Fault-tolerant finite-time dynamical consensus of double-integrator multi-agent systems with partial agents subject to synchronous self-sensing function failure

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This paper investigates fault-tolerant finite-time dynamical consensus problems of double-integrator multi-agent systems (MASs) with partial agents subject to synchronous self-sensing function failure (SSFF). A strategy of recovering the connectivity of network topology among normal agents based on multi-hop communication and a fault-tolerant finite-time dynamical consensus protocol with time-varying gains are proposed to resist synchronous SSFF. It is proved that double-integrator MASs with partial agents subject to synchronous SSFF using the proposed strategy of network topology connectivity recovery and fault-tolerant finite-time dynamical consensus protocol with the proper time-varying gains can achieve finite-time dynamical consensus. Numerical simulations are given to illustrate the effectiveness of the theoretical results.

Keywords: multi-agent systems, synchronous self-sensing function failure, finite-time dynamical consensus, network topology connectivity recovery

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1. Introduction

During the past twenty years, consensus of multi-agent systems (MASs), owing to its wide applications in multiple robot systems, wireless sensor networks, intelligent transportation systems, smart grids, social networks, and so on, has received a lot of attention from researchers in different disciplines. Consensus means that all agents under distributed control inputs called consensus protocols eventually reach a common value. According to different convergence time, consensus is divided into asymptotical and finite-time consensus. Compared with asymptotical consensus,[1–8] finite-time consensus possesses more advantages such as faster convergence rate, better disturbance rejection, and robustness to uncertainties. Also, in real application, an agent, such as an underwater robot and an unmanned aerial vehicle, is controlled for its desired motion by its acceleration rather than by its speed. Therefore, finite-time consensus problems of double-integrator MASs have been well studied.[9–14]

Notice that successful implementation of all of the finite-time consensus protocols in Refs. [9–14] depends on the availability of the position and velocity state errors between an agent and its local neighbors. In general, to compute the state errors, an agent has to measure its own absolute states by satellite positioning systems such as GPS. However, in some complex work scenarios, it is hard for an agent to obtain its own absolute states based on satellite positioning systems. For example, a group of unmanned aerial vehicles are required to keep a particular formation, get into a thick forest, and complete a task of extinguishing a fire. Once a vehicle gets into the forest, which can be regarded as a typical indoor environment, its own absolute position and velocity states will be no longer measured by GPS. In this situation, the vehicle that has got into the forest is called to suffer from self-sensing function failure (SSFF). If the vehicle subject to SSFF is not controlled by a new measure, it will no longer keep the particular formation with other vehicles, which might result in the loss of timely completing the task of putting out the fire. One can see from this example that once an agent suffers from SSFF, its absolute states can be no longer measured by its sensors, making the state errors between itself and its local neighbours unavailable and further making itself unable to be controlled in a closed-loop manner. If an agent subject to SSFF is not given a novel control scheme, it cannot achieve consensus with other normal agents. Moreover, if an agent suffers from SSFF, its neighbours can no longer obtain its states in spite of keeping communicating with it. Hence, the occurrence of SSFF at an agent might destroy the connectivity of network topology among normal agents in the sense of obtaining information. If the connectivity of network topology among normal agents is not recovered, the subsystems composed of normal agents cannot reach consensus, let alone the whole MASs. In summary, SSFF can destroy the achievement of consensus among the whole MASs.

Therefore, we need to find an appropriate strategy of recovering the connectivity of network topology among normal agents and a proper fault-tolerant finite-time consensus proto-

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col to ensure that double-integrator MASs with partial agents subject to SSFF can still reach finite-time consensus. As far as we know, there is no open result on fault-tolerant finite-time consensus of double-integrator MASs with partial agents subject to SSFF.

Based on the above considerations, in this paper we investigate the fault-tolerant finite-time consensus problems of double-integrator MASs with partial agents subject to SSFF. First, a strategy of recovering the connectivity of network topology among normal agents is proposed based on multi-hop communication together with agents subject to SSFF as routing nodes. Second, a fault-tolerant finite-time dynamical consensus protocol with time-varying gains is proposed. Third, convergence analysis is made in theory. Last, numerical simulations are provided to illustrate the effectiveness of the theoretical results.

