RAPID COMMUNICATION

Tunable phonon–atom interaction in a hybrid optomechanical system

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We theoretically analyze a hybrid system consisting of a levitated neutral atom and a nanoparticle coupled to a cavity. The mechanical oscillator and the atom are effectively coupled to each other through the cavity photons as a bus. By adjusting the driving lasers, we can conveniently switch the phonon–atom coupling between Jaynes–Cummings (JC) and anti-JC forms, which can be used to manipulate the motional states of the mechanical oscillator. As an application, we prepare a superposition state of the mechanical oscillator via the effective phonon–atom interaction and investigate the effects of dissipation on the state generation.

Keywords: optomechanics, hybrid system, optical tweezers, Fock state

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1. Introduction

Following the pioneering work by Ashkin and Dziedzic,[1–3] levitation of dielectric objects has become a widely used technique in science[4–6] and engineering.[7–9] Owing to the high isolation from thermal and environmental sources of noise possible in high-vacuum, optically levitated objects can provide ultrahigh force and acceleration sensitivity, which are fueling both the precise measurement[10–14] and the study of fundamental physics.[15–24] Recently, the precise manipulation of motion of the nano/micro-scale objects is attracting increasing attention.[25–28]

By far, both optical cavity-based[29] and active-feedback[30,31] schemes have successfully cooled the motion of a levitated dielectric nanoparticle into the quantum ground state that indicates levitated optomechanics has entered into the quantum regime. In the quantum regime, the quantum states of the motion of the nanoparticle, for instance, the superposition states and the squeezed states are the key resources that are useful for precise measurement, quantum information processing, and the research of fundamental quantum physics.[32,33] Therefore, the generation and manipulation of the quantum motional states of a levitated nanoparticle become an important issue worth studying.

Hybrid optomechanical systems, which integrate the advantages of several systems, have become a field of interest.[34–39] In this paper, we theoretically study a hybrid optomechanical system consisting of a neutral atom and a nanoparticle levitated in an optical cavity (see Fig. 1). The atomic transition and the motion of the nanoparticle are simultaneously coupled to the same cavity mode. The cavity photon connecting the atom and the nanoparticle leads to an effective phonon–atom coupling which can be switched between Jaynes–Cummings (JC) and anti-JC forms by adjusting the driving laser. This effective coupling can be used to generate and manipulate the motional states of the nanoparticle. As an application, we generate a motional superposition state of a nanoparticle and study the effects of dissipation. The rest of the paper is organized as follows. In Section 2, we present the physical model and the deviation of the effective Hamiltonian. In Section 3, the numerical results and the discussion of the effects of dissipation are presented. Finally, we conclude this paper in Section 4.

2. The physical model and the effective Hamiltonians

The Hamiltonian We consider a hybrid optomechanical system as displayed in Fig. 1, where a Λ-type three-level atom and a nanoparticle are simultaneously trapped and levitated in an optical cavity that is driven by a pumping laser. The three-level atom has an excited state $|s\rangle$ and two ground states $|e\rangle$ and $|g\rangle$. The two transitions $|s\rangle \leftrightarrow |e\rangle$ and $|s\rangle \leftrightarrow |g\rangle$ are pumped by a classical field and coupled to a cavity mode, respectively. Meanwhile, the nanoparticle is coupled to the same cavity mode. The total Hamiltonian of the hybrid op-

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tomechanical system reads ($\hbar = 1$)

$$H = \omega_{\rm c}\hat{a}^{\dagger}\hat{a} + \omega_{\rm m}\hat{b}^{\dagger}\hat{b} + \sum_{i=s,e,g}\omega_i|i\rangle\langle i| + H_{\rm ac}^{\rm int} + H_{\rm oc}^{\rm int} + H_{\rm d}, \tag{1}$$

where $\hat{a}$ and $\hat{b}$ stand for the annihilation operators of the cavity mode and the mechanical oscillator, respectively. $\omega_{\rm c}$, $\omega_{\rm m}$, $\omega_{i=s,e,g}$ are respectively the frequencies of the cavity mode, the mechanical oscillator, and the atomic energy levels. The atom–cavity interaction Hamiltonian under the rotating wave approximation (RWA) is given by

