Inverse stochastic resonance in modular neural network with synaptic plasticity

Yong-Tao Yu(于永涛) and Xiao-Li Yang(杨晓丽)†

School of Mathematics and Statistics, Shaanxi Normal University, Xi'an 710062, *China*

(Received 3 September 2022; revised manuscript received 12 November 2022; accepted manuscript online 25 November 2022)

This work explores the inverse stochastic resonance (ISR) induced by bounded noise and the multiple inverse stochastic resonance induced by time delay by constructing a modular neural network, where the modified Oja's synaptic learning rule is employed to characterize synaptic plasticity in this network. Meanwhile, the effects of synaptic plasticity on the ISR dynamics are investigated. Through numerical simulations, it is found that the mean firing rate curve under the influence of bounded noise has an inverted bell-like shape, which implies the appearance of ISR. Moreover, synaptic plasticity with smaller learning rate strengthens this ISR phenomenon, while synaptic plasticity with larger learning rate weakens or even destroys it. On the other hand, the mean firing rate curve under the influence of time delay is found to exhibit a decaying oscillatory process, which represents the emergence of multiple ISR. However, the multiple ISR phenomenon gradually weakens until it disappears with increasing noise amplitude. On the same time, synaptic plasticity with smaller learning rate also weakens this multiple ISR phenomenon, while synaptic plasticity with larger learning rate strengthens it. Furthermore, we find that changes of synaptic learning rate can induce the emergence of ISR phenomenon. We hope these obtained results would provide new insights into the study of ISR in neuroscience.

Keywords: inverse stochastic resonance, synaptic plasticity, modular neural network

PACS: 02.50.–r, 05.40.–a, 87.85.dq, 02.30.Ks **DOI:** 10.1088/1674-1056/aca602

1. Introduction

It is well known that noise is unavoidable in the neurobiological dynamical system. Various interesting phenomena induced by noise, such as synchronization and transition,[1–4] stochastic resonance, and coherent resonance,[5–9] have been extensively studied in neurobiological dynamical systems.[10,11] Interestingly, Paydarfar *et al*. discovered[12] a new phenomenon induced by noise when studying the effect of noise on neuronal pacemakers in a vitro preparation of squid giant axons, which showed that noise can suppress or turn off repetitive neuronal activity. Since the dependence of the neuronal response on noise is opposite to that in the stochastic resonance mechanism, this phenomenon is called inverse stochastic resonance (ISR). Once this phenomenon came out, it attracted a lot of research attention. For example, Uzuntarla *et al*. investigated ISR induced by ion channel noise[13] and they extended this research to the neural network.[14] Besides, the influence of electrical synapses and chemical synapses on ISR in single neuron and neural network were investigated.[15] The related experimental work of Anatoly *et al*. on ISR, such as the study of cerebellar Purkinje cells, provided preliminary experimental proof for ISR,[16] and showed the important role of ISR in cerebellar information processing. Up to now, the study of ISR has been reported mainly in individual neurons or simple networks. However, in recent years, experiments have discovered that in the cerebral cortex of cats and macaques,[17,18] the structure of the neural network has modular characteristics. A network with high modularity has dense connections between nodes within a module, but sparse connections between nodes in different modules. Compared with classical complex networks, this structure allows modular networks to accurately represent some collective behaviors in the real world. To further understand the role of ISR in the brain, it is imperative to study ISR in modular neural network.

Noise is a key factor that causes the phenomenon of ISR. In the previous numerical simulations of ISR, Gaussian noise and non-Gaussian noise were usually used instead in order to simulate the noise in reality. Among them, Tuckwell *et al*. studied the ISR of Hodgkin–Huxley (HH) neurons under white Gaussian noise and performed long-term firing analysis.[19] Guo found that Gaussian color noise has a stronger inhibitory effect on neuronal firing than Gaussian white noise.[20] Li *et al*. tested whether non-Gaussian colored noise can induce ISR.[21] Lu *et al*. compared the ISR induced by Gaussian and non-Gaussian colored noise and found that under the condition of non-Gaussian colored noise, the transition of the mean firing rate was smoother, the ISR duration was longer, and the ISR phenomenon was more obvious.[22] In recent years, bounded noise as a generic non-Gaussian colored noise has attracted a lot of attention from scholars.[23–25] Compared with other noises, bounded noise not only considers the amplitude and frequency of the noise, but also the random phase, which is more suitable for simulating the realistic noise. Wang *et al*. studied the inhibitory effect of bounded noise on neuronal firing, and found that the larger the amplitude of the bounded

†Corresponding author. E-mail: yangxiaoli@snnu.edu.cn

© 2023 Chinese Physical Society and IOP Publishing Ltd

noise and the smaller the frequency, the more obvious the inhibitory effect.[26]

In addition, in the real neural network system, since neurons cannot process and transmit the information received in an instant,[27] that is to say, we need to consider the influence of coupling time delay on neuronal dynamics in the neural network system. The effect of time delay on the ISR of a single neuron has been confirmed,[28] and there are related reports that time delay can induce multiple ISR phenomena on a single neuron. Therefore, it will be an interesting topic to study the effect of time delay on ISR in modular neural network.