The main innovations of this paper are summarized as follows. First, different from sensor faults in Refs. [15–19], where the states or outputs of an agent mixed with sensor faults can be directly measured for designing consensus protocols, SSFF discussed in this paper means that if an agent suffers from SSFF, its states can be no longer directly measured for designing consensus protocols. Second, unlike the research idea on dynamical network topology in Ref. [20], where the conditions on dynamical network topology guaranteeing consensus are derived from the perspective of analysis, the method of actively recovering the network topology connectivity is proposed in this paper. Third, different from the consensus protocols in Refs. [9–14], where a closed-loop control scheme is always put on each agent, a fault-tolerant finite-time consensus protocol is proposed to cope with SSFF in this paper, where an agent is controlled in a closed-/open-loop manner before/after suffering from SSFF, respectively.

The rest of this paper is organized as follows. In Section 2, some preliminaries are provided and the problem is formulated. The strategy of recovering the connectivity of network topology among all normal agents is proposed in Section 3. The fault-tolerant finite-time dynamical consensus protocol is presented, and its effectiveness to guarantee finite-time dynamical consensus is demonstrated theoretically in Section 4. Numerical simulations are provided to illustrate the effectiveness of the theoretical results in Section 5. Conclusions with future research directions are included in Section 6.

2. Preliminaries and problem formulation

2.1. Algebraic graph theory

Let $G=(V,E,A)$ be a weighted undirected graph with the set of nodes $V=\{v_1,v_2,\ldots,v_N\}$, the set of edges $E\subseteq V\times V$, and the weighted adjacency matrix $A=[a_{ij}]$ with nonnegative adjacency elements a_{ij}. The node indexes of G belong to a finite index set $I=\{1,2,\ldots,N\}$. An edge of G is denoted by $e_{ij}=(v_i,v_j)$. The adjacency elements associated with the edges are positive, *i.e.*, $e_{ij}\in E\Leftrightarrow a_{ij}>0$. For a weighted undirected graph G, $e_{ij}\in E\Leftrightarrow e_{ji}\in E$. Assume that $a_{ij}=a_{ji},\forall i,j\in I$ and $a_{ii}=0,\forall i\in I$. Thus, the adjacency matrix A is symmetric. The set of neighbours of node v_i is denoted by $N_i=\{v_j\in V: e_{ij}\in E\}$. The degree of node v_i is defined as $d_i=\sum\limits_{j\in N_i}a_{ij}$. The Laplacian matrix of G is defined as $L=D-A$, where $D=\mathrm{diag}(d_1,d_2,\ldots,d_N)$ is the degree matrix of G with diagonal elements d_i and zero off-diagonal elements. An important property of L is that all of row sums are zero and thus L has a right eigenvector with all elements equal to 1. A path between two distinct nodes v_i and v_j refers to a sequence of distinct edges of the form $(v_i,v_{k_1}),(v_{k_1},v_{k_2}),\ldots,(v_{k_l},v_j)$. An undirected graph is called connected if there is a path between any two distinct nodes of the graph. An undirected graph is called a complete graph if there is an edge between any two distinct nodes of the graph.

2.2. Problem formulation

For MASs with N agents, agent i and the available information flow between agent i and agent j are regarded as node v_i and edge e_{ij} in an undirected graph G, respectively. For double-integrator MASs, the dynamics of agent i can be described by

$$\begin{cases}\dot{p}_i(t)=q_i(t),\\ \dot{q}_i(t)=u_i(t),\end{cases}\quad i\in I,\tag{1}$$

where $p_i(t)\in R$, $q_i(t)\in R$, and $u_i(t)\in R$ are the position state, the velocity state, and the control input of agent i, respectively. It was proved in Ref. [9] that double-integrator MASs (1) with a fixed, undirected, and connected network topology under the consensus protocol

