Generation of microwave photon perfect *W* states of three coupled superconducting resonators

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We propose an efficient method for the generation of perfect *W* states on three microwave superconducting resonators, of which the two nearest neighbors are coupled by a symmetric direct current superconducting quantum interference device (dc-SQUID). With suitable external magnetic fluxes applied to the dc-SQUID symmetry loops, on-chip tunable interactions between neighboring resonators can be realized, and different perfect *W* states can be deterministically created on-demand in one step. Numerical simulations show that high-fidelity target states can be generated and our scheme is robust against imperfect parameter tuning and environment-induced decoherence. The present work may have potential applications for implementing quantum computation and quantum information processing based on microwave photons.

Keywords: quantum entanglement, perfect *W* state, circuit QED

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1. Introduction

With the development of superconducting quantum technologies, circuit quantum electrodynamics (circuit QED) has become one of the most outstanding platforms for the implementation of quantum information and quantum computation, as well as experimentation on the foundations of quantum mechanics.[1,2] In general, due to the unequal spacing between energy levels arising from the strong nonlinearity of Josephson junctions, superconducting qubits can be straightforwardly fully controlled by classical means,[3,4] and information can be stored in their two lowest levels.[5–7] Their numerous advantages, e.g., level spacing controllability,[8–10] microfabrication technology scalability, ever-increasing qubit coherence times,[11–14] and easy realization of strong coupling even ultra-strong coupling between qubits and resonant cavities,[15–17] allow for efficient and robust quantum manipulation, storage, and readout.[18–22] Significant progress has been made in quantum information processing with superconducting qubits in the past decade, for instance, the generation of two- and multi-qubit entanglement,[23–30] demonstration of efficient and high-fidelity quantum gate,[31–34] and implementation of elementary quantum algorithms.[35,36] In particular, recent experiments have created entangled Greenberger–Horne–Zeilinger states with up to 18 qubits and multicomponent atomic Schrödinger cat states of up to 20 qubits in circuit QED systems,[37,38] which are substantial steps towards practical quantum information processing.

Although superconducting resonators with infinite Hilbert space have extraordinarily rich quantum states, they are difficult to access through classical signals owing to the intrinsic energy level degeneracy associated with linearity. However, recent studies show that such devices can also be efficiently addressed and manipulated by using extra nonlinear elements, which enabling quantum information encoding, protection, and manipulation.[39,40] By coupling to a superconducting qubit to realize the robust control of a single superconducting resonator, Schrödinger cat states,[41] Fock states[42,43] and their arbitrarily superposed states[44] have been efficiently created in experiments. Thanks to the excellent coherence properties of superconducting resonators and the capacity of being permanently coupled to single nonlinear superconducting circuits, much attention has recently been paid to large-scale quantum information processing involving two or more spatially separated resonators interacting with each other via Josephson-junction-embedded superconducting circuits. The generation of entangled Fock states,[45,46] NOON states,[47–49] entangled coherent states,[50] and arbitrary entangled states[51,52] of two or more superconducting resonators has been proposed in many theoretical works. Experimentally, the demonstration of quantum state transfer[53] and quantum logic gates,[54] and the generation of NOON states[55] and Schrödinger cat states[56] in two separated resonators have been reported. Moreover, the coherent transfer of microwave Fock states between three resonators has been implemented in a superconducting solid-state system.[57] Circuit QED offers new possibilities for the implementation of quantum information processing schemes via microwave photons.

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Multipartite entangled states are central for plenty of applications in quantum information science.[37] Among them, maximally entangled W states show great robustness against particle loss and bit flip noise and are immune to global dephasing,[58] thus they have been used in many important fields, such as quantum teleportation, quantum telecloning, and quantum key distribution.[59,60] However, they cannot be used for deterministic teleportation, superdense coding, and quantum information splitting.[60–62] To address such issues, a special class of asymmetric W states, named perfect W states, are introduced, which can be denoted as[62,63]

$$|W_m\rangle = \frac{1}{\sqrt{2(1+m)}}\big(|1,0,0\rangle + \sqrt{m}\,\mathrm{e}^{\mathrm{i}\psi_1}|0,1,0\rangle + \sqrt{1+m}\,\mathrm{e}^{\mathrm{i}\psi_2}|0,0,1\rangle\big), \tag{1}$$