In the neurobiological system, it has been found that in the process of neurons growth and learning and memory, the weights of synapses connecting neurons are constantly updated, rather than remain unchanged.[29–31] The biological process by which this neural system modulates the strength of connections between neurons is called synaptic plasticity.[32] However, most previous studies of ISR did not consider synaptic plasticity in order to simplify the neural network model. To make our study more physiologically meaningful, synaptic plasticity needs to be included in neuronal model. Now, some simplified learning rules, such as Hebbian learning rule,[33] Oja's rule,[34] STDP rule,[35] have been proposed to regulate synaptic weights. As a modified Hebb learning rule, the Oja's learning rule can make the updated synaptic weights have good convergence. In recent years, many scholars have devoted themselves to using Oja's learning rule to study synaptic plasticity in biological neural networks. For example, Yao *et al*. discussed the effect of synaptic learning rate on spike synchronization in neural networks.[36] Through numerical studies, Yang *et al*. found that an increase in synaptic learning rate slightly inhibits burst synchronization between interacting neurons.[37]

In view of the fact that previous works of ISR have been mainly studied in single neurons or simple networks and have not considered the effect of synaptic plasticity, the purpose of this work is to extend the study of ISR to modular neural network. In order to make the network model more physiologically meaningful, synaptic plasticity is also considered in the connections of the modular neural network. Inspired by the above findings, this work constructs a modular neural network model with synaptic plasticity, and studies the effects of bounded noise, time delay, and synaptic plasticity on inverse stochastic resonance through numerical simulation. The rest of this work is arranged as follows: Section 2 presents the mathematical model of a modular neural network with synaptic plasticity. Section 3 illustrates the numerical simulation results. Finally, a conclusion is drawn in Section 4.

2. Model

The modular network in this work can be generated as following: Firstly, it is assumed that the modular network consists of M submodules. The submodules are all small-world networks that obey the random boundary process proposed by Newman and Watts.[38] Among them, the small-world network is generated by adding shortcuts between non-nearest nodes with intra-cluster probability p_{intra} in a regular ring-like network of N nodes (each node is connected to its K nearest neighbors). After generating all the submodules, nodes in different submodules are randomly connected with inter-cluster probability p_{inter}. The framework of the modular network is thus constructed. The network configuration parameters are set as $M = 2$, $N = 50$, $K = 6$, $p_{\text{intra1}} = 0.1$, $p_{\text{intra2}} = 0.09$, $p_{\text{inter}} = 0.01$ where p_{intra1} and p_{intra2} denote the intra-cluster probabilities of the two submodules, respectively.

The classical HH neuron model,[39] which reproduces spiking behavior similar to that of many real neurons, is often used to study the effect of noise on neuronal dynamics, which can be described by the following equation:

$$\begin{aligned} C\frac{\mathrm{d}V}{\mathrm{d}t} &= I_0 + g_{\mathrm{K}}n^4(V_{\mathrm{K}} - V) + g_{\mathrm{Na}}m^3h(V_{\mathrm{Na}} - V) \\ &\quad + g_{\mathrm{L}}(V_{\mathrm{L}} - V) + \xi(t). \end{aligned}$$

Here, V is the membrane potential and C is the membrane capacitance. g_{K}, g_{Na}, and g_{L} are the maximum conductance for potassium, sodium, and leak current with corresponding equilibrium potentials V_{K}, V_{Na}, and V_{L}. I_0 is external current and $\xi(t)$ represents the noisy environment of neurons.

The gating variables m and h represent the activation and inactivation variables of sodium channel, respectively, and the gating variable n represents the activation variable of potassium channel in neurons. They simulate activation and inactivation of sodium channel and activation of potassium channel respectively through the following differential equation:[39]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \alpha_x(1-x) - \beta_x x, \quad (x = m, n, h).$$

The voltage-dependent coefficients α_x and β_x ($x = m$, n, h) are determined experimentally and follows the following equation:[39,40]

$$\begin{aligned} \alpha_n(V) &= \frac{0.01(10-V)}{\exp[(10-V)/10]-1}, \\ \beta_n(V) &= 0.125\exp(-V/80), \\ \alpha_m(V) &= \frac{0.1(25-V)}{\exp[(25-V)/10]-1}, \\ \beta_m(V) &= 4\exp(-V/18), \\ \alpha_h(V) &= 0.07\exp(-V/20), \\ \beta_h(V) &= \frac{1}{\exp[(30-V)/10]+1}. \end{aligned}$$

On the basis of the HH neuron model, this work builds a mathematical model of modular neural network with synaptic plasticity under bounded noise and time delay environments, as shown in the following:

$$\begin{aligned} C\frac{\mathrm{d}V_{I,i}(t)}{\mathrm{d}t} &= I_0 + g_{\mathrm{K}} n_{I,i}^4 (V_{\mathrm{K}} - V_{I,i}(t)) \\ &\quad + g_{\mathrm{Na}} m_{I,i}^3 h_{I,i} (V_{\mathrm{Na}} - V_{I,i}(t)) + g_{\mathrm{L}} (V_{\mathrm{L}} - V_{I,i}(t)) \\ &\quad + g \sum_{J=1}^{M} \sum_{j=1}^{N} A_{I,J}(i,j) W_{I,J}(i,j,t) \\ &\quad \times [V_{I,i}(t) - V_{J,j}(t-\tau)] + \xi(t), \\ \frac{\mathrm{d}W_{I,J}(i,j,t)}{\mathrm{d}t} &= L \cdot \arctan\{V_{I,i}(t)[V_{J,j}(t) - V_{I,i}(t) W_{I,J}(i,j,t)]\}, \\ \frac{\mathrm{d}x_{I,i}}{\mathrm{d}t} &= \alpha_x (1 - x_{I,i}) - \beta_x x_{I,i}, \quad (x = m,n,h) \end{aligned} \tag{1}$$

