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Jianfeng Chen(陈剑锋), and Zhi-Yuan Li(李志远)

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TOPICAL REVIEW — Celebrating 30 Years of *Chinese Physics B*

Topological photonic states in gyromagnetic photonic crystals: Physics, properties, and applications

Jianfeng Chen(陈剑锋) and Zhi-Yuan Li(李志远)[†]

School of Physics and Optoelectronics, South China University of Technology, Guangzhou 510640, *China*

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Topological photonic states (TPSs) as a new type of waveguide state with one-way transport property can resist backscattering and are impervious to defects, disorders and metallic obstacles. Gyromagnetic photonic crystal (GPC) is the first artificial microstructure to implement TPSs, and it is also one of the most important platforms for generating truly one-way TPSs and exploring their novel physical properties, transport phenomena, and advanced applications. Herein, we present a brief review of the fundamental physics, novel properties, and practical applications of TPSs based on GPCs. We first examine chiral one-way edge states existing in uniformly magnetized GPCs of ordered and disordered lattices, antichiral one-way edge states in cross magnetized GPCs, and robust one-way bulk states in heterogeneously magnetized GPCs. Then, we discuss the strongly coupling effect between two co-propagating (or counter-propagating) TPSs and the resulting physical phenomena and device applications. Finally, we analyze the key issues and prospect the future development trends for TPSs in GPCs. The purpose of this brief review is to provide an overview of the main features of TPSs in GPC systems and offer a useful guidance and motivation for interested scientists and engineers working in related scientific and technological areas.

Keywords: topological photonic states, gyromagnetic photonic crystal, one-way edge states

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1. Introduction

Photonic crystals (PCs) are artificial microstructures composed of materials with different refractive indexes arranged in a certain period.[1–4] Its emergence provides a powerful and effective platform to control the transport of light and electromagnetic (EM) waves and the interactions between light and matter. However, according to the traditional photonics theory, the reciprocity (*i.e.*, light-transport reversibility) principle naturally implies the existence of strong backscattering when light and EM waves transmit through a bent PC waveguide. Besides, limited by modern lithography technology, the structural imperfections caused by fabrication errors will further cause serious backscattering loss. Consequently, it has become an urgent problem to explore and find a new physical mechanism and scheme to solve the backscattering loss in PC systems. It turns out that topological photonics can provide an effective way to attack this critical issue and become the promising answer to the question.[5,6]

Topology is a subject that studies the global properties of geometric figures or structures that can remain unchanged after continuously deforming.[7] The concept of topology was extended to photonics, which has opened up a new frontier field called topological photonics.[5,8–10] It is found that topological photonic states (TPSs) possess one-way transport property, and they are strongly robust against defects, obstacles, and disorders on the transport path, because these imperfections can only induce the change of local properties instead of global properties of topological PCs.[11–14] Therefore, compared with traditional nontopological PCs, the transport of light and EM waves in topological PCs shows a strong robustness against defects and impurities, and also has a very high tolerance to preparation imperfections.[15–19]

Topological photonics has remained an active field of research in science and engineering over the past 14 years after the first reports of TPSs, and some milestone works in various topological photonic systems have been reported and achieved great successes[20–25] For example, in 2008, Haldane *et al.* first theoretically proposed that chiral one-way edge states can be constructed by analogy to the integer quantum Hall effect in two-dimensional (2D) electron gas system,[26] and then some studies experimentally realized such one-way edge states in square and honeycomb gyromagnetic photonic crystals (GPCs).[27–31] These edge states display chirality, *i.e.*, unidirectionally transporting along opposite directions at two parallel edges of bulk GPC. Subsequently, by analogy with quantum spin-Hall effect, Hafezi *et al.* created photonic pseudo-spin by using the coupled ring resonator and realized spin-Hall TPSs by tuning the coupling coefficient of adjacent resonators.[32] Furthermore, inspired by quantum valley-Hall effect, Dong *et al.* introduced valley freedom into dielectric PCs based on electromagnetic duality theory and implemented TPSs with valley locking.[33] It is noteworthy that distinct with

[†]Corresponding author. E-mail: phzyli@scut.edu.cn

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the TPSs in other nonmagnetic systems, *e.g.*, valley-Hall PCs, spin-Hall PCs, TPSs based on GPCs with time-reversal symmetry breaking are still the most reliable solutions for robustly nonreciprocal EM wave transport, because only such systems support the truly one-way transport.

In addition to realizing TPSs in different systems, a plenty of studies have also focused on verifying basic topological concepts, such as large Chen number,[34–36] Weyl point,[37–40] higher-order corner state,[41–44] three-dimensional (3D) photonic topological insulator,[45,46] *etc*. Some topological photonic functional devices with excellent performance also have been designed and implemented, such as multimode one-way waveguides,[34,35] topological lasers,[47] topological delay lines,[48,49] topological beam splitters,[50–52] *etc*. Moreover, nonlinear effect also can be introduced into the field of topological photonics to develop new platform of nonlinear topological photonics, which provides an effective implementation scheme for realizing laser frequency conversion, stable mode frequency selection, no feedback parametric amplification, ultrafast optical switching, and other functionalities.[53–59] Recently, some results have shown the possibility of non-Hermitian control over TPSs, which may find applications in active topological photonic devices.[60–65]

GPC is the first system to observe the existence of TPSs, and is also one of the most powerful platforms to study the generation and interaction of TPSs and explore novel topological phenomena in photonics and electromagnetics. Herein in this mini review paper, we wish to focus on the research progress of TPSs in GPCs in combination with our recent works. In Section 2, chiral one-way edge states in uniformly magnetized GPCs of ordered and disordered lattices are reviewed. In Section 3, antichiral one-way edge states in cross magnetized GPCs and robust one-way bulk states in heterogeneously magnetized GPCs are introduced. In Section 4, the strongly coupling between two co-propagating/counter-propagating TPSs and the resulting functional devices are discussed. In Section 5, some novel topological applications are briefly described. Finally, we intend to summarize the key challenges and opportunities of TPSs in GPCs from both perspectives of fundamental physics and practical applications.