$$H_{\rm ac}^{\rm int} = g_{\rm a}^{0}\left(\hat{a}|s\rangle\langle g| + |g\rangle\langle s|\hat{a}^{\dagger}\right) + \Omega\left(|s\rangle\langle e|{\rm e}^{-{\rm i}\omega_{\Omega}t} + |e\rangle\langle s|{\rm e}^{{\rm i}\omega_{\Omega}t}\right), \tag{2}$$

where $g_{\rm a}^{0}$ is the coupling strength between the atom and the cavity mode. ω_{Ω} and Ω are respectively the driving frequency and strength of the classical field which drives the transition $|s\rangle \leftrightarrow |e\rangle$. $H_{\rm oc}^{\rm int}$ denotes the oscillator–cavity interaction Hamiltonian that originates from their dipole–dipole interaction. In the case the radius of the particle is much smaller than the cavity mode wavelength, i.e., $r \ll \lambda_{\rm c}$, the dipole force potential created by the cavity mode can be approximated as[40] $\hat{U}_{\rm m}^{\rm dipole} = -g_{\rm m}^{0}\hat{a}^{\dagger}\hat{a}\cos\left[2k(q^{\rm eq}+\hat{q})\right]$, where $k = 2\pi/\lambda_{\rm c}$ denotes the wave number of cavity field, $q^{\rm eq}$ is the equilibrium position of the particle, and $\hat{q} = \sqrt{\frac{1}{2m\omega_{\rm m}}}(\hat{b}^{\dagger}+\hat{b})$ is the position operator with the particle's mass m. The single photon optomechanical coupling strength is given by $g_{\rm m}^{0} = \frac{3V_{\rm m}}{4V_{\rm c}}\frac{\varepsilon_0-1}{\varepsilon_0+2}\omega_{\rm c}$, where $V_{\rm m}$ and $V_{\rm c}$ are the volumes of the particle and the cavity, respectively, and ε_0 the electric permittivity. Assuming the oscillation of the particle around its equilibrium position is small, one can expand the dipole force potential to the first order and obtain a linear-coupling interaction Hamiltonian

$$H_{\rm oc}^{\rm int} = g_{\rm m}\hat{a}^{\dagger}\hat{a}\left(\hat{b}+\hat{b}^{\dagger}\right), \tag{3}$$

where $g_{\rm m} = -2g_{\rm m}^{0}k\sin[2kq^{\rm eq}]\sqrt{\frac{1}{2m\omega_{\rm m}}}$ is the coupling strength, whose value can be adjusted by shifting the equilibrium position $q^{\rm eq}$. The driving Hamiltonian is

$$H_{\rm d} = \eta\left(\hat{a}{\rm e}^{{\rm i}\omega_{\rm L}t} + \hat{a}^{\dagger}{\rm e}^{-{\rm i}\omega_{\rm L}t}\right), \tag{4}$$

where $\omega_{\rm L}$ and η are the frequency and strength of the pumping laser, respectively.

Applying the unitary operator

$$\hat{U} = \exp\left(-{\rm i}\omega_{\rm L}\hat{a}^{\dagger}\hat{a}t + {\rm i}\omega_{\rm L}|g\rangle\langle g|t + {\rm i}\omega_{\Omega}|e\rangle\langle e|t\right)$$

to the total Hamiltonian in Eq. (1), we obtain a time-independent Hamiltonian in the rotating frame

$$H' = \Delta_{\rm c}\hat{a}^{\dagger}\hat{a} + \omega_{\rm m}\hat{b}^{\dagger}\hat{b} + \omega_s|s\rangle\langle s| + \Delta_e|e\rangle\langle e| + \Delta_g|g\rangle\langle g| + g_{\rm a}^{0}\left(\hat{a}^{\dagger}|g\rangle\langle s| + |s\rangle\langle g|\hat{a}\right) + \Omega\left(|e\rangle\langle s| + |s\rangle\langle e|\right) + g_{\rm m}\hat{a}^{\dagger}\hat{a}\left(\hat{b}+\hat{b}^{\dagger}\right) + \eta(\hat{a}^{\dagger}+\hat{a}), \tag{5}$$