in which $V_{I,i}(t)$ $(i = 1,2,\ldots,N, I = 1,\ldots,M)$ represents the membrane potential of the i-th neuron in the I-th submodule. $A_{I,J}(i,j)$ is the connection matrix of the modular network. $A_{I,J}(i,j) = A_{J,I}(j,i) = 1$ if neuron i in the I-th submodule is connected with neuron j in the J-th submodule, otherwise $A_{I,J}(i,j) = A_{J,I}(j,i) = 0$. In this model, we use a modified Oja's learning rule to describe the variation of synaptic weights between neurons. $W_{I,J}(i,j,t)$ represents the synaptic weight form neuron j in the J-th submodule to neuron i in the I-th submodule at time t. L is the synaptic learning rate, which reflects the speed at which the synaptic weights between neurons are updated. $x_{I,i}$ represents the gating variables of the i-th neuron in the I-th submodule. The bounded noise $\xi(t)$ can be expressed as $\xi(t) = D\sin(\Omega t + \sigma B(t) + \Gamma)$, where D and Ω represent the amplitude and frequency of the bounded noise, respectively. $B(t)$ is a standard Wiener process, Γ is a random variable uniformly distributed in $[0, 2\pi)$. In simulations, the parameters of the bounded noise are fixed at $\Omega = 6.0$ and $\sigma = 3.0$. In order to make the isolated HH neuron produce spiking activity, the values used for all parameters are listed in Table 1. respectively. In this work, the Euler algorithm is used to numerically simulate the above model with the time step 0.02, the initial value of $V_{I,i}(t)$ is set to be 0.0, and the initial value of the gating variables ($m_{I,i}$, $n_{I,i}$, $h_{I,i}$) is set by $x_{I,i}(0) = \alpha_{x(0)}/(\alpha_{x(0)} + \beta_{x(0)})$, $(x = m,n,h)$.

Table 1. Neuron model parameters.

Symbol	Description	Value	Units
C	membrane capacitance	1	μF/cm^2
g_{K}	maximum potassium conductance	36	mS/cm^2
g_{Na}	maximum sodium conductance	120	mS/cm^2
g_{L}	maximum sodium conductance	0.3	mS/cm^2
V_{K}	sodium reversal potential	−12	mV
V_{Na}	sodium reversal potential	115	mV
V_{L}	leakage reversal potential	10	mV
I_0	injected bias current	6.8	μA/cm^2
T	time interval	1000	ms

To quantify the ISR, first we count the number of spikes generated by each neuron in the modular network within the time interval T, and then calculate the mean firing rate of the modular network according to the following formula:

$$R_{\mathrm{mod}} = \frac{1}{NMT}\left(\sum_{I=1}^{M}\sum_{i=1}^{N} n_{\mathrm{spikes}}^{I,i}\right). \tag{2}$$

Here, $n_{\mathrm{spikes}}^{I,i}$ denotes the number of spikes of neuron i in the I-th submodule in the time interval T. This R_{mod} can effectively reflect the firing frequency of the modular neural network. A smaller R_{mod} represents a lower firing frequency of the modular neural network. If there appears a minimal value in the evolution of R_{mod}, one can say that ISR occurs in the modular neural network. Due to the random character of both the generation of the modular network and the generation of noise, the results obtained below are averaged over 50 realizations, with the exception of the membrane potential map, in order to ensure the real reliability of the numerical calculations.

3. Results

In this section, we mainly investigate ISR induced by bounded noise and multiple ISR induced by time delay in this modular neural network. Moreover, the effect of synaptic plasticity on ISR and multiple ISR are further discussed.

3.1. Inverse stochastic resonance induced by bounded noise

Firstly, we investigate how bounded noise affects HH neuron firing in the modular neural network without considering synaptic plasticity and time delay. For this purpose, we set both the synaptic learning rate L and the delay τ in Eq. (1) to be 0.

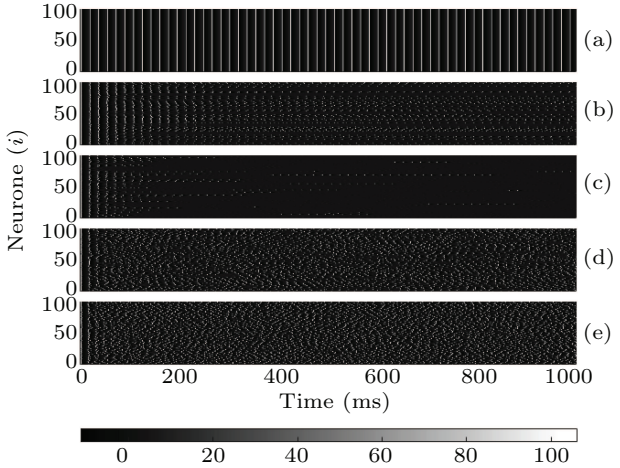


Fig. 1. The membrane potentials of modular neural network under different noise amplitudes: (a) $D = 0.0$, (b) $D = 0.5$, (c) $D = 1.5$, (d) $D = 4.0$, (e) $D = 8.0$.

Under the above preconditions, figure 1 shows the membrane potentials of the modular neural network at different noise amplitudes. In Fig. 2, the interspike interval histograms (ISIHs) of the modular neural network is further depicted to

more vividly describe the firing rhythm of collective neurons. Combining Figs. 1 and 2, we clearly find that the modular neural network is in a regular spiking state in the absence of noise, and the total number of firing spikes is 5700 in this the modular neural network. When the noise amplitude increases to 0.5, the firing spikes in the modular neural network become sparse, and the number of its firing spikes is significantly reduced to 2638. When the noise amplitude increases further, as shown in Fig. 2(c) the firing frequency of neurons decreases again and only a small number of firing spikes appear in the modular neural network, with the number of firing spikes decreasing to 603. However, for larger noise amplitudes, the number of firing spikes in the modular neural network no longer decreases. On the contrary, the number of firing spikes increases with increasing noise amplitude. For example, when $D = 4.0$ and $D = 8.0$ (see Figs. 2(d) and 2(e)), the number of firing spikes in the modular neural network increases to 4181 and 5495, respectively. Furthermore, we find from Fig. 2 that the range of the distribution of interspike intervals (ISIs) of the modular neural network gradually expands as the noise amplitude increases. This indicates that the distribution and regularity of neuronal firing are changed by the noise amplitude. Overall, one can observe that the firing frequency of the modular neural network first decreases and then increases with increasing noise amplitude, which suggests that bounded noise can induce ISR in this modular neural network.