2. Chiral topological photonic states

We start with summarizing the chiral TPSs (or one-way edge states) in uniformly magnetized GPCs of 2D ordered photonic lattices of gyromagnetic cylinders. In 2008, Haldane *et al.* first theoretically predicted that by analogy with quantum Hall effect in electronics,[26] by applying external magnetic field along off-plane vertical direction, the time-reversal symmetry of a PC composed of gyromagnetic material is broken, as a result of which the degeneration of two Dirac points is lifted up and a complete topological bandgap supporting a pair of chiral one-way edge states emerges. Besides, the topological property of band can be characterized by topological invariants (or Chern number), which is defined as[66]

$$C_n = \frac{1}{2\pi}\int_{\mathrm{BZ}} \boldsymbol{\Omega}(\boldsymbol{k})\,\mathrm{d}^2k, \tag{1}$$

where $\boldsymbol{\Omega}(\boldsymbol{k}) = \nabla_{\boldsymbol{k}} \times \boldsymbol{A}^{nm}(\boldsymbol{k})$ is the Berry curvature, $\boldsymbol{A}^{nm}(\boldsymbol{k}) = \mathrm{i}\langle u_{n\boldsymbol{k}}|\nabla_{\boldsymbol{k}}|u_{n\boldsymbol{k}}\rangle$ is the Berry connection, and $u_{n\boldsymbol{k}}$ is the periodic part of the Bloch function of eigenmodes in the n-th photonic band. The detailed calculation methods for Chern number in PC systems can be seen in references.[67,68] Subsequently, Wang *et al.* found that the chiral one-way edge states induced by special topological property of band also exist in a square-lattice GPC without the restriction of Dirac points.[27] By applying the external magnetic field along the off-plane vertical direction, the degeneracy between the second and third bands at the M point was broken, and a complete bandgap characterized by a nonzero Chern number is obtained. As illustrated in Fig. 1(a), they calculated and measured the reflection-free one-way transport of microwave signal in a line-defect waveguide between a square GPC and a metal plate.[28] Furthermore, Fu *et al.* also verified such chiral one-way edge states at the interface between a square GPC with a nontrivial bandgap and an Al_2O_3 PC of a triangular lattice with a complete trivial bandgap.[29] The experimental results illustrated that the edge states only propagate in one direction along the boundary of GPCs, even if there exist metallic obstacles and defects on the transport path.

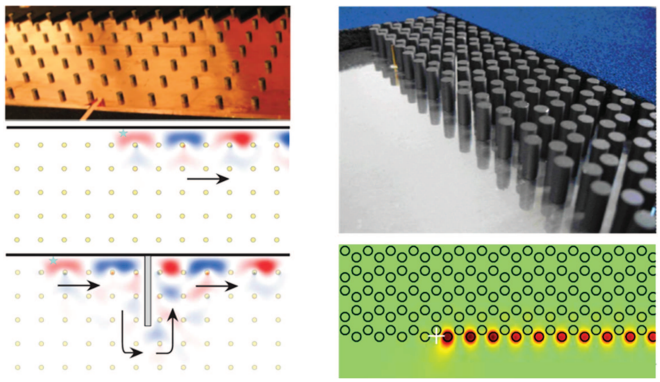


Fig. 1. Chiral one-way edge states in uniformly magnetized gyromagnetic photonic crystal of ordered lattices. (a) Square lattice.[27,28] Reproduced with permission from Refs. [27,28]. (b) Honeycomb lattice.[30] Reproduced with permission from Refs. [30].

The gyromagnetic material is commercially available, *e.g.*, with yttrium iron garnet (YIG) being a prominent candidate, which has measured relative permittivity about $\varepsilon = 14.5$–16.0 and $\mu = 1$ respectively in the absence of external magnetic field. When an external magnetic field is applied along the out-of-plane ($+z$) direction, there induces a strong gyromagnetic anisotropy in YIG cylinders so that their permeability becomes a tensor as follows:[27,29,30]

$$\mu = \begin{pmatrix} \mu_r & \mathrm{i}\mu_k & 0 \\ -\mathrm{i}\mu_k & \mu_r & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2}$$

where

$$\mu_r = 1 + \frac{(\omega_0 + \mathrm{i}\alpha\omega)\,\omega_m}{(\omega_0 + \mathrm{i}\alpha\omega)^2 - \omega^2}, \quad \mu_k = \frac{\omega\omega_m}{(\omega_0 + \mathrm{i}\alpha\omega)^2 - \omega^2},$$

$\omega_m = \gamma M_s$, $\omega_0 = \gamma H_0$, H_0 is the external magnetic field, M_s is the saturation magnetization, γ is the gyromagnetic ratio, α is the damping coefficient, and ω is the operating frequency.

Since the early theoretical proposal and experimental demonstration, chiral one-way edge states have been extended to other GPC structures. For instance, in 2009, Ao *et al.* theoretically designed a GPC of a honeycomb lattice to make the dispersion curve of chiral one-way edge states fall below the light cone, so as to realize the chiral one-way edge states in which the energy fluxes are completely localized on the edge.[31] Later on, Poo *et al.* completed the experimental measurement of such perfect chiral one-way edge states, and verified that the chiral one-way edge states was insensitive to the perturbations on the edge even if the edge was exposed to free space,[30] as shown in Fig. 1(b). Besides, in 2012, Liu *et al.* proposed that the chiral one-way edge state also could be sustained by the edge of a uniformly magnetized GPC slab formed by a triangular lattice of air holes in a gyromagnetic slab.[69] The calculation results indicated that the transport of chiral one-way edge states is concentrated horizontally to the slab edge, and there is no need to place metal plates on the upper and lower parts of GPC slab to prevent the leakage of EM waves along the vertical direction.

In addition to the chiral TPSs realized in ordered lattice, an amount of studies have realized chiral TPSs in different types of disordered GPCs. This is attributed to the fact that the unique properties of PCs are not only determined by long-range Bragg scattering produced from periodic lattice, but also affected by short-range local EM response of unit cell. For PCs made of conventional dielectric materials (such as silicon, silicon carbide, *etc.*), the local EM response of unit cell is negligible and Bragg scattering is dominant. However, when unit cell is composed of some materials with strong EM response (such as gyroelectric and gyromagnetic materials), the local EM response will play a critical role in some scenarios. At the early stage, most of works focused on the chiral one-way edge states in a topological bandgap generated by Bragg scattering.

