# Fast population transfer with a superconducting qutrit via non-Hermitian shortcut to adiabaticity

Xin-Ping Dong(董新平), Zhi-Bo Feng(冯志波)†, Xiao-Jing Lu(路晓静), Ming Li(李明), and Zheng-Yin Zhao(赵正印)

*School of Science, Xuchang University, Xuchang* 461000, *China*

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Non-Hermitian dissipation dynamics, capable of turning the conventionally detrimental decoherence effects to useful resources for state engineering, is highly attractive to quantum information processing. In this work, an effective scheme is developed for implementing fast population transfer with a superconducting qutrit via the non-Hermitian shortcut to adiabaticity (STA). We first deal with a Λ-configuration interaction between the qutrit and microwave drivings, in which the dephasing-assisted qubit state inversion requiring an overlarge dephasing rate is constructed non-adiabatically. After introducing a feasible ancillary driving that directly acts upon the qubit states, the target state transfer can be well realized but with an accessible qubit dephasing rate. Moreover, a high fidelity could be numerically obtained in the considered system. The strategy could provide a new route towards the non-Hermitian shortcut operations on superconducting quantum circuits.

**Keywords:** population transfer, superconducting qutrit, non-Hermitian shortcut to adiabaticity

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## 1. Introduction

Behaving as artificial atoms, solid-state superconducting circuits based on Josephson junctions possess some discrete energy levels, from which one can define qubit states for quantum information applications.[1–6] Superconducting qubits, holding many distinctive advantages (such as desirable level structure, convenient microwave control, and easily-realized scalability), have attracted increasing attention in recent years.[7–13] Although some remarkable progress has been made using superconducting qubits, the construction of high-fidelity quantum operations is still a crucial topic in the current field of information processing and state engineering.[14–18] The primary reasons for it are as follows. On the one hand, superconducting qubits are inevitably subjected to their environmental noises, which induce the adverse effect of decoherence in qubits. On the other hand, external drivings are indispensable for performing information processing tasks, in which the fluctuations of control variables unavoidably cause errors to target operations.

To implement fast and robust quantum operation, the technique of STA[19] has been put forward in the Hermitian systems. Compared with the adiabatic case, the powerful method of STA not only accomplishes manipulations in an accelerated way, but also remains the robustness of operations against the fluctuations of control variables and timing errors.[20,21] Due to these inherent advantages, the STA-based operations on superconducting qubits have been investigated both theoretically and experimentally.[22–34] Considering the system dissipation effect, much attention has been further devoted to the technique of non-Hermitian STA theoretically,[35–38] by which the detrimental dissipation could be especially utilized for serving the dynamical evolutions of interest.[39–41] However, in order to make the non-Hermitian effects significantly helpful to dynamical behaviors, the system dissipation generally requires a large rate, giving rise to a severe restriction for some systems without desirable dissipation rates. Therefore, how to realize the STA-based quantum operations on the non-Hermitian superconducting artificial atoms with accessible dissipation rates is an interesting issue.

Motivated by these aspects, we present a theoretical scheme for implementing fast population transfer in a superconducting qutrit by the technique of non-Hermitian STA. Firstly, based on a Λ-configuration interaction between the qutrit and microwave drivings, a non-adiabatic state inversion can be constructed within the framework of non-Hermitian STA, however, the needed qubit dephasing rate is inaccessibly large. To this end, we exert an ancillary driving acting on the qubit states and then achieve a Δ-configuration interaction, in which the target state transfer can be rapidly completed. Owing to the complementarity between the dephasing effect and the ancillary driving, the dephasing rate can be reduced remarkably to be available within the current system. Numerically, we analyze the feasibility of ancillary driving and estimate the operation fidelity. The scheme gives a potential possibility of fast implementing state operations on superconducting qubits by means of the non-Hermitian dissipation dynamics.

†Corresponding author. E-mail: zbfeng010@163.com

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This paper is structured as follows. We present a superconducting quantum circuit driven by microwave fields in Section 2. By the method of non-Hermitian STA, in Section 3 we first address a state transfer within a Λ-type qutrit with a overlarge qubit dephasing rate. By adding a feasible ancillary driving, Section 4 focuses on the qubit state transfer within a Δ-type qutrit with an accessible dephasing rate. Discussion and a conclusion are finally drawn in Section 5.