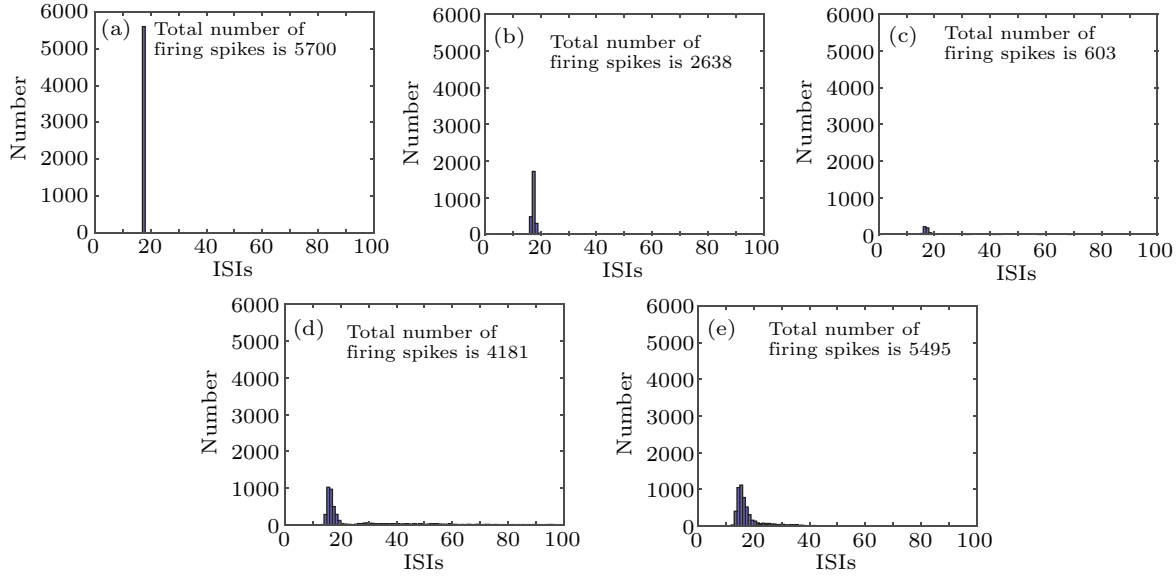


Fig. 2. ISIHs of modular neural network at different noise amplitudes: (a) $D = 0.0$, (b) $D = 0.5$, (c) $D = 1.5$, (d) $D = 4.0$, (e) $D = 8.0$.

To quantify the ISR phenomenon in the modular neural network, the mean firing rate of the modular neural network on the different noise amplitude is calculated in Fig. 3. As the noise amplitude increases, the mean firing rate first decreases and then increases, reaching a minimum around $D = 1.5$, which shows an inverted bell-like shape. This implies that modular neural network does show a phenomenon of ISR under the influence of bounded noise.

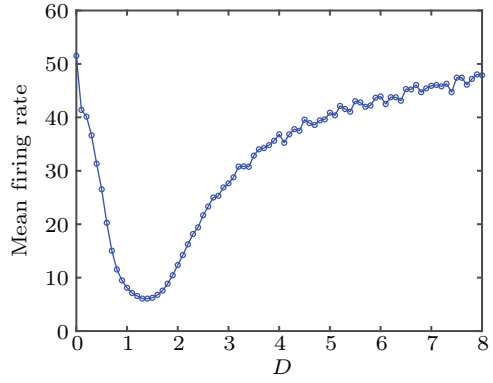


Fig. 3. The variation of mean firing rate with the bounded noise amplitude D.

3.2. Effect of synaptic plasticity on noise-induced ISR

In the following, we investigate how the synaptic plasticity affects the above bounded noise-induced inverse stochastic resonance. First, we depict the effect of synaptic plasticity on the ISR phenomenon by characterizing the change curve of mean firing rate with noise amplitude under different synaptic learning rates in Fig. 4. The results in Fig. 4 show that for relatively weak synaptic learning rate, there is a minimum of the mean firing rate, while for stronger synaptic learning rate (*e.g.*, $L = 8$ and $L = 9$), there is no significant minimum in the evolution of mean firing rate. This indicates that within a certain range of the synaptic learning rate L, the modular neural network will still experience ISR, and when the synaptic learning rate L is too large, the modular neural network will not experience ISR. Furthermore, for smaller synaptic learning rate, the shift in the mean firing rate is gentler, while for slightly larger synaptic learning rate (*e.g.*, $L = 1, 3$), the shift will be sharper. In Fig. 4, the mean firing rate stays at a lower state for the appropriate noise amplitude. Compared to different synaptic

learning rates, and it can be seen that the range of noise amplitude that keeps the mean firing rate low becomes larger as the synaptic learning rate increases. As the synaptic learning rate increases further, this range narrows again. Therefore, we can choose the appropriate synaptic learning rate so that the range of noise amplitudes that can inhibit neuronal firing activity becomes wider. On the other hand, we can also find that with increasing of the synaptic learning rate, the minimum value of the mean firing rate keeps decreasing. However, further increasing of the synaptic learning rate, we find that the minimum value of the mean firing rate starts to increase until ISR no longer occurs in the module neural network. In order to illustrate this situation more intuitively, we extract the minimal value of each curve, as shown in Fig. 5(a). By comparing with the case when the synaptic learning rate $L = 0$, it can be found that when L is relatively small ($L < 6.5$), synaptic plasticity will strengthen the inhibitory effect of noise, thus strengthening the ISR phenomenon; when L is relatively large ($L > 6.5$), synaptic plasticity weakens the inhibitory effect of noise, so that the ISR phenomenon in the modular neural network is weakened or even disappeared.