Until 2011, Liu *et al.* first theoretically proposed that chiral one-way edge states can also exist near the resonance frequency of magnetic surface plasmons of gyromagnetic materials, and demonstrated the transport of chiral one-way edge states in a disordered GPC,[76] as shown in Fig. 2(a). Subsequently, Fu *et al.* first experimentally measured the chiral one-way edge states based on the magnetic surface plasmon resonance, and analyzed the transport properties of the chiral one-way edge states generated by long-range Bragg scattering and short-range local magnetic surface plasmon resonance.[71] Their simulation and measurement results indicated that the chiral one-way edge states originated from these two effects both exhibit the reflection-free properties. However, the chiral one-way edge states generated by long-range Bragg scattering are very sensitive to lattice perturbations, while the chiral one-way edge states induced by short-range local magnetic surface plasmon resonance are very robust against the lattice disorder. This feature is attributed to the fact that EM waves can transport forwards through the magnetic surface plasmon resonance modes between adjacent gyromagnetic cylinders. As shown in Fig. 2(b), even if there is only a row of disordered gyromagnetic cylinders, chiral one-way edge states generated by short-range local magnetic surface plasmon resonance can still show one-way transport property. Besides, the chiral one-way edge states based on magnetic surface plasmon resonance are more easily modulated by external magnetic field, while the chiral one-way edge states generated by Bragg scattering have a weak response to external magnetic field.

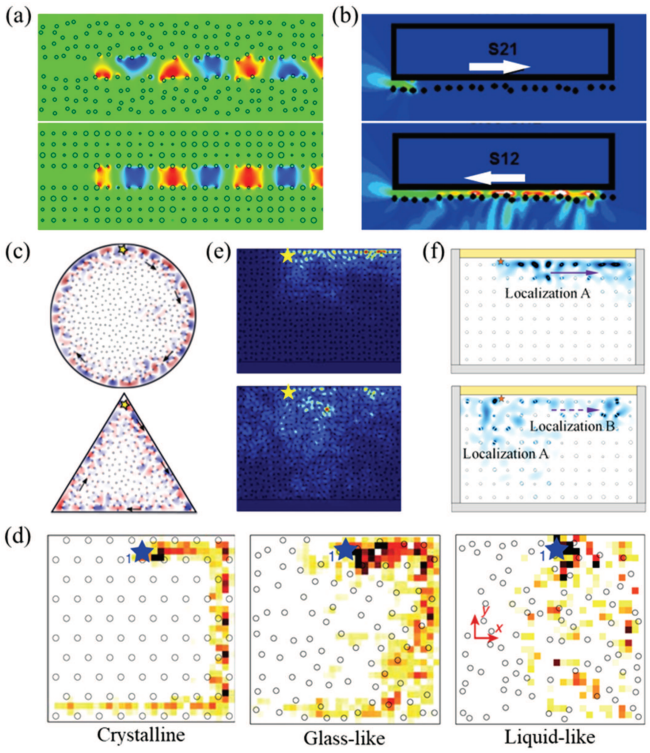


Fig. 2. Chiral one-way edge states in uniformly magnetized gyromagnetic photonic crystal of disordered lattices. (a) Position-disordered and radius-fluctuated GPCs.[70] Reproduced with permission from Ref. [70]. (b) Single-row disordered gyromagnetic cylinders.[71] Reproduced with permission from Ref. [71]. (c) Circular and triangular amorphous GPCs.[72] Reproduced with permission from Ref. [72]. (d) Crystalline, glass-like and liquid-like GPC.[73] Reproduced with permission from Ref. [73]. (e) Position-disordered GPCs.[74] Reproduced with permission from Ref. [74]. (f) Radius-fluctuated GPCs.[75] Reproduced with permission from Ref. [75].

In 2019, Yang *et al.* designed a supercell with internal disorders, and then used such a supercell as a unit cell to construct a GPC.[74] The calculation results illustrated that such GPC also produces a bandgap supporting the chiral one-way

edge states, and the single-mode or multi-mode one-way edge states can be realized. Besides, as plotted in Fig. 2(c), Mansha *et al.* used molecular dynamics algorithm to generate circular and triangular amorphous GPC and observed chiral one-way edge states with strong transport robustness when the short-range order is sufficiently high.[72] Subsequently, Zhou *et al.* further observed the solid–liquid phase transition from crystalline state (with both long-range order and short-range order) to amorphous glass state (only short-range order) and then to liquid state (highly disordered).[73] As illustrated in Fig. 2(d), by gradually deforming the amorphous lattice into a liquid-like lattice through the glass transition, they directly measured the closing of the mobility gap and the disappearance of chiral one-way edge states. These results showed the key role of short-range order in the formation of TPSs. Moreover, as plotted in Figs. 2(e) and 2(f), Yang *et al.*[77] and Chen *et al.*[75] respectively demonstrated that chiral one-way edge states can exist in position-disordered and radius-fluctuated GPCs. The calculation results showed that photonic topological bandgaps are far more sensitive to disorders with a radius fluctuation than with a position randomness. Very recently, as illustrated in Fig. 3, Liu *et al.* also revealed a counterintuitive case, *i.e.* through randomly rotating the dielectric pillars in each unit cell to add disorders one can turn a trivial PC into a nontrivial PC, called as photonic topological Anderson insulator.[78] These theoretical and experimental results have drawn enormous interests in the role of disorder playing in GPCs.

