

Quantum routing of few photons using a nonlinear cavity coupled to two chiral waveguides

Jian-Shuang Liu(刘建双), Ya Yang(杨亚), Jing Lu(卢竞), and Lan Zhou(周兰)

Citation: Chin. Phys. B, 2022, 31 (**11**): 110301. DOI: 10.1088/1674-1056/ac7f8f

What follows is a list of articles you may be interested in

Improvement of a continuous-variable measurement-device-independent quantum key distribution system via quantum scissors

Lingzhi Kong(孔令志), Weiqi Liu(刘维琪), Fan Jing(荆凡), Zhe-Kun Zhang(张哲坤), Jin Qi(齐锦), and Chen He(贺晨)

Chin. Phys. B, 2022, 31 (**9**): 090304. DOI: 10.1088/1674-1056/ac6dba

Finite-key analysis of practical time-bin high-dimensional quantum key distribution with afterpulse effect

Yu Zhou(周雨), Chun Zhou(周淳), Yang Wang(汪洋), Yi-Fei Lu(陆宜飞), Mu-Sheng Jiang(江木生), Xiao-Xu Zhang(张晓旭), and Wan-Su Bao(鲍皖苏)

Chin. Phys. B, 2022, 31 (**8**): 080303. DOI: 10.1088/1674-1056/ac6017

Practical security analysis of continuous-variable quantum key distribution with an unbalanced heterodyne detector

Lingzhi Kong(孔令志), Weiqi Liu(刘维琪), Fan Jing(荆凡), and Chen He(贺晨)

Chin. Phys. B, 2022, 31 (**7**): 070303. DOI: 10.1088/1674-1056/ac4102

Error-detected single-photon quantum routing using a quantum dot and a double-sided microcavity system

A-Peng Liu(刘阿鹏), Liu-Yong Cheng(程留永), Qi Guo(郭奇), Shi-Lei Su(苏石磊), Hong-Fu Wang(王洪福), Shou Zhang(张寿)

Chin. Phys. B, 2019, 28 (**2**): 020301. DOI: 10.1088/1674-1056/28/2/020301

Distributed wireless quantum communication networks with partially entangled pairs

Yu Xu-Tao (余旭涛), Zhang Zai-Chen (张在琛), Xu Jin (徐进)

Chin. Phys. B, 2014, 23 (**1**): 010303. DOI: 10.1088/1674-1056/23/1/010303

Quantum routing of few photons using a nonlinear cavity coupled to two chiral waveguides

Jian-Shuang Liu(刘建双), Ya Yang(杨亚), Jing Lu(卢竞), and Lan Zhou(周兰)†

Affiliation Synergetic Innovation Center for Quantum Effects and Applications, Key Laboratory for Matter Microstructure and Function of Hunan Province, Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, School of Physics and Electronics, Hunan Normal University, Changsha 410081, *China*

(Received 11 April 2022; revised manuscript received 27 June 2022; accepted manuscript online 8 July 2022)

We investigate few-photon scattering properties in two one-dimensional waveguides chirally coupled to a nonlinear cavity. The quantum states of scattered few photons are solved analytically via a real-space approach, and the solution indicates the few-photon reflection and transmission properties. When inputting two photons of equal energy to resonate with the cavity, the propagation characteristics of the two photons will be interesting, which is different from the previous anti-bunching effects with a quantum emitter. More importantly, when the total energy of the two incident photons equals the energy of a nonlinear cavity accommodating two photons, influence of the bound state will become larger to result in disappearance of antibunching effect. However, the bound state has no effect on probability of routing to another waveguide.

Keywords: quantum routing, few-photon scattering, nonlinear cavity, chirally couple

PACS: 03.67.Dd, 03.67.Hk **DOI:** 10.1088/1674-1056/ac7f8f

1. Introduction

Quantum networks are an important part of quantum information processing and quantum communication, which is composed of quantum channels (to ensure low dissipation of quantum signals transmitted along the channels) and quantum nodes (to control the transmission of quantum signals). Photons become an ideal candidate for carriers of quantum information,[1–10] because they travel fast and are not significantly affected by decoherence effects over long distance. Waveguide can naturally guide photons, so waveguide quantum electrodynamics (QED) is a hot research topic in recent years. One-dimensional (1D) waveguides exhibit diversity as they can modulate the transmission of photonic states by coupling with quantum emitters.[11–15] There are various types of quantum emitters, including quantum dots,[16–18] two-level atoms,[19–25] three-level atoms,[26–28] side optical cavities,[29–31] and cavities with an atom[32–36] or nonlinear Kerr mediums.[37–39] Many physical phenomena have been studied, such as electromagnetically induced transparency (EIT),[40–42] Fano resonance,[43–45] polarization effect,[46] slow light behavior,[47] and multi-photon transport.[48,49] However, these research works mainly focus on the structure of a single 1D waveguide coupled with a two-level system or single-photon qubit.