$$\begin{aligned}u_i(t)=&\sum_{j\in N_i}a_{ij}\big(\mathrm{sgn}\,(p_j(t)-p_i(t))\,|p_j(t)-p_i(t)|^{\sigma}\\&+\mathrm{sgn}\,(q_j(t)-q_i(t))\,|q_j(t)-q_i(t)|^{2\sigma/1+\sigma}\big),\quad i\in I,\end{aligned}\tag{2}$$

have the property

$$\begin{cases}p_i(t)=\dfrac{\sum\limits_{j\in I}(p_j(0)+q_j(0)t)}{N},\\ q_i(t)=\dfrac{\sum\limits_{j\in I}q_j(0)}{N},\end{cases}\quad \forall t\geq T,\ \forall i\in I,$$

i.e., reach finite-time dynamical consensus, where $\mathrm{sgn}(\cdot)$ is the sign function, $\sigma\in(0,1)$, and T is a finite time relying on $(p_i(0),q_i(0)),i\in I$.

Definition 1 Agent i with double-integrator dynamics (1) is called to suffer from SSFF at t_i^+, if its position state $p_i(t)$ and velocity state $q_i(t)$ can be measured only in $[0,t_i]$ but cannot be done in $(t_i,+\infty)$, where $0\leq t_i<+\infty$.

One can see from Eqs. (1) and (2) that to reach finite-time dynamical consensus among the whole MASs, agent i needs

to get the states $(p_j(t), q_j(t))$ of its neighbours $j \in N_i$ and its own states $(p_i(t), q_i(t))$. However, just as we said above, once agent i suffers from SSFF, the closed-loop control scheme in Eqs. (1) and (2) can be no longer carried out, and the connectivity of network topology among normal agents might be destroyed, leading to the loss of finite-time dynamical consensus among the whole MASs.

In the following, we will propose a strategy of recovering the connectivity of network topology among normal agents and a fault-tolerant finite-time dynamical consensus protocol to tackle SSFF and simultaneously guarantee the achievement of finite-time dynamical consensus among the whole MASs.

Before moving on, we need to present the following assumptions.

Assumption 1 SSFF is synchronous, *i.e.*, $t_{\mathrm{F}} \triangleq t_i = t_j$, $t_{\mathrm{F}}^+ \triangleq t_i^+ = t_j^+, \forall i, j \in F$, where F represents the set of all agents subject to SSFF, and the number $|F|$ of agents subject to SSFF satisfies $1 \leq |F| < N$.

Assumption 2 The network topology before SSFF is fixed, undirected, and connected, and there is no factor bringing about the change of network topology except SSFF and carrying out the strategy of recovering the connectivity of network topology among normal agents to be presented.

3. Network topology connectivity recovery

In this section, a strategy of recovering the connectivity of network topology among normal agents is proposed, where Assumption 1 and Assumption 2 are satisfied. A specific example is used to better show the processes of the network topology connectivity recovery.

For MASs with ten agents and the initial network topology shown in Fig. 1, without loss of generality, assume that agents 3, 5, 10 suffer from synchronous SSFF at t_{F}^+.

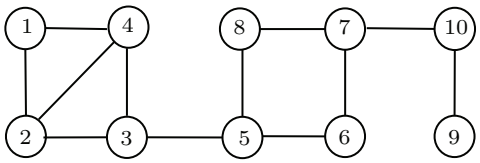


Fig. 1. Network topology among all agents in $[0, t_{\mathrm{F}}]$.

At t_{F}^+, because agent 3 can no longer measure its own states by its own sensors, agent 3 knows that itself suffers from SSFF and immediately relays the states received from agent 2, agent 4, and agent 5 to agents 4, 5, agents 2, 5, and agents 2, 4, respectively. However, because agent 5 also suffers from SSFF at t_{F}^+, agent 3 cannot receive the states of agent 5, knows that agent 5 also suffers from SSFF, informs agent 5 to repay the states received from agents 6, 8 to it, *i.e.*, agent 3, and immediately relays the states received from agent 2, agent 4, agent 6, and agent 8 to agents 6, 8, agents 6, 8, agents 2, 4, 8, and agents 2, 4, 6, respectively. For agents 5, 10 subject to SSFF at t_{F}^+, similar procedures are implemented simultaneously. It follows from the above procedures that when agents 3, 5, 10 suffer from synchronous SSFF at t_{F}^+, new undirected edges (v_2, v_6), (v_2, v_8), (v_4, v_6), (v_4, v_8), (v_6, v_8), and (v_7, v_9) are added to the network topology among normal agents from the viewpoint of obtaining information, where multi-hop communication technology together with agents 3, 5, 10 as routing nodes is utilized. For these newly added edges, without loss of generality, all of their weights are set as 1. The new network topology among normal agents shown in Fig. 2 is established via the above procedures, which is obviously connected in $(t_{\mathrm{F}}, +\infty)$.