where the related detunings are defined as $\Delta_{\rm c} = \omega_{\rm c} - \omega_{\rm L}$, $\Delta_g = \omega_g + \omega_{\rm L}$, and $\Delta_e = \omega_e + \omega_{\Omega}$. In the strong driving regime, the operators of the cavity mode and the mechanical oscillator can be rewritten as the sum of its classical steady-state value and a small quantum fluctuation operator, i.e., $\hat{a} \to \alpha + \hat{a}$ and $\hat{b} \to \beta + \hat{b}$. By employing the standard linearization procedure,[41,42] we obtain the atom–oscillator–cavity (AOC) Hamiltonian in terms of the quantum fluctuation operators as

$$H_{\rm AOC} = \Delta_{\rm c}'\hat{a}^{\dagger}\hat{a} + \omega_{\rm m}\hat{b}^{\dagger}\hat{b} + \omega_s|s\rangle\langle s| + \Delta_e|e\rangle\langle e| + \Delta_g|g\rangle\langle g| + g_{\rm a}\left(\hat{a}^{\dagger}|g\rangle\langle s| + |s\rangle\langle g|\hat{a}\right) + \Omega\left(|e\rangle\langle s| + |s\rangle\langle e|\right) + G_{\rm m}\left(\hat{a}^{\dagger}+\hat{a}\right)\left(\hat{b}^{\dagger}+\hat{b}\right), \tag{6}$$

where $\Delta_{\rm c}'$, $g_{\rm a}$, and $G_{\rm m}$ are respectively the effective detuning and the coupling strength that are determined by the classical steady-state values α and β. In experiments, both the coupling between the atom and cavity $g_{\rm a}$ and the coupling between the nanoparticle and cavity $G_{\rm m}$ have been achieved.[43,44]

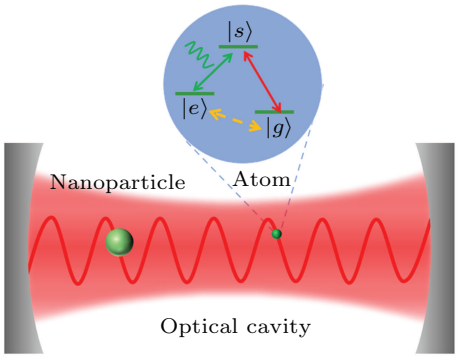


Fig. 1. The simplified diagram of the hybrid optomechanical system: a Λ-type three-level atom and a nanoparticle are simultaneously trapped and levitated in a driven optical cavity. The three-level atom has an excited state $|s\rangle$ and two ground states $|e\rangle$ and $|g\rangle$. The two atomic transitions $|s\rangle \leftrightarrow |e\rangle$ and $|s\rangle \leftrightarrow |g\rangle$ are respectively pumped by a classical field and coupled to a cavity mode. Meanwhile, the nanoparticle is coupled to the same cavity mode.

The effective two-level configuration of the atom When the atomic energy differences are very large, the atomic excited state $|s\rangle$ can be adiabatically eliminated and the three-level configuration reduces to an effective two-level configuration. Employing the effective operator method,[45] we obtain the effective Hamiltonian that describes a two-level atom interacting with the cavity mode

$$H_{\rm AOC}' = \Delta_{\rm c}'\hat{a}^{\dagger}\hat{a} + \omega_{\rm m}\hat{b}^{\dagger}\hat{b} + \Delta_e'|e\rangle\langle e| + \Delta_g|g\rangle\langle g| + \Delta_g'\hat{a}^{\dagger}\hat{a}|g\rangle\langle g| + g_{\rm a}'\left(\hat{a}^{\dagger}|g\rangle\langle e| + |e\rangle\langle g|\hat{a}\right) + G_{\rm m}\left(\hat{a}^{\dagger}+\hat{a}\right)\left(\hat{b}^{\dagger}+\hat{b}\right), \tag{7}$$

where the effective detunings and coupling strength are given by

$$\Delta_e' = \Delta_e - \frac{\Omega^2}{\omega_s - \Delta_e},$$

$$\Delta_g' = -\frac{g^2}{\omega_s - \Delta_g},$$
$$g_a' = -\frac{g_a G_m(\omega_s - \Delta_e + \omega_s - \Delta_g)}{2(\omega_s - \Delta_e)(\omega_s - \Delta_g)}.$$

Here, we point out that the three-level configuration in our scheme can be replaced by a two-level configuration. However, due to the virtual excitation of the excited state, the three-level atom has a longer coherent time which is advantageous for the generation and manipulation of the quantum state of the mechanical oscillator. Therefore, we used the three-level atom in our scheme.

