All-optical switches based on three-soliton inelastic interaction and its application in optical communication systems

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In high-speed optical communication systems, in order to improve the communication rate, the distance between pulses must be compressed, which will cause the problem of the interaction between optical pulses in optical communication systems, which has been widely concerned by researches. In this paper, the bilinear method will be used to analyze the coupled high-order nonlinear Schrödinger equations and obtain their three-soliton solutions. Then, the influence of the relevant parameters in the three-soliton solution on the soliton inelastic interaction is studied. In addition, the constraint conditions of each parameter in the three-soliton solution are analyzed, the inelastic interaction properties of optical solitons under different parameter conditions are obtained, and the relevant laws of the inelastic interaction of solitons are studied. The results will have potential applications in the soliton control, all-optical switching and optical computing.

Keywords: optical solitons, solitons interaction, nonlinear Schrödinger equation, soliton solution

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1. Introduction

In the anomalous dispersion region of optical fibers, due to the interaction of dispersion and nonlinear effects, a very attractive phenomenon, soliton, can be produced.[1–6] It can achieve long-distance transmission without deformation and is used in many branches of physics. At present, the formation mechanism of solitons and their nonlinear dynamic characteristics such as propagation and interaction in nonlinear media have been extensively studied.[7–10]

On the other hand, in optical communication systems, information cannot be transmitted in the form of a single soliton, but must be transmitted in the form of soliton string. In this case, the interaction between solitons will also occur, which may affect the transmission bandwidth.[11,12] In order to improve the optical communication capacity, the distance between adjacent solitons is always reduced as much as possible. However, when the soliton spacing becomes smaller, interaction will occur, which will lead to soliton waveform distortion, transmission deterioration, and ultimately affect the capacity and communication quality of the optical communication system.[13,14] Therefore, with the improvement of the transmission rate of optical communication systems, in order to acquire the maximum transmission distance at the same bit rate, studying the soliton interaction is necessary.[15,16]

The interaction between solitons is often very complex, they are either attracted or repelled each other. When multiple solitons interact, some new phenomena may occur, such as fission, fusion and annihilation.[17,18] With the development of research, more and more characteristics of soliton interactions are revealed. When certain parameter conditions are met, elastic interactions will occur between optical solitons.[19,20] Otherwise, inelastic interaction may occur, and the interaction behavior is different. Different parameters have different behaviors.[21,22]

In this paper, we will study the interaction of optical solitons and find the interaction rules of inelastic interactions. The propagation of solitons in birefringent fibers can be described by the nonlinear Schrödinger (NLS) equation[2]

$$\begin{aligned}
&\mathrm{i}A_{1,t}+\beta(t)A_{1,xx}+2\beta(t)\left(\rho_1|A_1|^2+\rho_2|A_2|^2\right)A_1\\
&+\mathrm{i}\omega(t)\left[A_{1,xxx}+3\left(\rho_1|A_1|^2+\rho_2|A_2|^2\right)A_{1,x}\right.\\
&\left.+3\left(\rho_1A_1^*A_{1,x}+\rho_2A_2^*A_{2,x}\right)A_1\right]=0,\\
&\mathrm{i}A_{2,t}+\beta(t)A_{2,xx}+2\beta(t)\left(\rho_1|A_1|^2+\rho_2|A_2|^2\right)A_2\\
&+\mathrm{i}\omega(t)\left[A_{2,xxx}+3\left(\rho_1|A_1|^2+\rho_2|A_2|^2\right)A_{2,x}\right.\\
&\left.+3\left(\rho_1A_1^*A_{1,x}+\rho_2A_2^*A_{2,x}\right)A_2\right]=0.
\end{aligned}\tag{1}$$

For Eq. (1), the elastic interactions have been discussed.[2] However, the properties of all-optical switches based on three-soliton inelastic interaction have not been reported before.

In this paper, we will discuss the three-soliton inelastic interaction, and analyze the related characteristics of them in Section 2. The conclusion will be given in Section 3.

2. Three-soliton solutions and inelastic interaction discussion

Assuming that[15–17]

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$$A_1(x,t)=\frac{P(x,t)}{Q(x,t)},\quad A_2(x,t)=\frac{R(x,t)}{Q(x,t)}, \tag{2}$$

and substituting them into Eq. (1), we have

$$\begin{aligned}
&\left[\mathrm{i}D_t+\beta(t)D_x^2+\mathrm{i}\omega(t)D_x^3\right]P\cdot Q=0,\\
&\left[\mathrm{i}D_t+\beta(t)D_x^2+\mathrm{i}\omega(t)D_x^3\right]R\cdot Q=0,\\
&D_x^2(Q\cdot Q)-2\left[\rho_1|P|^2+\rho_2|R|^2\right]=0.
\end{aligned} \tag{3}$$

Equation (3) is the bilinear forms of Eq. (1). To get the three-soliton solutions, we assume $P(x,t)$, $R(x,t)$ and $Q(x,t)$ have the forms

$$\begin{aligned}
&P(x,t)=\varepsilon P_1(x,t)+\varepsilon^3P_3(x,t)+\varepsilon^5P_5(x,t),\\
&R(x,t)=\varepsilon R_1(x,t)+\varepsilon^3R_3(x,t)+\varepsilon^5R_5(x,t),\\
&Q(x,t)=1+\varepsilon^2Q_2(x,t)+\varepsilon^4Q_4(x,t)+\varepsilon^6Q_6(x,t).
\end{aligned} \tag{4}$$

