INSTRUMENTATION AND MEASUREMENT

Measurement of remanent magnetic moment using a torsion pendulum with single frequency modulation method

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In TianQin spaceborne gravitational-wave detectors, the stringent requirements on the magnetic cleanliness of the test masses demand the high resolution ground-based characterization measurement of their magnetic properties. Here we present a single frequency modulation method based on a torsion pendulum to measure the remanent magnetic moment $m_{\rm r}$ of 1.1 kg dummy copper test mass, and the measurement result is $(6.45 \pm 0.04(\text{stat}) \pm 0.07(\text{syst})) \times 10^{-8}$ A$\cdot$m^2. The measurement precision of the $m_{\rm r}$ is about 0.9 nA$\cdot$m^2, well below the present measurement requirement of TianQin. The method is particularly useful for measuring extremely low magnetic properties of the materials for use in the construction of space-borne gravitational wave detection and other precision scientific apparatus.

Keywords: remanent magnetic moment, torsion pendulum, single frequency modulation method

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1. Introduction

TianQin is a planned space-based gravitational-wave (GW) observatory, aiming to detect GWs between 10^{-4} Hz and 1 Hz that can be generated by a wide variety of important astrophysical and cosmological sources.[1,2]

TianQin is comprised of three Earth-orbiting spacecrafts in a nearly equilateral triangle formation with 1.7×10^5 km arm. There are two quasi-free-falling cubic test masses (nominally 2-kg) inside each spacecraft and the GW signals are detected by laser interferometry with the test masses as the end mirrors. The scientific detection depends on the purity of test mass (TM) free-falling motion at low frequencies, which demands the remnant acceleration noise of the TMs must be below 1×10^{-15} m/s$^2 \cdot$Hz$^{1/2}$ around mHz band under present concept of TianQin. There are many noise sources of the remnant acceleration noise such as the thermal fluctuation, magnetic disturbance and so on.[1] The magnetic forces are one of the largest contributors to the remnant acceleration noise with budget 24% in TianQin. The magnetic forces could be described by the coupling of the remanent magnetic moment $\boldsymbol{m}_{\rm r}$ and the induced magnetic moment $\boldsymbol{m}_{\rm i}$ of TM with its fluctuated ambient magnetic field $\boldsymbol{B}$. In the limit of weakly magnetic materials, the magnetic forces can be expressed as[3]

$$\boldsymbol{F}_m = [(\boldsymbol{m}_{\rm r} + \boldsymbol{m}_{\rm i}) \cdot \nabla]\boldsymbol{B}. \tag{1}$$

To suppress the forces, one can decrease the ambient magnetic field composed of satellite magnetic field and interplanetary magnetic field, and reduce both of $m_{\rm r}$ and $m_{\rm i}$ of TMs. Under the current magnetic environment of the TMs, the requirement of $m_{\rm r}$ of Au–Pt alloy TMs is < 20 nA$\cdot$m^2 in the TianQin.[4] For verifying this very stringent remanent magnetic moment cleanliness of TMs, the measurement precision of $m_{\rm r}$ is preferred to be a order of higher than the requirement, i.e., 2 nA$\cdot$m^2, in the ground based test.

There are many instruments and methods applied to measure the magnetic moment of a substance. The traditional instruments such as physical property measurement system (PPMS), magnetic property measurement system (MPMS) and vibrating sample magnetometer (VSM) have satisfied resolution for the measurement,[5] whereas the specimen chamber of the instruments could not accommodate the cm-scale TMs sample used in TianQin. The traditional torque magnetometry using a cantilever and the force magnetometry using a balance lack sufficient sensitivity.[6] The precision balance and torsion pendulum have been verified as the effective and convenient tools for investigating the magnetic properties.[6–11]

Therefore, in this paper we present a project based on a torsion pendulum to measure the remanent magnetic moment of a cm-scale solid TM with a single frequency modulation method. The method can effectively avoid interference noises

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in measurement such as the potential thermal noise and the intrinsic signals of the pendulum. The remanent magnetic moment of a solid copper cubic TM was measured to demonstrate the sensitivity of the instrument. Compared with the traditional instruments and methods, the instrument based on torsion pendulum with the single frequency modulation method further improves the resolution of the measurement.