In recent years, chiral waveguide QED[50,51] has opened up exciting new ways to study interaction between light and matter, which has attracted wide attention. Many quantum systems have realized chiral coupling,[52–58] and it is found that the reflection and transmission of photons in these systems will become asymmetric, and the propagation of photons is irreversible. Therefore, the study of structures composed of multiple one-dimensional chiral waveguides has attracted our attention because of its valuable applications in quantum networks.

In this paper, we investigate the scattering problem of few photons in two one-dimensional waveguides chirally coupled by a nonlinear optical cavity. Note that it is difficult to obtain both strong Kerr nonlinearity and low loss. However, compared with a waveguide coupled atom, intuitively speaking, the atom can at most absorb only one photon at a time, and the optical cavity we study can absorb multiple photons, so it is very interesting to study the transport characteristics of multiple photons. In real space, we give an explicit construction of the exact n-photon scattering eigenstates and then discuss the transport properties. In the analytic expression of amplitude for single photon, we show that in the case of chiral coupling, the reflection and transmission of single photon become asymmetric due to the addition of a waveguide (quantum channel), realizing the non-reciprocity of a single photon. Then, by analyzing the scattering eigenstates of two photons, on account of the possibility of that the nonlinear cavity can absorb two photons at the same time, we discuss two special cases. These show rich characteristics, including the influence of the two-photon bound state, which are valuable for many experiments of multi-photon quantum routing. The rest of the paper is organized as follows. In Section 2, we present the theoretical model of the chiral coupling of a nonlinear cavity to two one-dimensional (1D) waveguides, and give an effective real space Hamiltonian of the system. In Section 3, we use the decomposition of scattering matrix (S matrix) to greatly sim-

†Corresponding author. E-mail: zhoulan@hunnu.edu.cn

© 2022 Chinese Physical Society and IOP Publishing Ltd

plify the calculation process, and discuss the transport properties of two-photon in detail. Immediately, obtain the output wave function for the photon pair incident from one port of a waveguide, the anti-bunch and bunch effects are discussed for photon-pair appearing to the same port, and the conditions of incident photons routed to desired ports are studied. Finally, we conclude with a brief summary of the results.

2. Model and hamiltonian

We consider a cavity-waveguides system, illustrated in Fig. 1(b), consisting of two infinite 1D waveguides (labeled by a and b, respectively) chirally coupled with a nonlinear cavity. The 1D continuous structure of the waveguide is formed by the right and left modes that propagate photons in two directions, and is referred to as the "two-mode" waveguide. The real-space field operators $\hat{R}_p(x)$ and $\hat{L}_p(x)$ [$\hat{R}_p^\dagger(x)$ and $\hat{L}_p^\dagger(x)$] annihilate (create) a left- or right-mode photon at position x in the waveguide p, and satisfy the bosonic commutation rules $[\hat{R}_p(x),\hat{R}_{p'}^\dagger(x')]=[\hat{L}_p(x),\hat{L}_{p'}^\dagger(x')]=\delta_{pp'}\delta(x-x')$, $[\hat{R}_p(x),\hat{L}_{p'}^\dagger(x')]=[\hat{L}_p(x),\hat{R}_{p'}^\dagger(x')]=0$, while the rest of the commutators vanish. The Hamiltonian of free photons propagating in waveguides is given by

$$\hat{H}_p=-\mathrm{i}v\sum_{p=\mathrm{a,b}}\int \mathrm{d}x\left[\hat{R}_p^\dagger(x)\,\partial_x\hat{R}_p(x)-\hat{L}_p^\dagger(x)\,\partial_x\hat{L}_p(x)\right], \quad (1)$$

where v is the group velocity of the photons, for simplicity, we set $v_\mathrm{a}=v_\mathrm{a}=v$. The free Hamiltonian for a nonlinear cavity at coordinate $x=0$ is reformulated as

$$\hat{H}_d=\Omega\hat{d}^\dagger\hat{d}+\frac{U}{2}\hat{d}^\dagger\hat{d}^\dagger\hat{d}\hat{d}, \quad (2)$$

where $\hat{d}$ ($\hat{d}^\dagger$) is the annihilation (creation) operator of the cavity mode with frequency Ω, which is far away from the cutoff frequency of the waveguide modes. The second term in Eq. (2) describes the Kerr nonlinear interaction with strength U. For the sake of simplicity, we first neglect the photon loss in our calculation. For finite U, an addition energy $U>0$ is required to add a further boson to the d mode once the d mode is occupied by one photon.