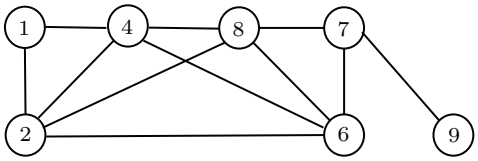


Fig. 2. Network topology among normal agents in $(t_{\mathrm{F}}, +\infty)$.

For general cases, it is obvious that under Assumption 1 and Assumption 2, the above strategy of recovering the connectivity of network topology among normal agents can guarantee that the new network topology among normal agents is connected in $(t_{\mathrm{F}}, +\infty)$.

For the convenience of the following expression, we present the following definitions.

Definition 2 Set V' is called maximum failure connected subset (MFCS) of graph G, if the following conditions are simultaneously satisfied: (i) $V' \subseteq F$; (ii) when any normal node is not used as a routing node, any two distinct nodes of V' are connected; (iii) when any normal node is not used as a routing node, $\forall v_i \in F - V'$, $\forall v_j \in V'$, node v_i and node v_j are not connected.

According to Definition 2, G in the above example only has two MFCSs, *i.e.*, $\{v_3, v_5\}$ and $\{v_{10}\}$.

Definition 3 Set V'' is called maximum normal neighbor set (MNNS) of MFCS V', if V'' is only composed of all normal neighbor nodes of each node in MFCS V'.

According to Definition 3, $\{v_2, v_4, v_6, v_8\}$ and $\{v_7, v_9\}$are MNNSs of MFCSs $\{v_3, v_5\}$ and $\{v_{10}\}$, respectively.

Definition 4 Set V''' is called minimum-hop normal neighbour set (MHNNS) of node v_i suffering from SSFF, if the following conditions are simultaneously satisfied: (i) if node v_i belongs to MFCS V', then V''' is a subset of MNNS V'' of MFCS V'; (ii) $v_j \in V'''$ if and only if $\mathrm{Dis}(v_j, v_i) = \min_{k \in V''} \mathrm{Dis}(v_k, v_i)$, where $\mathrm{Dis}(v_j, v_i)$ is the length of the shortest path between node v_j and node v_i.

According to Definition 4, MHNNSs of node v_3, node v_5, and node v_{10} are $\{v_2, v_4\}$, $\{v_6, v_8\}$, and $\{v_7, v_9\}$, respectively.

Remark 1 The core of the above strategy of recovering the connectivity of network topology among normal agents is to use multi-hop communication technology together with all agents subject to SSFF as routing nodes for reconnecting any two distinct nodes of each MNNS and ensuring that the subgraph composed of all nodes in each MNNS is a complete graph.

Remark 2 Multi-hop communication technology has been successfully used in topology control and routing protocol design of wireless sensor networks, which are typical MASs, and has been used for designing consensus protocols of MASs to improve the convergence rate of achieving consensus in Refs. [21–25]. Besides, when the number of agents in an MFCS is not large, carrying out the proposed strategy of network topology connectivity recovery will not quite damage the distributed property of MASs.

4. Fault-tolerant finite-time dynamical consensus protocol and convergence analysis

In this section, a fault-tolerant finite-time dynamical consensus protocol is proposed for double-integrator MASs with partial agents subject to synchronous SSFF and its effectiveness to ensure finite-time dynamical consensus is theoretically demonstrated.