The JC and anti-JC phonon–atom interaction In Eq. (7), the oscillator–cavity interaction term $G_m\left(\hat{a}^\dagger + \hat{a}\right)\left(\hat{b}^\dagger + \hat{b}\right)$ contains both the rotating and counter-rotating interactions. When the cavity detuning is set to $\Delta_c' \approx \omega_m$, only the rotating coupling term is retained and the AOC Hamiltonian is simplified as

$$\begin{aligned} H_{AOC}'' \approx\ & \Delta_c'\hat{a}^\dagger\hat{a} + \omega_m\hat{b}^\dagger\hat{b} + \Delta_e'|e\rangle\langle e| + \Delta_g|g\rangle\langle g| \\ & + \Delta_g'\hat{a}^\dagger\hat{a}|g\rangle\langle g| + g_a'\left(\hat{a}^\dagger|g\rangle\langle e| + |e\rangle\langle g|\hat{a}\right) \\ & + G_m\left(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger\right). \end{aligned} \tag{8}$$

In the large detuning condition, we adiabatically eliminate the cavity mode and obtain the effective atom–oscillator (AO) interaction Hamiltonian[45,46]

$$\begin{aligned} H_{OA}^{JC} =\ & \omega_m'\hat{b}^\dagger\hat{b} + \Delta_e''|e\rangle\langle e| + \Delta_g|g\rangle\langle g| \\ & + G_{JC}\left(\hat{b}^\dagger|g\rangle\langle e| + |e\rangle\langle g|\hat{b}\right), \end{aligned} \tag{9}$$

where the related parameters are defined as

$$\Delta_e'' = \Delta_e' - \frac{g_a'^2}{\Delta_c - (\Delta' - \Delta_g)},$$
$$\omega_m' = \omega_m - \frac{G_m^2}{\Delta_c' - \omega_m},$$
$$G_{JC} = \frac{g_a' G_m}{2}\left(\frac{1}{\Delta_c - (\Delta_e' - \Delta_g)} - \frac{1}{\Delta_c - \omega_m}\right).$$

Similarly, when the detuning is set to $\Delta_c' \approx -\omega_m$, we obtain the effective AO Hamiltonian that only contains the counter-rotating interaction

$$\begin{aligned} H_{OA}^{AJC} =\ & \omega_m''\hat{b}^\dagger\hat{b} + \Delta_e''|e\rangle\langle e| + \Delta_g|g\rangle\langle g| \\ & + G_{AJC}\left(\hat{b}^\dagger|e\rangle\langle g| + |g\rangle\langle e|\hat{b}\right), \end{aligned} \tag{10}$$

where the related parameters are given by

$$\omega_m'' = \omega_m - \frac{G_m^2}{\Delta_c' + \omega_m},$$
$$G_{AJC} = \frac{g_a' G_m}{2}\left(\frac{1}{\Delta_c - (\Delta_e' - \Delta_g)} - \frac{1}{\Delta_c + \omega_m}\right).$$

The derivation above shows that the original AOC Hamiltonian in Eq. (1) can be reduced to an effective AO Hamiltonian in the JC- or anti-JC-form[34] by simply adjusting the detuning of the driving laser.