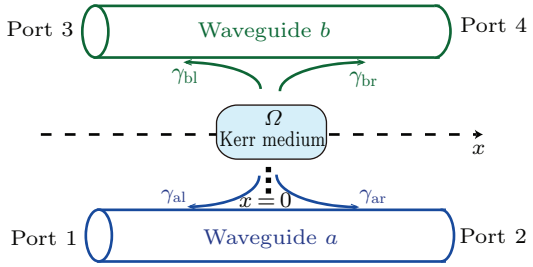


Fig. 1. Schematic of a few-photon router with four ports: A nonlinear cavity system interacts with two independent waveguides labeled by a and b. The blue (green) line represents the chiral coupling between the nonlinear cavity with Kerr medium U and the left and right going photons of the bottom waveguide a (b), and the coupling rates are γ_ar (γ_br) and γ_al (γ_bl), respectively.

The chiral coupling between the cavity and the waveguide modes reads

$$\begin{aligned}\hat{H}_\mathrm{int}=&\sum_{p=\mathrm{a,b}}\sqrt{v\gamma_\mathrm{pr}}\int \mathrm{d}x\delta(x)\left[\hat{R}_p^\dagger(x)\,\hat{d}+\mathrm{H.c.}\right]\\&+\sum_{p=\mathrm{a,b}}\sqrt{v\gamma_\mathrm{pl}}\int \mathrm{d}x\delta(x)\left[\hat{L}_p^\dagger(x)\,\hat{d}+\mathrm{H.c.}\right], \quad (3)\end{aligned}$$

where the coupling strengths $\sqrt{v\gamma_\mathrm{pr}}$ and $\sqrt{v\gamma_\mathrm{pl}}$ are much smaller than the excitation energy Ω. The Hamiltonian of the total system reads

$$\hat{H}=\hat{H}_p+\hat{H}_d+\hat{H}_\mathrm{int}. \quad (4)$$

This Hamiltonian describes the case in which a propagating photon can travel in both directions of two one-dimensional waveguides, so that the nonlinear cavity interacts with the four modes.

We employ the following transformations:

$$\hat{E}_p^\dagger(x)=\sqrt{\frac{\gamma_\mathrm{pr}}{\gamma_p}}\hat{R}_p^\dagger(x)+\sqrt{\frac{\gamma_\mathrm{pl}}{\gamma_p}}\hat{L}_p^\dagger(-x), \quad (5a)$$

$$\hat{O}_p^\dagger(x)=\sqrt{\frac{\gamma_\mathrm{pl}}{\gamma_p}}\hat{R}_p^\dagger(x)-\sqrt{\frac{\gamma_\mathrm{pr}}{\gamma_p}}\hat{L}_p^\dagger(-x), \quad (5b)$$

$$\hat{C}^\dagger(x)=\sqrt{\frac{\gamma_\mathrm{a}}{\gamma}}\hat{E}_\mathrm{a}^\dagger(x)+\sqrt{\frac{\gamma_\mathrm{b}}{\gamma}}\hat{E}_\mathrm{b}^\dagger(x), \quad (5c)$$

$$\hat{D}^\dagger(x)=\sqrt{\frac{\gamma_\mathrm{b}}{\gamma}}\hat{E}_\mathrm{a}^\dagger(x)-\sqrt{\frac{\gamma_\mathrm{a}}{\gamma}}\hat{E}_\mathrm{b}^\dagger(x), \quad (5d)$$

where $\gamma_p=\gamma_\mathrm{pr}+\gamma_\mathrm{pl}$, $\gamma=\gamma_\mathrm{a}+\gamma_\mathrm{b}$, the original Hamiltonian is transformed into four decoupled "one-mode" Hamiltonians, i.e., $\hat{H}=\hat{H}_\mathrm{C}+\hat{H}_\mathrm{DO}$, where