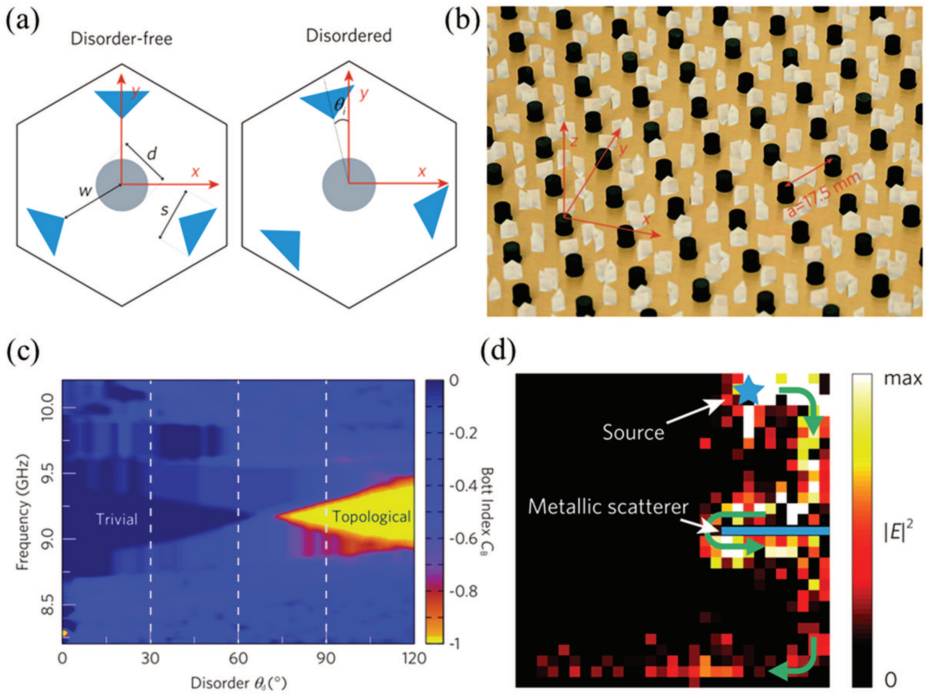


Fig. 3. Photonic topological Anderson insulator.[78] (a) Unit cell of the disorder-free and disordered GPCs. (b) Photograph showing a portion of the sample. (c) Simulated Bott index C_B. (d) One-way transport in a sample with a large obstacle. Reproduced with permission from Ref. [78].

3. Antichiral topological photonic states

The TPSs discussed in above section possess chirality that the one-way edge states can only propagate clockwise or counterclockwise along the boundary of uniformly magnetized GPCs. For instance, when the whole GPC is a rectangular or parallelogram, the one-way edge states propagate in opposite directions at two parallel boundaries but not in the same direction. In 2020, Chen *et al.* reported another possibility: the one-way edge states at two opposite parallel boundaries can propagate in the same direction, and these peculiar states are called antichiral one-way edge states,[79] as shown in Fig. 4(a). They theoretically proposed that such antichiral one-way edge states can be realized in a GPC of a honeycomb composed of two interpenetrating triangular sublattices A and B. When the sublattices A and B are immersed in external magnetic fields of opposite directions (*i.e.* cross magnetized), respectively, two Dirac points of GPC move up and down respectively but without opening a bandgap, leading to the tilt of overall band structure and the generation of antichiral one-way edge states. The calculated projected band structure indicated that these two one-way edge states propagate in the same direction at two parallel edges. Note that, as required by the law of energy power conservation, where the number of leftward and rightward states of the whole system must be the same, there also emerge two counterpropagating associated one-way bulk states that belong to the bulk of GPC and are spatially separated from the edge states. Therefore, when a line source is set at the boundary, almost only the antichiral one-way edge

states will be excited. Since the slope of dispersion curve of antichiral one-way edge state is positive, so EM waves will only transmit rightwards and are strongly robust against the metallic obstacles and defects on the transport path. This work also indicated that a deeper exploration of TPSs in antichiral GPC systems can help to find more rich, novel, and meaningful topological physics, and provide useful guidance for the design of novel topological photonic devices with excellent performance. For example, a compact three-channel one-way waveguide can be constructed by using two antichiral GPCs, which cannot be achieved in two chiral GPCs. Subsequently, as plotted in Figs. 4(b) and 4(c), Zhou *et al.* directly observed the existence of antichiral one-way edge states in GPCs in an experimental sample[80] which is similar to the scheme theoretically designed by Chen *et al.*[79]

In 2022, Chen *et al.* further theoretically and experimentally observed that antichiral one-way edge states exist only at the zigzag edge but not at the armchair edge of antichiral GPC of a honeycomb lattice.[52] Using this feature, they combined two rectangular antichiral GPCs holding left- and right-propagating antichiral one-way edge states respectively to realize the bidirectionally radiating one-way edge states at two parallel zigzag edges. Such unique bidirectionally radiating phenomenon can be utilized to design a topological beam splitting with the easily adjustable right-to-left ratio, as shown in Fig. 5(a). Their results revealed that such a splitter is compact and configurable, has high transmission efficiency, allows for multi-channel utilization, crosstalk-proof, and robust against defects and obstacles. More importantly, it is not possible to realize such a splitter in other well-studied PC systems, such as trivial PC,[81] valley-Hall PC,[82] spin-Hall PC,[83] and chiral GPC.[84] Besides, Zhang *et al.* also theoretically realized reconfigurable light imaging in an antichiral GPC with both broken time-reversal and inversion symmetries,[85] as plotted in Fig. 5(b). Consequently, the realization of antichiral one-way edge states not only broadens the current understanding in the field of topological photonics, but also provides important guidance for further exploration of novel topological photonic devices based on antichiral one-way edge states.

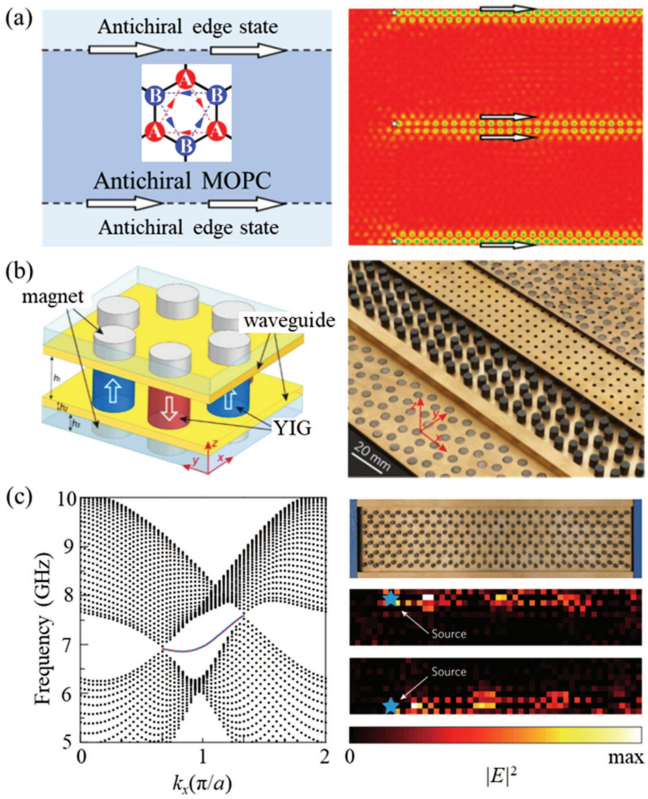


Fig. 4. Theoretically proposed and experimentally observed antichiral one-way edge states in a cross magnetized GPC. (a) Schematic diagram of antichiral one-way edge states and a compact three-channel one-way waveguide.[79] Reproduced with permission from Ref. [79]. (b) Design and implementation of antichiral GPC.[80] Reproduced with permission from Ref. [80]. (c) Projected band structure and measured antichiral one-way edge states.[80] Reproduced with permission from Ref. [80].