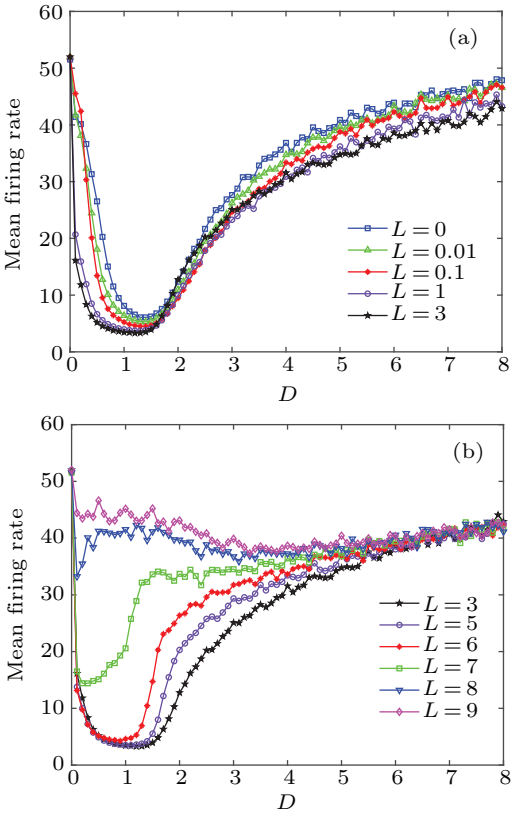


Fig. 4. The variation of mean firing rate with the noise amplitudes D under different synaptic learning rates.

Next, we further investigate the effect of synaptic plasticity on ISR by fixing the noise amplitude. Figure 5(b) shows the variation of the mean firing rate with the synaptic learning rate for different case of noise amplitudes. As can be seen in Fig. 5(b), each curve falls and then rises upon increasing synaptic learning rate. Furthermore, the fact that these curves fall and then rise implies that the mean firing rate of the modular neural network decreases and then increases as the synaptic learning rate increases, which is precisely the characteristic of ISR. We thus conclude that, under a suitable noise, changes in the learning rate of synaptic plasticity can also induce ISR in modular neural network.

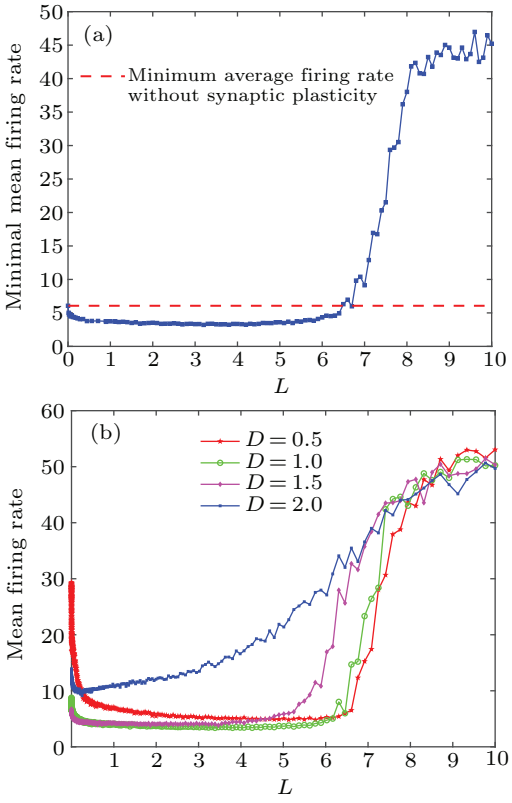


Fig. 5. (a) Minimal mean firing rate as a function of the synaptic learning rate L. (b) The variation of mean firing rate with the synaptic learning rate under different noise amplitudes D.

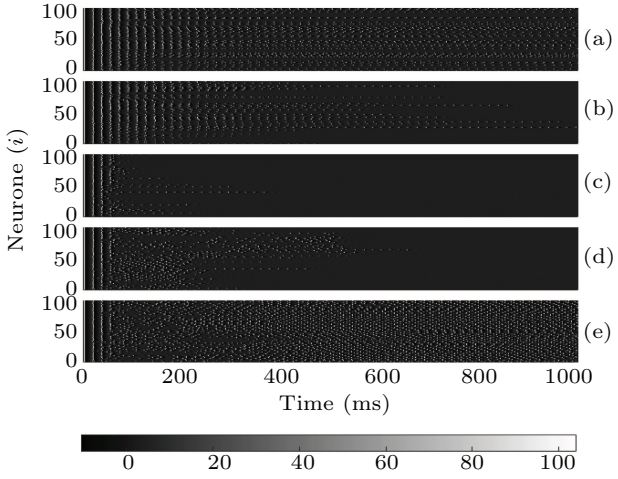


Fig. 6. The membrane potentials of modular neural network with the condition of $D = 0.5$ under different synaptic learning rates L: (a) $L = 0.0$, (b) $L = 0.1$, (c) $L = 6$, (d) $L = 7$, (e) $L = 8$.

To verify whether changes in the learning rate of synaptic plasticity can induce ISR in the modular neural network, we depict some membrane potentials of the modular neural network and their corresponding ISIHs in Figs. 6 and 7. Figure 6

shows some membrane potentials of the modular neural network under the condition of noise amplitude $D = 0.5$. From the figure we note that without considering synaptic plasticity, *i.e.*, when the synaptic learning rate $L = 0$, the modular neural network still has a large number of firing spikes, and combined with Fig. 7 we know that the number of firing spikes at this time is 2638. When considering synaptic plasticity and setting the synaptic learning rate to 0.1, the firing spikes of the modular neural network becomes sparse and the number of firing spikes is reduced to 1410. When the synaptic learning rate is increased to 6, the modular neural network has only a few firing spikes at the beginning, and then the network is in a quiescent state, and the number of firing spikes is only 429. When the synaptic learning rate is further increased, the modular neural network starts to resume firing and the number of firing spikes gradually increases. When the synaptic learning rate is 7, the modular neural network starts to gradually resume firing during the first 500 ms, and the number of firing spikes increases to 996 at this time. When the synaptic learning rate is 8, the modular neural network is basically in a spiking state, and there are 4541 firing spikes at that time. From Figs. 6 and 7, we can clearly see that with the increase of the synaptic learning rate L, the number of firing spikes of the modular neural network has obviously experienced a process of first decreasing and then increasing. Thus we can say more definitely that changes of learning rate in synaptic plasticity can induce ISR in modular neural network.

