\cos(v_j(t))$. Taking the transformation matrix to be $H = \mathrm{diag}\{1,1,1,1,-1,-1,-1\}$, then it can be calculated that $\lambda_2\left(\mathcal{L}_{g_1}\right) = 1.382$. Based on Theorem 1, one gets the settling time $T \le 1.21$.

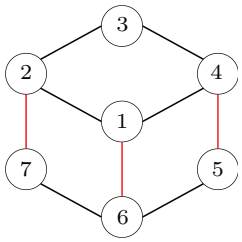


Fig. 1. The interaction topology $\mathcal{G}_1$.

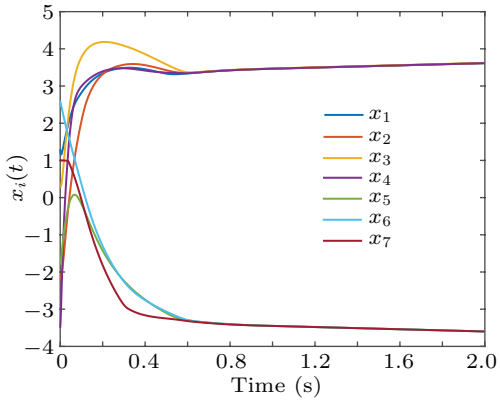


Fig. 2. Trajectories of the position state.

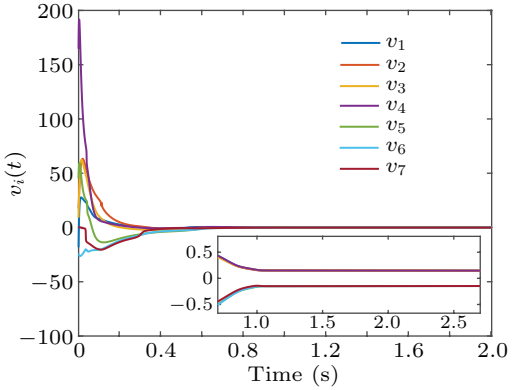


Fig. 3. Trajectories of the velocity state.

For MAS (1) over $\mathcal{G}_1$, Figs. 2 and 3 give the position and velocity state trajectories of agents, which shows that agents 1, 2, 3, and 4 achieve to the same state, and agents 5, 6, and

7 tend to a different state. The system could achieve convergence within the settling time $T_{\max} = 2.5$. Therefore, the desired goal of fixed-time group consensus is achieved.

Example 2 The initial positions of agents are selected as $x(0) = (1, 2.5, -3, 1.5, -1, 0.2, -0.6)^{\mathrm{T}}$ and initial velocities are selected as $v(0) = (2, -1.6, 0.5, 3.5, -3, 2, -0.1)^{\mathrm{T}}$. Under control protocol (11), set $\sigma_1 = 0.5, \sigma_2 = 2.0$, $\sigma_3 = 2.5$, $\sigma_4 = 5.3$, $\sigma_5 = 5.5$, $\sigma = 10$, $m = 5/7$, $n = 3$, $r = 9/7$, $d = 0.8$, $\delta = 1.2$, $\iota = 0.3$, and the disturbance $\mu(x_j(t), v_j(t), t) = 0.3\sin(x_j(t)) + 0.9\cos(v_j(t))$. The transformation matrix is $H = \mathrm{diag}\{1, 1, 1, 1, -1, -1, -1\}$. It could be calculated that $\lambda_2(\mathcal{L}_{g_1}) = 1.382$. Based on Theorem 2, one obtains the settling time $T \leq 4.4$.

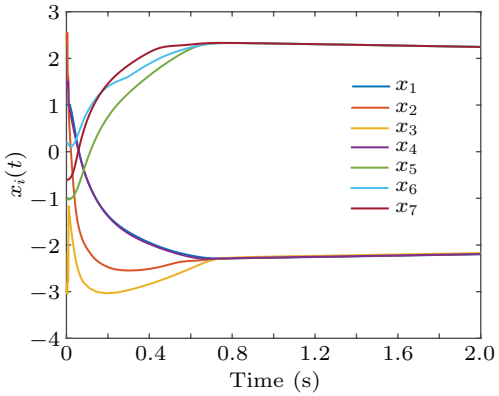


Fig. 4. Trajectories of the position state.

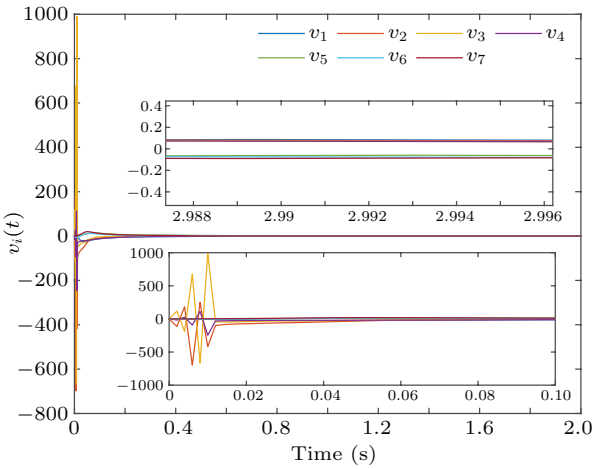


Fig. 5. Trajectories of the velocity state.

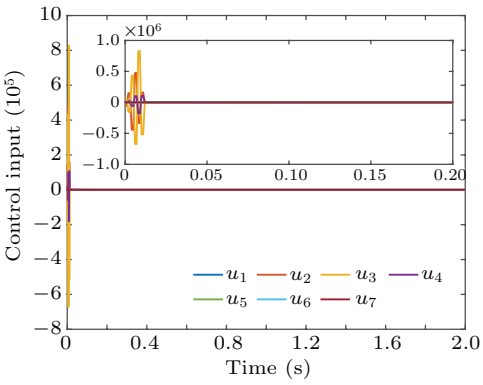


Fig. 6. Control inputs of agents.