$$\begin{aligned}\hat{H}_\mathrm{C}=&-\mathrm{i}v\int \mathrm{d}x\hat{C}^\dagger(x)\frac{\partial}{\partial x}\hat{C}(x)+\Omega\hat{d}^\dagger\hat{d}+\frac{U}{2}\hat{d}^\dagger\hat{d}^\dagger\hat{d}\hat{d}\\&+\sqrt{v\gamma}\int \mathrm{d}x\delta(x)\left[\hat{C}^\dagger(x)\,\hat{d}+\hat{d}^\dagger\hat{C}(x)\right], \quad (6)\\ \hat{H}_\mathrm{DO}=&-\mathrm{i}v\int \mathrm{d}x\hat{D}^\dagger(x)\frac{\partial}{\partial x}\hat{D}(x)\\&-\mathrm{i}v\sum_{p=\mathrm{a,b}}\int \mathrm{d}x\hat{O}_p^\dagger(x)\frac{\partial}{\partial x}\hat{O}_p(x). \quad (7)\end{aligned}$$

The $\hat{H}_\mathrm{DO}$ is an interaction-free three-mode Hamiltonian, while $\hat{H}_\mathrm{C}$ describes a nontrivial one-mode interacting model with coupling strength $\sqrt{v\gamma}$. Note that the number operator for the three-mode bosons is

$$\hat{n}_\mathrm{DO}=\int \mathrm{d}x\hat{D}^\dagger(x)\,\hat{D}(x)+\sum_{p=\mathrm{a,b}}\int \mathrm{d}x\hat{O}_p^\dagger(x)\,\hat{O}_p(x), \quad (8)$$

for C-mode bosons it is

$$\hat{n}_\mathrm{C}=\int \mathrm{d}x\hat{C}^\dagger(x)\,\hat{C}(x), \quad (9)$$

and the occupation number of the cavity is

$$\hat{n}_d=\hat{d}^\dagger\hat{d}. \quad (10)$$

Because H commutes with certain number operators, $[\hat{H},\hat{n}_{\rm C}+\hat{n}_d]=[\hat{H},\hat{n}_{\rm DO}]=0$, the total numbers of excitations in both the three-mode and the C-mode spaces are separately conserved. In other words, the three modes evolve freely in the waveguide. Therefore we shall focus on the calculation of the transport properties of the photons in C-mode, and then transform back to the left (right) representation.

3. Scattering in two-photon excitation

Assume that there are two incident photons in the waveguide from the left side with momenta k and p, respectively, the frequency $\omega=v(k+p)$ and the energy difference between two photons $\Delta=v(k-p)/2$. Physically, there may be three cases of two photons: (i) Both photons are in waveguide C-mode. (ii) One photon is in waveguide C-mode and the other photon is absorbed into nonlinear cavity. (iii) Both photons are absorbed into nonlinear cavity. In the free two-photon Hilbert space, its real-space representation consists of a symmetric function of the coordinates of the photons x_1,x_2, and is defined as $\phi_{\rm CC}(x_1,x_2)$. The general two-photon scattering state of the system in the spaces with C-mode takes the following form:

$$\begin{aligned}\left|\Psi_{k,p}\right\rangle_{\rm CC} &= \int\int {\rm d}x_1{\rm d}x_2\phi_{\rm CC}(x_1,x_2)\frac{1}{\sqrt{2}}\hat{C}^\dagger(x_1)\hat{C}^\dagger(x_2)\left|\emptyset\right\rangle\\ &+\int {\rm d}x e(x)\hat{C}^\dagger(x)\hat{d}^\dagger\left|\emptyset\right\rangle+\phi_{\rm dd}\frac{1}{\sqrt{2}}\hat{d}^\dagger\hat{d}^\dagger\left|\emptyset\right\rangle,\end{aligned}\tag{11}$$

where $e(x)$ is the probability amplitude function of one photon while the nonlinear cavity contains the other photon; $\phi_{\rm dd}$ is the probability amplitude with two photons in the cavity. In order to satisfy the statistical property of photons, the amplitudes satisfy the relation $\phi_{\rm CC}(x_1,x_2)=\phi_{\rm CC}(x_2,x_1)$, and it is continuous on the line $x_1=x_2$.