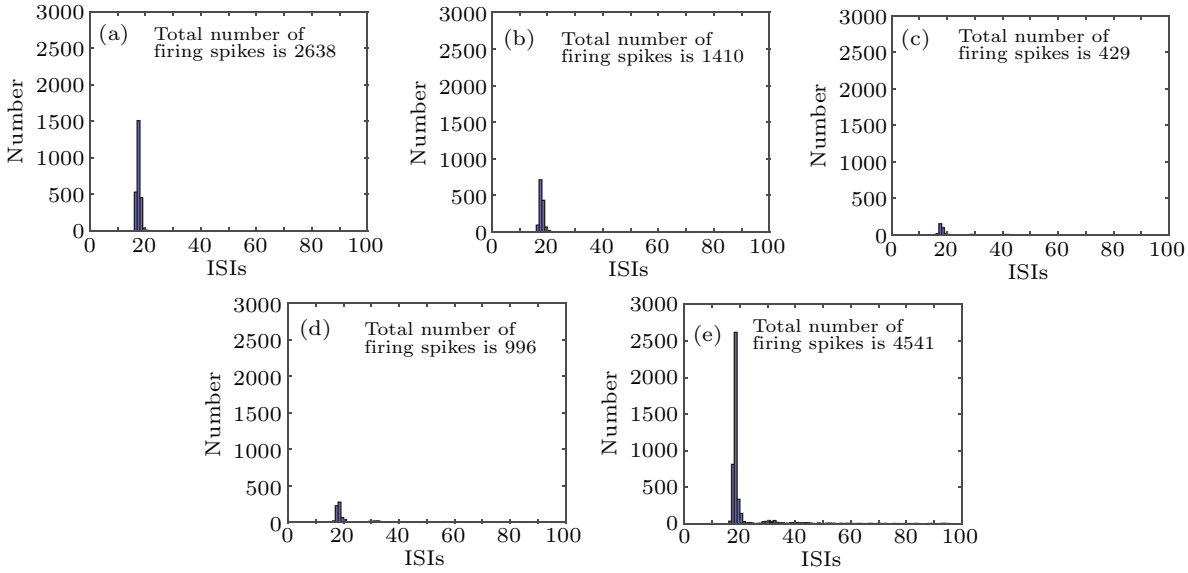


Fig. 7. The ISIHs of modular neural network with the condition of $D = 0.5$ under different synaptic learning rates L: (a) $L = 0.0$, (b) $L = 0.1$, (c) $L = 6$, (d) $L = 7$, (e) $L = 8$.

3.3. Multiple inverse stochastic resonances induced by time delay

This subsection builds on the previous subsection and further investigates the effect of time delay on bounded noise-induced ISR by introducing the coupling time delay into the coupling term. Without considering synaptic plasticity, we first show the membrane potentials of the modular neural network at different time delay τ. The membrane potential in Fig. 8(a) does not take into account the effect of time delay, and it can be seen that there are only a few firing spikes in the modular neural network at this case, which is due to the noise inhibiting the neuronal firing. When the effect of time delay is considered and the time delay is set to be 10, the modular neural network shifts to a spiking state under the effect of time delay. When the time delay is increased to 14, the neuronal firing is again suppressed and a large number of firing spikes disappear. Similarly, when the time delay continues to increase to 27, a large number of firing spikes appear in the modular neural network. And when the time delay is further increased to 31, these firing spikes almost disappear again. From these findings we get a conclusion that multiple ISR phenomenon can occur in the modular neural network as the time delay τ increases.

To further verify the above conclusion, we depict the mean firing rate of the modular neural network as a function of time delay in Fig. 9 when $D = 1.0$. As can be seen from the curve, the mean firing rate exhibits a decaying oscillation process with respect to the time delay τ. First, the firing of the modular neural network is already in a suppressed state under the effect of noise, and as the time delay τ increases, the network resumes firing so that the mean firing rate reaches its first maximum. Further increasing the time delay τ, the mean firing rate rapidly decreases to a minimum. Increasing the time delay again, the mean firing rate reaches the second maximum. The following process is analogous. This means that the multiple ISR phenomenon does appear in the modular neural network with the change of the delay τ. On the

other hand, we find that as the delay τ increases, the maximal value on the curve gradually decreases and the minimal value gradually increases, which means that the amplitude of the oscillation becomes smaller. This result indicates that the multiple ISR phenomenon induced by the time delay gradually weakens with the increase of the time delay τ. Without loss of generality, we also depict the mean firing rate for small noise amplitude such as $D = 0.5$ and 1.5, from which the above similar results are obtained. In addition, the variations of mean firing rate with time delay τ for large noise amplitude such as $D = 2.0$, 2.5, 3.0, and 5.0 are illustrated in Fig. 9. One can find that as the noise amplitude D increases, the amplitude of the oscillations of the mean firing rate curve decreases until the oscillations disappear. This indicates that the multiple ISR phenomenon gradually weakens until it disappears with continuous increasing of noise amplitude D.

