Modulational instability of a resonantly polariton condensate in discrete lattices

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We study modulational instability of a resonantly polariton condensate in a discrete lattice. Employing a discrete gain-saturation model, we derive the dispersion relation for the modulational instability by means of the linear-stability analysis. Effects of the pumping strength, the nonlinearity, the strength of the detuning, and the coupling strength on the modulation instability are investigated. It is found that the interplay between these parameters will dramatically change the modulational instability condition. We believe that the predicted results in this work can be useful for future possible experiment of exciton-polariton condensate in lattices.

Keywords: modulational instability, polariton condensate, discrete lattices

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1. Introduction

Recently, degenerate bosonic states have been achieved in semiconductor microcavities, in which photons are confined and strongly coupled to excitons.[1] The superposition between a photon mode and a quantum well exciton is called exciton polariton, these quasiparticles are characterized by an effective mass orders of magnitude smaller than the mass of the electron,[2] this property makes condensation at room temperature possible.[3] Polariton systems turn out to be a very convenient test bench for theoretical and experimental studies of nonlinear waves such as solitons and quantized vortices.[4–6]

In the meantime, following the fast development of the semiconductor technology, modern lithography techniques open large possibilities in potential engineering,[7–9] which can provide a well-defined artificial potential for polaritons. One of the simplest and yet nontrivial types of potentials is the periodic one,[10] meanwhile, the polariton condensation in deep periodic potentials has already been observed.[11] Theoretically, the mean-field description with the generalized Gross–Pitaevskii equation (GPE) has proved to be adequate to explain many experiments. Especially for the deep periodic potentials, based on the tight-binding approximation, which leads to the discrete version of the GPE (or the discrete nonlinear Schrödinger equation, DNLSE)[12] to describe the exciton-polariton Bose–Einstein condensate in lattice. Dynamics of exciton-polaritons in discrete lattices,[13] solitons and patterns in exciton-polariton lattice are also investigated.[14] Gap soliton,[15] topological phase transition,[16] topological soliton,[17] Lieb polariton topological insulators,[18] nonreciprocal transport phenomenon,[19] and corner modes coupling[20] are all discussed in exciton-polariton discrete lattice.

A fundamental ingredient of the matter-wave dynamics is the modulation instability (MI), which is the key mechanism for the formation of soliton trains[21–23] in diverse physical media, as a result of the interplay between the intrinsic nonlinearity and the kinetic-energy term. The MI in continuous exciton-polariton systems has been addressed in some works.[24,25] However, to the best of our knowledge, the MI of resonantly exciton-polariton in discrete lattice is still lacking. In this paper, we study the MI of a driven resonantly exciton-polariton condensate in discrete lattices. Within the mean-field level, we model such a system with the DNLSE. Using the linear stability analysis, we derive the MI criterion of our model system. Our results show that the pumping strength, the nonlinearity, the strength of the detuning, and the coupling strength can dramatically change the modulation instability condition.

The paper is organized as follows. In Section 2, we present the physical model and the analytical results of MI. In Section 3, the results and discussions are presented. In Section 4, we give our main conclusions.

2. The model and modulational-instability analysis

Our starting point is a general gain-saturation DNLSE that describes the exciton-polariton condensate in a one-dimensional chain:[26,27]

$$\begin{aligned}\dot{\psi}_n = {} & \psi_n\left(\frac{P}{1+|\psi_n|^2}-1\right)-\mathrm{i}s|\psi_n|^2\psi_n \\ & +(1-\mathrm{i}g)J(\psi_{n+1}+\psi_{n-1}),\end{aligned} \tag{1}$$

where ψ_n is the complex amplitude of the n-th cite, s represents the nonlinear interaction strength arising from the inter-

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action between polaritons, and P is the pumping strength, g is the strength of the detuning, and J is the coupling strength.