Here,

$$\begin{aligned}
P_1(x,t) &= A_1\mathrm{e}^{\varphi_1}+A_2\mathrm{e}^{\varphi_2}+A_3\mathrm{e}^{\varphi_3},\\
R_1(x,t) &= B_1\mathrm{e}^{\varphi_1}+B_2\mathrm{e}^{\varphi_2}+B_3\mathrm{e}^{\varphi_3},\\
Q_2(x,t) &= C_1\mathrm{e}^{\varphi_1+\varphi_1^*}+C_2\mathrm{e}^{\varphi_1+\varphi_2^*}+C_3\mathrm{e}^{\varphi_1+\varphi_3^*}\\
&\quad+C_4\mathrm{e}^{\varphi_2+\varphi_1^*}+C_5\mathrm{e}^{\varphi_2+\varphi_2^*}+C_6\mathrm{e}^{\varphi_2+\varphi_3^*}\\
&\quad+C_7\mathrm{e}^{\varphi_3+\varphi_1^*}+C_8\mathrm{e}^{\varphi_3+\varphi_2^*}+C_9\mathrm{e}^{\varphi_3+\varphi_3^*},\\
P_3(x,t) &= D_1\mathrm{e}^{\varphi_1+\varphi_2+\varphi_1^*}+D_2\mathrm{e}^{\varphi_1+\varphi_2+\varphi_2^*}+D_3\mathrm{e}^{\varphi_1+\varphi_2+\varphi_3^*}\\
&\quad+D_4\mathrm{e}^{\varphi_1+\varphi_3+\varphi_1^*}+D_5\mathrm{e}^{\varphi_1+\varphi_3+\varphi_2^*}+D_6\mathrm{e}^{\varphi_1+\varphi_3+\varphi_3^*}\\
&\quad+D_7\mathrm{e}^{\varphi_2+\varphi_3+\varphi_1^*}+D_8\mathrm{e}^{\varphi_2+\varphi_3+\varphi_2^*}+D_9\mathrm{e}^{\varphi_2+\varphi_3+\varphi_3^*},\\
R_3(x,t) &= E_1\mathrm{e}^{\varphi_1+\varphi_2+\varphi_1^*}+E_2\mathrm{e}^{\varphi_1+\varphi_2+\varphi_2^*}+E_3\mathrm{e}^{\varphi_1+\varphi_2+\varphi_3^*}\\
&\quad+E_4\mathrm{e}^{\varphi_1+\varphi_3+\varphi_1^*}+E_5\mathrm{e}^{\varphi_1+\varphi_3+\varphi_2^*}+E_6\mathrm{e}^{\varphi_1+\varphi_3+\varphi_3^*}\\
&\quad+E_7\mathrm{e}^{\varphi_2+\varphi_3+\varphi_1^*}+E_8\mathrm{e}^{\varphi_2+\varphi_3+\varphi_2^*}+E_9\mathrm{e}^{\varphi_2+\varphi_3+\varphi_3^*},\\
Q_4(x,t) &= G_1\mathrm{e}^{\varphi_1+\varphi_1^*+\varphi_2+\varphi_2^*}+G_2\mathrm{e}^{\varphi_1+\varphi_1^*\varphi_2+\varphi_3^*}\\
&\quad+G_3\mathrm{e}^{\varphi_1+\varphi_2^*+\varphi_2+\varphi_3^*}+G_4\mathrm{e}^{\varphi_1+\varphi_1^*+\varphi_3+\varphi_2^*}\\
&\quad+G_5\mathrm{e}^{\varphi_1+\varphi_1^*+\varphi_3+\varphi_3^*}+G_6\mathrm{e}^{\varphi_1+\varphi_2^*+\varphi_3+\varphi_3^*}\\
&\quad+G_7\mathrm{e}^{\varphi_2+\varphi_1^*+\varphi_3+\varphi_2^*}+G_8\mathrm{e}^{\varphi_2+\varphi_1^*+\varphi_3+\varphi_3^*}\\
&\quad+G_9\mathrm{e}^{\varphi_2+\varphi_2^*+\varphi_3+\varphi_3^*},
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
P_5(x,t) &= H_1\mathrm{e}^{\varphi_1+\varphi_2+\varphi_3+\varphi_1^*+\varphi_2^*}+H_2\mathrm{e}^{\varphi_1+\varphi_2+\varphi_3+\varphi_1^*+\varphi_3^*}\\
&\quad+H_3\mathrm{e}^{\varphi_1+\varphi_2+\varphi_3+\varphi_2^*+\varphi_3^*},\\
R_5(x,t) &= J_1\mathrm{e}^{\varphi_1+\varphi_2+\varphi_3+\varphi_1^*+\varphi_2^*}+J_2\mathrm{e}^{\varphi_1+\varphi_2+\varphi_3+\varphi_1^*+\varphi_3^*}\\
&\quad+J_3\mathrm{e}^{\varphi_1+\varphi_2+\varphi_3+\varphi_2^*+\varphi_3^*},\\
Q_6(x,t) &= L\mathrm{e}^{\varphi_1+\varphi_2+\varphi_3+\varphi_1^*+\varphi_2^*+\varphi_3^*}.
\end{aligned} \tag{7}$$

Assume that

$$\begin{aligned}
\varphi_j &= \lambda_j x+\omega_j(t)+\xi_j=\lambda_{j1}x+\mathrm{i}\lambda_{j2}x+\omega_{j1}(t)\\
&\quad+\mathrm{i}\omega_{j2}(t)+\xi_{j1}+\mathrm{i}\xi_{j2},\quad(j=1,2,3).
\end{aligned} \tag{8}$$

Substituting Eqs. (5) and (11) into Eq. (3), we can get

$$\begin{aligned}
\omega_{j1}(t) &= \int\left[3\lambda_{j1}\lambda_{j2}^2\omega(t)-\lambda_{j1}^3\omega(t)-2\lambda_{j1}\lambda_{j2}\beta(t)\right]\mathrm{d}t,\\
\omega_{j2}(t) &= \int\left[\lambda_{j2}^3\omega(t)-3\lambda_{j1}^2\lambda_{j2}\omega(t)+\left(\lambda_{j1}^2-\lambda_{j2}^2\right)\beta(t)\right]\mathrm{d}t,
\end{aligned} \tag{9}$$

and

$$\begin{aligned}
&C_1=\frac{A_1A_1^*\rho_1+B_1B_1^*\rho_2}{4\lambda_{11}^2},\quad C_2=\frac{A_1A_2^*\rho_1+B_1B_2^*\rho_2}{\left(\lambda_1+\lambda_2^*\right)^2},\\
&C_3=\frac{A_1A_3^*\rho_1+B_1B_3^*\rho_2}{\left(\lambda_1+\lambda_3^*\right)^2},\\
&C_4=\frac{A_1^*A_2\rho_1+B_1^*B_2\rho_2}{\left(\lambda_1^*+\lambda_2\right)^2},\quad C_5=\frac{A_2A_2^*\rho_1+B_2B_2^*\rho_2}{4\lambda_{21}^2},\\
&C_6=\frac{A_2A_3^*\rho_1+B_2B_3^*\rho_2}{\left(\lambda_2+\lambda_3^*\right)^2},\\
&C_7=\frac{A_1^*A_3\rho_1+B_1^*B_3\rho_2}{\left(\lambda_1^*+\lambda_3\right)^2},\quad C_8=\frac{A_2^*A_3\rho_1+B_2^*B_3\rho_2}{\left(\lambda_2^*+\lambda_3\right)^2},\\
&C_9=\frac{A_3A_3^*\rho_1+B_3B_3^*\rho_2}{4\lambda_{31}^2}.
\end{aligned} \tag{10}$$

