Spin transport properties in ferromagnet/superconductor junctions on topological insulator

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The spin-dependent Andreev reflection is investigated theoretically by analyzing the electronic transport in a thin-film topological insulator (TI) ferromagnet/superconductor (FM/SC) junction. The tunneling conductance and shot noise are calculated based on the Dirac–Bogoliubov–de Gennes equation and Blonder–Tinkham–Klapwijk theory. It is found that the magnetic gap in ferromagnet can enhance the Andreev retro-reflection but suppress the specular Andreev reflection. The gate potential applied to the electrode on top of superconductor can enhance the two types of reflections. There is a transition between the two types of reflections at which both the tunneling conductance and differential shot noise become zero. These results provide a method to realize and detect experimentally the intra-band specular Andreev reflection in thin film TI-based FM/SC structures.

Keywords: TI-based FM/SC junctions, magnetic gap effect, Andreev retro-reflection, specular Andreev reflection

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1. Introduction

Andreev reflection (AR)[1] is an important phenomenon in superconducting tunnel junctions, through which the dissipative electrical current can converse into non-dissipative supercurrent. Recently, Beenakker[2] predicted that two different types of AR may happen in graphene superconducting junctions. Usually, reflected hole at the normal metal/superconductor (NM/SC) interface goes back along the path of the incident electron. This process is called Andreev retro-reflection.[3] In undoped graphene, however, the reflected hole may travel like the normal reflected electron which is called specular AR. The specular AR process is absent in ordinary NM/SC junctions.[4,5] From the band structure point of view, the Andreev retro-reflection occurs where the incident electron and the reflected hole come from the same band (conduction band or valence band). While, specular AR happens when the incident electron and the reflected hole are from conduction and valence bands, respectively.[6–8] Very recently, Zhang and co-workers[9] reported a theoretical investigation of the spin-dependent AR process in a graphene-based superconducting junction. They provided a method to experimentally distinguish the specular AR from the conventional retro-reflection by measuring the tunneling conductance and shot noise in graphene-based ferromagnet/superconductor (FM/SC) junctions.

On the other hand, the transport properties in topological insulator (TI)-based superconducting junctions have also attracted great interests due to the peculiar band structures of TI.[10–20] For example, the tunneling conductance in TI-based FM/SC junctions is studied in order to investigate the effect of both the magneto and Fermi energy mismatch between FM and SC.[16] The interplay between unconventional SC and ferromagnetism in TI-based superconducting junctions is studied in Refs. [18,19]. Recently, Majidi and Asgari[21] investigated precisely the AR process by analyzing the quasiparticles transport through TI-based NM/SC junctions. Their results theoretically revealed the possibility of generating specular AR in the presence of a gate electric field. Complete contrast to the inter-band type in graphene, the coupling between the top and bottom surface of an ultrathin TI can lead to the intra-band specular AR. Later, Islam *et al.*[22] studied the crossed AR in a TI-based NM/SC/NM junction in the presence of a gate electric field. How to experimentally distinguish the specular AR from the conventional retro-reflection in TI-based superconducting junctions is a natural question. This is one of the motives of our work.

In this paper, we investigate theoretically the spin-dependent AR by analyzing the electronic transport in a TI-based FM/SC junction. The tunneling conductance and the shot noise are calculated by the Dirac–Bogoliubov–de Gennes (DBdG) equation and Blonder–Tinkham–Klapwijk (BTK) theory. It is found that both the specular AR and the conventional retro-reflection are strongly affected by the ferromagnetic exchange energy and the applied electric field. With the change of the exchange interaction and electric field, there will be a transition from one reflection to another.

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2. Theoretical model

Our model is a three-dimensional TI-based FM/SC junction where carriers are confined in the xy-plane. The applied magnetization along the z direction is in the region for $x \leq 0$ and the SC lies in the region for $x > 0$. The magnetization can give rise to the magnetic energy gap $\sim 2m$ in the density of states of the carriers. An external potential U of a gate is placed over the superconducting region. Therefore, the Fermi energy in SC equals $E_{\rm F}+U$ with $E_{\rm F}$ the Fermi energy in FM.