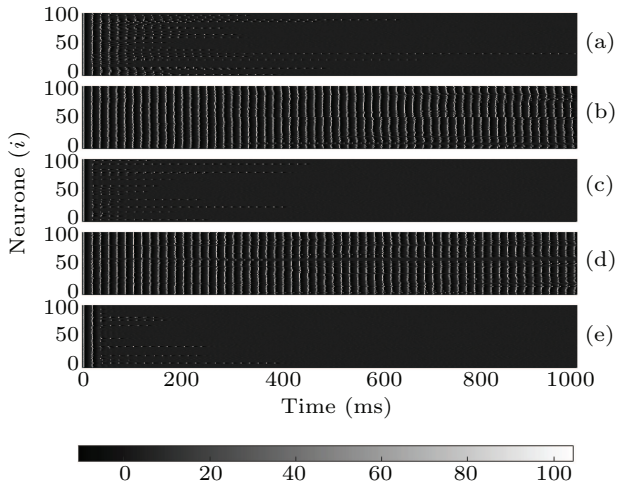


Fig. 8. The membrane potentials of the modular neural network with the condition of noise amplitude $D = 1.0$ under different time delay τ: (a) $\tau = 0$, (b) $\tau = 10$, (c) $\tau = 14$, (d) $\tau = 27$, (e) $\tau = 31$.

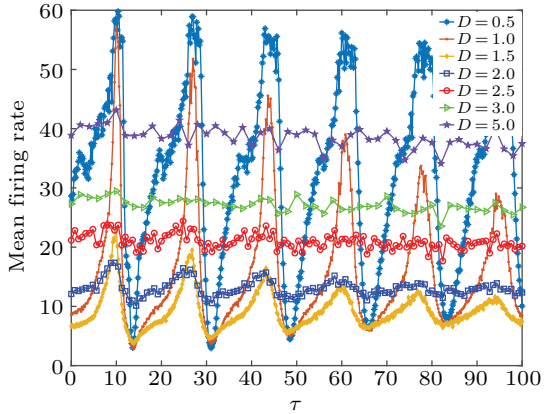


Fig. 9. The mean firing rate as a function of time delay for different noise amplitudes.

3.4. Effect of synaptic plasticity on delay-induced multiple ISR

In this subsection, we fix the noise amplitude at $D = 1.0$, and then investigate the effect of synaptic plasticity on time delay induced multiple ISR. Figure 10 shows the mean firing rate as a function of time delay for different synaptic learning rates with the condition of specific D value. It can be seen from the figure that regardless of the value of the synaptic learning rate, the mean firing rate still show a decaying oscillatory process with respect to the time delay τ, *i.e.*, the amplitude of the oscillation decreases as the time delay τ increases. These results indicate that the modular neural network still shows multiple ISR phenomena when considering synaptic plasticity, and its property of weakening with increasing time delay τ does not change. However, the amplitude of the oscillations in the mean firing rate changes as the synaptic learning rate increases. Figure 10(a) shows that the amplitude of the oscillations decreases as the synaptic learning rate increases. Further increasing the synaptic learning rate, the amplitude of the oscillations starts to increase as shown in Fig. 10(b).

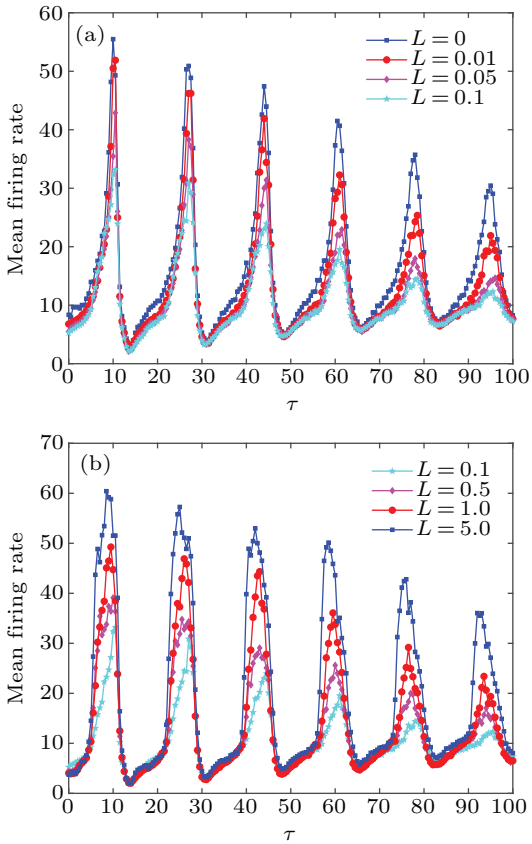


Fig. 10. The mean firing rate as a function of time delay with the condition of $D = 1.0$ under different synaptic learning rates L.

To illustrate the change in the amplitude of oscillations in a clearer way, figure 11 depicts the difference between the first maximum value and the first minimum value of the mean firing rate curve for different synaptic learning rates. As the synaptic learning rate increases, the amplitude of the oscillation first decreases and then increases. This suggests that the multiple ISR phenomenon in the modular neural network first weakens and then strengthens as the synaptic learning rate increases. By comparing with the situation when the synaptic learning rate $L = 0$, it can be found that when L is relatively small ($L < 1.5$), synaptic plasticity weakens the multiple ISR

phenomenon; when L is larger ($L > 1.5$), synaptic plasticity strengthens the multiple ISR phenomenon.

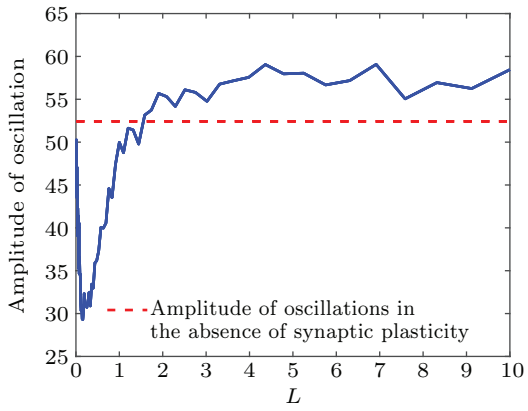


Fig. 11. The amplitude of the oscillations as a function of synaptic learning rates L.