For $P_3(x,t)$, we can get its coefficients as

$$\begin{aligned}
D_1&=\frac{S_{12}^{(1)}\left[3(\mathrm{i}A_1C_4\lambda_{12}S_{12}^{(2)}+A_2C_1\lambda_{11}S_{12}^{(3)})\omega(t)+(2A_2C_1\lambda_{11}-A_1C_4S_{12}^{(2)})\mathrm{i}\beta(t)\right]}{\lambda_{11}S_{12}^{(2)}\left[3S_{12}^{(4)}\omega(t)-2\mathrm{i}\beta(t)\right]},\\
D_2&=\frac{S_{12}^{(1)}\left[3(A_1C_5\lambda_{21}S_{12}^{(6)}-\mathrm{i}A_2C_2\lambda_{22}S_{12}^{(5)})\omega(t)+(A_2C_2S_{12}^{(5)}-2A_1C_5\lambda_{21})\mathrm{i}\beta(t)\right]}{\lambda_{21}S_{12}^{(5)}\left[3S_{12}^{(4)}\omega(t)-2\mathrm{i}\beta(t)\right]},\\
D_3&=\frac{S_{12}^{(1)}\left[3\mathrm{i}(A_1C_6S_{23}^{(5)}S_{13}^{(6)}-A_2C_3S_{13}^{(5)}S_{23}^{(6)})\omega(t)+(A_1C_6S_{23}^{(5)}-2A_2C_3s_{13}^{(5)})\mathrm{i}\beta(t)\right]}{\mathrm{i}S_{23}^{(5)}S_{13}^{(5)}\left[3S_{12}^{(4)}\omega(t)-2\mathrm{i}\beta(t)\right]},\\
D_4&=\frac{S_{13}^{(1)}\left[3(\mathrm{i}A_1C_7\lambda_{12}S_{13}^{(2)}+A_3C_1\lambda_{11}S_{13}^{(3)})\omega(t)+(2A_3C_1\lambda_{11}-A_1C_7S_{13}^{(2)})\mathrm{i}\beta(t)\right]}{\lambda_{11}S_{13}^{(2)}\left[3S_{13}^{(4)}\omega(t)-2\mathrm{i}\beta(t)\right]},
\end{aligned} \tag{11}$$

$$
\begin{aligned}
D_5 &= \frac{S_{13}^{(3)}\left[3\mathrm{i}(A_1C_8S_{12}^{(6)}S_{23}^{(2)}+A_3C_2S_{12}^{(5)}S_{23}^{(3)})\omega(t)+2(A_1C_8S_{23}^{(2)}-A_3C_2S_{12}^{(5)})\beta(t)\right]}{S_{12}^{(5)}S_{23}^{(2)}\left[3\mathrm{i}S_{13}^{(4)}\omega(t)+2\beta(t)\right]},\\
D_6 &= \frac{S_{13}^{(1)}\left[3(A_1C_9\lambda_{31}S_{13}^{(6)}-\mathrm{i}A_3C_3\lambda_{32}S_{13}^{(5)})\omega(t)+(A_3C_3S_{13}^{(5)}-2A_1C_9\lambda_{31})\mathrm{i}\beta(t)\right]}{\lambda_{31}S_{13}^{(5)}\left[3S_{13}^{(4)}\omega(t)-2\mathrm{i}\beta(t)\right]},\\
D_7 &= \frac{S_{23}^{(1)}\left[3\mathrm{i}(A_3C_4S_{12}^{(2)}S_{13}^{(3)}-A_2C_7S_{12}^{(3)}S_{13}^{(2)})\omega(t)+2(A_2C_7S_{13}^{(2)}-A_3C_4S_{12}^{(2)})\beta(t)\right]}{S_{12}^{(2)}S_{13}^{(2)}\left[3\mathrm{i}S_{23}^{(4)}\omega(t)+2\beta(t)\right]},\\
D_8 &= \frac{S_{23}^{(1)}\left[3(\mathrm{i}A_2C_8\lambda_{22}S_{23}^{(2)}+A_3C_5\lambda_{21}S_{23}^{(3)})\omega(t)+(2A_3C_5\lambda_{21}-A_2C_8S_{23}^{(2)})\mathrm{i}\beta(t)\right]}{\lambda_{21}S_{23}^{(2)}\left[3S_{23}^{(4)}\omega(t)-2\mathrm{i}\beta(t)\right]},\\
D_9 &= \frac{S_{23}^{(1)}\left[3(A_2C_9\lambda_{31}S_{23}^{(6)}-\mathrm{i}A_3C_6\lambda_{32}S_{23}^{(5)})\omega(t)+(A_3C_6S_{23}^{(5)}-2A_2C_9\lambda_{31})\mathrm{i}\beta(t)\right]}{\lambda_{31}S_{23}^{(5)}\left[3S_{23}^{(4)}\omega(t)-2\mathrm{i}\beta(t)\right]}.
\end{aligned}
\tag{12}
$$

For $R_3(x,t)$, we can get the coefficients E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8 and E_9 when A_1, A_2 and A_3 are replaced by B_1, B_2 and B_3. For $Q_4(x,t)$, its coefficients can be obtained as

$$
\begin{aligned}
G_1 &= \frac{-4C_1C_5(\lambda_{11}-\lambda_{21})^2+4C_2C_4(\lambda_{12}-\lambda_{22})^2+(A_2^*D_1+A_1^*D_2)\rho_1+(B_2^*E_1+B_2E_1^*+B_1^*E_2+B_1E_2^*)\rho_2}{4(\lambda_{11}+\lambda_{21})^2},\\
G_2 &= \frac{-C_1C_6(S_{23}^{(5)}-2\lambda_{11})^2+C_3C_4(-2\lambda_{12}-\mathrm{i}S_{23}^{(6)})^2+(A_3^*D_1+A_1^*D_3)\rho_1+(B_3^*E_1+B_1^*E_3+B_2E_4^*+B_1E_5^*)\rho_2}{(2\lambda_{11}+S_{23}^{(5)})^2},\\
G_3 &= \frac{-C_3C_5(S_{13}^{(5)}-2\lambda_{21})^2+C_2C_6(-2\mathrm{i}\lambda_{22}+S_{13}^{(6)})^2+(A_3^*D_2+A_2^*D_3)\rho_1+(B_3^*E_2+B_2^*E_3+B_2E_7^*+B_1E_8^*)\rho_2}{(2\lambda_{21}+S_{13}^{(5)})^2},\\
G_4 &= \frac{-C_1C_8(S_{23}^{(2)}-2\lambda_{11})^2+C_2C_7(-2\lambda_{12}+\mathrm{i}S_{23}^{(3)})^2+(A_2^*D_4+A_1^*D_5)\rho_1+(B_3E_1^*+B_1E_3^*+B_2^*E_4+B_1^*E_5)\rho_2}{(2\lambda_{11}+S_{23}^{(2)})^2},\\
G_5 &= \frac{-4C_1C_9(\lambda_{11}-\lambda_{31})^2+4C_3C_7(\lambda_{12}-\lambda_{32})^2+(A_3^*D_4+A_1^*D_6)\rho_1+(B_3^*E_4+B_3E_4^*+B_1^*E_6+B_1E_6^*)\rho_2}{4(\lambda_{11}+\lambda_{31})^2},\\
G_6 &= \frac{-C_2C_9(S_{12}^{(5)}-2\lambda_{31})^2+C_3C_8(-2\mathrm{i}\lambda_{32}+S_{12}^{(6)})^2+(A_3^*D_5+A_2^*D_6)\rho_1+(B_3^*E_5+B_2^*E_6+B_3E_7^*+B_1E_9^*)\rho_2}{(2\lambda_{31}+S_{12}^{(5)})^2},\\
G_7 &= \frac{-C_5C_7(S_{13}^{(2)}-2\lambda_{21})^2-C_4C_8(2\mathrm{i}\lambda_{22}+S_{13}^{(3)})^2+(A_2^*D_7+A_1^*D_8)\rho_1+(B_3E_2^*+B_2E_3^*+B_2^*E_7+B_1^*E_8)\rho_2}{(2\lambda_{21}+S_{13}^{(2)})^2},\\
G_8 &= \frac{-C_4C_9(S_{12}^{(2)}-2\lambda_{31})^2-C_6C_7(2\mathrm{i}\lambda_{32}+S_{12}^{(3)})^2+(A_3^*D_7+A_1^*D_9)\rho_1+(B_3E_5^*+B_2E_6^*+B_3^*E_7+B_1^*E_9)\rho_2}{(2\lambda_{31}+S_{12}^{(2)})^2},\\
G_9 &= \frac{-4C_5C_9(\lambda_{21}-\lambda_{31})^2+4C_6C_8(\lambda_{22}-\lambda_{32})^2+(A_3^*D_8+A_2^*D_9)\rho_1+(B_3^*E_8+B_3E_8^*+B_2^*E_9+B_2E_9^*)\rho_2}{4(\lambda_{21}+\lambda_{31})^2}.
\end{aligned}
\tag{13}
$$