## 2. A superconducting circuit interacting with microwave drivings

As depicted in Fig. 1, a Cooper-pair box (CPB) circuit under consideration contains a superconducting island with $n$ extra Cooper pairs, in which the charging energy scale of the system is $E_c$. Through two symmetric Josephson junctions (with the identical coupling energies $E_J$ and capacitances $C_J$), the CPB is connected to a segment of a superconducting ring. In the charge-phase regime,[42,43] the characteristic system parameters $E_J$ and $E_c$ have the same order of magnitude, $E_J \sim E_c$. The CPB is biased by a static bias voltage $V_d$ through a gate capacitance $C_d$. A static magnetic flux $\Phi_d$ in the ring aims to adjust the effective Josephson coupling $E_{Jd}$. Meanwhile, ac gate voltages $\tilde{V}_{s,a}$ are respectively applied to the box through gate capacitances $C_{s,a}$, and a time-dependent flux $\tilde{\Phi}_p$ threads the ring as well. These microwave drivings, interacting with the CPB circuit effectively, are used to induce state transitions.[44]

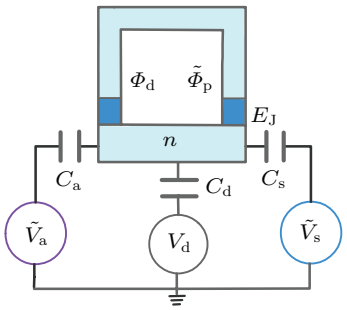



**Fig. 1.** Schematic demonstration of the considered Josephson circuit driven by microwave fields.

The static Hamiltonian of the CPB system is given by $H_0 = E_c(n-n_d)^2 - E_{Jd}\cos\theta$, where the first term stands for the charging energy, and the second one represents the Josephson coupling. The charging energy scale reads $E_c = 2e^2/C_t$, with $C_t(=2C_J + C_d + C_s + C_a)$ being the total capacitance, and the number of polarized gate charge induced by $V_d$ is $n_d = C_d V_d/(2e)$. The effective Josephson coupling is $E_{Jd} = 2E_J\cos(\pi\frac{\Phi_d}{\Phi_0})$, in which $\Phi_0 = h/(2e)$ indicates the flux quantum. Here $\theta$ denotes the average phase difference of the two junctions, which is canonically conjugate to $n$, namely, $[\theta, n] = \mathrm{i}$. Within the basis of Cooper-pair number states $\{|n\rangle, |n+1\rangle\}$ (with $n$ being integer numbers), the Hamiltonian can be rewritten as

$$H_0 = \sum_n \left[E_c(n-n_d)^2 |n\rangle\langle n| - \frac{E_{Jd}}{2}(|n\rangle\langle n+1| + \mathrm{H.c.})\right]. \quad (1)$$

With an effective Josephson coupling $E_{Jd} = 1.5E_c$, the first three eigenenergies $E_k$ and eigenstates $|k\rangle$ at a magic point $n_d = 0.5$ can be derived in light of Eq. (1), with $k = q_0$, $q_1$, and $f$. Each level state can be expressed as a superposition of Cooper-pair number states, $|k\rangle = \sum_n c_{kn}|n\rangle$, with $c_{kn}$ being the superposition coefficients. Notice that level anharmonicity of the qutrit is enough for the present circuit in the charge-phase regime.