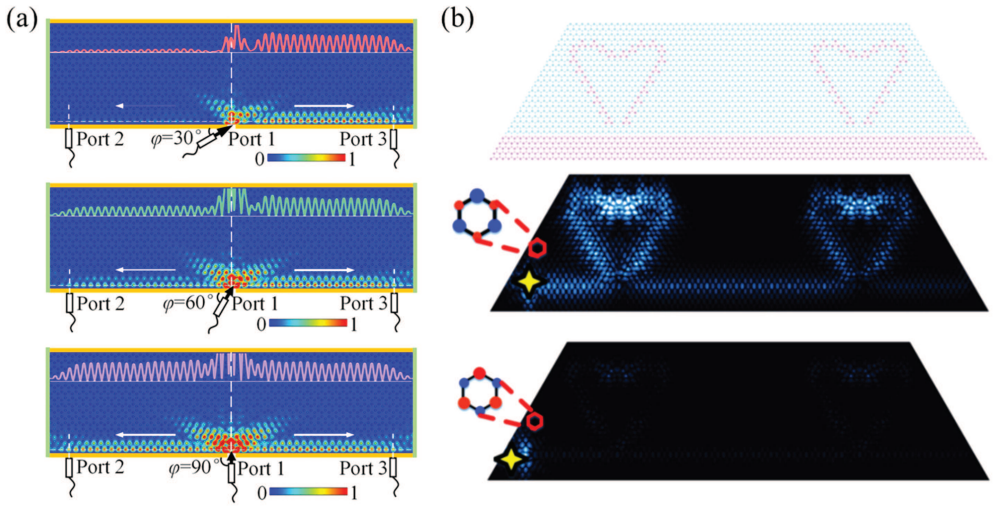


Fig. 5. Topological beam splitting and reconfigurable light imaging. (a) Electric field distributions in simulation of variable-ratio topological beam splitting.[52] Reproduced with permission from Ref. [52]. (b) Versatile reconfigurable light imaging in a two-heart complex pattern.[85] Reproduced with permission from Ref. [85].

As mentioned above, chiral and antichiral one-way edge states brought via uniformly and cross magnetized GPCs with time-reversal symmetry breaking are still the most reliable solutions for the robust one-way transport, because they provide truly one-way, reflection-free transport at the edges of GPCs. However, such behavior inherently limits the high-throughput robust one-way transport to a low level, because only a small area around the edge is utilized to collect and transfer EM waves, which also greatly sacrifices the space utilization of sample. To implement the robust one-way transport in a large cross-sectional area rather than only limited at an edge, Chen *et al.* established a heterogeneous magnetized GPC to realize a robust one-way bulk states and verify their transport robustness against metallic obstacles intruded in the bulk of GPC,[86] as shown in Fig. 6(a). The calculated projected band structure showed that by applying heterogeneous magnetization to GPC, the antichiral one-way edge states and associated one-way bulk states can be separates from the trivial bidirectional bulk states appeared in cross magnetized GPCs, as illustrated in Fig. 6(b). As a result, the antichiral one-way edge states and associated one-way bulk states can be selectively excited under suitable excitation conditions. Beyond the demonstration of one-way transport, they also theoretically demonstrated a robust one-way transmission line with long-distance, large-area, and high-throughput in a 2D open space, as plotted in Fig. 6(c).

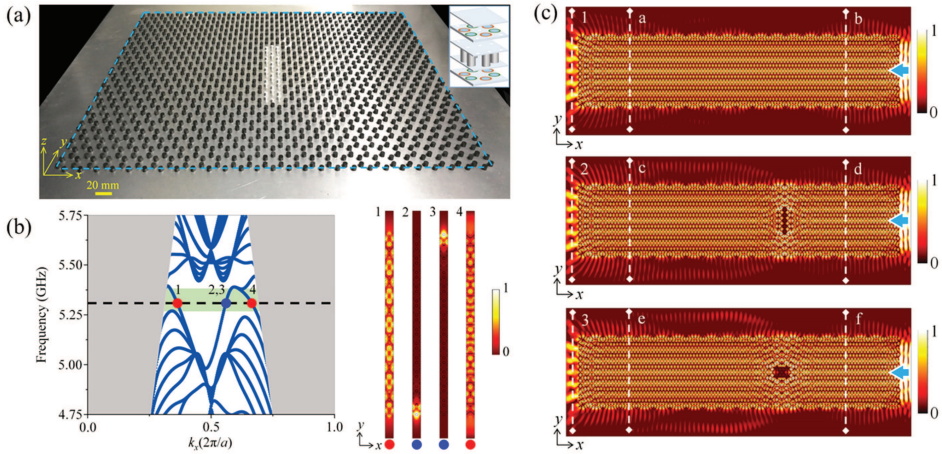


Fig. 6. Robust one-way edge states in a heterogeneous magnetized GPC.[86] (a) Photograph showing a portion of the sample. (b) Projected band structure and eigenmodal field distributions. (c) Long-distance, large area, high throughput, and robust one-way bulk states. Reproduced with permission from Ref. [86].

4. Strong coupling between two chiral one-way edge states

To further explore the rich physics, novel properties, and potential applications of TPSs in GPCs, many functionalities and prototypes of photonic devices derived from the interactions between TPSs have been presented in recent years, *e.g.*, topological dispersionless slow light,[87,88] switchable slow light rainbow trapping,[89] and other functionalities.[90–92] It has been gradually understood and appreciated that the GPCs possess three unique advantages for investigating the interactions between TPSs: (i) each edge supports the single-mode one-way edge states, which is conducive to realize the coupling between TPSs of distinct configurations; (ii) it can produce imperfect localized edge states so that each GPC can couple part of energy fluxes of another GPC to its own edge and propagate forwards; (iii) the transport directions of one-way edge states can be tuned by reversing the external magnetic field, so distinct coupling types between TPSs can be constructed in one GPC system. Here we will mainly give a brief introduction to review two typical configurations, *i.e.*, the coupling between two co-propagating chiral one-way edge states or two counter-propagating chiral one-way edge states in GPC waveguides, and discuss the related physical phenomena and functional applications resulting from these coupling effects.