Using Eqs. (9)–(13), we can get

$$
\begin{aligned}
&H_1=\frac{-3R_1\omega(t)-\mathrm{i}R_2\beta(t)}{3T_1\omega t+2T_2\beta(t)},\quad H_2=\frac{-3R_3\omega(t)-\mathrm{i}R_4\beta(t)}{3T_3\omega t-2\mathrm{i}T_4\beta(t)},\quad H_3=\frac{3R_5\omega(t)+\mathrm{i}R_6\beta(t)}{3T_5\omega t+2\mathrm{i}T_6\beta(t)},\quad J_1=\frac{-3R_1'\omega(t)-\mathrm{i}R_2'\beta(t)}{3T_1'\omega t+2T_2'\beta(t)},\\
&J_2=\frac{-3R_3'\omega(t)-\mathrm{i}R_4'\beta(t)}{3T_3'\omega t-2\mathrm{i}T_4'\beta(t)},\quad J_3=\frac{3R_5'\omega(t)+\mathrm{i}R_6'\beta(t)}{3T_5'\omega t+2\mathrm{i}T_6'\beta(t)},\quad L=\frac{4L_1+(A_3^*H_1+A_2^*H_3+A_1^*H_3)\rho_1+L_2\rho_2}{4(\lambda_{11}+\lambda_{21}+\lambda_{31})^2}
\end{aligned}
\tag{14}
$$

with

$$
\begin{aligned}
&S_{ij}^{(1)}=\lambda_{i1}-\lambda_{ji}+\mathrm{i}(\lambda_{i2}-\lambda_{j2}),\ S_{ij}^{(2)}=\lambda_{i1}+\lambda_{ji}+\mathrm{i}(-\lambda_{i2}+\lambda_{j2}),\ S_{ij}^{(3)}=\lambda_{i1}-\lambda_{ji}+\mathrm{i}(-\lambda_{i2}-\lambda_{j2}),\\
&S_{ij}^{(4)}=\lambda_{i1}+\lambda_{ji}+\mathrm{i}(\lambda_{i2}+\lambda_{j2}),\ S_{ij}^{(5)}=\lambda_{i1}+\lambda_{ji}+\mathrm{i}(\lambda_{i2}-\lambda_{j2}),\ S_{ij}^{(6)}=\lambda_{i1}-\lambda_{ji}+\mathrm{i}(\lambda_{i2}+\lambda_{j2}),\quad (i=1,2,3)
\end{aligned}
\tag{15}
$$

and

$$
R_1 = U_{11}C_8D_1+U_{12}C_1D_8+U_{13}C_5D_4+U_{14}C_4D_5+U_{15}C_2D_7+U_{16}C_7D_2+U_{17}A_3G_1+U_{18}A_2G_4+U_{19}A_1G_7,
$$