2. Experimental principle and instrument

As shown in Fig. 1, the torsion pendulum is a side 5-cm copper cubic mimetic TM that is suspended by a 90-cm long, 125-μm diameter tungsten fiber. The tungsten fiber is connected to a magnetic damper that is used to suppress the unwanted modes of the pendulum.[12] The magnetic damper is then suspended by a 400-μm diameter tungsten fiber, which is connected with the feedthrough that is mounted on top of the vacuum chamber. The distance between the damper and the TM is about 0.9 m, and the damper's parasitic magnetic field around the TM is less 0.5 μT. In addition, the direction of the parasitic field is along the fiber and the field will not contribute torque to the fiber. Hence, the parasitic effect of the damper could be ignored. A rotary vane pump and a turbomolecular pump are used to acquire vacuum, while an ion pump is used to maintain the chamber at a pressure of 10^{-5} Pa during the experimental measurement. Since the ion pump works without vibration, it will not bring additional vibrational influence. The pendulum body twist $\theta(t)$ is monitored by an autocollimator that reflects the light off the reflector continuously every 40 ms. The calculated moment of inertia I about fiber equals to (527.1 ± 0.5) kg·mm^2.

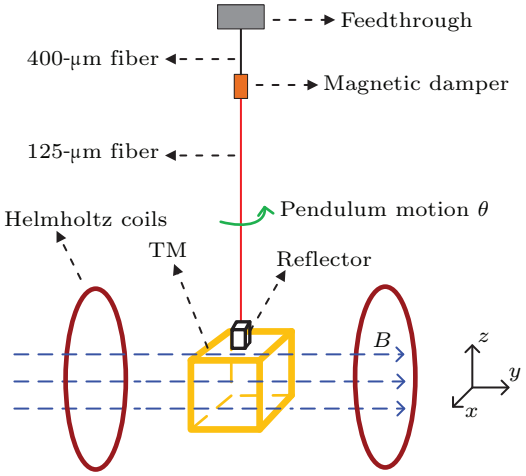


Fig. 1. Schematic diagram of the device. The yellow cube is TM, and the purple annulus is Helmholtz coils. The origin of the coordinate at the center of the cube, and horizontal coordinates paralleled with the sides of the cube respectively (the y-axis is set along the axes of coils, and the z-axis is set parallel to the fiber). The vacuum chamber around the torsion pendulum and two pumps are not shown for concision.

The TM is suspended at the center of a Helmholtz coil ((250 ± 2) mm in diameter, 511 turns) which is applied to produce a uniform magnetic field $\boldsymbol{B}$ in total volume of the TM along the y axis. Then, the TM will sense a torque $\boldsymbol{N} = \boldsymbol{m} \times \boldsymbol{B}$ due to the coupling between the remanent magnetic moment of the TM and the uniform magnetic field. Since the torsion pendulum has only one twist mode along the fibre (z axis), the torque could be expressed as

$$N_{\rm s} = m_x B_y + m_y B_x \simeq m_x B_y, \tag{2}$$

where m_x and B_y are the x-component of $\boldsymbol{m}_{\rm r}$ and dominated y-component of $\boldsymbol{B}$. The x-component of $\boldsymbol{B}$ is much less than B_y due to the character of Helmholtz coil, and the B_x can be ignored.

By using the single frequency modulation method, the magnetic field will be modulated at a single frequency $\omega_{\rm s}$. In this way, the measured torque can be written as $N_{\rm s}(t) = m_x \cdot B_y \cos(\omega_{\rm s} t)$.

It is common to take a simplified pendulum model for analysis. Considering the angle fluctuation subject to the velocity damping, the equation of motion of the pendulum in a laboratory frame can be written as

$$I\ddot{\theta} + 2\beta\dot{\theta} + k\theta = N_{\rm s}(t) + N_{\rm th}(t), \tag{3}$$

where θ is the torsion fiber twist angle, $\beta = I\omega_0/Q$ is the damping factor (Q is the quality factor of the pendulum), I is the pendulum moment of inertia about the fiber and k is the elastic constant of fiber. $N_{\rm th}(t)$ is the thermal fluctuation torque on the torsion pendulum.[13] By solving Eq. (3), as with the linear system, the motion of the torsion pendulum can be noted by

$$\theta(t) = \theta_{\rm s}(t) + \theta_{\rm th}(t) + \theta_{\rm i}(t) + \gamma(t), \tag{4}$$

where the targeted steady-state signal $\theta_{\rm s}(t)$ is

$$\theta_{\rm s}(t) = \frac{m_x \cdot B_y}{\sqrt{(I\omega_{\rm s}^2 - k)^2 + 4\beta^2\omega_{\rm s}^2}} \cos(\omega_{\rm s} t), \tag{5}$$