where m represents a real number, ψ_1 and ψ_2 refer to phases, and $|0\rangle$ and $|1\rangle$ are vacuum and single-photon Fock states of a resonator, respectively. Such states possess many unique merits and are proposed to act as key resources for perfect teleportation, superdense coding, and quantum information splitting, sharing, and securely transmitting.[62,64–66] Therefore, the efficient preparation of such states is of great interest and is essential for quantum computation and quantum information processing tasks. To date, various physical systems have been proposed as possible candidates to generate perfect W states, such as linear optical systems,[63] Cavity QED systems,[67] and quantum dots.[68] However, the proposed schemes usually need multiple steps and subtle adjustment, suffering from sensitivity to decoherence and no scalability.

In this paper, we present an efficient scheme to generate microwave photon perfect W states in superconducting quantum circuits, where three transmission line resonators are linearly coupled by two dc-SQUIDs. The interactions between neighboring resonators can be tuned on-chip by choosing appropriate magnetic fluxes applied to the dc-SQUIDs to match the frequency differences between the resonators, and the system can selectively evolve to the perfect W states with a certain value of m in one step, enabling deterministic, high fidelity, and robustness state preparation. The discussion on the chosen parameters implies that our scheme is feasible with the present circuit QED technology.

The remaining part of this article is organized as follows. In Section 2, the model describing three superconducting resonators coupled by dc-SQUIDs is introduced. In Section 3, the deterministic generation of microwave photon perfect W states on the three resonators is presented. The effects of parameter mismatch and environment-induced decoherence are demonstrated by numerical simulations and the experimental feasibility of the present scheme is discussed in Section 4. Finally, a summary of the main results is given in Section 5.

2. Model

As illustrated in Fig. 1(a), we consider a circuit QED setup composed of three largely detuned and nearest-neighbor interacting one-dimensional transmission line resonators. Such kind of microwave resonators consist of a narrow central conductor and two nearby lateral ground planes. The breaks in the central conductor act as capacitors, in analogy to mirrors in cavity QED. It is essentially an LC resonator with distributed inductor L and capacitance C.[69–71] Here we just consider the fundamental modes of the resonators, which can be simply modeled as harmonic oscillators. The effective circuit diagram for the considered setup is plotted in Fig. 1(b), where the red crosses represent Josephson junctions. The corresponding free Hamiltonian of the resonators takes the form of (hereafter let $\hbar = 1$)

$$H_0 = \sum_{k=1}^{3} \omega_k a_k^{\dagger} a_k, \tag{2}$$

where $\omega_k/2\pi = 1/\sqrt{L_k C_k}$ is the frequency of the fundamental mode, and $a_k^{\dagger}$ and a_k are the creation and annihilation operators of the microwave field of the resonator k, respectively.

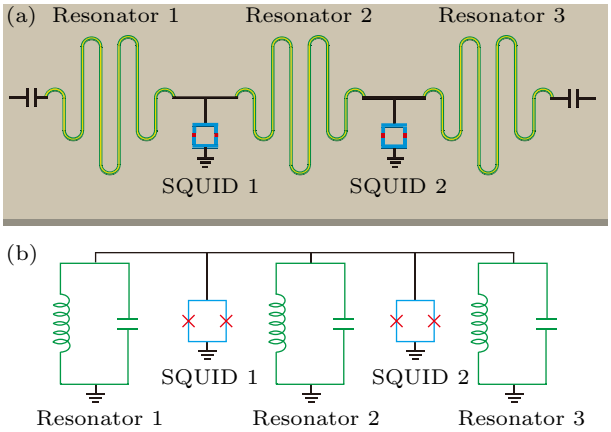


Fig. 1. (a) Schematic diagram of three coupled transmission line resonators. (b) Circuit diagram for the previous setup with the resonators effectively represented by LC resonators.