In 2019, Chen *et al.* investigated the strong coupling of two counter-propagating TPSs in a line-defect GPC waveguide channel constructed by bringing close two identical GPCs which each supports a counter-propagating one-way edge state,[93] as shown in Fig. 7(a). They found that with the decreasing of waveguide width, the counter-coupling strengths between two TPSs increases, so that the dispersion curves of guided modes in the middle of the first Brillouin zone exhibit a continuous deformation of concave-band, flat-band and convex-band, as illustrated in Fig. 7(b). Especially, as the critical point of band inversion, the flat-band dispersion curve can produce a very small group velocity $v_g = 1.89 \times 10^{-3}c$ (or a very large group index about $n_g = 529.2$) and a zero/near-zero group velocity dispersion (GVD) simultaneously, meaning that an EM wave pulse can slowly transport along the waveguide channel in a distortionless way, as plotted in Figs. 7(c) and 7(d). They also revealed that such a unique group-dispersionless slow-light state originates from the complete exchange and transfer of energy flow between two counter-propagating chiral one-way edge states. In addition to the

slow-light property, they also disclosed that the frequency of the flat-band can be easily modulated by tuning the external magnetic field. When the external magnetic field increases from 1000 Gs to 1900 Gs (1 Gs = 10^{-4} T), the flat-band moves up from 0.465 to 0.567 ($2\pi c/a$) and the tunable normalized bandwidth reaches 19.8%. Such an apparent modulation effect is very beneficial to ease the contradiction between slow-light transport and broad bandwidth to some extent.

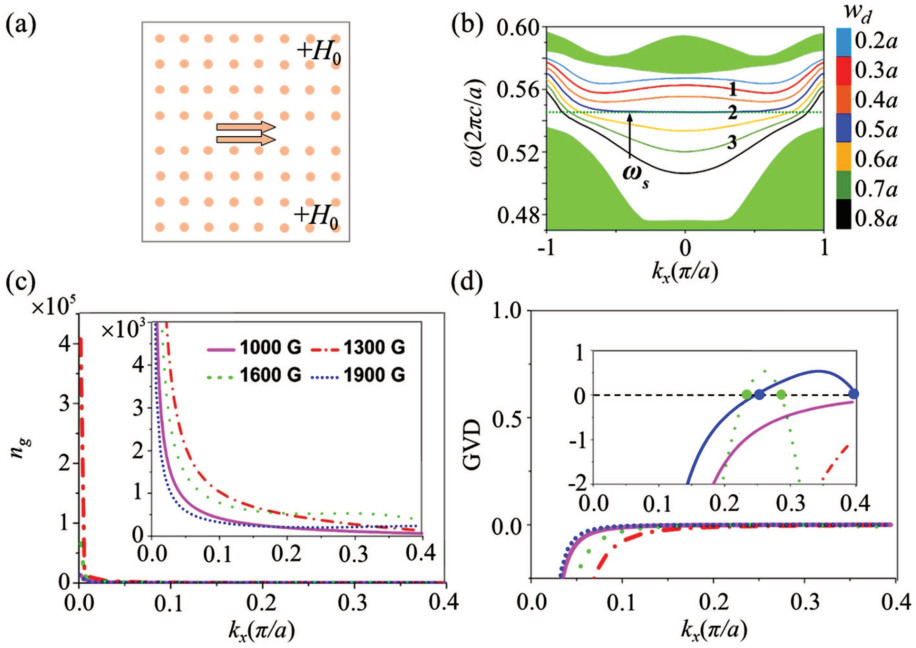


Fig. 7. Group-dispersionless slow-light via strongly counter-coupling between two chiral one-way edge states.[93] (a) Schematic diagram. (b) Projected band structures with various waveguide widths. (c) Group index. (d) Group velocity dispersion. Reproduced with permission from Ref. [93].

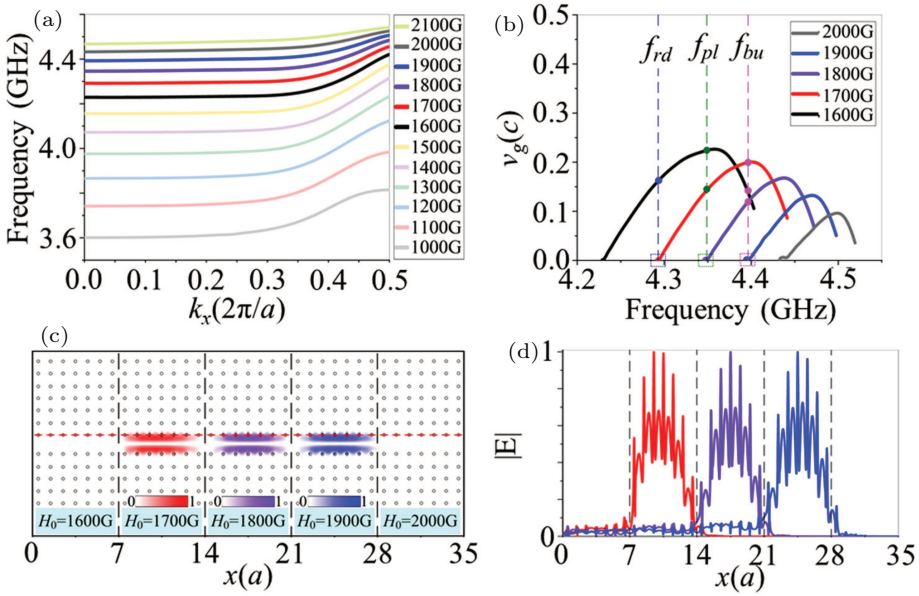


Fig. 8. Slow-light rainbow trapping.[89] (a) Flat band under different H in the half-Brillouin zone. (b) Group velocities of different flat bands. (c) Electric field profiles at three frequencies. (d) Electric field amplitudes along the upper boundary as shown in panel (c). Reproduced with permission from Ref. [89].