We study the modulational instability of Eq. (1) that has plane wave solutions of the form

$$\psi_n = A\exp[\mathrm{i}(kn-\omega t)], \tag{2}$$

with amplitude A, wave numbers k and the frequency ω satisfying the following nonlinear dispersion relation:

$$\omega = \mathrm{i}\Big(\frac{P}{1+A^2}-1\Big)+sA^2+2(\mathrm{i}+g)J\cos k. \tag{3}$$

Due to the frequency ω is a positive real number, the relations $A^2=\frac{P}{1-2J\cos k}-1$ and $k\neq\pm\arccos(sA^2+2gJ)\pm\pi$ must be satisfied. To analyze MI in Eq. (1) we introduce the perturbation ansatz

$$\psi_n(t) = [A+\phi_n(t)]\exp[\mathrm{i}(kn-\omega t)], \tag{4}$$

where $\phi_n(t)$ is the perturbed wave function. By inserting this ansatz into Eq. (1) and linearizing $\phi_n(t)$, we obtain the evolution equation for the perturbation

$$\begin{aligned}\dot{\phi}_n \;=\; & (\mathrm{i}\omega-2J\cos k)\phi_n-\frac{PA^2}{(1+A^2)^2}(\phi_n+\phi_n^*)\\ & -\mathrm{i}sA^2(2\phi_n+\phi_n^*)+(1-\mathrm{i}g)\\ & \times J(\phi_{n+1}\mathrm{e}^{\mathrm{i}k}+\phi_{n-1}\mathrm{e}^{-\mathrm{i}k}).\end{aligned} \tag{5}$$

We pose $\phi_n(t)$ in the form

$$\phi_n(t) = B\mathrm{e}^{\mathrm{i}(Qn-\Omega t)}+C^*\mathrm{e}^{-\mathrm{i}(Qn-\Omega^* t)}. \tag{6}$$

Here, B and C are complex numbers, Q is the modulational wave vector, and the perturbation frequency Ω can be determined by the following linear system:

$$\begin{pmatrix} M^+-2\mathrm{i}sA^2-L+\mathrm{i}\Omega & -D+\mathrm{i}sA^2\\ -D-\mathrm{i}sA^2 & M^-+2\mathrm{i}sA^2-L+\mathrm{i}\Omega\end{pmatrix}\begin{pmatrix}B\\C\end{pmatrix} = 0, \tag{7}$$

where we define $D\equiv\frac{PA^2}{(1+A^2)^2}$, $M^{\pm}\equiv 2(1\mp\mathrm{i}g)J\cos(k\pm Q)\pm\mathrm{i}\omega$, $L\equiv 2J\cos k+D$, $\Delta\equiv\cos k\cos Q$, $N\equiv\sin k\sin Q$. We can obtain the eigenfrequency

$$\Omega = \frac{1}{2}\big[\Lambda_1\pm\sqrt{\Lambda_1^2+4\Lambda_2}\big], \tag{8}$$

with defining

$$\Lambda_1 = 4\mathrm{i}J(\Delta+\mathrm{i}gN)-2\mathrm{i}L, \tag{9}$$

and

$$\begin{aligned}\Lambda_2 \;=\; & 4(1+g^2)J^2(\Delta^2-N^2)+4\Delta J[g(2sA^2-\omega)-L]\\ & +4\mathrm{i}JN[\omega-2sA^2-Lg]+\omega(\omega-4sA^2)\\ & +L^2-D^2+3s^2A^4.\end{aligned} \tag{10}$$

The carrier plane wave of the exciton-polariton condensate is modulationally unstable when the eigenfrequencies Ω become imaginary. Thus, we define the instability gain $G=|\mathrm{Im}[\Omega]|$, if $G\neq 0$, the nonlinear plane wave experiences MI. Step further, the MI gain $G=|\mathrm{Im}[\Omega]|$ is readily obtained as follows:

$$G = \Big|2J\Delta-L+\frac{1}{\sqrt{2}}\sqrt{\sqrt{F^2+U^2}-F}\Big|, \tag{11}$$

with the functions F and U given by

$$\begin{aligned}F \;=\; & 4J^2(g^2\Delta^2-N^2)+4\Delta Jg(2sA^2-\omega)\\ & +\omega(\omega-4sA^2)+3s^2A^4-D^2,\end{aligned} \tag{12}$$

$$U \;=\; 4JN(\omega-2sA^2-2Jg\Delta). \tag{13}$$

In the following, based on Eq. (11), we study the importance of ingredient of pumping strength P, self-interactions strength s, the strength of the detuning g and the coupling strength J on the MI.