The two nearest-neighbour resonators can be coupled inductively by an inductance or capacitively by a capacitor, so that microwave photons can hop from one resonator to the other. In our setup, a dc-SQUID is used as a coupler, which consists of two Josephson junctions connected in parallel and forming a loop.[72,73] The dc-SQUID behaves as a parametric inductor to induce controllable inductive coupling between the two resonators by changing the external magnetic flux injected into the symmetric dc-SQUID loop, allowing us to produce time-dependent boundary conditions. In this case, the resonator–resonator interaction can be described as[74]

$$H_{\mathrm{int}} = \sum_{k=1}^{2} \alpha_k(\phi_k^{\mathrm{ext}})(a_k + a_k^{\dagger})(a_{k+1} + a_{k+1}^{\dagger}), \tag{3}$$

where $\phi_k^{\rm ext}$ is the external flux penetrating the k-th dc-SQUID, and

$$\alpha_k(\phi_k^{\rm ext}) = \frac{\Phi_0^2}{4\pi^2 E_{\rm J}(\phi_k^{\rm ext})}\sqrt{\frac{\omega_k\omega_{k+1}}{C_kC_{k+1}}}\frac{1}{Z_kZ_{k+1}}$$

is the coupling strength between resonator k and $k+1$, with the effective Josephson coupling energy $E_{\rm J}(\phi_k^{\rm ext}) = 2E_{\rm J}^0\cos(\pi\phi_k^{\rm ext}/\Phi_0)$, $E_{\rm J}^0$ being the Josephson junction energy of a single Josephson junction embedded into the dc-SQUIDs, $Z_k = \sqrt{L_k/C_k}$ being the impedance of the k-th resonator, $\Phi_0 = h/2e$ being the magnetic flux quantum, and h being the Planck constant. For more details on the derivation of the system Hamiltonian, see the supplementary information.

3. Deterministic preparation of perfect W states

In this section, we study how to couple three largely detuned resonators under periodically modulated boundary conditions and then deterministically generate the target perfect W states when the system is prepared in a certain initial state.

To detail the procedure of the state preparation, we start with the entire Hamiltonian of the system given by

$$H = H_0 + H_{\rm int}. \tag{4}$$

Switching to the interaction picture with respective to H_0, we have

$$H_{\rm I} = \sum_{k=1}^{2}\alpha_k\left(\phi_k^{\rm ext}\right)\left(a_k\,{\rm e}^{-{\rm i}\omega_k t} + a_k^\dagger\,{\rm e}^{{\rm i}\omega_k t}\right) \times\left(a_{k+1}\,{\rm e}^{-{\rm i}\omega_{k+1}t} + a_{k+1}^\dagger\,{\rm e}^{{\rm i}\omega_{k+1}t}\right). \tag{5}$$

We can easily see from the above equation that when the detunings between the neighboring resonators fulfill $\Delta_k = |\omega_{k+1} - \omega_k| \gg \alpha_k(\phi_k^{\rm ext})$, all of the terms in the above Hamiltonian are fast-oscillating and can be neglected under the rotating-wave approximation. Therefore, the whole system has no evolution in such a situation.

In order to make further progress, we consider the case that the external flux $\phi_k^{\rm ext}$ is split into two parts, namely, a perturbation part oscillating at the driving frequency $\omega_k^{\rm d}$ and a fixed offset ϕ_0, i.e., $\phi_k^{\rm ext} = \phi_0 + \phi_k\cos(\omega_k^{\rm d}t)$ with $\phi_k \ll \phi_0$. In this way, with suitable parameters under consideration, the SQUID plasma frequencies will far exceed the driving frequency $\omega_k^{\rm d}$ for every value of $\phi_k^{\rm ext}$ during the time evolution.[75,76] The term associated with the effective Josephson coupling energy in the coupling strength can be expanded via the Taylor expansion

$$\frac{1}{E_{\rm J}(\phi_k^{\rm ext})} = \frac{1}{2E_{\rm J}^0}\left(\frac{1}{\cos(\pi\phi_0/\Phi_0)} + \frac{\pi}{\Phi_0}\frac{\sin(\pi\phi_0/\Phi_0)}{\cos^2(\pi\phi_0/\Phi_0)}\phi_k\cos(\omega_k^{\rm d}t) + O(\phi_k)\right). \tag{6}$$