Through the magnetically tunable property of such flat-band shown in Figs. 8(a) and 8(b), in 2019, Chen *et al.* applied a gradient magnetic field to a long GPC waveguide, so that different frequency components of a broadband wave packet can be separated at different positions to form a slow-light rainbow trapping.[89] Interestingly, such rainbow trapping not only can store the EM waves at the waveguide channel for a long temporal duration, but also possesses a high spatial precision so that there is almost no crosstalk and overlap between the trapped field of different frequencies, as shown in Figs. 8(c) and 8(d). Besides, by tuning the external magnetic field, the slow-light rainbow trapping state also can be easily switched among forbidden state, trapped state and releasing state. Nevertheless, from a practical point of view, the gradient magnetic field is difficult to implement in practice, so it is particularly important to provide a more convenient solution to

form a broadband tunable slow light rainbow trapping. To attack this issue, in 2021, Chen *et al.* presented a novel scheme to realize a slow-light rainbow trapping in a waveguide channel consisting of hybrid GPCs, which are of distinct geometric configurations but immersed in a uniform external magnetic field.[91] Besides, they also demonstrated that the slow-light rainbow trapping can be remotely and nonreciprocally controlled. These results show that in GPC waveguide system, the strong coupling between two TPSs can be apparently and arbitrarily regulated by a series of means, such as modulating the radius of gyromagnetic cylinder and the width of waveguide channel, tuning the direction and intensity of external magnetic field.

Unfortunately, such slow-light waveguide supports two symmetric opposite propagating one-way edge states with mutual coupling, and it is not topologically protected, so the transport of EM waves is not robust against backscattering from the imperfections on the transport path. In 2020, Chen *et al.* further proposed a scheme to implement a topologically protected one-way slow-light state by harnessing the strong coupling between two co-propagating TPSs in a GPC waveguide.[88] They also revealed that such unique one-way slow light originates from an eight-shaped energy flowing loop within each unit cell of waveguide, leading to a low group velocity ($v_g = 7.5 \times 10^{-2}c$), a vanishing group velocity dispersion, a broad bandwidth of 3.08% and a large normalized delay-bandwidth product (about 0.409). These results illustrate that in such a waveguide, broadband electromagnetic pulse signals can pass through metallic obstacles and transmit slowly forwards without any backscattering and distortion.

5. Device applications

To further explore the potential applications of TPSs in various topological photonic systems, an amount of functionalities and prototypes of topological photonic devices have been implemented, including but not limited to multimode one-way waveguides,[34,35] high-throughput one-way waveguides,[86,94] topological antennas,[95–97] topological one-way delay-lines,[48,88,90,98,99] topological beam splitters,[52,84] topological laser,[47] and topological one-way fiber.[100] In this section, we will mainly give a brief introduction of two typical topological photonic applications in GPCs, topological laser[47] and topological one-way fiber.[100]

5.1. Topological laser

For nearly 60 years since the advent of laser, it has benefited from the rapid development of modern scientific and technological progress and the breakthrough progress of various related technologies, materials, and processes. Laser has played an indispensable role in the national economic development of countries around the world, especially in the technological application fields of optical communication, optical sensing, optical data storage, optical medical treatment, and so on. In 2017, Bahari *et al.* drew the idea of topological photonics into the design of microcavity, constructed an irregular closed ring microcavity that can support one-way transport of light and EM waves, and realized a single-mode topological laser,[47] as illustrated in Fig. 9.

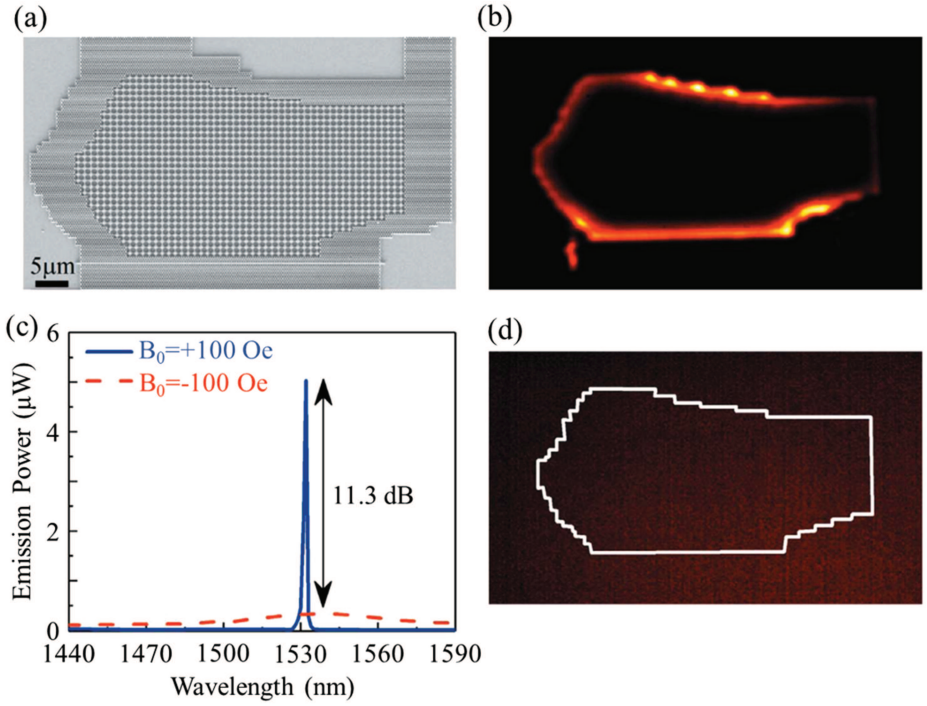


Fig. 9. Topological laser.[47] (a) Top-view SEM of a fabricated arbitrarily shaped topological cavity. (b) Real-space camera image of the top of the device under high-power optical pumping. (c) Photoluminescence spectrum of the topological cavity. (d) Edge mode disappears when the external magnetic field is turned off. Reproduced with permission from Ref. [47].

Such a topological microcavity is composed of two GPCs of square and triangular lattices respectively. By applying an external magnetic field along the vertical direction, the inner irregular closed ring microcavity composed of a square GPC shows nontrivial topological property, while the external photonic crystal of triangular lattice is trivial, so that the interface between these two types of GPCs can form a cavity of topological laser. Their experimental results indicated that the TPSs in such a topological microcavity are immune to defects, impurities, and geometric configurations, and the isolation ratio of TPSs coupled to the output waveguide exceeds 10 dB. This landmark work provides important guidance for various topological lasers emerging in recent years, and paves the way for the realization of diversified topological photonic devices with high performance in the future.