Two ac gate voltages $\tilde{V}_s = V_s\cos(\omega_s t)$ and $\tilde{V}_a = V_a\cos(\omega_a t)$ are coupled to the CPB, in which $V_{s,a}$ are the voltage amplitudes, and $\omega_{s,a}$ act as the microwave frequencies. The Hamiltonians describing electrical interactions have the forms $H_{s,a} = -2E_c\tilde{n}_{s,a}\sum_n(n-n_d)|n\rangle\langle n|$, where $\tilde{n}_{s,a} = n_{s,a}\cos(\omega_{s,a}t)$, with $n_{s,a} = C_{s,a}V_{s,a}/(2e)$. The transition matrix elements between $|q_1\rangle$ and $|f\rangle$ ($|q_0\rangle$) induced by $\tilde{V}_s$ ($\tilde{V}_a$) take $t_{q_1 f} = \Omega_s\cos(\omega_s t)$ and $t_{q_1 q_0} = \Omega_a\cos(\omega_a t)$, respectively, in which $\Omega_{s,a}$ are the coupling strengthes. Differently, the magnetic interaction Hamiltonian between the time-dependent flux driving $\tilde{\Phi}_p = \Phi_p\cos(\omega_p t)$ and the CPB is of form $H_p = \frac{E_{Jp}}{2}\sum_n(|n\rangle\langle n+1| + \mathrm{H.c.})$. For the amplitude of $\tilde{\Phi}_p$ much smaller than $\Phi_0$, one has $\sin(\pi\frac{\tilde{\Phi}_p}{\Phi_0}) \approx \pi\frac{\tilde{\Phi}_p}{\Phi_0}$ and $\cos(\pi\frac{\tilde{\Phi}_p}{\Phi_0}) \approx 1$, leading to $E_{Jp} = 2E_J\frac{\pi\tilde{\Phi}_p}{\Phi_0}\sin(\pi\frac{\Phi_d}{\Phi_0})$. Allowed by the level transition rule, the interaction causes a transition between $|f\rangle$ and $|q_0\rangle$ (at $n_d = 0.5$). The transition matrix element reads $t_{fq_0} = \Omega_p\cos(\omega_p t)$, where $\Omega_p$ denotes the relevant Rabi rate.[43,44]

## 3. Qubit state inversion within a Λ-type qutrit via non-Hermitian STA

We first treat a Λ-type interaction between the qutrit and the microwave drivings. As demonstrated in Fig. 2(a), two classical microwave fields $\tilde{V}_s$ and $\tilde{\Phi}_p$, behaving as the corresponding Stokes and pump drivings, are employed to induce the desired level couplings $|f\rangle \longleftrightarrow |q_1\rangle$ ($|q_0\rangle$), and their Rabi frequencies are $\Omega_s$ and $\Omega_p$, respectively. Here $|q_0\rangle$ and $|q_1\rangle$ are encoded as qubit states, while $|f\rangle$ is an auxiliary state. The microwave frequency $\omega_s$ ($\omega_p$) is largely detuned to the transition frequency $\omega_{fq_1}$ ($\omega_{fq_0}$). In addition, due to the sufficient level anharmonicity, the $\tilde{V}_s$-caused transition between $|q_1\rangle$ and $|q_0\rangle$ gets vanished nearly. Behaving as non-Hermitian decoherence terms, $\pm\mathrm{i}\gamma$ represent the dephasing effect regarding qubit states, as mentioned in the following.

Within the bare state basis $\{|q_0\rangle, |f\rangle, |q_1\rangle\}$ and with no dephasing effect $\pm\mathrm{i}\gamma$, a Hamiltonian characterizing the two-

photon resonance interaction is[41]

$$H_1 = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_\mathrm{p} & 0 \\ \Omega_\mathrm{p} & 2\delta & \Omega_\mathrm{s} \\ 0 & \Omega_\mathrm{s} & 0 \end{pmatrix}, \tag{2}$$

where $\delta = \omega_{fq_0} - \omega_\mathrm{p} = \omega_{fq_1} - \omega_\mathrm{s}$ is a common detuning. The instantaneous eigenstates of the Hamiltonian $H_1$ are denoted as $|\psi_+\rangle$, $|\psi_0\rangle$ and $|\psi_-\rangle$, which are connected to the bare states $|q_0\rangle$, $|f\rangle$ and $|q_1\rangle$ as follows:

$$\begin{aligned} |\psi_+\rangle &= \sin\varphi\sin\theta\,|q_0\rangle + \cos\varphi\,|f\rangle + \sin\varphi\cos\theta\,|q_1\rangle, \\ |\psi_0\rangle &= \cos\theta\,|q_0\rangle - \sin\theta\,|q_1\rangle, \\ |\psi_-\rangle &= \cos\varphi\sin\theta\,|q_0\rangle - \sin\varphi\,|f\rangle + \cos\varphi\cos\theta\,|q_1\rangle, \end{aligned} \tag{3}$$