Keeping ϕ_k up to the first order, we obtain

$$H_{\rm I}' = \sum_{k=1}^{2}\left(\beta_k + J_k\left({\rm e}^{{\rm i}\omega_k^{\rm d}t} + {\rm e}^{-{\rm i}\omega_k^{\rm d}t}\right)\right)\left(a_k\,{\rm e}^{-{\rm i}\omega_k t} + a_k^\dagger\,{\rm e}^{{\rm i}\omega_k t}\right) \times\left(a_{k+1}\,{\rm e}^{-{\rm i}\omega_{k+1}t} + a_{k+1}^\dagger\,{\rm e}^{{\rm i}\omega_{k+1}t}\right), \tag{7}$$

where

$$\beta_k = \frac{\Phi_0^2}{8\pi^2 E_{\rm J}^0\cos(\pi\phi_0/\Phi_0)}\sqrt{\frac{\omega_k\omega_{k+1}}{C_kC_{k+1}}}\frac{1}{Z_kZ_{k+1}},$$
$$J_k = \frac{\Phi_0\sin(\pi\phi_0/\Phi_0)}{16\pi E_{\rm J}^0\cos^2(\pi\phi_0/\Phi_0)}\sqrt{\frac{\omega_k\omega_{k+1}}{C_kC_{k+1}}}\frac{1}{Z_kZ_{k+1}}\phi_k.$$

To select the desired couplings between the neighbouring resonators, we choose the detunings Δ_1 and Δ_2 to match $\Delta_1 = \Delta_2 = \omega_1^{\rm d} = \omega_2^{\rm d}$. If the condition $\Delta_k \gg \beta_k,\ J_k$ is satisfied, we can discard all the fast-oscillating terms and keep only the resonant terms under the rotating-wave approximation. Thus, an effective Hamiltonian can be obtained as

$$H_{\rm eff} = J(a_1^\dagger a_2 + a_2^\dagger a_1) + rJ(a_2^\dagger a_3 + a_3^\dagger a_2), \tag{8}$$

where $J = J_1$ is the effective coupling strength between the resonators 1 and 2, rJ is that between the resonators 2 and 3, and $r = \phi_2/\phi_1$ is the ratio of the two effective coupling strengths. We can see that the ratio r can be tuned by controlling the two small flux amplitudes ϕ_1 and ϕ_2. It can be found that the considered regime of parameters in our work guarantees that the boundary conditions produced by the SQUIDs do not destroy the structure of the normal modes of the resonators, hence they can still be considered as single mode ones.

Next, we exploit this interaction to prepare perfect W states on the three coupled microwave resonators. To this end, we assume that the resonator 2 is prepared in Fock state $|1\rangle$ and the others two in vacuum states at the initial time, i.e., the initial state of the system is $|\Psi(0)\rangle = |0,1,0\rangle$. Since the dynamics of the system is restricted to the single-excitation subspace, after a period of time t, the state of the resonators takes the form of $|\Psi(t)\rangle = c_1(t)|1,0,0\rangle + c_2(t)|0,1,0\rangle + c_3(t)|0,0,1\rangle$. According to the Schrödinger equation ${\rm i}\partial|\Psi(t)\rangle/\partial t = H_{\rm eff}|\Psi(t)\rangle$, we can obtain three coupled differential equations

$${\rm i}\frac{{\rm d}c_1(t)}{{\rm d}t} = c_2(t)J, \tag{9}$$

$${\rm i}\frac{{\rm d}c_2(t)}{{\rm d}t} = c_1(t)J + c_3(t)rJ, \tag{10}$$

$${\rm i}\frac{{\rm d}c_3(t)}{{\rm d}t} = c_2(t)rJ. \tag{11}$$

The state of the system at arbitrary time t can be obtained by solving these equations. To solve for $c_1(t)$, $c_2(t)$, and $c_3(t)$, it is convenient to rewrite Eqs. (9)–(11) in the following matrix form:

$${\rm i}\frac{{\rm d}R(t)}{{\rm d}t} = MR(t), \tag{12}$$

where

$$R(t)=\begin{pmatrix} c_1(t)\\ c_2(t)\\ c_3(t)\end{pmatrix},\quad M=\begin{pmatrix} 0 & J & 0\\ J & 0 & rJ\\ 0 & rJ & 0\end{pmatrix}. \tag{13}$$