The manipulation of the quantum states of the mechanical oscillator Both the JC- and anti-JC-form Hamiltonians can generate the Rabi oscillation and thus can be used to prepare and manipulate the quantum states of the oscillator. For example, transforming the JC AO Hamiltonian in Eq. (9) into the interaction picture in the resonance condition $\omega_m' = \Delta_e'' - \Delta_g$, we obtain

$$H_{JC}^{I} = G_{JC}\left(\hat{b}^\dagger|g\rangle\langle e| + |e\rangle\langle g|\hat{b}\right). \tag{11}$$

By solving the Schrödinger equation $i|\psi(t)\rangle = H_{JC}^{I}|\psi(t)\rangle$ with the initial state $|\psi_{ini}\rangle = |e,n\rangle$ in the subspace $\{|e,n\rangle, |g,n+1\rangle\}$, we obtain the wave function at time t as

$$|\psi(t)\rangle = \cos\left(\frac{\Omega_n}{2}t\right)|e,n\rangle - i\sin\left(\frac{\Omega_n}{2}t\right)|g,n+1\rangle, \tag{12}$$

where $\Omega_n = 2G_{JC}\sqrt{n+1}$ is the Rabi frequency. Correspondingly, the populations of states $|e,n\rangle$ and $|g,n+1\rangle$ are given by

$$P_{e,n} = \frac{1+\cos(\Omega_n t)}{2}, \quad P_{g,n+1} = \frac{1-\cos(\Omega_n t)}{2}. \tag{13}$$

The equations above show that the JC OA Hamiltonian leads to a Rabi oscillation between the states $|e,n\rangle$ and $|g,n+1\rangle$ with the period $\tau_{Rabi} = 2\pi/\Omega_n$. Similarly, the anti-JC AO Hamiltonian in Eq. (10) leads to a Rabi oscillation between the states $|g,n\rangle$ and $|e,n+1\rangle$.

As an application, we prepare a motional superposition state of the mechanical oscillator via the JC OA Hamiltonian. We first initialize the atom and the oscillator in the state $|\psi_{ini}\rangle = (c_g|g\rangle + c_e|e\rangle)\otimes|n\rangle$ with the complex coefficients c_e and c_g satisfying $|c_e|^2 + |c_g|^2 = 1$, then turn on JC OA Hamiltonian for a duration $\tau_n = \pi/\Omega_n$, and finally get a superposition state $|\psi_{sp}\rangle = |g\rangle\otimes(c_g|n\rangle - ic_e|n+1\rangle)$.

3. The numerical results and the effects of dissipation

We numerically simulate the dynamics of the whole hybrid-optomechanical system by employing the standard Markovian master equation

$$\begin{aligned} \dot{\hat{\rho}} =\ & -i\left[\hat{H}_{AOC}, \hat{\rho}\right] + \kappa\mathcal{L}(\hat{a}, \hat{\rho}) \\ & + \lambda\mathcal{L}(\hat{\sigma}_{es}, \hat{\rho}) + \lambda\mathcal{L}(\hat{\sigma}_{gs}, \hat{\rho}) \\ & + \gamma(n_{th}+1)\mathcal{L}(\hat{b}, \hat{\rho}) + \gamma n_{th}\mathcal{L}(\hat{b}^\dagger, \hat{\rho}), \end{aligned} \tag{14}$$

where $\hat{\rho}$ is the density operator of the whole system, $\hat{H}_{AOC}$ is the system Hamiltonian in Eq. (6), $\hat{\sigma}_{ms} = |m\rangle\langle s|$ $(m = e, g)$ is the atomic transition operator, and $\mathcal{L}\left(\hat{O}, \hat{\rho}\right) = \hat{O}\hat{\rho}\hat{O}^\dagger - \frac{1}{2}\hat{O}^\dagger\hat{O}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{O}^\dagger\hat{O}$ is the Lindblad superoperator which phenomenally describes the effects of dissipation. The parameters κ, λ, and γ denote the decay rates of the cavity field, the atom (atom spontaneous emission), and the mechanical oscillator,

respectively. Without loss of generality, we have assumed that the two decay channels of atom spontaneous emission have the same decay rate λ. $n_{\rm th}$ is the mean phonon number of the thermal reservoir. The equilibrium mean photon number of thermal bath is $\bar{n}_{\rm c} = (\exp\{\hbar\omega_{\rm c}/k_{\rm B}T\}-1)^{-1} \simeq 0$ due to the high optical frequency $\hbar\omega_{\rm c}/k_{\rm B}T \gg 1$ with $k_{\rm B}$ the Boltzmann constant and T the environment temperature.