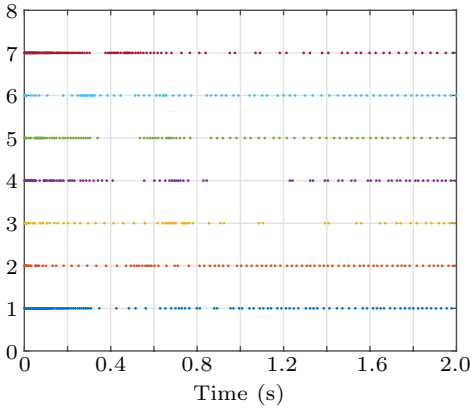


Fig. 7. The trigger instant of agents.

For MAS (1) over the $\mathcal{G}_1$, the state trajectories of agents are given in Figs. 4 and 5, the control inputs of agents are shown in Fig. 6, and the trigger instant of agents in Fig. 7. Under event-triggered control (11), the system could achieve convergence within the settling time $T_{\max} = 0.9$, which illustrates that the desired goal of fixed-time event-triggered group consensus for second-order MAS (1) is achieved.

5. Conclusions

The group consensus problem of second-order MASs with uncertain disturbances is explored in this paper. Separately, the fixed-time consensus with event-triggered control and without event-triggered control is investigated. Sufficient conditions for MASs to achieve group consensus are obtained. It is proved that no Zeno behavior is present under the designed event-triggered controller. The presented numerical simulations verify the potential correctness of the main results and the effectiveness of methods. The adaptive event-triggered group consensus for heterogeneous MASs will be studied in the future.

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References

[1] Li Y F, Tang C C, Peeta S and Wang Y B 2019 *IEEE Trans. Intell. Transp. Syst.* **20** 2209
[2] Chen L M, Li C J, Sun Y C and Ma G F 2017 *Chin. Phys. B* **26** 068703
[3] Zhang H, Zhou X, Wang Z P, Yan H C and Sun J 2019 *IEEE Trans. Cybern.* **49** 1580
[4] Hu X, Liu Z W, Wen G H, Yu X H and Liu C 2020 *IEEE Trans. Smart Grid* **11** 4017
[5] Sun F L, Wu X S, Kurths J and Zhu W 2023 *IEEE Trans. Syst. Man Cybern. Syst.* **53** 2572

[6] Shi H T, Hou M X, Wu Y H, Guo J and Feng D K 2020 *Int. J. Control Autom. Syst.* **18** 2765
[7] Shan L and Liu C L 2018 *J. Franklin Inst.* **355** 8957
[8] Gao H Y, Hu A H, Shen W Q and Jiang Z X 2019 *Chin. Phys. B* **28** 060501
[9] Liu Z W, Wen G H, Yu X H, Guan Z H and Huang T W 2020 *IEEE Trans. Cybern.* **50** 3045
[10] Han Y Y and Li C D 2018 *Neurocomputing* **286** 51
[11] Wu Z H and Xie L B 2022 *Chin. Phys. B* **31** 128902
[12] Han Z Y, Jia Q and Tang W K 2020 *Int. J. Syst. Sci.* **51** 2192
[13] Ren H W and Deng F Q 2017 *Chin. Phys. B* **26** 100506
[14] Shi S, Feng H Y, Liu W H and Zhuang G M 2019 *Nonlinear Dyn.* **96** 1317
[15] Wang H, Yu W W, Ren W and Lü J H 2021 *IEEE Trans. Cybern.* **51** 1347
[16] Ke J, Huang W C, Wang J Y and Zeng J P 2022 *ISA Trans.* **119** 135
[17] Zou W C, Qian K W and Xiang Z R 2020 *IEEE Trans. Circuits Syst. II Express Briefs.* **67** 1279
[18] Du H B, Wen G H, Wu D, Cheng Y Y and Lü J H 2020 *Automatica* **113** 108797
[19] Shi S, Xu S Y and Feng H Y 2021 *IEEE Trans. Syst. Man Cybern. Syst.* **51** 1869
[20] Sun F L, Wang F, Liu P Y and Kurths J 2022 *Int. J. Robust Nonlinear Control* **32** 1469
[21] Ji L H, Gao T and Liao X F 2021 *IEEE Trans. Syst. Man Cybern. Syst.* **51** 5367
[22] Guo X, Lu J Q, Alsaedi A and Alsaadi F E 2018 *Physica A* **495** 488
[23] Ning B D, Han Q L and Zuo Z Y 2021 *IEEE Trans. Automat. Control* **66** 2739
[24] Sun F L, Wu X S, Kurths J and Zhu W 2022 *Nonlinear Dyn.* **110** 3447
[25] Liu J, Zhang Y L, Yu Y and Sun C Y 2019 *IEEE Trans. Syst. Man Cybern. Syst.* **49** 2221
[26] Wang J H, Xu Y L, Zhang J and Yang D D 2018 *Chin. Phys. B* **27** 040504
[27] Liu J, Zhang Y L, Sun C Y and Yu Y 2019 *Inform. Sci.* **480** 261
[28] Zhang A, Zhou D, Yang P and Yang M 2019 *Int. J. Control Autom. Syst.* **17** 836
[29] Altafini C 2013 *IEEE Trans. Automat. Control* **58** 935
[30] Polyakov A 2012 *IEEE Trans. Automat. Control* **57** 2106
[31] Zuo Z Y and Tie L 2016 *Int. J. Syst. Sci.* **47** 1366