4. Conclusion

As described above, ISR is a phenomenon in which the firing of neurons is inhibited. At present, the function and role of ISR on information transmission in brain neural networks gradually emerges. Recent neuroanatomical studies have shown that the connections in brain neural networks have modular characteristics. In addition, synaptic plasticity is a factor that cannot be ignored in neural system. Therefore, it is desirable to study ISR in modular neural network as well as the influence of synaptic plasticity on ISR. To this end, this work mainly explores bounded noise-induced ISR and time delay-induced multiple ISR in a modular neural network and discusses the effect of synaptic plasticity on the above two ISR phenomena.

First, a modular neural network model with synaptic plasticity is constructed. Then, bounded noise-induced ISR and time delay-induced multiple ISR are investigated in this modular neural network. Finally, the effect of synaptic learning rate on the ISR is investigated. Simulation results show that the ISR phenomenon occurs in the modular neural networks under the influence of bounded noise, and that synaptic plasticity with a smaller learning rate can strengthen this ISR phenomenon, while synaptic plasticity with a larger learning rate can weaken or even destroy it. In addition, we also find that changes in synaptic learning rate can also induce the ISR phenomenon in the modular neural network. On the other hand, the modular neural network also shows multiple ISR phenomena under the influence of time delay, and it is found that this phenomenon gradually decreases as the time delay τ increases. Besides, synaptic plasticity does not affect the occurrence of multiple ISR phenomenon, but only the strength of multiple ISR is changed, *i.e.*, synaptic plasticity with a small learning rate weakens multiple ISR phenomena, while synaptic plasticity with a large learning rate strengthens it.

Acknowledgement

Project supported by the National Natural Science Foundation of China (Grant No. 11972217).

References

[1] Zhang H, Wang Q, He X and Chen G 2014 *Neural Netw.* **49** 107
[2] Boccaletti S, Kurths J, Osipov G, Valladares D L and Zhou C S 2002 *Phys. Rep.* **366** 101
[3] Pikovsky A, Rosenblum M and Kurths J 2001 *Am. J. Phys.* **70** 655
[4] Liu S, Sun Z, Zhao N and Xu W 2022 *Int. J. Bifurcat. Chaos* **32** 2250018
[5] Yang X, Li N and Sun Z 2019 *Nonlinear Dyn.* **98** 1029
[6] Gammaitoni L, Hänggi P, Jung P and Marchesoni F 2009 *Eur. Phys. J. B* **69** 3
[7] Ozer M, Perc M and Uzuntarla M 2009 *Phys. Lett. A* **373** 964
[8] Wang Q, Zhang H, Perc M and Chen G 2012 *Commun. Nonlinear Sci.* **17** 3979
[9] Masoliver J, Robinson A and Weiss G H 1995 *Phys. Rev. E* **51** 4021
[10] Zheng Y, Wang Q and Danca M F 2014 *Cogn. Neurodyn.* **8** 143
[11] Liu Y, Sun Z and Yang X 2021 *Appl. Math. Comput.* **409** 126384
[12] Paydarfar D, Forger D B and Clay J R 2006 *J. Neurophysiol.* **96** 3338
[13] Uzuntarla M, Cressman J R, Ozer M and Ernest B 2012 *BMC Neurosci.* **13** 181
[14] Uzuntarla M, Barreto E and Torres J J 2017 *PLoS Comput. Biol.* **13** 1005646
[15] Uzuntarla M 2013 *Phys. Lett. A* **377** 2585
[16] Buchin A J, Roth A, Hausser M and Rieubland S 2016 *PLoS Comput. Biol.* **12** 1005000
[17] Hilgetag C C, Burns G A P C, O'Neill M A, Scannell J W and Young M P 2000 *Philos. Trans. R. Soc. Lond., B, Biol. Sci.* **355** 91
[18] Zamora-López G, Zhou C and Kurths J 2009 *Chaos* **4** 1
[19] Tuckwell H C and Jost J 2012 *Physica A* **391** 5311
[20] Guo D 2011 *Cogn. Neurodyn.* **5** 293
[21] Li D, Cui X and Yang Y 2018 *Neurocomputing* **287** 52
[22] Lu L, Jia Y, Ge M, Xu Y and Li A 2020 *Nonlinear Dyn.* **100** 877
[23] Feng Z H, Lan X J and Zhu X D 2007 *Int. J. Nonlinear Mech.* **42** 1170
[24] Cai G Q and Wu C 2004 *Probabilistic Eng. Mech.* **19** 197
[25] Kai-Leung Y and Lei Y M 2010 *Chin. Phys. B* **19** 010503
[26] Wang J and Li D X 2017 *Journal of North University of China (Natural Science Edition)* **38** 31
[27] Kandel E R, Schwartz J H and Jessell T M 2014 *Principles of Neural Science*, 5th edn. (New York: McGraw-Hill)
[28] Zhang N, Li D and Xing Y 2021 *Eur. Phys. J. B* **94** 1
[29] Holtmaat A and Svoboda K 2009 *Nat. Rev. Neurosci.* **10** 647
[30] Schmidt-Hieber C, Jonas P and Bischofberger J 2004 *Nature* **10** 244
[31] Munakata Y and Pfaffly J 2010 *Dev Sci.* **7** 141
[32] Martin S J, Grimwood P D and Morris R 2000 *Annu. Rev. Neurosci.* **23** 649
[33] Hebb D O 2022 *The Organization of Behavior: A Neuropsychological Theory* (New York: Psychology Press)
[34] Oja E 1982 *J. Math. Biol.* **15** 267
[35] Gerstner W, Kempter R, Hemmen J and Wagner H 1996 *Nature* **383** 76
[36] Yao Z L, Yang X L and Sun Z K 2018 *Chaos* **28** 083120
[37] Yang X L, Wang J Y and Sun Z K 2017 *Nonlinear Dyn.* **89** 2593
[38] Newman M and Watts D J 1999 *Phys. Lett. A* **263** 341
[39] Hodgkin A L and Huxley A F 1990 *Bull. Math. Biol.* **52** 25
[40] Pankratova E V, Polovinkin A V and Mosekilde E 2005 *Eur. Phys. J. B* **45** 391