in which the angles $\theta$ and $\varphi$ fulfill $\tan\theta = \Omega_\mathrm{p}/\Omega_\mathrm{s}$ and $\tan(2\varphi) = \Omega/\delta$, respectively, with $\Omega = \sqrt{\Omega_\mathrm{p}^2 + \Omega_\mathrm{s}^2}$. The probability amplitudes $a(t) = [a_+(t), a_0(t), a_-(t)]^\mathrm{T}$ of the eigenstates are linked to $b(t) = \left[b_{q_0}(t), b_f(t), b_{q_1}(t)\right]^\mathrm{T}$ of the bare states by the unitary transformation

$$b(t) = Wa(t),$$

where the superscript T indicates matrix transposition, and the transformation matrix $W$ satisfies

$$W = \begin{pmatrix} \sin\varphi\sin\theta & \cos\theta & \cos\varphi\sin\theta \\ \cos\varphi & 0 & -\sin\varphi \\ \sin\varphi\cos\theta & -\sin\theta & \cos\varphi\cos\theta \end{pmatrix}. \tag{4}$$

By making use of $H_1^{(a)} = W^{-1}H_1W - \mathrm{i}\hbar W^{-1}\dot{W}$, the system evolution within the eigenstate basis can be addressed according to the Schrödinger equation, $\mathrm{i}\hbar\dot{a}(t) = H_1^{(a)}a(t)$. The choice of eigenstate basis is convenient to judge the adiabaticity of system evolution.

After introducing the non-Hermitian dephasing term $\pm\mathrm{i}\gamma$ to the Hamiltonian $H_1$, we then have

$$H_{1\gamma} = \frac{\hbar}{2}\begin{pmatrix} -\mathrm{i}\gamma & \Omega_\mathrm{p} & 0 \\ \Omega_\mathrm{p} & 2\delta & \Omega_\mathrm{s} \\ 0 & \Omega_\mathrm{s} & \mathrm{i}\gamma \end{pmatrix}. \tag{5}$$

As mentioned in Ref. [45], the dephasing could physically originate from environmental effects or the fluctuating frequencies of the external fields. Within the eigenstate basis $\{|\psi_+\rangle, |\psi_0\rangle, |\psi_-\rangle\}$, the Hamiltonian $H_{1\gamma}$ can be expressed as

$$H_{1\gamma}^{(a)} = W^{-1}H_{1\gamma}W - \mathrm{i}\hbar W^{-1}\dot{W}, \tag{6}$$

whose matrix elements of interest are

$$\begin{aligned} [H_{1\gamma}^{(a)}]_{12} &= -\mathrm{i}\sin\varphi[\gamma\sin(2\theta) - 2\dot{\theta}]/2, \\ [H_{1\gamma}^{(a)}]_{32} &= -\mathrm{i}\cos\varphi[\gamma\sin(2\theta) - 2\dot{\theta}]/2, \end{aligned}$$

which give the transition strengthes from $|\psi_0\rangle$ to $|\psi_\pm\rangle$, respectively. To make the system evolution only along an initial state $|\psi_0\rangle$, these two non-adiabatic coupling terms need to be $[H_{1\gamma}^{(a)}]_{12} = [H_{1\gamma}^{(a)}]_{32} = 0$, meaning

$$\gamma = \frac{2\dot{\theta}}{\sin(2\theta)}. \tag{7}$$

In this situation, the state transitions from $|\psi_0\rangle$ to $|\psi_\pm\rangle$ are cut off completely.[40]