To verify the approximation in the derivation of the effective Hamiltonians, we plot the populations of the states $|e,0\rangle$, $|g,1\rangle$, and $|e,2\rangle$ as a function of time respectively determined by the original AOC Hamiltonian in Eq. (6) and the effective (a) JC and (b) anti-JC AO Hamiltonians in Eqs. (9) and (10) in Fig. 2. The Rabi oscillations between $|e,0\rangle$ and $|g,1\rangle$ and between $|g,1\rangle$ and $|e,2\rangle$ are seen. The time evolution of the populations governed by the effective AO Hamiltonians agrees well with that governed by the original AOC Hamiltonian.

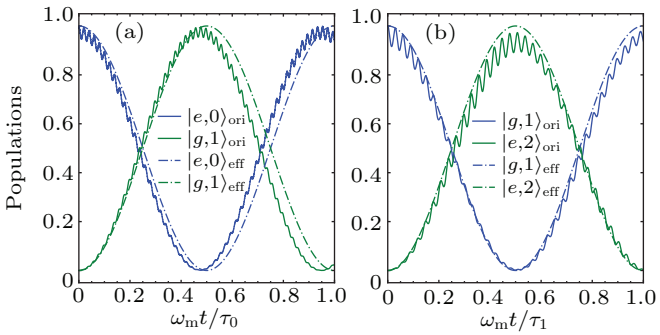


Fig. 2. The populations of the states $|e,0\rangle$, $|g,1\rangle$, and $|e,2\rangle$ as a function of time respectively determined by the original AOC Hamiltonian in Eq. (6) and the effective (a) JC and (b) anti-JC AO Hamiltonians in Eqs. (9) and (10). The parameters are set as follows: (a) $\omega_{\rm b}=1$, $\Delta_{\rm c}'=1.1$, $\omega_s=0$, $\Delta_e=-11.4688$, $\Delta_g=-12.5$, $g_{\rm a}=0.2$, $\Omega=0.5981$, $G_{\rm m}=0.01$ and (b) $\omega_{\rm b}=1$, $\Delta_{\rm c}'=-0.9$, $\omega_s=0$, $\Delta_e=-10.2748$, $\Delta_g=-9.3$, $g_{\rm a}=0.2$, $\Omega=0.4882$, $G_{\rm m}=0.01$.

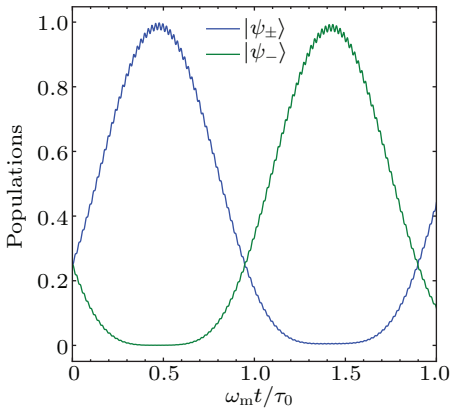


Fig. 3. The fidelity of the superposition states $|\psi_\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$ as a function of time. The parameters are set as follows: $\omega_{\rm b}=1$, $\Delta_{\rm c}'=1.1$, $\omega_s=0$, $\Delta_e=-11.4688$, $\Delta_g=-12.5$, $g_{\rm a}=0.2$, $\Omega=0.5981$, $G_{\rm m}=0.01$.

As an application, the generation of the motional superposition state $|\psi_\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$ is studied. We initialize the hybrid-optomechanical system in the state $|\psi_{\rm ini}\rangle = (|e\rangle - {\rm i}|g\rangle)\otimes|0_b,0_a\rangle/\sqrt{2}$ (where the subscripts a and b respectively stand for the cavity mode and mechanical mode) and set the detuning as $\Delta_{\rm c}' \approx \omega_{\rm m}$ so that the JC-form interaction Hamiltonian works. We simulate the dynamical evolution via the original AOC Hamiltonian without considering the dissipation and plot the fidelity of the superposition states $|\psi_\pm\rangle$ as a function of time in Fig. 3. As expected, the superposition states $|\psi_\pm\rangle$ are generated at the time $t_+ = \tau_0/2$ and $t_- = 3\tau_0/2$, respectively.