3. Results and discussion

From Eq. (11), it is shown that the interplay of the pumping strength (P), the nonlinearity (s), the strength of the detuning (g), and the coupling strength (J) will dramatically influence the modulational stability criterion. Figure 1 shows the stability gain G as a function of plane wave number k and perturbation wave number Q. Notice that the white open regions visible in the figures correspond to the lines $k\neq\pm\arccos(sA^2+2gJ)\pm\pi$, on which the wavevector is not defined (see Eq. (3)). From this analysis the existence of nonlinear localized and extended solutions of Eq. (1) is expected. The former is existence in the regions of MI gain $G(k,Q)\neq 0$, and the later is formed in the MI gain $G(k,Q)=0$ regimes.

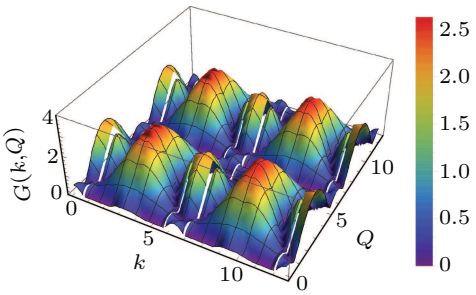


Fig. 1. Modulation instability condition of $G(k,Q)$ in Eq. (11) for $P=2$, $g=0.1$, $s=0.01$, and $J=0.6$.

For the purpose of how the strength of detuning (g) influences the MI, Fig. 2 shows the MI gain G in the parameter of nonlinear (s) and perturbed wave number (Q) plane. We can see that for different values of detuning parameter g from negative to positive, the stability region will dramatically changed, which means that the detuning will play a critical role in the MI. For example, for the negative detuning, the MI mainly occurs in the focusing region as clearly shown in Fig. 2(a). In the absence of the detuning ($g=0$), the MI will occur not only in focusing region ($s>0$) but also in defocusing region ($s<0$)

except for the perturbed wave number $Q=(2m+1)\pi$, with m being a integer number, as clearly shown in Fig. 2(b). However, if the detuning is positive, the MI can also occur in the defocusing cases as shown in Figs. 2(c) and 2(d). Meanwhile, the strength of detuning can only influence the amplitude of MI gain G, but does not change the regions where the MI can set in, the tendencies are also reflected in Figs. 2(c) and 2(d).

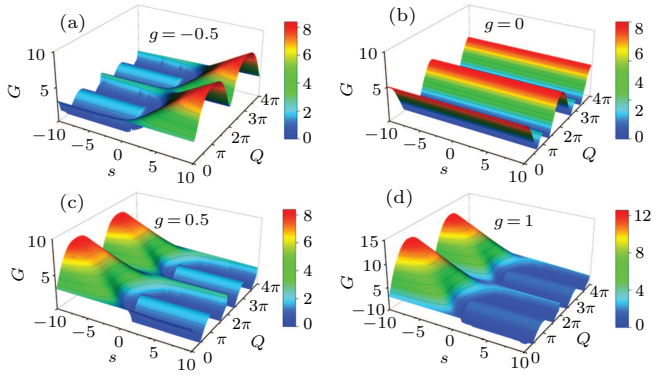


Fig. 2. The MI gain versus s for fixed $k=0$ with different values of g by taking $P=2$ and $J=1$.