$$
\begin{aligned}
R_2 &= U_{21}C_8D_1+U_{22}C_1D_8+U_{23}C_5D_4+U_{24}C_4D_5+U_{25}C_2D_7+U_{26}C_7D_2+U_{27}A_3G_1+U_{28}A_2G_4+U_{29}A_1G_7,\\
R_3 &= U_{31}C_9D_1+U_{32}C_1D_9+U_{33}C_6D_4+U_{34}C_4D_6+U_{35}C_3D_7+U_{36}C_7D_3+U_{37}A_3G_2+U_{38}A_2G_5+U_{39}A_1G_8,\\
R_4 &= U_{41}C_9D_1+U_{42}C_1D_9+U_{43}C_6D_4+U_{44}C_4D_6+U_{45}C_3D_7+U_{46}C_7D_3+U_{47}A_3G_2+U_{48}A_2G_5+U_{49}A_1G_8,\\
R_5 &= U_{51}C_9D_2+U_{52}C_2D_9+U_{53}C_6D_5+U_{54}C_5D_6+U_{55}C_3D_8+U_{56}C_8D_3+U_{57}A_3G_3+U_{58}A_2G_6+U_{59}A_1G_9,\\
R_6 &= U_{61}C_9D_2+U_{62}C_2D_9+U_{63}C_6D_5+U_{64}C_5D_6+U_{65}C_3D_8+U_{66}C_8D_3+U_{67}A_3G_3+U_{68}A_2G_6+U_{69}A_1G_9,\\
L_1 &= U_{71}C_1G_9+U_{72}C_2G_8+U_{73}C_3G_7+U_{74}C_4G_6+U_{75}C_5G_5+U_{76}C_6G_4+U_{77}C_7G_3+U_{78}C_8G_2+U_{79}C_9G_1,\\
L_2 &= E_3E_3^*+E_5E_5^*+E_6E_2^*+E_2E_6^*+E_7E_7^*+E_8E_4^*+E_4E_8^*+E_9E_1^*+E_1E_9^*+J_3B_3^*+J_2B_2^*+J_3B_1^*,\\
U_{11} &= \lambda_{11}^3-\mathrm{i}\lambda_{22}(\lambda_{21}^2+(\lambda_{22}+\mathrm{i}\lambda_3)^2)+2\mathrm{i}\lambda_{11}^2(2\lambda_{22}+\mathrm{i}\lambda_3)+\lambda_{11}(\lambda_{12}^2+(2\lambda_{22}+\mathrm{i}\lambda_3)^2),\\
U_{12} &= \lambda_{21}(\lambda_{22}^2+(\lambda_{21}+\lambda_3)^2)-\lambda_{11}^3+\lambda_{11}(\lambda_{12}^2+(2\lambda_{21}+\lambda_3)^2)+2\lambda_{11}^2(2\lambda_{21}+\lambda_3),\\
U_{13} &= \lambda_{11}^3+\lambda_{11}(\lambda_{12}^2+(\lambda_3-2\lambda_{21})^2)+\lambda_{21}(\lambda_{22}^2+(\lambda_3-\lambda_{21})^2)+2\lambda_{11}^2(\lambda_3-2\lambda_{21}),\\
U_{14} &= -\mathrm{i}(\lambda_{11}^2\lambda_{12}+\lambda_{12}^3-\lambda_{22}(\lambda_{21}^2+(\lambda_{22}+\mathrm{i}\lambda_3)^2)-2\lambda_{12}^2(2\lambda_{22}+\mathrm{i}\lambda_3)+\lambda_{12}(2\lambda_{22}+\mathrm{i}\lambda_3)^2),\\
U_{15} &= \mathrm{i}(\lambda_{11}^2\lambda_{12}+\lambda_{12}^3-2\lambda_{12}(2\lambda_{22}-\mathrm{i}\lambda_3)+\lambda_{12}(2\lambda_{22}-\mathrm{i}\lambda_3)^2-\lambda_{22}(\lambda_{21}^2+(\lambda_{22}+\mathrm{i}\lambda_3)^2)),\\
U_{16} &= \lambda_{21}(\lambda_{22}^2+(\lambda_3-\lambda_{21})^2)+2\lambda_{12}^2(\lambda_3-2\lambda_{21})+\mathrm{i}\lambda_{12}(\lambda_3-2\lambda_{21})^2-\mathrm{i}\lambda_{11}^2\lambda_{12}-\mathrm{i}\lambda_{12}^3,\\
U_{17} &= \lambda_{21}(\lambda_{22}^2+(\lambda_3-\lambda_{21})^2)+2\lambda_{11}^2(\lambda_3-2\lambda_{21})-\lambda_{11}^3-\lambda_{11}(\lambda_{12}^2+(\lambda_3-2\lambda_{21})^2),\\
U_{18} &= 2\mathrm{i}\lambda_{11}^2(2\lambda_{22}+\mathrm{i}\lambda_3)-\lambda_{11}^3-\mathrm{i}\lambda_{22}(\lambda_{21}^2+(\lambda_{22}+\mathrm{i}\lambda_3)^2)-\lambda_{11}(\lambda_{12}^2-(2\lambda_{22}+\mathrm{i}\lambda_3)^2),\\
U_{19} &= 2\lambda_{12}^2(2\lambda_{21}+\lambda_3)+\mathrm{i}\lambda_{12}(2\lambda_{21}+\lambda_3)^2-\mathrm{i}\lambda_{11}^2\lambda_{12}-\mathrm{i}\lambda_{12}^3-\lambda_{21}(\lambda_{22}^2+(\lambda_{21}+\lambda_3)^2),\\
U_{21} &= \lambda_{21}^2+(\lambda_{22}+\mathrm{i}\lambda_3)^2-2\lambda_{11}(\lambda_1^*+2\mathrm{i}\lambda_{22}-\lambda_3),\quad U_{22}=2(\lambda_{11}(2\lambda_{21}+\lambda_3)-\lambda_1\lambda_{11}-\lambda_{21}(\lambda_2^*+\lambda_3)),\\
U_{23} &= 2(\lambda_{21}(\lambda_3-\lambda_2)-\lambda_{11}(\lambda_1^*+\lambda_3-2\lambda_{21})),\ U_{24}=\lambda_{11}^2-\lambda_{21}^2+(\lambda_{12}-\lambda_{22})(\lambda_{12}-3\lambda_{22}-2\mathrm{i}\lambda_3),\\
U_{25} &= \lambda_{21}^2-\lambda_{11}^2+(\lambda_{12}-\lambda_{22})(3\lambda_{12}-\lambda_{22}+2\mathrm{i}\lambda_3),\ U_{26}=\lambda_{11}^2+\lambda_{12}^2+2\mathrm{i}\lambda_{12}(\lambda_3-2\mathrm{i}\lambda_{21})-\lambda_3^2-2\lambda_{21}(\lambda_{21}^*-\lambda_3),\\
U_{27} &= 2\lambda_3(\lambda_{11}+\lambda_{21})-2(\lambda_1\lambda_{11}+2\lambda_{11}\lambda_{21}+\lambda_2\lambda_{21}),\ U_{28}=\lambda_{21}^2+(\lambda_{22}+\mathrm{i}\lambda_3)^2-2\lambda_{11}(\lambda_1-2\mathrm{i}\lambda_{22}+\lambda_3),\\
U_{29} &= \lambda_{11}^2+\lambda_{12}^2+2\mathrm{i}\lambda_{12}(2\lambda_{21}+\lambda_3)-\lambda_3^2-2\lambda_{21}(\lambda_2+\lambda_3),\\
U_{31} &= \lambda_{11}^3+\lambda_{11}(\lambda_{12}^2+(\lambda_2-2\lambda_{31})^2)+2\lambda_{11}^2(\lambda_2-2\lambda_{31})-\lambda_{31}(\lambda_{32}^2-(\lambda_2-2\lambda_{31})^2),\\
U_{32} &= \lambda_{31}(\lambda_{32}^2+(\lambda_2+\lambda_{31})^2)+2\lambda_{11}^2(\lambda_2+2\lambda_{31})-\lambda_{11}^3-\lambda_{11}(\lambda_{12}^2+(\lambda_2+2\lambda_{31})^2),\\
U_{33} &= \lambda_{11}^3+\lambda_{11}(\lambda_{12}^2+(\lambda_2-2\mathrm{i}\lambda_{32})^2)-\mathrm{i}\lambda_{32}(\lambda_{31}^2-(\lambda_2-\mathrm{i}\lambda_{32})^2)-2\lambda_{11}^2(\lambda_2-2\mathrm{i}\lambda_{32}),\\
U_{34} &= \lambda_{31}(\lambda_{32}^2+(\lambda_2-\lambda_{31})^2)+2\lambda_{12}^2(\lambda_2-2\lambda_{31})+\mathrm{i}\lambda_{12}(\lambda_2-2\lambda_{31})^2-\mathrm{i}\lambda_{11}^2\lambda_{12}-\mathrm{i}\lambda_{12}^3,\\
U_{35} &= \mathrm{i}(\lambda_{11}^2\lambda_{12}+\lambda_{12}^3-\lambda_{12}^2(2\mathrm{i}\lambda_2-4\lambda_{32})-\lambda_{12}(\lambda_2+2\mathrm{i}\lambda_{32})^2-\lambda_{32}(\lambda_{31}^2-(\lambda_2+\mathrm{i}\lambda_{32}))^2)),\\
U_{37} &= -\lambda_{11}^3-\lambda_{11}(\lambda_{12}^2+(\lambda_2-2\mathrm{i}\lambda_{32})^2)-\mathrm{i}\lambda_{32}(\lambda_{31}^2-(\lambda_{21}+\mathrm{i}(\lambda_{31}-\lambda_{32}))^2)-2\lambda_{11}^2(\lambda_{21}+\mathrm{i}(\lambda_{31}-2\lambda_{32})),\\
U_{38} &= 2\lambda_{11}^2(\lambda_2-2\lambda_{31})-\lambda_{31}(\lambda_{32}^2+(\lambda_2-\lambda_{31})^2)-\lambda_{11}^3-\lambda_{11}(\lambda_{12}^2+(\lambda_2-2\lambda_{31})^2),\\
U_{39} &= 2\lambda_{12}^2(\lambda_2+2\lambda_{31})+\mathrm{i}\lambda_{12}(\lambda_2+2\lambda_{31})^2-\mathrm{i}\lambda_{11}^2\lambda_{12}-\mathrm{i}\lambda_{12}^3-\lambda_{31}(\lambda_{32}^2+(\lambda_2+\lambda_{31})^2),\\
U_{41} &= -2(\lambda_{11}(\lambda_1^*+\lambda_2-2\lambda_{31})+\lambda_{31}(\lambda_3-\lambda_2)),\ U_{43}=2\lambda_{11}(\lambda_2-\lambda_1^*)-\lambda_2^2+\lambda_{31}^2+\lambda_{32}(\lambda_{32}-4\mathrm{i}\lambda_{11}+2\mathrm{i}\lambda_2),\\
U_{42} &= 2\lambda_{11}(\lambda_2-\lambda_1)+2\lambda_{31}(2\lambda_{11}-\lambda_2-\lambda_3^*),\ U_{44}=\lambda_{11}^2+2\mathrm{i}(\lambda_{12}\lambda_2-\lambda_{21}\lambda_{22})+\lambda_{22}^2+2\lambda_2-2\lambda_{31}(\lambda_3^*+2\mathrm{i}\lambda_{12}),\\
U_{45} &= 3\lambda_{21}^2-\lambda_{11}^2+2\mathrm{i}\lambda_{12}\lambda_2+\lambda_{31}^2+2\mathrm{i}\lambda_{32}(2\mathrm{i}\lambda_{12}-\lambda_2)+\lambda_{32}^2,\ U_{46}=\lambda_{11}^2-\lambda_{31}^2+(\lambda_{12}-\lambda_{32})(\lambda_{12}-2\mathrm{i}\lambda_2-3\lambda_{32}),\\
U_{47} &= \lambda_{31}^2-2\mathrm{i}\lambda_{32}(2\lambda_{11}+\lambda_2)+\lambda_{32}^2-2\lambda_{11}(\lambda_1+\lambda_2)-\lambda_2^{*2},\ U_{48}=2\lambda_{11}(\lambda_2-\lambda_1-2\lambda_{31})+2\lambda_{31}(\lambda_2-\lambda_3),\\