and $\theta_{\rm th}(t)$ is the response to the thermal fluctuation torque, and the response is the basic limit for our actual test according to the fluctuation-dissipation theorem.[14] The transient response is

$$\theta_{\rm i}(t) = A_0 \, {\rm e}^{-\beta t} \cos(\omega_0 t + \phi_0), \tag{6}$$

which depends on the initial conditions (A_0 and ϕ_0) and the damping factor β. The $\omega_0 = 2\pi/T_0$ is the natural frequency of the system. The response torsion at different frequency ω_0 is non-interference to the targeted signal. The $\gamma(t)$ is a low frequency drift item caused by some complex sources such as the creep of the fiber material, the fluctuations of ambient temperature, and so on.[15]

According to Eq. (5), the remanent magnetic moment m_x can be determined by

$$m_x = \frac{\Theta \cdot \sqrt{(I\omega_{\rm s}^2 - k)^2 + 4\beta^2\omega_{\rm s}^2}}{B_y}, \tag{7}$$

where Θ is the amplitude of the targeted signal at ω_s estimated from the twist motion of the pendulum.

It is visible that the targeted signal $\theta_s(t)$ is modulated at ω_s reposefully which is separated with the extensive interference signal $\theta_{th}(t)$ and $\theta_i(t)$ in frequency domain. Furthermore, the temperature noise due to the radiation of the Helmholtz coil at $2\omega_s$ and the almost constant effect from the geomagnetic field are also avoided. Besides, the frequency of the drift item is far below the applied $\theta_s(t)$, and interference of the substantial drift can be avoided. Therefore, the single frequency modulation method can suppress some important interference signals and hence improve the measurement precision in principle.

3. Data analysis

To measure the remanent magnetic moment m_x, one should precisely estimate the amplitude Θ of the targeted signal at ω_s. The typical twist of the pendulum $\theta(t)$ can be expressed as

$$\theta(t) = \Theta\cos(\omega_s t) + A_0\, e^{-\beta t}\cos(\omega_0 t + \phi_0) + \gamma(t), \qquad (8)$$

where $\gamma(t) = dt$ is a slow drift and can be regarded as linear with the coefficient d in the data analysis.[16] The tiny damping factor β can be neglected within a period of $\sim T_0$. The norm data processing is performed to estimate the amplitude at the frequency ω_s. Firstly, we use a torsional filter to process the raw twist data in order to eliminate the ω_0 signal. The filter can be expressed as

$$\begin{aligned}\theta_f(t) &= \theta(t) - \theta(t+T_0)\\ &= \Theta\sin(\omega_s T_0)\cos\left(\omega_s t + \frac{\omega_s T_0}{2}\right) + dT_0, \qquad (9)\end{aligned}$$

where t is the sample sequence. The $\theta(t+T_0)$ is acquired by linear interpolation with two sample data close to the moment $t+T_0$. Secondly, the whole filtered data $\theta_f(t)$ are cut into some segments, and each segment sustains 2000 s typically. Then, we adopt a nonlinear fitting method to determine the amplitude Θ_i and its uncertainty σ_i for the ith segment.[17]

Since the existence of the stochastic noise in each segment, the result of Θ and its uncertainty σ can be given by

$$\Theta = \sum_i p_i\Theta_i \Big/ \sum_i p_i, \quad \sigma = \sigma_i\sqrt{p_i \Big/ \sum_i p_i}, \qquad (10)$$

where $p_i = 1/\sigma_i^2$ is the weighting factor of the fitted Θ_i in the ith segment. And later the m_x can be estimated with Eq. (7).

4. Experimental results

Before formal experiment, the calibration test of the intensity and uniformity of the magnetic field produced by the coils were performed. As Fig. 2 shows, we measured the filed in a 60-mm-side cubic at the center of the coils by a commercial magnetic probe.

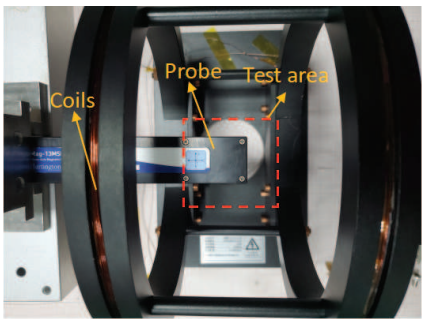


Fig. 2. Schematic diagram of the calibration test. The three-axis fluxgate magnetic probe installed where its axis y coincides with the axes of the coils and its center is located at the test area. The probe is installed at a three-axis displacement mechanism. The probe can scan the test area marked by red dotted frame with 6 cm side.