At last, we consider the effects of the dissipation on the generation of the motional superposition state. Figure 4(a) shows the Rabi oscillation between $|e,0\rangle$ and $|g,1\rangle$ under the JC OA Hamiltonian in the presence of the dissipation of the cavity field, the atom, and the mechanical oscillator. The populations show an oscillation decay due to the dissipation of the environments. To study the effects of each decay channel on the generation of the superposition state, we individually simulate the evolution of the system in the presence of merely one decay channel.

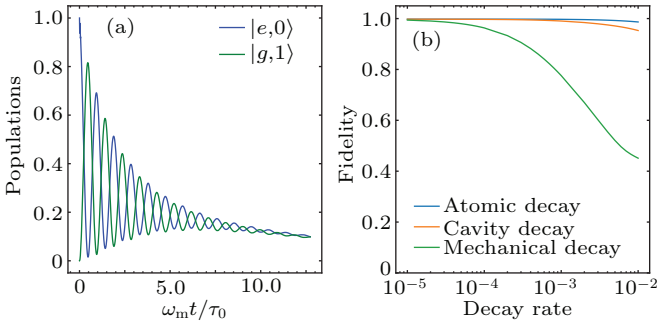


Fig. 4. (a) The Rabi oscillation between $|e,0\rangle$ and $|g,1\rangle$ under the JC OA Hamiltonian in the presence of the dissipation of the cavity field, the atom, and the oscillator. (b) The fidelity of the state $|\psi_+\rangle$ at the time $\omega_{\rm m}t=\tau_0/2$ as a function of the decay rate. The parameters are set as follows: (a) $\omega_{\rm b}=1$, $\Delta_{\rm c}'=1.1$, $\omega_s=0$, $\Delta_e=-11.4688$, $\Delta_g=-12.5$, $g_{\rm a}=0.2$, $\Omega=0.5981$, $G_{\rm m}=0.01$, $\kappa=0.01$, $\lambda=0.01$, $\gamma=0.00001$ and (b) $\omega_{\rm b}=1$, $\Delta_{\rm c}'=1.1$, $\omega_s=0$, $\Delta_e=-11.4688$, $\Delta_g=-12.5$, $g_{\rm a}=0.2$, $\Omega=0.5981$, $G_{\rm m}=0.01$.

Figure 4(b) shows the fidelity of the state $|\psi_+\rangle$ at the time $\omega_{\rm m}t=\tau_0/2$ as a function of the decay rate. The results show that the fidelity is insensitive to the atomic decay because the atomic excited state $|s\rangle$ is only virtually excited in the large detuning condition. Similarly, the fidelity is also insensitive to the decay of the cavity field since the cavity mode is almost always in the vacuum state during the whole generation procedure. On the contrary, the fidelity is very sensitive to the mechanical decay, that is, the fidelity decreases rapidly with increasing mechanical decay rate. The mechanical decay rate scales with the pressure which is well kept at 10^{-8} mbar in the current optomechanical experiments. The mechanical decay rate at this pressure is about 10 order of magnitude smaller than the frequency of the mechanical oscillator, i.e., $\gamma \approx 10^{-10}$. Therefore, the mechanical decay almost has no effect on the generation of the motional superposition state in experiments.

4. Conclusion

We theoretically analyze a hybrid optomechanical system where a three-level neutral atom and a nanoparticle are simultaneously levitated in an optical cavity. The atomic transition is coupled to a cavity mode, at the same time, the motion of the mechanical oscillator is also coupled to the same cavity mode. As a result, the mechanical oscillator and the atom are effectively coupled to each other through the cavity photons as a bus. By adjusting the detuning of the driving laser, we can conveniently switch the phonon–atom interaction between JC and anti-JC forms, both of which can produce the Rabi oscillations between the motional states of the mechanical oscillator. As an application, we prepare the motional superposition state of the mechanical oscillator via the effective phonon–atom interaction and discuss the effects of the dissipation of the cavity field, the atom, and the mechanical oscillator.

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