The considered system satisfies the following DBdG equation:[23]

$$\begin{pmatrix} v_{\rm F}\hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma}+e\boldsymbol{A}_{\rm eff}\cdot\boldsymbol{\sigma}-E_{\rm F}+U & \hat{\Delta}_{\rm S} \\ -\hat{\Delta}_{\rm S}^{*} & -(v_{\rm F}\hat{\boldsymbol{p}}\cdot\boldsymbol{\sigma}+e\boldsymbol{A}_{\rm eff}\cdot\boldsymbol{\sigma}-E_{\rm F}+U)^{*} \end{pmatrix} \psi(x)\,{\rm e}^{{\rm i}p_y y/\hbar v_{\rm F}} = E\psi(x)\,{\rm e}^{{\rm i}p_y y/\hbar v_{\rm F}}, \tag{1}$$

where $e\boldsymbol{A}_{\rm eff}$, $v_{\rm F}$, and E are the exchange-energy-like effective vector potential, the Fermi velocity, and the excited energy of quasiparticles, respectively. The vector of the Pauli spin matrices is represented by $\boldsymbol{\sigma}$. The momentum operator is defined by $\boldsymbol{p} = \left\langle -\boldsymbol{i}\frac{\partial}{\partial x}, -\boldsymbol{j}\frac{\partial}{\partial y}, 0 \right\rangle$. The superconducting gap matrix of s-wave is given as $\hat{\Delta}_{\rm S} = {\rm i}\Delta_{\rm S}\hat{\sigma}_y$ with $\Delta_{\rm S} = \Delta_0$.

Considering $e\boldsymbol{A}_{\rm eff}$ in Eq. (1) is proportional to the applied magnetization, *i.e.*, $e\boldsymbol{A}_{\rm eff} \propto \boldsymbol{M} = \langle 0,0,m\rangle$, giving rise to $eA_{\rm eff(z)} \sim m$ and assuming that $E \ll E_{\rm F}$ and $E_{\rm F}+U$, the wave functions in SC and FM regions can be given by

$$\begin{aligned}
\psi_{\rm S} &= t_{\rm e}\begin{bmatrix} u \\ u{\rm e}^{{\rm i}\theta_{\rm S}} \\ -v{\rm e}^{{\rm i}\theta_{\rm S}}{\rm e}^{-{\rm i}\varphi_+} \\ v{\rm e}^{-{\rm i}\varphi_+} \end{bmatrix}{\rm e}^{{\rm i}(E_{\rm F}+U)x\cos\theta_{\rm S}/\hbar v_{\rm F}} \\
&\quad + t_{\rm h}\begin{bmatrix} v \\ -v{\rm e}^{-{\rm i}\theta_{\rm S}} \\ u{\rm e}^{-{\rm i}\theta_{\rm S}}{\rm e}^{-{\rm i}\varphi_-} \\ u{\rm e}^{-{\rm i}\varphi_-} \end{bmatrix}{\rm e}^{-{\rm i}(E_{\rm F}+U)x\cos\theta_{\rm S}/\hbar v_{\rm F}}, \\
\psi_{\rm F} &= \begin{bmatrix} 1 \\ \frac{E_{\rm F}-m}{\sqrt{E_{\rm F}^2-m^2}}{\rm e}^{{\rm i}\theta} \\ 0 \\ 0 \end{bmatrix}{\rm e}^{{\rm i}\sqrt{E_{\rm F}^2-m^2}x\cos\theta/\hbar v_{\rm F}} \\
&\quad + r_{\rm e}\begin{bmatrix} 1 \\ -\frac{E_{\rm F}-m}{\sqrt{E_{\rm F}^2-m^2}}{\rm e}^{-{\rm i}\theta} \\ 0 \\ 0 \end{bmatrix}{\rm e}^{-{\rm i}\sqrt{E_{\rm F}^2-m^2}x\cos\theta/\hbar v_{\rm F}} \\
&\quad + r_{\rm A}\begin{bmatrix} 0 \\ 0 \\ 1 \\ -\frac{E_{\rm F}-m}{\sqrt{E_{\rm F}^2-m^2}}{\rm e}^{-{\rm i}\theta} \end{bmatrix}{\rm e}^{{\rm i}\sqrt{E_{\rm F}^2-m^2}x\cos\theta/\hbar v_{\rm F}},
\end{aligned} \tag{2}$$

where

$$u = \sqrt{1+\sqrt{1-\Delta_0^2/E^2}},\quad v = \sqrt{1-\sqrt{1-\Delta_0^2/E^2}},$$

θ is the incident angle of quasiparticles in FM region, and $\theta_{\rm S}$ is the incident angle of the electron in SC satisfies the following relationship:

$$\sin\theta\sqrt{E_{\rm F}^2-m^2}/\hbar v_{\rm F} = \sin\theta_{\rm S}(E_{\rm F}+U)/\hbar v_{\rm F}.$$

The coefficients of normal reflection $r_{\rm e}$, Andreev reflection $r_{\rm A}$, transmission as electron-like quasiparticle $t_{\rm e}$, and transmission as hole-like quasiparticle $t_{\rm h}$ in Eq. (2) can be obtained by using the condition for continuity of wave functions at boundary. That is $\psi_{\rm S}|_{x=0} = \psi_{\rm F}|_{x=0}$.