Since the coupling matrix M is independent of time t, the solution can be written as

$$R(t)=\mathrm{e}^{-\mathrm{i}Mt}R(0). \tag{14}$$

The evolution matrix $\mathrm{e}^{-\mathrm{i}Mt}$ can be calculated as

$$\mathrm{e}^{-\mathrm{i}Mt}=\begin{pmatrix} \frac{r^2+\cos(J\sqrt{1+r^2}t)}{1+r^2} & -\frac{\mathrm{i}\sin(J\sqrt{1+r^2}t)}{\sqrt{1+r^2}} & \frac{r(\cos(J\sqrt{1+r^2})-1)}{1+r^2}\\ -\frac{\mathrm{i}\sin(J\sqrt{1+r^2}t)}{\sqrt{1+r^2}} & \cos\left(J\sqrt{1+r^2}t\right) & -\frac{\mathrm{i}r\sin(J\sqrt{1+r^2}t)}{\sqrt{1+r^2}}\\ \frac{r(\cos(J\sqrt{1+r^2})-1)}{1+r^2} & -\frac{\mathrm{i}r\sin(J\sqrt{1+r^2}t)}{\sqrt{1+r^2}} & \frac{1+r^2\cos(J\sqrt{1+r^2}t)}{1+r^2}\end{pmatrix}. \tag{15}$$

Substituting the evolution matrix and the initial state of the system into Eq. (14), we can get the solutions of Eqs. (9)–(11) as follows:

$$c_1(t)=-\frac{\mathrm{i}\sin(J\sqrt{1+r^2}t)}{\sqrt{1+r^2}}, \tag{16}$$

$$c_2(t)=\cos(J\sqrt{1+r^2}t), \tag{17}$$

$$c_3(t)=-\frac{\mathrm{i}r\sin(J\sqrt{1+r^2}t)}{\sqrt{1+r^2}}. \tag{18}$$

If the parameters are chosen to be $r=\sqrt{1+m}$ at the time $Jt=\arcsin(\sqrt{(2+m)/2(1+m)})/\sqrt{2+m}$, the evolved state of the system takes the form

$$\begin{aligned}|\Psi(t)\rangle\ =\ &-\frac{\mathrm{i}}{\sqrt{2(1+m)}}\big(|1,0,0\rangle+\mathrm{i}\sqrt{m}|0,1,0\rangle\\ &+\sqrt{1+m}|0,0,1\rangle\big).\end{aligned} \tag{19}$$

It can be found that the created state (19) is just the desired microwave photon perfect W state given by Eq. (1) for $\psi_1=\pi/2$ and $\psi_2=0$ with a global phase factor. When the small flux amplitudes ϕ_1 and ϕ_2 are tuned, the ratio r takes different values, and thus various perfect W states with different values of m can be created on-demand.

4. Numerical simulations

In this section, we verify the validity of our protocol in practical applications. The effects of the imperfect tuning of the ratio r and the decoherence induced by the environment are discussed via the study of the system dynamics based on numerical simulations.[77] When taking the coupling of the whole system to a harmonic oscillator environment in the Markovian approximation into consideration, the dynamics of the system is governed by the following master equation:

$$\begin{aligned}\frac{\mathrm{d}\rho}{\mathrm{d}t}\ =\ &-\mathrm{i}[H_{\mathrm{eff}},\rho]+\frac{\kappa}{2}\sum_{k=1}^{3}\left(\bar{n}_{\mathrm{th}}^{(k)}+1\right)D[a_k]\rho\\ &+\frac{\kappa}{2}\sum_{k=1}^{3}\bar{n}_{\mathrm{th}}^{(k)}D[a_k^{\dagger}]\rho,\end{aligned} \tag{20}$$

where ρ is the system density operator, $D[o]\rho=2o\rho o^{\dagger}-\rho o^{\dagger}o-o^{\dagger}o\rho$ denotes the standard Lindblad operator, κ represents the decay rate of the resonators, and $\bar{n}_{\mathrm{th}}^{(k)}=1/(\exp(\hbar\omega_k/k_{\mathrm{B}}T)-1)$ is the thermal average photon number with k_{B} and T being the Boltzmann constant and the working temperature for the superconducting circuit, respectively. For the case of the working temperature below 20 mK and the resonant frequencies of the resonators about several gigahertz, e.g., $\omega_k/2\pi\sim 6$ GHz, the thermal average photon number can be calculated as $\bar{n}_{\mathrm{th}}^{(k)}\sim 10^{-7}$, which is so small that it can be neglected in the state preparation process. In the following discussions, we make use of fidelity F to measure the overlap of the created state and the target state given by Eq. (19). Fidelity is defined as

$$F=\mathrm{Tr}[\rho_t\rho(t)], \tag{21}$$

where $\rho(t)$ is the density matrix of the system and $\rho_t=|\Psi(t)\rangle\langle\Psi(t)|$ is the target state.