U_{49} &= \lambda_{11}^2+2\mathrm{i}\lambda_{12}\lambda_2-2\mathrm{i}\lambda_{21}\lambda_{22}+\lambda_{22}^2+4\mathrm{i}\lambda_{12}\lambda_{31}-2\lambda_{31}(\lambda_2+\lambda_3),\\
U_{51} &= -\lambda_{21}^3-\lambda_{21}\lambda_{22}^2+\lambda_{12}^2(\lambda_{21}-\lambda_{31})-2\mathrm{i}\lambda_{12}(\lambda_{21}-\lambda_{31})^2-2\lambda_{11}(\lambda_{21}-\lambda_{31})(\mathrm{i}\lambda_{12}+\lambda_{21}-\lambda_{31})\\
&\quad+4\lambda_{21}^2\lambda_{31}-4\lambda_{21}\lambda_{31}^2+\lambda_{31}^3+\lambda_{11}^2(-\lambda_{21}+\lambda_{31})+\lambda_{31}\lambda_{32}^2,\\
U_{52} &= \lambda_{11}^2(-\mathrm{i}\lambda_{22}-\lambda_{31})+2\lambda_{11}(\lambda_{22}-\mathrm{i}\lambda_{31})(\lambda_{12}-\lambda_{22}+\mathrm{i}\lambda_{31})+\mathrm{i}(\lambda_{21}^2\lambda_{22}+\lambda_{22}^3+\lambda_{12}^2(\lambda_{22}-\mathrm{i}\lambda_{31})\\
&\quad-2\lambda_{12}(\lambda_{22}-\mathrm{i}\lambda_{31})^2-4\mathrm{i}\lambda_{22}^2\lambda_{31}-4\lambda_{22}\lambda_{31}^2+\mathrm{i}\lambda_{31}^3+\mathrm{i}\lambda_{31}\lambda_{32}^2),\\
U_{53} &= \mathrm{i}\lambda_{11}^2(\lambda_{22}-\lambda_{32})-2\lambda_{11}(\lambda_{22}-\lambda_{32})(\lambda_{12}-\lambda_{22}+\lambda_{32})-\mathrm{i}((\lambda_{21}^2+(\lambda_{12}-\lambda_{22})^2)\lambda_{22}\\
&\quad-((\lambda_{12}-2\lambda_{22})^2+\lambda_{31}^2)\lambda_{32}-2(\lambda_{12}-2\lambda_{22})\lambda_{32}^2-\lambda_{32}^3),\\
U_{54} &= \lambda_{21}^3+\lambda_{21}\lambda_{22}^2+\lambda_{11}^2(\lambda_{21}-\lambda_{31})-2\mathrm{i}\lambda_{12}(\lambda_{21}-\lambda_{31})^2+2\lambda_{11}(\lambda_{21}-\lambda_{31})(\mathrm{i}\lambda_{12}-\lambda_{21}+\lambda_{31})\\
&\quad-4\lambda_{21}^2\lambda_{31}+4\lambda_{21}\lambda_{31}^2-\lambda_{31}^3+\lambda_{12}^2(-\lambda_{21}+\lambda_{31})-\lambda_{31}\lambda_{32}^2,\\
U_{55} &= \lambda_{21}((\lambda_{12}+\mathrm{i}\lambda_{21})^2-\lambda_{22}^2)-\lambda_{11}^2(\lambda_{21}+\mathrm{i}\lambda_{32})+2\lambda_{11}(\lambda_{21}+\mathrm{i}\lambda_{32})(-\mathrm{i}\lambda_{12}+\lambda_{21}+\mathrm{i}\lambda_{32})\\
&\quad+\mathrm{i}((\lambda_{12}+2\mathrm{i}\lambda_{21})^2+\lambda_{31}^2)\lambda_{32}+2(-\mathrm{i}\lambda_{12}+2\lambda_{21})\lambda_{32}^2+\mathrm{i}\lambda_{32}^3,
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
U_{56} &= -\mathrm{i}\lambda_{11}^2(\lambda_{22}-\lambda_{32})+2\lambda_{11}(\lambda_{22}-\lambda_{32})(\lambda_{12}+\lambda_{22}-\lambda_{32})+\mathrm{i}((\lambda_{21}^2+(\lambda_{12}-\lambda_{22})^2)\lambda_{22}\\
&\quad -((\lambda_{12}-2\lambda_{22})^2+\lambda_{31}^2)\lambda_{32}+2(\lambda_{12}+2\lambda_{22})\lambda_{32}^2-\lambda_{32}^3),\\
U_{57} &= \lambda_{21}^3+\lambda_{21}\lambda_{22}^2+\lambda_{11}^2(\lambda_{21}-\mathrm{i}\lambda_{32})-2\mathrm{i}\lambda_{12}(\lambda_{21}-\lambda_{32})^2+2\lambda_{11}(\lambda_{21}-\mathrm{i}\lambda_{32})(\mathrm{i}\lambda_{12}+\lambda_{21}-\mathrm{i}\lambda_{31})\\
&\quad -4\mathrm{i}\lambda_{21}^2\lambda_{31}-4\lambda_{21}\lambda_{32}^2+\mathrm{i}\lambda_{32}^3-\lambda_{12}^2(\lambda_{21}-\mathrm{i}\lambda_{32})-\mathrm{i}\lambda_{31}^2\lambda_{32},\\
U_{58} &= \mathrm{i}\lambda_{22}^3+\mathrm{i}\lambda_{21}^2\lambda_{22}+\mathrm{i}\lambda_{12}^2(\lambda_{22}+\mathrm{i}\lambda_{31})+2\lambda_{11}(\lambda_{22}+\mathrm{i}\lambda_{31})(\lambda_{12}-\lambda_{22}-\mathrm{i}\lambda_{31})-2\mathrm{i}\lambda_{12}(\lambda_{22}+\mathrm{i}\lambda_{31})^2\\
&\quad -4\lambda_{22}^2\lambda_{31}-4\mathrm{i}\lambda_{22}\lambda_{31}^2+\lambda_{31}^3+\lambda_{11}^2(-\mathrm{i}\lambda_{22}+\lambda_{31})+\lambda_{31}\lambda_{32}^2,\\
U_{59} &= \lambda_{21}^3+\lambda_{21}\lambda_{22}^2+\lambda_{11}^2(\lambda_{21}+\lambda_{31})-2\mathrm{i}\lambda_{12}(\lambda_{21}+\lambda_{31})^2-2\lambda_{11}(\lambda_{21}+\lambda_{31})(-\mathrm{i}\lambda_{12}+\lambda_{21}+\lambda_{31})\\
&\quad +4\lambda_{21}^2\lambda_{31}+4\lambda_{21}\lambda_{31}^2+\lambda_{31}^3-\lambda_{12}^2(\lambda_{21}+\lambda_{31})+\lambda_{31}\lambda_{32}^2,\\
U_{61} &= 2(\lambda_{21}\lambda_2^*+\lambda_1\lambda_{21}-\lambda_{31}\lambda_1-2\lambda_{21}\lambda_{31}+\lambda_{31}\lambda_3),\ U_{62}=\lambda_1^2-\lambda_2\lambda_2^*-2\mathrm{i}\lambda_1(\lambda_{22}-\mathrm{i}\lambda_{31})+4\mathrm{i}\lambda_{22}\lambda_{31}+2\lambda_{31}\lambda_3^*,\\
U_{63} &= \lambda_{21}^2-\lambda_{31}^2+(\lambda_{22}-\lambda_{32})(\lambda_{32}-2\mathrm{i}\lambda_1-3\lambda_{22}),\ U_{64}=2(\lambda_{21}\lambda_2-\lambda_1(\lambda_{21}-\lambda_{31})-2\lambda_{21}\lambda_{31}+\lambda_{31}\lambda_3^*),\\
U_{65} &= \lambda_1^2+2\lambda_{21}(\lambda_2^*+2\mathrm{i}\lambda_{32})-\lambda_3\lambda_3^*-2\lambda_1(\lambda_{21}+\mathrm{i}\lambda_{32}),\ U_{66}=\lambda_{31}^2-\lambda_{21}^2-(\lambda_{22}-\lambda_{32})(\lambda_{22}-3\lambda_{32}-2\mathrm{i}\lambda_1),\\
U_{67} &= \lambda_1^2+2\lambda_{21}(\lambda_2-2\mathrm{i}\lambda_{32})-\lambda_3\lambda_3^*+2\lambda_1(\lambda_{21}-\mathrm{i}\lambda_{32}),\ U_{69}=2(\lambda_2\lambda_{21}+\lambda_{11}^*(\lambda_{21}+\lambda_{31})+2\lambda_{21}\lambda_{31}+\lambda_3\lambda_{31}),\\
U_{68} &= \lambda_1^2-\lambda_2\lambda_2^*-2\mathrm{i}\lambda_1(\lambda_{22}+\mathrm{i}\lambda_{31})+2\lambda_{31}(\lambda_3-2\mathrm{i}\lambda_{22}),
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
U_{71} &= -(\lambda_{11}-\lambda_{21}-\lambda_{31})^2,\ U_{72}=(\lambda_{12}-\lambda_{22}+\mathrm{i}\lambda_{31})^2,\ U_{73}=(\lambda_{12}+\mathrm{i}\lambda_{21}-\lambda_{32})^2,\\
U_{74} &= (\lambda_{12}-\lambda_{22}-\mathrm{i}\lambda_{31})^2,\ U_{75}=-(\lambda_{11}-\lambda_{21}+\lambda_{31})^2,\ U_{76}=-(\lambda_{11}-\mathrm{i}(\lambda_{22}-\lambda_{32}))^2,\\
U_{77} &= (\lambda_{12}-\mathrm{i}\lambda_{21}-\lambda_{32})^2,\ U_{78}=-4(\lambda_{11}+\mathrm{i}(\lambda_{22}-\lambda_{32}))^2,\ U_{79}=-(\lambda_{11}+\lambda_{21}-\lambda_{31})^2,\\
T_1 &= \lambda_{21}(\lambda_{22}^2+(\lambda_{21}+\lambda_3)^2)+\lambda_{11}(\lambda_1\lambda_1^*+(2\lambda_{21}+\lambda_3)^2)+2\lambda_{11}^2(2\lambda_{21}+\lambda_3),\\
T_2 &= -\mathrm{i}(\lambda_{11}\lambda_3+\lambda_{21}\lambda_3+\lambda_{11}\lambda_1^*+2\lambda_{11}\lambda_{21}+\lambda_{21}\lambda_2^*),\ T_4=\lambda_{11}(\lambda_1^*+\lambda_2+2\lambda_{31})+\lambda_{31}(\lambda_2+\lambda_3^*),\\
T_3 &= \lambda_{11}^3+2\lambda_{11}^2(\lambda_2+2\lambda_{31})+\lambda_{11}(\lambda_{12}^2+(\lambda_2+2\lambda_{31})^2)+\lambda_{31}((\lambda_2+\lambda_{31})^2+\lambda_{32}^2),\\
T_5 &= \lambda_{21}\lambda_2\lambda_2^*+(\lambda_1^2+4\lambda_{21}\lambda_{31})(\lambda_{21}+\lambda_{31})+2\lambda_1(\lambda_{21}+\lambda_{31})^2+\lambda_{31}\lambda_3\lambda_3^*,\\
T_6 &= \lambda_{21}(\lambda_2^*+2\lambda_{31}+\lambda_1)+\lambda_{31}(\lambda_3^*+\lambda_1).
\end{aligned}
\tag{18}
$$