To further study the nonlinear influence on the MI, Fig. 3 shows the nonlinear strength parameter s from negative [Fig. 3(a)] to positive [Figs. 3(c) and 3(d)], even in the limitation of nonlinearity is zero as shown in Fig. 3(b). We can see that the MI can occur in major regions in (g,Q) plane, which is the detuning and perturbed wave number plane. Only in the region where the perturbed wave number Q satisfies the condition $Q=(2m+1)\pi$ or in the absence of detuning ($g=0$), the MI will not set in.

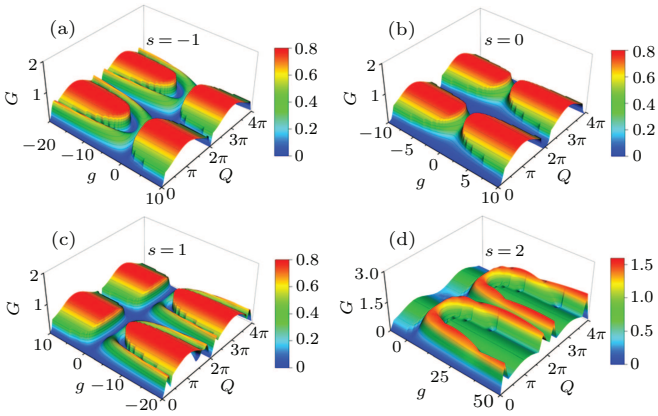


Fig. 3. The MI gain $G(g,Q)$ in Eq. (11) versus g for fixed $k=0$ with different values of s by taking $P=1.6$ and $J=0.1$.

Finally, for the purpose of studying the influences of interplay of the detuning parameter g, pumping strength P and coupling parameter J on the MI, Fig. 4 shows the MI gain G in g,P plane with different values of coupling parameter J. It is clearly shown that the strength of coupling parameter J plays a critical role on the MI, following the parameter J from small to large, which dramatically changes the region where the MI will set in, these tendencies are clearly shown in Fig. 4.

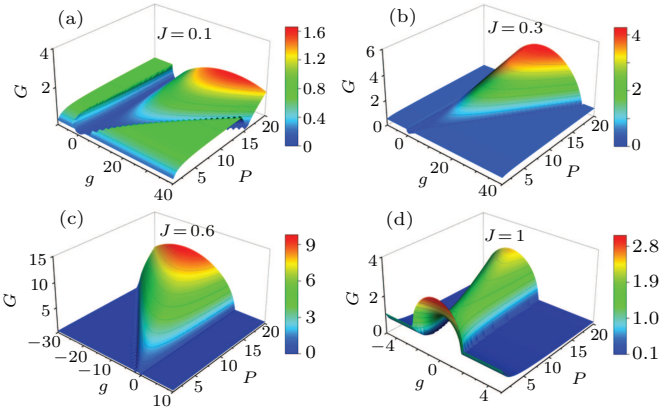


Fig. 4. The MI gain $G(g,P)$ in Eq. (11) with (a) $J=0.1$, (b) $J=0.3$, (c) $J=0.6$, (d) $J=1$, for $s=0.1$, $k=0$, and $Q=1$.

4. Conclusion

In this work, we show that the exciton-polariton condensate is arranged in a simple one-dimensional lattice line, the modulational instability is investigated in detail. The results indicate that the interplay among the pumping strength, the nonlinearity, the strength of the detuning and the coupling strength will dramatically influence the MI conditions. In the experiment, a one-dimensional lattice of coupled micropillars can be achieved in GaAs-based microcavity with a long cavity photon lifetime exceeding 100 ps.[28] Each micropillar contains a quantum well and is assumed to host a tightly bound exciton-polariton mode. The pumping strength will be tuned by the external laser power, the nonlinearity and the coupling strength can be tuned through biexcitonic Feshbach resonance technology[29] and distance between micropillars, respectively. The experiments are carried out by slightly varying positions on the sample, which corresponds to tuning the strength of detuning. Our theoretical prediction may be observed in future experiments.

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