The probe was fixed on a three-axis displacement mechanism, and the probe could reach the whole volume of the cubic. We firstly set the probe at the centre of the coils, and then injected a 10 mA current with a period of 100 s into the coils. As Fig. 3 shows, the required applied filed $B_y = (42.95 \pm 0.01)$ μT, which is much larger than the undesired components $B_x = (0.50 \pm 0.01)$ μT and $B_z = (0.30 \pm 0.01)$ μT. It indicates the dominated component is B_y and the undesired component B_x in Eq. (2) can be ignored. We also performed calibration test for the uniformity of field. The probe was fixed about 3-minutes at each measuring points in the whole volume of the cubic. The calibration value of B_y is (42.95 ± 0.40) μT while the electric current of the coils equals to 10 mA. Here the uncertainty 0.40 μT is mainly due to the uniformity of magnetic field in the test area. Hence, the calibration uniformity is superior to 0.9% and also coincident with the calculation of uniformity of the magnetic field in the volume.[18] The uniformity and intensity of the applied magnetic field are adoptable to the design of the measurement.

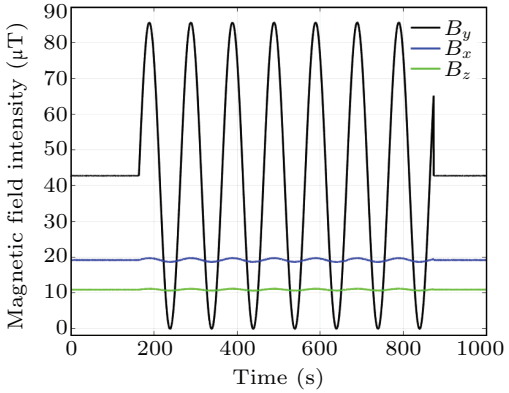


Fig. 3. Magnetic field intensity at the center of coils. The current is not injected in the duration 0–160 s and 880–1000 s, and the probe measures the background magnetic field. In the medial duration the current is injected, the probe measures the magnetic field produced by the coils.

In formal experiment, we injected a 60 mA sinusoidal electric current at $\omega_s = 1$ mHz into the Helmholtz coils. Ac-

cordingly, a uniform sinusoidal magnetic field B_y in total volume of the TM was produced. An optical detection system was installed to monitor the pendulum twist with about 7-hours duration, and the power spectral density (PSD) is shown in Fig. 4. The targeted signal $\theta_s(t)$ is remarkable in the PSD of the raw data. The signal-noise-ratio of the $\theta_s(t)$ is about 200, and the noise background level of raw data is higher than the read noise of the autocollimator and the theoretical thermal noise by about 3 times and two orders, respectively. Although the read noise could be a significant limit of the measurement, the measurement resolution of remanent magnetic moment can reach $\sim 2\times10^{-10}$ A·m^2 with 13.6-hour measurement duration and the magnetic field of $B_y \sim 250$ μT. Contrasting with the measurement requirements of TianQin, this performance of the instrument is satisfactory.

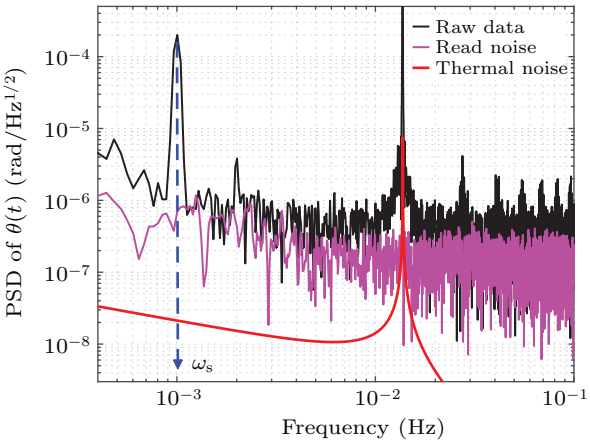


Fig. 4. The black line is the PSD of the monitor twist $\theta(t)$ of the torsion pendulum. The signal at 1 mHz is targeted $\theta_s(t)$, and the irrelevant signal at 2 mHz is caused by the temperature effect of coil, the imperfect of the uniformity of field or irrelevant induced magnetic moment potentially. The pink line is the PSD of the basic read noise of autocollimator with the same duration with $\theta(t)$. The red line is the theoretical thermal noise of the pendulum system. Spectra for those 13.6-h measurements are calculated with a Hanning window.