Numerically, we consider the dynamical behavior based on two pulses $\Omega_{\mathrm{p,s}}$ as the Gaussian functions of time $t \in [0, t_\mathrm{f}]$, as demonstrated in Fig. 2(b),

$$\begin{aligned} \Omega_\mathrm{p} &= \Omega_0 \exp[-(t - t_\mathrm{f}/2 - \tau)^2/\sigma^2], \\ \Omega_\mathrm{s} &= \Omega_0 \exp[-(t - t_\mathrm{f}/2 + \tau)^2/\sigma^2], \end{aligned}$$

in which $\Omega_0$ is the peak amplitude, $\tau$ and $\sigma$ are the pulse-related parameters, and $t_\mathrm{f}$ is the duration time. From Eq. (7), the non-adiabatic coupling condition yields $\gamma = 4\tau/\sigma^2$. Note that the dephasing rate $\gamma$ becomes a constant, which is extremely consistent with the usual quantum system having a fixed decoherence rate. Quantitatively, these parameters are selected as $\Omega_0/2\pi = 0.2$ GHz, $\delta/2\pi = 2.0$ GHz, $\tau = t_\mathrm{f}/14$, and $\sigma = t_\mathrm{f}/4.7$, with $t_\mathrm{f} = 20$ ns. When the system is initially in $|\psi_0\rangle$, the system evolution is only along the eigenstate during the whole time domain $t \in [0, t_\mathrm{f}]$, see Fig. 2(c). Since the states $|\psi_\pm\rangle$ are not populated all the time, the non-adiabatic excitations from $|\psi_0\rangle$ to $|\psi_\pm\rangle$ do not exist completely. Unlike the unitary case, the probability of $|\psi_0\rangle$ keeps non-conserving in the middle stage of evolution process. Correspondingly, we have qubit state transfer from $|q_0\rangle$ to $|q_1\rangle$ within the basis of bare states $\{|q_0\rangle, |f\rangle, |q_1\rangle\}$, as demonstrated in Fig. 2(d). Although the state transfer has a high inversion probability, the operation needs a very large dephasing rate $\gamma/2\pi = 50.2$ MHz, which is not applicable to superconducting quantum circuits.

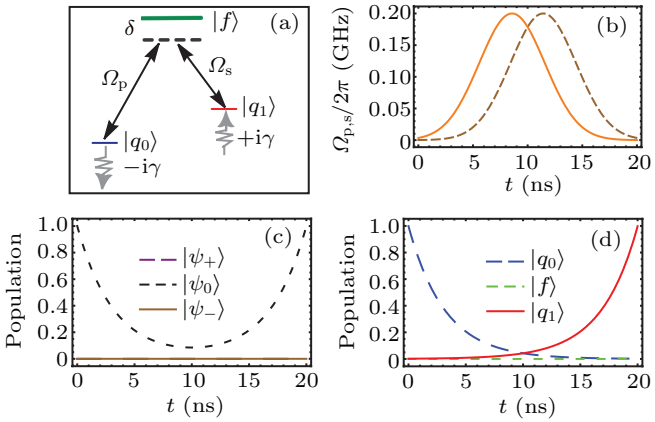



**Fig. 2.** (a) Schematic diagram of a Λ-configuration interaction between the microwave drivings and the qutrit with non-Hermitian qubit dephasing. (b) Rabi rates $\Omega_\mathrm{p}/2\pi$ (dashed line) and $\Omega_\mathrm{s}/2\pi$ (solid line) as Gaussian functions of time $t \in [0, 20$ ns$]$. (c) Population of three eigenstates $|\psi_0\rangle$ and $|\psi_\pm\rangle$ versus time $t$. (d) Qubit state inversion from $|q_0\rangle$ to $|q_1\rangle$ within the time duration.

We further consider the dependence of $t_\mathrm{f}$ on the dephasing rate $\gamma$. As shown in Table 1, with the decrease of $\gamma$, the

operation time $t_f$ becomes longer. If one chooses $\gamma$ with a relatively small value, for instance, $\gamma/2\pi = 2.0$ MHz, the time $t_f$ could reach up to 500 ns, which is highly contrary to the construction of rapid information processing. The dependence implies that the operation with a shorter duration time requires a larger dephasing rate. Obviously, the direct application of the method proposed in Ref. [40] to investigate fast operation on non-Hermitian superconducting qubits could pose a sever challenge. Thereby, with the accessible dephasing rate $\gamma$, the exploration of fast operations on superconducting systems via non-Hermitian STA is an active area of research in quantum science and technology.