For double-integrator MASs (1) satisfying Assumption 1 and Assumption 2, we propose the following fault-tolerant finite-time dynamical consensus protocol:

$$u_i(t)=\begin{cases}\displaystyle\sum_{j\in N_i(0)}a_{ij}\left(\operatorname{sgn}\left(p_j(t)-p_i(t)\right)\left|p_j(t)-p_i(t)\right|^{\sigma}+\operatorname{sgn}\left(q_j(t)-q_i(t)\right)\left|q_j(t)-q_i(t)\right|^{2\sigma/(1+\sigma)}\right), & t\in[0,t_{\mathrm{F}}],i\in I,\\ \displaystyle\sum_{j\in N_i(t_{\mathrm{F}}^+)}a_{ij}\left(\operatorname{sgn}\left(p_j(t)-p_i(t)\right)\left|p_j(t)-p_i(t)\right|^{\sigma}+\operatorname{sgn}\left(q_j(t)-q_i(t)\right)\left|q_j(t)-q_i(t)\right|^{2\sigma/(1+\sigma)}\right), & t\in(t_{\mathrm{F}},+\infty),i\in I-F,\\ \dfrac{\displaystyle\sum_{j\in N_i(t_{\mathrm{F}}^+)_{\mathrm{hop}}}\sum_{m\in N_j(t_{\mathrm{F}}^+)}a_{jm}\left(\operatorname{sgn}\left(p_m(t)-p_j(t)\right)\left|p_m(t)-p_j(t)\right|^{\sigma}+\operatorname{sgn}\left(q_m(t)-q_j(t)\right)\left|q_m(t)-q_j(t)\right|^{2\sigma/(1+\sigma)}\right)}{\left|N_i(t_{\mathrm{F}}^+)_{\mathrm{hop}}\right|}\\ -\alpha_i(t)\left(p_i(t_{\mathrm{F}})-\dfrac{\displaystyle\sum_{j\in N_i(t_{\mathrm{F}}^+)_{\mathrm{hop}}}p_j(t_{\mathrm{F}})}{\left|N_i(t_{\mathrm{F}}^+)_{\mathrm{hop}}\right|}\right)-\beta_i(t)\left(q_i(t_{\mathrm{F}})-\dfrac{\displaystyle\sum_{j\in N_i(t_{\mathrm{F}}^+)_{\mathrm{hop}}}q_j(t_{\mathrm{F}})}{\left|N_i(t_{\mathrm{F}}^+)_{\mathrm{hop}}\right|}\right), & t\in(t_{\mathrm{F}},\ +\infty),\ i\in F,\end{cases}\tag{3}$$

where $N_i(0)$ is the neighbour set of agent i $(i\in I)$ at $t=0$, $N_i(t_{\mathrm{F}}^+)$ is the neighbor set of agent i $(i\in I-F)$ at $t=t_{\mathrm{F}}^+$, $N_i(t_{\mathrm{F}}^+)_{\mathrm{hop}}$ is the MHNNS of agent i $(i\in F)$, $\alpha_i(t)$ and $\beta_i(t)$ are time-varying gains defined in $(t_{\mathrm{F}},+\infty)$, $\sigma\in(0,1)$, and other notations are the same to those above.

Theorem 1 Under Assumption 1 and Assumption 2, double-integrator MASs (1) applying the proposed strategy of recovering the connectivity of network topology among normal agents and the fault-tolerant finite-time dynamical consensus protocol (3) reach finite-time dynamical consensus, provided that

$$\int_{t_{\mathrm{F}}}^{t}\alpha_i(t)\,\mathrm{d}t=0,\quad\int_{t_{\mathrm{F}}}^{t}\left(\int_{t_{\mathrm{F}}}^{t}\alpha_i(t)\,\mathrm{d}t\right)\mathrm{d}t=1,\quad\forall t\geq t_{\mathrm{F}}+\Delta,\ \ \forall i\in F,\tag{4}$$

$$\int_{t_{\mathrm{F}}}^{t}\beta_i(t)\,\mathrm{d}t=1,\quad\int_{t_{\mathrm{F}}}^{t}\left(1-\int_{t_{\mathrm{F}}}^{t}\beta_i(t)\,\mathrm{d}t\right)\mathrm{d}t=0,\quad\forall t\geq t_{\mathrm{F}}+\Delta,\ \ \forall i\in F,\tag{5}$$

where Δ is a positive constant that can be arbitrarily chosen.