As discussed in Section 3, to generate a certain perfect W state, the ratio of the two effective coupling strengths must satisfy the corresponding condition $r=\sqrt{1+m}$. Unfortunately, it is hardly possible to control the effective coupling strengths meeting this condition exactly in practice, resulting in inevitable damage to the state generation. Here we consider the situation when the ratio r has a small deviation from the ideal value. The evolution of fidelity F against the ratio r and dimensionless parameter Jt is depicted in Figs. 2(a) and 3(a), for the target states with $m=1$ and $m=2$, respectively. The parameters are chosen as $\omega_1/2\pi=6$ GHz, $\omega_2/2\pi=8$ GHz, $\omega_3/2\pi=6$ GHz, $J=5$ MHz, and $\kappa=0$. One can see that the target states with high fidelities greater than 0.98 still could be obtained when the ratio r varies in a considerably large region, though the time when the fidelity reaches its maximum has a slight shift according to the variation of the ratio r. This

implies that our scheme is very robust against the imperfect tuning of the effective coupling strengths.

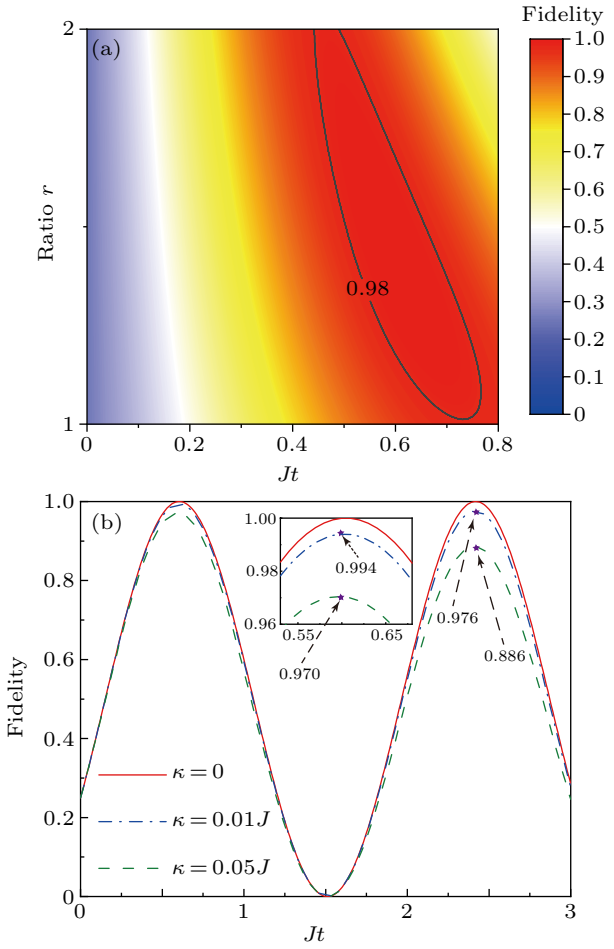


Fig. 2. (a) Fidelity F as a function of the ratio r and the dimensionless parameter Jt and (b) time dependence of fidelity F for different decay rates of the resonators versus the dimensionless function Jt for $m = 1$.

Next, we study the state creation including the environment coupling and investigate the effect of photon leakage of the resonators. In Figs. 2(b) and 3(b), we plot the time evolution of fidelities F for the target states with $m = 1$ and $m = 2$, respectively, for different decay rates κ. Two enlarged insets are depicted corresponding to the fidelities in the vicinity of preparation time. In addition, exact fidelity values at some peak positions are listed. It could be found that the fidelities oscillate with time and reach their maximums at $Jt \simeq 0.61$ and $Jt \simeq 0.48$ in Figs. 3(a) and 3(b) for the first time, respectively, agreeing very well with the expected theoretical results. Even in the large decay situations ($\kappa = 0.1J$, green dashed curves), the fidelities can reach very high values close to 1 (0.996 and 0.998, respectively) at the preparation time. Therefore, our scheme is insensitive to environment-induced decoherence.