**Table 1.** The operation duration time $t_f$ versus the qubit dephasing rate $\gamma/2\pi$.

| $t_f$ (ns) | 10 | 20 | 50 | 100 | 200 | 500 |
|---|---|---|---|---|---|---|
| $\gamma/2\pi$ (MHz) | 100.4 | 50.2 | 20.1 | 10.0 | 5.0 | 2.0 |

## 4. Population transfer within a Δ-type qutrit via non-Hermitian STA

As depicted in Fig. 3, after introducing an ancillary driving $\Omega_a$ that is used to directly couple $|q_0\rangle$ to $|q_1\rangle$, we have the following Hamiltonian within the basis of $\{|q_0\rangle, |f\rangle, |q_1\rangle\}$,

$$H_{2\gamma} = \frac{\hbar}{2}\begin{pmatrix} -i\gamma & \Omega_p & i\Omega_a \\ \Omega_p & 2\delta & \Omega_s \\ -i\Omega_a & \Omega_s & i\gamma \end{pmatrix}. \tag{8}$$

Mention that the ac gate voltage $\tilde{V}_a$ (having an appropriate frequency $\omega_a$ and amplitude $V_a$) is capable of resonantly inducing the desirable coupling $\Omega_a$. By means of the transformation $W$, the Hamiltonian $H_{2\gamma}$ can be expressed as $H_{2\gamma}^{(a)}$ within the basis $\{|\psi_+\rangle, |\psi_0\rangle, |\psi_-\rangle\}$,

$$H_{2\gamma}^{(a)} = W^{-1}H_{2\gamma}W - i\hbar W^{-1}\dot{W}. \tag{9}$$

We concentrate its two matrix elements:

$$\begin{aligned} [H_{2\gamma}^{(a)}]_{12} &= -i\sin\varphi[\gamma\sin(2\theta) - 2\dot{\theta} - \Omega_a]/2, \\ [H_{2\gamma}^{(a)}]_{32} &= -i\cos\varphi[\gamma\sin(2\theta) - 2\dot{\theta} - \Omega_a]/2, \end{aligned}$$

which characterize the couplings from $|\psi_0\rangle$ to $|\psi_\pm\rangle$, respectively, as mentioned above. When the non-adiabatic coupling terms meet $[H_{2\gamma}^{(a)}]_{12} = [H_{2\gamma}^{(a)}]_{32} = 0$, we have

$$\gamma = \frac{\Omega_a + 2\dot{\theta}}{\sin(2\theta)}. \tag{10}$$

Compared to the restriction shown in Eq. (7), the dephasing rate is related to both the Gaussian pulses $\Omega_{p,s}$ and the ancillary driving $\Omega_a$. Thereby, for two chosen pulses $\Omega_{p,s}$, the variation of $\gamma$ can be obtained using the adjustable driving $\Omega_a$.

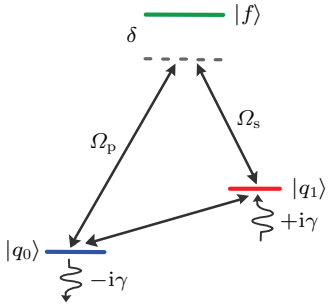



**Fig. 3.** A Δ-configuration interaction between three microwave drivings and the three-level system with non-Hermitian qubit dephasing.

Without loss of generality, we assume that $\gamma$ is still a constant rate. The Rabi frequency caused by ancillary driving can be derived from Eq. (10),