Based on the BTK theory[24] at zero temperature, the differential conductance of the considered system is written as[16]

$$\begin{aligned}
G &= \frac{G_0}{2}\int_{-\theta_{\rm t}}^{\theta_{\rm t}}\frac{\sqrt{E_{\rm F}^2-m^2}}{E_{\rm F}} \\
&\quad\times\left(1+|r_{\rm A}|^2\frac{\cos\theta_{\rm S}}{\cos\theta}-|r_{\rm e}|^2\right)\cos\theta\,{\rm d}\theta,
\end{aligned} \tag{3}$$

where $\theta_{\rm t} = \sin^{-1}\frac{E_{\rm F}+U}{\sqrt{E_{\rm F}^2-m^2}}$, $G_0 = N(E_{\rm F})We^2/\pi\hbar^2 v_{\rm F}$ with W the width of the junction and $N(E_{\rm F}) \sim E_{\rm F}/2\pi(\hbar v_{\rm F})^2$ the normal density of state.

The zero-frequency shot noise is defined by $S = 2\int_{-\infty}^{+\infty}{\rm d}t\left\langle[\hat{I}(t)-\langle I\rangle]\times[\hat{I}(0)-\langle I\rangle]\right\rangle$[25,26] with I the tunneling current and $\hat{I}(t)$ the current operator. Considering the important role of the AR, the differential shot noise for the present system is given by

$$\begin{aligned}
S &= \frac{2e^3}{h}N(0)\int_{-\pi/2}^{\pi/2}\frac{\sqrt{E_{\rm F}^2-m^2}}{E_{\rm F}}|r_{\rm A}|^2\frac{\cos\theta_{\rm S}}{\cos\theta} \\
&\quad\times\left(1-|r_{\rm A}|^2\frac{\cos\theta_{\rm S}}{\cos\theta}\right)\cos\theta\,{\rm d}\theta.
\end{aligned} \tag{4}$$

3. Results and discussion

We first study the effect of the magnetic gap m on the spin-dependent AR process in TI-based FM/SC junctions. The tunneling conductance as a function of bias voltage for different m with $U = 10E_{\rm F}$ is given in Fig. 1. It can be seen that, in the case of $m/E_{\rm F} < 1$, the subgap conductance increases with increasing bias voltage. When $m = 0$ a conductance minimum due to the Fermi-level mismatch between FM and SC induced by the gate electric field appears at zero energy and a peak caused by the Andreev resonance occurs at the point where the applied bias voltage is equal to the amplitude of the super-

conducting gap, *i.e.*, $eV = \Delta_0$.[16] As increasing m, the amplitude of the conductance in the whole energy range decreases. When $m/E_F = 1$, the tunneling conductance vanishes. In the case of $m/E_F > 1$, the tunneling conductance only exists in the subgap energy range and the subgap conductance increases with m increasing. These behaviors may be physically explained as follows. In the case of $m/E_F < 1$, the electron–hole conversion is predominant retro-reflection correspondingly the tunneling conductance is also mainly determined by Andreev retro-reflection. In contrast, for $m/E_F > 1$, *i.e.*, $E_F < m$, the electron–hole conversion and the corresponding conductance are predominant specular reflection. When $m/E_F = 1$, there is no AR for any angle of incidence. Therefore, it can be reasonably inferred that the magnetic gap can enhance the specular AR and suppress the Andreev retro-reflection. This can be used to a criterion for distinguishing between the specular AR and the conventional Andreev retro-reflection. It is worth mentioning that this criterion is also applicable to the graphene-based FM/SC junctions.[9]

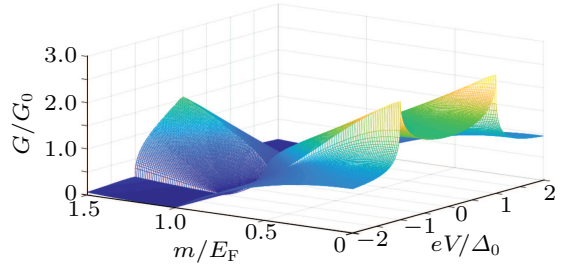


Fig. 1. Tunneling conductance as a function of bias voltage for different magnetic gaps with $U = 10E_F$.