Replacing A_k and D_l of R_i and T_i into B_k and E_l, we can obtain the expressions of R_i' and T_i'.

Thus, the three-soliton solutions can be presented as

$$A_1(x,t)=\frac{\varepsilon P_1(x,t)+\varepsilon^3 P_3(x,t)+\varepsilon^5 P_5(x,t)}{1+\varepsilon^2 Q_2(x,t)+\varepsilon^4 Q_4(x,t)+\varepsilon^6 Q_6(x,t)}, \tag{19}$$

$$A_2(x,t)=\frac{\varepsilon R_1(x,t)+\varepsilon^3 R_3(x,t)+\varepsilon^5 R_5(x,t)}{1+\varepsilon^2 Q_2(x,t)+\varepsilon^4 Q_4(x,t)+\varepsilon^6 Q_6(x,t)}. \tag{20}$$

For solutions (19), $A_1(x,t)$ is proportional to $A_2(x,t)$, they have the same interaction characteristics. Hence, we will mainly discuss the interaction characteristics of three-optical solitons for $A_1(x,t)$. In Fig. 1, optical soliton amplifications are demonstrated through the inelastic interactions. We can realize the amplifications among different solitons by choosing the different values of λ_{j2}. This is mainly due to the different incident phases of optical solitons before interactions.