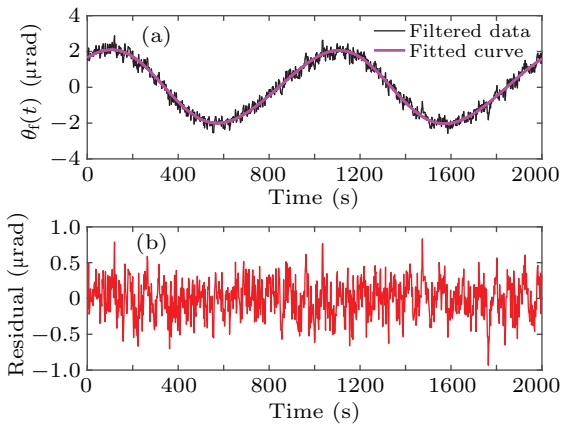


Fig. 5. (a) The fitting of $\theta_f(t)$ with Eq. (9). (b) The fitting residual error.

By using the traditional Fourier transform methods,[19,20] the natural frequency ω_0 and the damping factor β are estimated to (13.70 ± 0.04) mHz and $\beta = (3.93\pm0.03)\times10^{-8}$, respectively. The raw data $\theta(t)$ are filtered by Eq. (9). The filtered data $\theta_f(t)$ are cut into 23 segments with 2000 s duration. We use a nonlinear fitting method to fit the $\theta_f(t)$, and the results from the 1st segment are shown in Fig. 5. The estimated Θ_1 is 2.14 μrad with the uncertainty $\sigma_1 = 0.03$ μrad by this segment. In this way, we can acquire 23 arrays $(\hat{\Theta}_s, \sigma)$. Then, we take the arrays into Eq. (10), and the estimated

$$\Theta = (2.13\pm0.01)\ \mu\text{rad}. \tag{11}$$

According to Eq. (7), the remanent magnetic moment m_x is given by

$$m_x = (3.44\pm0.02)\times10^{-8}\ \text{A}\cdot\text{m}^2.$$

The systemic errors of remanent magnetic moment are fully evaluated in Table 1. The constant systematic relative error is about 0.9% mainly due to the imperfect uniformity of the fields.

Table 1. One σ uncertainty of systemic error budget (in units of 10^{-3}).

Error sources	$\Delta m_r/m_r$
Uniformity of B_y	9
Geomagnetic field	<0.1 (corrected)
Moment of inertia	<0.3
Fluctuation of applied fields	<0.1
Error of damping coefficient β	<0.1
Error of natural frequency ω_0	<0.05
Combined systemic relative error	9
Total	9

By mounting TM with different orientations relative to the coils, the other two components can also be measured based on this method. We measured the remanent magnetic moment components $m_y = (1.84\pm0.02)\times10^{-8}$ A·m^2 and $m_z = (5.13\pm0.03)\times10^{-8}$ A·m^2 with the same method and process. Then the composited module value of the remanent magnetic moment is compounded by

$$\begin{aligned} m_r &= \sqrt{m_x^2+m_y^2+m_z^2} \\ &= (6.45\pm0.04(\text{stat})\pm0.07(\text{syst}))\times10^{-8}\ \text{A}\cdot\text{m}^2 \end{aligned} \tag{12}$$

with measurement precision of m_r better than 1 nA·m^2, well below the measurement requirements of TianQin.

5. Summary

Remanent magnetic moment cleanliness of TM is strictly required for TianQin, and therefore precision test for the TM in the laboratory is needed prior to the launch of TianQin. In this paper, a torsion pendulum with single frequency modulation method is used to measure the remanent magnetic moment of the mimetic TM, and its precision could reach to 1 nA·m^2, well below the requirements of the TianQin mission. Therefore, this method is demonstrated to be a reliable way for estimating the remanent magnetic moment of TMs in other space detection missions.[21,22] Besides, we believe this precision

measuring method could also be applied in rock magnetism of geophysics[23,24] and metrology field.[25,26] Since the uniformity of magnetic filed is relatively uneven under present design and the distribution of remnant magnetic moments in TM volume has not been fully understood so far, there are still many works needed to be performed in this measurement.

Acknowledgments

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