In a steady state, we have $\hat{H}_{\rm C}\left|\Psi_{k,p}\right\rangle_{\rm CC}=\omega\left|\Psi_{k,p}\right\rangle_{\rm CC}$. We obtain the following linear equations for the amplitudes in the scattering state:

$$\begin{aligned}\omega\phi_{\rm CC}(x_1,x_2) &= -{\rm i}v(\partial_{x_1}+\partial_{x_2})\phi_{\rm CC}(x_1,x_2)\\ &+\sqrt{\frac{v\gamma}{2}}[\delta(x_1)e(x_2)+\delta(x_2)e(x_1)],\end{aligned}\tag{12a}$$

$$\begin{aligned}(\omega-\Omega)e(x) &= -{\rm i}v\partial_x e(x)+\sqrt{2v\gamma}\delta(x)\phi_{\rm dd}\\ &+\sqrt{\frac{v\gamma}{2}}[\phi_{\rm CC}(0,x)+\phi_{\rm CC}(x,0)],\end{aligned}\tag{12b}$$

$$(\omega-U)\phi_{\rm dd}=2\Omega\phi_{\rm dd}+\sqrt{2v\gamma}e(0).\tag{12c}$$

We use $\phi_{\rm CC}(0,x)\equiv(1/2)[\phi_{\rm CC}(0^+,x)+\phi_{\rm CC}(0^-,x)]$. The functions $\phi_{\rm CC}(x_1,x_2)$ and $e(x)$ are piecewise continuous. Applying the bosonic symmetrical statistics, we can settle for $\phi_{\rm CC}(x_1,x_2)$ by first considering the half-space $x_1<x_2$ and then by interchanging $x_1\Longleftrightarrow x_2$ to extend the result to get the full space. In this case, there are three quadrants in real space: $x_1<x_2<0$, $x_1<0<x_2$, and $0<x_1<x_2$. In the region that all photons are away from the cavity, the plane wave with wave vectors k and p. We require that the incoming wave functions consist entirely of plane waves, that is, for $x_1<x_2<0$,

$$\phi_{\rm CC}(x_1,x_2)=\frac{1}{2\sqrt{2}\pi}\left({\rm e}^{{\rm i}kx_1+{\rm i}px_2}+{\rm e}^{{\rm i}kx_2+{\rm i}px_1}\right).\tag{13}$$

Using the boundary conditions and the continuity of the function, we can solve it naturally.

For $x_1<0<x_2$

$$\phi_{\rm CC}(x_1,x_2)=\frac{1}{2\sqrt{2}\pi}\left(\bar{t}_p{\rm e}^{{\rm i}kx_1+{\rm i}px_2}+\bar{t}_k{\rm e}^{{\rm i}kx_2+{\rm i}px_1}\right),\tag{14}$$

and for $0<x_1<x_2$

$$\begin{aligned}\phi_{\rm CC}(x_1,x_2) &= \frac{1}{2\sqrt{2}\pi}\Big[\bar{t}_k\bar{t}_p\left({\rm e}^{{\rm i}kx_1+{\rm i}px_2}+{\rm e}^{{\rm i}kx_2+{\rm i}px_1}\right)\\ &+B{\rm e}^{{\rm i}\omega x_{\rm c}/v}{\rm e}^{{\rm i}(\omega-2\Omega)|x|/2v}{\rm e}^{-\gamma|x|/2}\Big],\end{aligned}\tag{15}$$

where

$$\bar{t}_\varepsilon=\frac{v_{\rm c}\varepsilon-\Omega-{\rm i}\gamma/2}{v_{\rm c}\varepsilon-\Omega+{\rm i}\gamma/2}\tag{16}$$

is the single-photon transmission amplitudes (for $\varepsilon=k,p$), and $x_{\rm c}\equiv(x_1+x_2)/2$ and $x\equiv x_1-x_2$ are the center-of-mass coordinate and the relative coordinate, B is a constant to be determined. Again, applying the continuity of the function yields

$$\begin{aligned}B &= U\gamma^2\frac{1}{(\Omega-vk-{\rm i}\gamma/2)(\Omega-vp-{\rm i}\gamma/2)}\\ &\times\frac{1}{\Omega+(U-\omega-{\rm i}\gamma)/2}.\end{aligned}\tag{17}$$