For Fig. 2, we present the interactions between two parallel transmission solitons and a single soliton. Through the single soliton, we can effectively adjust the distance between two parallel solitons and their transmission amplitude. For Figs. 2(a) and 2(d), the two parallel solitons can be integrated into one soliton to achieve effective amplification of the soliton and realize the function of all-optical switches, which can be used to realize the relay free amplification in optical communication systems.

For Fig. 3, the three solitons form a confinement region in the interaction region. The boundary of this area can also be adjusted. After the interaction, the soliton can also be amplified or reduced. The amplification efficiency can be controlled by λ_{j2}. Thus, we can choose the different values of λ_{j2} to control the soliton amplification and realize all-optical switch function.

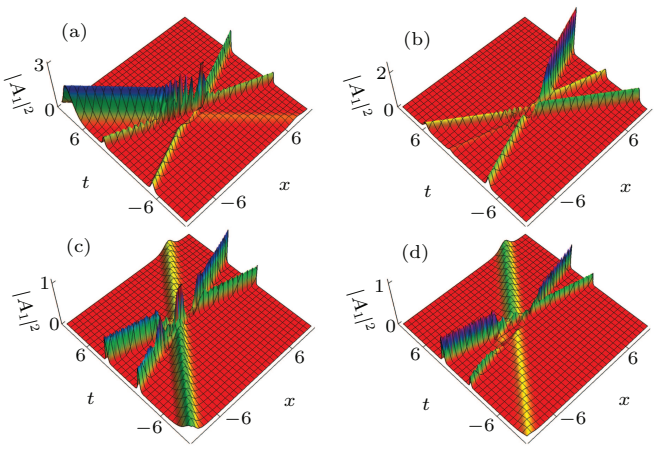


Fig. 1. The variation of optical soliton amplitude caused by the interaction of three solitons. Parameters are $\xi_{11}=1$, $\xi_{12}=1$, $\xi_{21}=2$, $\xi_{22}=1$, $\xi_{31}=3$, $\xi_{32}=1$, $A_1=2+\mathrm{i}$, $B_1=3+3\mathrm{i}$, $A_2=3+2\mathrm{i}$, $B_2=4+3\mathrm{i}$, $A_3=2$, $B_3=2+3\mathrm{i}$, $\beta(t)=-2$, $\omega(t)=1.2$, $\rho_1=\rho_2=1$, $\lambda_{11}=1$, $\lambda_{21}=-1.4$, $\lambda_{31}=1.8$ with (a) $\lambda_{12}=0.4$, $\lambda_{22}=-1.1$, $\lambda_{32}=-2$; (b) $\lambda_{12}=-1.8$, $\lambda_{22}=-1$, $\lambda_{32}=0.78$; (c) $\lambda_{12}=-1.2$, $\lambda_{22}=0.9$, $\lambda_{32}=0.2$; (d) $\lambda_{12}=-1.2$, $\lambda_{22}=0.9$, $\lambda_{32}=-0.2$.

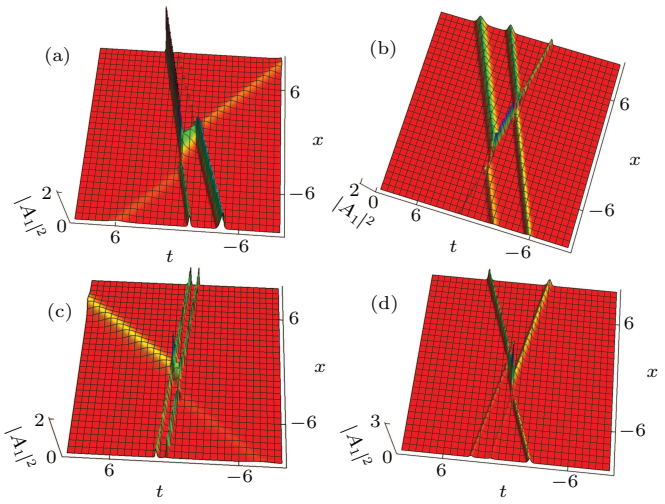


Fig. 2. The variation of optical soliton amplitude caused by the interaction of three solitons. (a) $\lambda_{12} = 0.4$, $\lambda_{22} = -0.56$, $\lambda_{32} = -0.44$; (b) $\lambda_{12} = -0.6$, $\lambda_{22} = -1.1$, $\lambda_{32} = 1.1$; (c) $\lambda_{12} = -1.2$, $\lambda_{22} = 1.3$, $\lambda_{32} = 1.4$; (d) $\lambda_{12} = 0.8$, $\lambda_{22} = -2$, $\lambda_{32} = 0.2$.

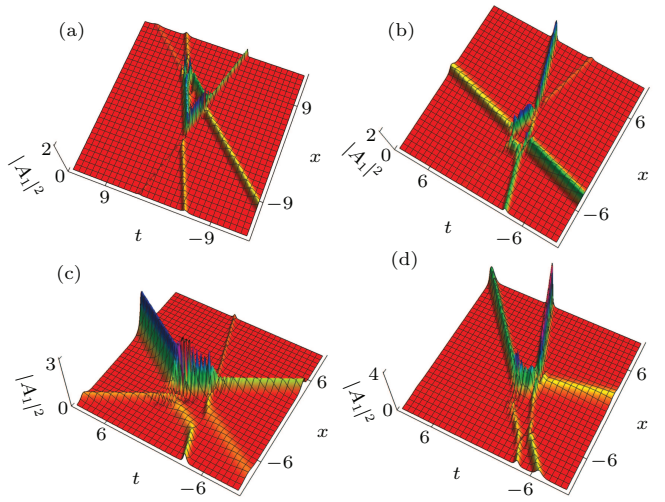


Fig. 3. The variation of optical soliton amplitude caused by the interaction of three solitons. (a) $\lambda_{12} = -0.6$, $\lambda_{22} = -1.4$, $\lambda_{32} = 1$; (b) $\lambda_{12} = 0.72$, $\lambda_{22} = -0.8$, $\lambda_{32} = -1.7$; (c) $\lambda_{12} = 0.13$, $\lambda_{22} = -0.25$, $\lambda_{32} = -1.8$; (d) $\lambda_{12} = -1.4$, $\lambda_{22} = -0.13$, $\lambda_{32} = 0.47$.

3. Conclusion

In this paper, the inelastic interaction laws of optical solitons have been studied. Through the analytical study of Eq. (19), the influence of the relevant parameters on the soliton interaction has been discussed. In the case of different incidence of three solitons, based on the interaction characteristics of different initial phase difference of them, the effective control of the interaction of three solitons has been realized. The related results will provide a theoretical reference for further understanding the application of soliton dynamics, soliton interaction and soliton control.

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