Proof Substituting Eq. (3) into Eq. (1) gives the closed-loop model of agent i $(i\in I-F)$ in $(t_{\mathrm{F}},+\infty)$ as follows:

$$\begin{cases}\dot{p}_i(t)=q_i(t),\\ \dot{q}_i(t)=\displaystyle\sum_{j\in N_i(t_M^+)}a_{ij}\left(\operatorname{sgn}\left(p_j(t)-p_i(t)\right)\left|p_j(t)-p_i(t)\right|^{\sigma}+\operatorname{sgn}\left(q_j(t)-q_i(t)\right)\left|q_j(t)-q_i(t)\right|^{2\sigma/(1+\sigma)}\right),\end{cases}\ t\in(t_{\mathrm{F}},\ +\infty),\ i\in I-F.\tag{6}$$

Using the analysis method in Ref. [9] yields that systems (6) satisfy

$$\begin{cases}p_i(t)=\dfrac{\displaystyle\sum_{j\in I-F}\left(p_j(t_{\mathrm{F}})+q_j(t_{\mathrm{F}})t\right)}{|I-F|},\\ q_i(t)=\dfrac{\displaystyle\sum_{j\in I-F}q_j(t_{\mathrm{F}})}{|I-F|},\end{cases}\quad\forall t\geq t_{\mathrm{F}}+T^*,\ \ \forall i\in I-F,\tag{7}$$

where T^* is a finite time relying on $(p_i(t_{\mathrm{F}}),q_i(t_{\mathrm{F}}))$, $i\in I-F$.

Substituting Eq. (3) into Eq. (1) can also yield the open-loop model of agent i ($i \in F$) in $(t_{\rm F}, +\infty)$ as follows:

$$\begin{cases} \dot{p}_i(t) = q_i(t), \\ \dot{q}_i(t) = \dfrac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} \sum\limits_{m\in N_j(t_{\rm F}^+)} a_{jm}\left(\operatorname{sgn}\left(p_m(t)-p_j(t)\right)\left|p_m(t)-p_j(t)\right|^{\sigma} + \operatorname{sgn}\left(q_m(t)-q_j(t)\right)\left|q_m(t)-q_j(t)\right|^{2\sigma/(1+\sigma)}\right)}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|} \\ \qquad -\alpha_i(t)\left(p_i(t_{\rm F}) - \dfrac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} p_j(t_{\rm F})}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|}\right) - \beta_i(t)\left(q_i(t_{\rm F}) - \dfrac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} q_j(t_{\rm F})}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|}\right), \quad t\in(t_{\rm F},+\infty), \quad i\in F. \end{cases} \tag{8}$$

It follows from Eqs. (6) and (8) that

$$\dot{q}_i(t) = \frac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} \dot{q}_j(t)}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|} - \alpha_i(t)\left(p_i(t_{\rm F}) - \frac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} p_j(t_{\rm F})}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|}\right) - \beta_i(t)\left(q_i(t_{\rm F}) - \frac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} q_j(t_{\rm F})}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|}\right), \quad t\in(t_{\rm F},+\infty), \quad i\in F. \tag{9}$$

Integrating both sides of Eq. (9) about t from $t_{\rm F}$ to t leads to

$$\begin{aligned} q_i(t) = {} & \frac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} q_j(t)}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|} - \left(\int_{t_{\rm F}}^{t}\alpha_i(t)\,{\rm d}t\right)\left(p_i(t_{\rm F}) - \frac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} p_j(t_{\rm F})}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|}\right) \\ & + \left(1-\int_{t_{\rm F}}^{t}\beta_i(t)\,{\rm d}t\right)\left(q_i(t_{\rm F}) - \frac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} q_j(t_{\rm F})}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|}\right), \quad t\in(t_{\rm F},+\infty), \quad i\in F. \end{aligned} \tag{10}$$