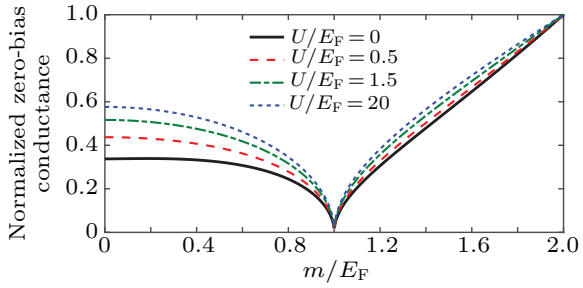


Fig. 2. Normalized zero-bias conductance as a function of magnetic gap for different U.

The influence of the electrostatic potential U on the AR process is shown in Fig. 2 where we plot the dependence of the normalized zero-bias conductance on the magnetic gap for different U. It is found that for all the values of m, the conductance is enhanced by increasing the gate potential when U is small. For values of $U \gg E_F$, in fact according to our calculation when $U/E_F \geq 20$, the conductance spectra remain unchanged. This indicates for large U, the conductance is independent of the gate potential. Even more interesting is that no matter what the value of U is, with the increase of m, the zero-bias conductance first is reduced to zero at $m/E_F = 1$ and then goes up. These results show that the gate-induced potential difference between the top and bottom surfaces of the TI can enhance the probability of specular AR and the specular electron–hole conversion can happen in a wide experimentally accessible range of the electric field[21] for the proposed TI-based FM/SC structure.

The behaviors of the shot noise are given in Figs. 3 and 4. Figure 3 shows the shot noise spectra as a function of bias voltage for different magnetic gap with $U = 10E_F$. One finds that the shot noise, especially in subgap energy, is strongly influenced by the exchange interaction. Just like the subgap conductance, with m increasing the shot noise in sub-gap region also decreases to zero at $m = E_F$ and then increases. In order to see this feature more clearly, we plot the normalized shot noise at zero-bias energy for different E_F in Fig. 4. It can be seen obviously that the zero-bias shot noise vanishes when $m = E_F$. It follows that in this case the Fermi surface just locates at the Dirac point (at which conduction and valence bands touch) and all of the conduction channels are closed. Then there no AR process occurs at the FM/SC interface in the in the same band. Therefore, both the zero-bias conductance and the shot noise vanishes. When $E_F > m$, from Eq. (4) we can know the numbers of the electron states contributing to the specular electron–hole conversion from two drops to one as E_F increases. Correspondingly, the dominant AR is retro type and the zero-bias conductance is reduced. The values of the differential shot noise and conductance are equal to zero at $E_F = m$ can be used to judge the presence of specular AR as well as to determine the spin-polarization.

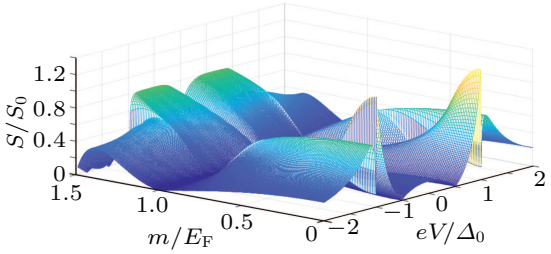


Fig. 3. Shot noise as a function of bias voltage for different magnetic gaps with $U = 10E_F$.

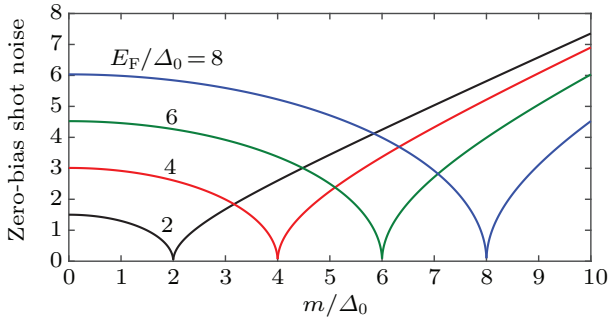


Fig. 4. Shot noise as a function of magnetic gap for different E_F with $U = 10E_F$.

4. Conclusions

In this paper, we investigated theoretically the two types of reflections by the tunneling conductance and shot noise in a TI-based FM/SC junction. It is found that both the magnetic gap (m) in FM region and the gate potential U applied to the electrode on top of SC can enhance the Andreev retroreflection. The specular Andreev reflection can be suppressed by m but enhanced by U. There is a transition between the two types of reflections at $m = E_{\rm F}$ where the amplitudes of the tunneling conductance and the differential shot noise become zero. For the proposed TI-based FM/SC structure the specular electron–hole conversion can happen in a wide experimentally accessible range of the electric field.

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