5.2. Topological one-way fiber

As an important and unique waveguide structure, optical fiber shows the excellent properties including but not limited to large transmission capacity, strong confidentiality, and long relay distance. However, there still are some fundamental problems in traditional optical fiber that need to be solved. For example, due to the existence of backscattering channels, the transport of light and EM waves will meet strong backscattering loss due to some inevitable reasons, such as nonuniformity or nonlinear effect of materials, unevenness of end face, which greatly reduces the transmission efficiency. To solve this fundamental problem in physical principle, in 2018, Lu *et al.* theoretically proposed and designed a topological one-way fiber supporting the nonreciprocal transport of light and EM waves by constructing a 3D Weyl GPC,[100] as shown in Fig. 10. Distinct from traditional fiber, the backscattering channel of topological one-way fiber is closed, so light and EM waves can bypass impurities, obstacles or defects of distinct shapes without any backscattering. The construction of such topological one-way fiber can be divided into two steps: (i) constructing a 3D Weyl GPC by spatially modulating the duty cycle of periodic structure; (ii) annihilating the Weyl points to create a 3D topological bandgap and separating the forward and backward modes, so that a line-defect is generated at the core of topological fiber to support unidirectional transmission. Note that, the structure of topological one-way fiber is much more complex than that of traditional fiber, and it is still difficult to manufacture practical topological fiber by current preparation technology. Besides, it is also difficult to expand its working frequency to optical band because the gyromagnetic effect of gyromagnetic materials in optical band is too weak. However, it is undeniable that as a typical example, topological one-way fiber skillfully combines various research achievements in the field of topological photonics, provides a unique solution to the actual transmission problems of light and EM waves, and further promotes the process of topological photonics from theoretical exploration to practical application.

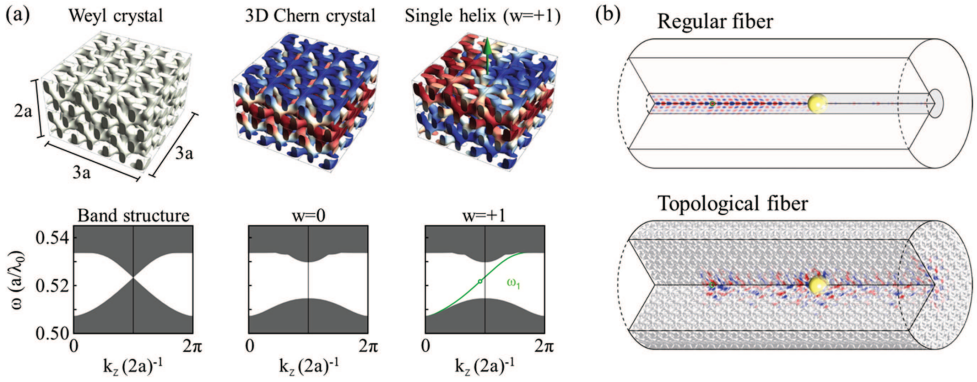


Fig. 10. Topological one-way fiber.[100] (a) Schematic diagram of distinct unit cells and the corresponding band structures. (b) Regular fiber and topological fiber. Reproduced with permission from Ref. [100].

6. Summary and outlook

In summary, we have given a brief review of fundamental physics, novel properties, and practical applications of TPSs based on GPCs. We have introduced chiral and antichiral one-way edge states in uniformly magnetized GPCs of ordered and disordered lattices. We also have discussed the realization of antichiral one-way edge states in a magnetized GPC where the sublattices A and B are immersed in external magnetic fields along the opposite directions, and described robust one-way bulk states in a heterogeneously magnetized GPC. Moreover, we have reviewed the strong coupling between two co-propagating/counter-propagating TPSs and related functional devices via the strong coupling. Furthermore, we have introduced two prototypical examples (topological laser and topological one-way fiber) to exhibit the novel topological applications based on GPCs.

Although in many issues encompassing the initial prediction, the observation, and the realization of novel topological photonic devices, the topological photonics communities have witnessed the great success in the study of TPSs in GPCs, there are still many key issues that need to be solved. For

instance, as the cyclotron resonance frequency of gyromagnetic materials is proportional to the intensity of external magnetic field so that optical materials with strong magnetic-optic effect are very rare, it is difficult to expand such topological systems to the optical band. Besides, although the material loss can be reduced by keeping the excitation frequency away from the resonance frequency of gyromagnetic materials, it also virtually narrows the available frequency window. Therefore, how to manufacture gyromagnetic materials with low loss still remains a very important issue. To solve these practical problems, it is urgent to find other new types of gyromagnetic materials with low loss and high frequency response that can replace current gyromagnetic materials like YIG. Moreover, currently, it mainly relies on a large magnet array or Helmholtz coil to provide uniform external magnetic field for GPCs, which greatly increases the structural volume and casts a shadow on the realization of compact or integrated system, so another urgent task is how to realize compact GPC system. Furthermore, how to achieve uniform magnetic field distribution in different regions within a limited range without mutual interference is also a stumbling block to further excavate novel topological phenomena in GPCs. Aiming to realize the compact and portable external magnetic field, a relatively effective scheme at present is to apply external magnetic field to each gyromagnetic cylinder by using a pair of magnets, although this scheme cannot produce an absolutely uniform external magnetic field.

Although the development of TPSs in GPCs still faces many key problems that need to be solved, this does not mean that the research on TPSs in GPCs will come to a stop. History has proved for many times that there are rich physical properties and physical phenomena behind the TPSs in GPC system. By deeply exploring novel topological photonic phenomena and comprehensively mastering their regulation mechanism, one can not only provide rich experience for other topological photonic systems, but also provide useful guidance for designing various topological photonic devices, such as topological one-way waveguides, topological lasers, topological one-way fiber, topological antenna, topological slow light, *etc*. Besides, it is expected that topological photonics can offer promising means to transform and upgrade the current key microwave optoelectronic devices (such as microwave circulator and splitter[101,102]) to obtain more excellent performance. Finally, on the fundamental physics aspects, although TPSs in GPCs are well studied in Hermitian and linear systems, there are still much to be explored by further introducing non-Hermitian and/or nonlinear effect.[103] It can be predicted that there are still a series of important problems worth further exploration, such as interactions between TPSs and other topological and nontopological photonic states,[90] gyromagnetic high-order TPSs,[41,85] 3D GPCs,[45] *etc*. Solving these important problems will not only help to deepen the understanding of some basic principles of topological photonic system, but also greatly enrich the research connotation of topological photonics. These will surely further expand novel phenomena, technologies and methods based on TPSs, and provide important guidance for the design of various topological photonic devices.

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