$$\Omega_a = \gamma\sin(2\theta) - 2\dot{\theta}, \tag{11}$$

as numerically plotted in Fig. 4(a), where we have chosen two Gaussian-typed drivings $\Omega_p$ and $\Omega_s$ again. It is found that $\Omega_a$ is a continuous and smoothed function of time $t$. Here the peak amplitude of $|\Omega_a|/2\pi$ is about 48.2 MHz. Thus, such a microwave driving is easily available under the current condition. By the assistance of the auxiliary driving, system evolution only along eigenstate $|\psi_0\rangle$ and then qubit state inversion from $|q_0\rangle$ to $|q_1\rangle$ can be well performed, see Figs. 4(b) and 4(c), where we take an accessible dephasing rate $\gamma/2\pi = 2.0$ MHz.[46] Different from the previous schemes in the case of Hermitian STA,[26] the driving $\Omega_a$ together with the dephasing effect plays the role of eliminating the non-adiabatic excitations from $|\psi_0\rangle$ to $|\psi_\pm\rangle$. Within the framework of non-Hermitian STA, the addition of $\Omega_a$ intends to assist dephasing effect to perform the non-adiabatic quantum operation. As a result, for a given dephasing rate $\gamma$ that is accessible to the present non-Hermitian system, one can construct faster manipulations by alternatively monitoring $\Omega_a$, which is highly beneficial to quantum information processing.

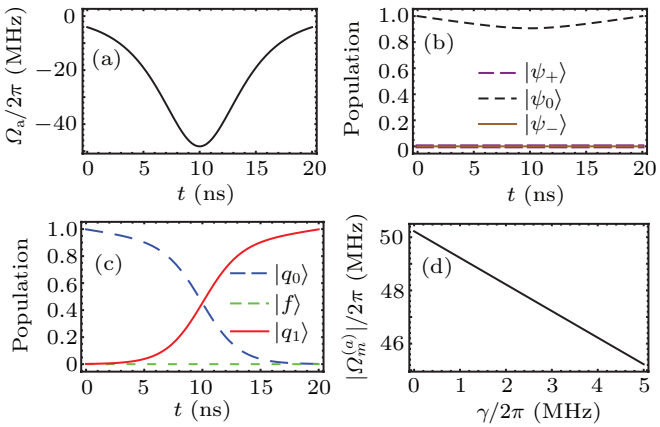



**Fig. 4.** (a) The dependance of auxiliary driving $\Omega_a/2\pi$ on time $t \in [0, 20$ ns$]$. (b) The occupied probabilities of three eigenstates versus time. (c) Qubit state inversion from $|q_0\rangle$ to $|q_1\rangle$. (d) For the considered state transfer, the peak amplitude $|\Omega_m^{(a)}|/2\pi$ of auxiliary driving versus the dephasing rate $\gamma/2\pi$.

We quantitatively analyze the relation between the ancillary driving $\Omega_{\rm a}$ and the dephasing rate $\gamma$. For the mentioned state transfer displayed in Fig. 4(c), the peak amplitude $|\Omega_{\rm a}^{(m)}|/2\pi$ versus dephasing rate $\gamma/2\pi$ is our concern. In terms of Eq. (11), $|\Omega_{\rm a}^{(m)}|/2\pi$ gets reduced linearly as the $\gamma/2\pi$ increases in a broad range, see Fig. 4(d). Physically, $|\Omega_{\rm a}^{(m)}|/2\pi$ and $\gamma/2\pi$ are complementary in the present case. By increasing the value of $|\Omega_{\rm a}^{(m)}|/2\pi$, one can diminish the dephasing rate $\gamma/2\pi$ to be completely available to the considered superconducting circuit system. We stress that the dephasing effect having a smaller rate gives a minor contribution to the non-Hermitian dissipation dynamics, as mentioned above. Moreover, for different dephasing rates, the corresponding ancillary drivings hold the time profiles similar to that demonstrated in Fig. 4(a), but have different peak amplitudes. Consequently, from the perspective of technical operation, the protocol could provide an effective approach to study dissipative dynamics by means of the non-Hermitian STA.

Now the fidelity of state inversion is our concern. Assume that the error of control variables can be neglected, the dominant factor that degrades the operation fidelity could be the qubit relaxation since the dephasing effect has been utilized for serving the non-adiabatic evolution within the present system. The reduced density matrix regarding the qutrit $(|q_0\rangle, |f\rangle, |q_1\rangle)$ is $\rho$, the dynamical evolution follows the Lindblad master equation:

$$\frac{{\rm d}\rho}{{\rm d}t} = -{\rm i}\left[H_{2\gamma}, \rho\right] + \gamma_r D[\sigma_-]\rho,$$