Finally, we discuss the experimental feasibility. The preparation times of the desired states are $t_1 \approx 19.4$ ns for $m = 1$ and $t_2 \approx 15.3$ ns for $m = 2$, respectively. The required quality factors for the resonators 1 and 3 are about 731 and 577, respectively, and these for the resonator 2 are about 975 and 769. For the decay rates $\kappa = 0.1J$ and $\kappa = 0.01J$, the required quality factors for the resonators 1 and 3 are 1.2×10^4 and 1.2×10^5, respectively, and these for the resonator 2 are 1.6×10^4 and 1.6×10^5. These conditions can be readily satisfied since the superconducting resonator with a quality factor $Q > 10^7$ has been fabricated[78] and a quality factor beyond 10^8 has been reported in three-dimensional resonators.[79,80] Experimentally, transmission line resonators usually work in the 2–15 GHz microwave regime.[81] The coupling strength between two superconducting resonators can be modulated ranging between 0 and hundreds of MHz by utilizing simple Josephson junctions circuits.[82] In our scheme, the coupling strengths can be readily tuned via both the fixed offsets ϕ_0 of the external fluxes and the small flux amplitudes ϕ_k. Thus all the parameters adopted in our work are typically achievable in superconducting circuit experiments. This suggests that the high-fidelity preparation of perfect W states is feasible within the-state-of-art technologies.

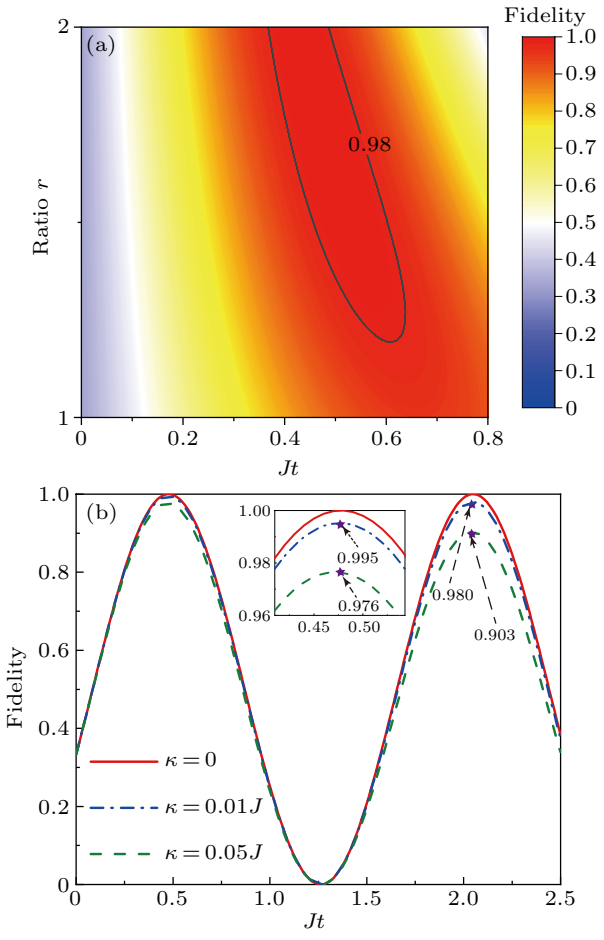


Fig. 3. (a) Fidelity F as a function of the ratio r and the dimensionless parameter Jt and (b) time dependence of fidelity F for different decay rates of the resonators versus the dimensionless function Jt for $m = 2$.

5. Conclusion

In conclusion, we have proposed an efficient scheme for the generation of microwave photon perfect W states in superconducting quantum circuits. Two dc-SQUIDs serve as

couplers to interconnect the two nearest-neighbors of the three resonators. By controlling the frequencies of the driving fields applied to the dc-SQUIDs, the desired interaction terms in the system dynamics can be selectively activated. With a special initial state, the engineered effective Jaynes–Cummings interactions can drive the system to various perfect W states depending on the choice of the ratio of the two effective coupling strengths. Numerical simulations and discussions on adopted parameters exhibit the successful creation of the target states and the practical feasibility of our protocol. By tuning system parameters and taking more resonators into consideration, our scheme can be extended to create maximally entangled W states and multi-partite W-type states.

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