Further integrating both sides of Eq. (10) about t from $t_{\rm F}$ to t derives

$$\begin{aligned} p_i(t) = {} & \frac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} p_j(t)}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|} + \left(1-\int_{t_{\rm F}}^{t}\left(\int_{t_{\rm F}}^{t}\alpha_i(t)\,{\rm d}t\right){\rm d}t\right)\left(p_i(t_{\rm F}) - \frac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} p_j(t_{\rm F})}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|}\right) \\ & + \left(\int_{t_{\rm F}}^{t}\left(1-\int_{t_{\rm F}}^{t}\beta_i(t)\,{\rm d}t\right){\rm d}t\right)\left(q_i(t_{\rm F}) - \frac{\sum\limits_{j\in N_i(t_{\rm F}^+)_{\rm hop}} q_j(t_{\rm F})}{\left|N_i(t_{\rm F}^+)_{\rm hop}\right|}\right), \quad t\in(t_{\rm F},+\infty), \quad i\in F. \end{aligned} \tag{11}$$

Substituting Eqs. (4), (5), and (7) into Eqs. (10) and (11) yields

$$\begin{cases} p_i(t) = \dfrac{\sum\limits_{j\in I-F}\left(p_j(t_{\rm F}) + q_j(t_{\rm F})t\right)}{|I-F|}, \\ q_i(t) = \dfrac{\sum\limits_{j\in I-F} q_j(t_{\rm F})}{|I-F|}, \end{cases} \quad \forall t \geq t_{\rm F} + \max\left\{T^*, \Delta\right\}, \ \forall i \in F. \tag{12}$$

It directly follows from Eqs. (7) and (12) that

$$\begin{cases} p_i(t) = \dfrac{\sum\limits_{j\in I-F}\left(p_j(t_{\rm F}) + q_j(t_{\rm F})t\right)}{|I-F|}, \\ q_i(t) = \dfrac{\sum\limits_{j\in I-F} q_j(t_{\rm F})}{|I-F|}, \end{cases} \quad \forall t \geq t_{\rm F} + \max\left\{T^*, \Delta\right\}, \ \forall i \in I.$$

This means that double-integrator MASs (1) under the proposed strategy of recovering the connectivity of network topology among normal agents and the fault-tolerant finite-time dynamical consensus protocol (3) achieve finite-time dynamical consensus. The proof is completed.

Remark 3 The effectiveness of the fault-tolerant finite-

time dynamical consensus protocol (3) depends on the existence of $\alpha_i(t)$ satisfying condition (4) and $\beta_i(t)$ satisfying condition (5). It is easy to see that any constant gain does not satisfy condition (4) or condition (5). Fortunately, both $\alpha_i(t)$ satisfying condition (4) and $\beta_i(t)$ satisfying condition (5) really exist. It can be verified that

$$\alpha_i(t)=\begin{cases}\dfrac{4}{\Delta^2}, & t\in\left(t_{\rm F},t_{\rm F}+\dfrac{\Delta}{2}\right],\\ -\dfrac{4}{\Delta^2}, & t\in\left(t_{\rm F}+\dfrac{\Delta}{2},t_{\rm F}+\Delta\right],\\ 0, & t\in(t_{\rm F}+\Delta,+\infty),\end{cases}\quad \forall i\in F, \tag{13}$$

$$\beta_i(t)=\begin{cases}\dfrac{3}{\Delta}, & t\in\left(t_{\rm F},t_{\rm F}+\dfrac{\Delta}{3}\right],\\ \dfrac{3}{2\Delta}, & t\in\left(t_{\rm F}+\dfrac{\Delta}{3},t_{\rm F}+\dfrac{2\Delta}{3}\right],\\ -\dfrac{3}{2\Delta}, & t\in\left(t_{\rm F}+\dfrac{2\Delta}{3},t_{\rm F}+\Delta\right],\\ 0, & t\in(t_{\rm F}+\Delta,+\infty),\end{cases}\quad \forall i\in F, \tag{14}$$

satisfy condition (4) and condition (5), respectively.