where the first term of the right side represents the ideal evolution governed by $H_{2\gamma}$, and the second term characterizes the effect of qubit relaxation with a rate $\gamma_r$. The superoperator is $D[\sigma_-]\rho = (2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-)/2$, with $\sigma_- = |q_0\rangle\langle q_1|$ and $\sigma_+ = |q_1\rangle\langle q_0|$ being inversion operators. To quantify the overlap of the resulting state with the ideal one, we adopt the fidelity $F = \sqrt{\langle\psi_{\rm i}(t_{\rm f})|\,\rho(t_{\rm f})\,|\psi_{\rm i}(t_{\rm f})\rangle}$,[47] in which $|\psi_{\rm i}(t_{\rm f})\rangle$ is an ideal state (at the final time $t = t_{\rm f}$) in the absence of relaxation effect, and $\rho(t_{\rm f}) = |\psi(t_{\rm f})\rangle\langle\psi(t_{\rm f})|$ is associated with the resulting state $|\psi(t_{\rm f})\rangle$. With a viable rate $\gamma_r/2\pi = 0.2$ MHz,[46] the fidelity $F$ could reach up to 99.93% numerically, implying that a high-fidelity transfer from $|q_0\rangle$ to $|q_1\rangle$, as given in Fig. 4(c), could be realized using the technique of non-Hermitian STA.

## 5. Discussion and conclusion

The scheme could have the following distinct merits: (i) For the coherent unitary dynamics, the qubit dephasing effect acting as a detrimental factor generally causes the system decoherence.[48] On the contrary, the dephasing effect in our device has been utilized to construct a non-adiabatic qubit state inversion by the technique of non-Hermitian STA, which is thus an appealing protocol to high-fidelity operations on dissipative Josephson qubits with large decoherence rates. (ii) Apart from the non-Hermitian effect of qubit dephasing, an ancillary driving (directly acting upon qubit states) is introduced for performing the fast operation within the framework of non-Hermitian STA. By taking advantage of complementation between the dephasing and the ancillary driving, the fast non-Hermitian operations with a given dephasing rate that is fully accessible to superconducting qubit can be achieved by means of the flexibly adjustable $\Omega_{\rm a}$. Compared with the previous case,[40] the strategy could broaden the avenue to investigate dissipative dynamics with superconducting systems. (iii) In contrast to the method of invariant-based STA,[43] Rabi couplings $\Omega_{\rm p,s}$ (behaving as Gaussian functions of time) and the ancillary driving $\Omega_{\rm a}$ (having the smooth and continuous waveform) are readily generated in our scenario, which are beneficial to precisely implement quantum operations.

As an open question of fast quantum control, the experimental investigation of the STA-based state transfers with superconducting artificial atoms needs to be concerned. By taking advantage of the approach of STA, some rapid operations on the Hermitian systems have been demonstrated in experiments.[25,28,29,31] However, the application of the non-Hermitian STA to constructing fast operations on superconducting qubits is less studied in the present stage, in which the main challenge could be the establishment of non-Hermitian Hamiltonian with an appropriate dissipation effect. In our scenario, with the help of an ancillary driving, the dephasing effect has been effectively utilized to perform a non-adiabatic qubit state inversion within the framework of non-Hermitian STA. For a viable dephasing rate, the key point to realizing the dynamical process could be the alternative control of external drivings, which could greatly relax the experimental requirement for the non-Hermitian shortcut-based manipulations.

In conclusion, we have proposed a promising scheme for implementing fast population transfer with a non-Hermitian superconducting qutrit. First, by utilizing a $\Lambda$-configuration interaction between the qutrit and microwave drivings, the dephasing-assisted qubit state transfer can be rapidly performed within the context of non-Hermitian STA, however, the dephasing rate is greatly beyond the range of dissipation strength. After adding an ancillary driving acting on the qubit states, the non-adiabatic state transfer can be well realized but with an applicable qubit dephasing rate. Moreover, we analyze the dependence of the ancillary driving on the dephasing rate and simulate the transfer with a high fidelity. Thus, thanks to the dephasing effect as a favorable factor, the strategy could provide a novel approach towards the shortcut-based state operations on the non-Hermitian superconducting artificial atoms experimentally.

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