Remark 4 The function of

$$\sum_{m\in N_j(t_{\rm F}^+)} a_{jm}\begin{pmatrix}\operatorname{sgn}(p_m(t)-p_j(t))\left|p_m(t)-p_j(t)\right|^{\sigma}\\ +\operatorname{sgn}(q_m(t)-q_j(t))\left|q_m(t)-q_j(t)\right|^{2\sigma/(1+\sigma)}\end{pmatrix}$$

in Eq. (3) is to compute control input $u_j(t)$ $(t\in(t_{\rm F},+\infty))$ of agent j $(j\in N_i(t_{\rm F}^+)_{\rm hop})$ by agent i $(i\in F)$. For agent i $(i\in F)$, both of $(p_j(t),q_j(t))$ $(j\in N_i(t_{\rm F}^+)_{\rm hop})$ and $(p_m(t),q_m(t))$ $(m\in N_j(t_{\rm F}^+))$, which are need while computing control input $u_j(t)$ $(t\in(t_{\rm F},+\infty))$, can be obtained by using multi-hop communication technology together with partial or all agents in an MFCS where agent i $(i\in F)$ is located as routing nodes. Similarly, when the number of nodes in an MFCS is not large, implementing the fault-tolerant finite-time dynamical consensus protocol (3) will not quite damage the distributed property of MASs.

5. Numerical simulations

Consider double-integrator MASs with ten agents and the initial network topology shown in Fig. 1. Without loss of generality, all weights of edges are 1, the initial states of all agents are randomly set, and agents 3, 5, 10 suffer from synchronous SSFF with $t_{\rm F}=3$ s. The new network topology among normal agents shown in Fig. 2 is constructed by using the proposed strategy of network topology connectivity recovery. Numerical results of double-integrator MASs (1) under protocol (3) with $\alpha_3(t)=\alpha_5(t)=\alpha_{10}(t)$ in Eq. (13), $\beta_3(t)=\beta_5(t)=\beta_{10}(t)$ in Eq. (14), $\Delta=3$, and $\sigma=0.5$ are shown in Figs. 3 and 4. It is easy to see from Figs. 3 and 4 that double-integrator MASs (1) using the fault-tolerant finite-time dynamical consensus protocol (3) achieve finite-time dynamical consensus. Therefore, the results of Theorem 1 are numerically verified.

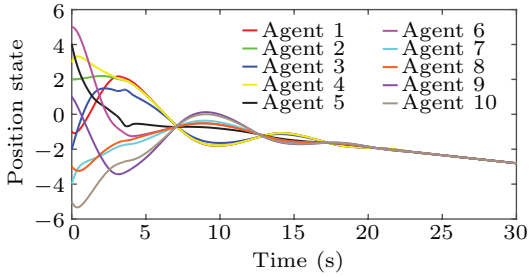


Fig. 3. Position states of systems (1) under protocol (3).

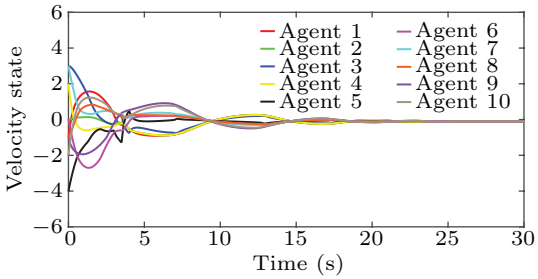


Fig. 4. Velocity states of systems (1) under protocol (3).

6. Conclusions

This paper studied the fault-tolerant finite-time dynamical consensus problems of double-integrator MASs with partial agents subject to synchronous SSFF. The strategy of network topology connectivity recovery and the fault-tolerant finite-time dynamical consensus protocol were proposed for resisting SSFF. Theoretical and numerical results showed that double-integrator MASs with partial agents subject to synchronous SSFF using the strategy of network topology connectivity recovery and the fault-tolerant finite-time dynamical consensus protocol can reach finite-time dynamical consensus. It should be noticed that SSFF was assumed to be synchronous in this paper. However, in real application, SSFF might be asynchronous. Therefore, in the future, we will investigate fault-tolerant finite-time dynamical consensus of double-integrator MASs with partial agents subject to asynchronous SSFF.

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