Obviously, B is dependent on the nonlinear interaction strength U. For later convenience, we introduce

$$X_{k,p}(x_{\rm c},x)\equiv\frac{1}{2\sqrt{2}\pi}{\rm e}^{{\rm i}kx_1+{\rm i}px_2},\tag{18a}$$

$$Y_{k,p}(x_{\rm c},x)\equiv\frac{1}{2\sqrt{2}\pi}{\rm e}^{{\rm i}kx_2+{\rm i}px_1},\tag{18b}$$

$$\begin{aligned}S_{k,p}(x_{\rm c},x) &\equiv X_{k,p}(x_{\rm c},x)+Y_{k,p}(x_{\rm c},x)\\ &=\frac{1}{\sqrt{2}\pi}{\rm e}^{{\rm i}\omega x_{\rm c}}\cos(\Delta x),\end{aligned}\tag{18c}$$

$$B_{\rm CC}(x_{\rm c},x)\equiv B{\rm e}^{{\rm i}\omega x_{\rm c}/v}{\rm e}^{{\rm i}(\omega-2\Omega)|x|/2v}{\rm e}^{-\gamma|x|/2}.\tag{18d}$$

The wave function for the two photons before scattering (incoming state) consists entirely of plane waves, so the in state can be written as

$$\begin{aligned}\left|\varphi_{\rm Ra}^{\rm in}\right\rangle &= \int\int {\rm d}x_1{\rm d}x_2\frac{1}{2\sqrt{2}\pi}\left[{\rm e}^{{\rm i}kx_1+{\rm i}px_2}+{\rm e}^{{\rm i}kx_2+{\rm i}px_1}\right]\\ &\times\frac{1}{\sqrt{2}}\hat{R}_{\rm a}^\dagger(x_1)\hat{R}_{\rm a}^\dagger(x_2)\left|\emptyset\right\rangle.\end{aligned}\tag{19}$$

Each of photons impinges on the quantum node, being eventually scattered off. Every one may be reflected (transmitted) to the left (right) side of the quantum node (denoted as port 1 (2)) in waveguide a, or be reflected (transmitted) to

the left (right) side of the quantum node (denoted as port 3 (4)) in waveguide b. Then the output state contains ten terms: Both photons are transmitted to port 2 or 4 with amplitudes $\rho_{22}(x_c,x)$ and $\rho_{44}(x_c,x)$,

$$\rho_{22}(x_c,x) = t^a_{ak}t^a_{ap}S_{k,p}(x_c,x)+\frac{\gamma^2_{ar}}{\gamma^2}B_{CC}(x_c,x), \tag{20a}$$

$$\rho_{44}(x_c,x) = t^b_{ak}t^b_{ap}S_{k,p}(x_c,x)-\frac{\gamma_{ar}\gamma_{br}}{\gamma^2}B_{CC}(x_c,x). \tag{20b}$$

Both photons are reflected to port 1 or 3 with amplitudes $\rho_{11}(x_1,x_2)$ or $\rho_{33}(x_1,x_2)$,

$$\rho_{11}(x_c,x) = r^a_{ak}r^a_{ap}S_{k,p}(-x_c,x) + \frac{\gamma_{ar}\gamma_{al}}{\gamma^2}B_{CC}(-x_c,x), \tag{21a}$$

$$\rho_{33}(x_c,x) = r^b_{ak}r^b_{ap}S_{k,p}(-x_c,x) - \frac{\gamma_{ar}\gamma_{bl}}{\gamma^2}B_{CC}(-x_c,x). \tag{21b}$$

One photon is reflected to port 1 and the other is transmitted to port 2 with amplitude

$$\rho_{12}(x_c,x) = \sqrt{2}t^a_{ak}r^a_{ap}X_{k,p}\left(\frac{x}{2},2x_c\right)+\sqrt{2}r^a_{ak}t^a_{ap}Y_{k,p}\left(\frac{x}{2},2x_c\right) + \sqrt{2}\frac{\sqrt{\gamma^3_{ar}\gamma_{al}}}{\gamma^2}B_{CC}\left(\frac{x}{2},2x_c\right). \tag{22}$$

One photon is directed to port 3 and the other is directed to port 4 with amplitude

$$\rho_{34}(x_c,x) = \sqrt{2}t^b_{ak}r^b_{ap}S_{k,p}\left(\frac{x}{2},2x_c\right) - \sqrt{2}\frac{\sqrt{\gamma^2_{ar}\gamma_{br}\gamma_{bl}}}{\gamma^2}B_{CC}\left(\frac{x}{2},2x_c\right). \tag{23}$$

One photon is transmitted to port 2 and the other is directed to port 3 or 4 with amplitudes $\rho_{23}(x_1,x_2)$ or $\rho_{24}(x_1,x_2)$,

$$\rho_{23}(x_c,x) = \sqrt{2}r^b_{ap}t^a_{ak}X_{k,p}\left(\frac{x}{2},2x_c\right)+\sqrt{2}r^b_{ak}t^a_{ap}Y_{k,p}\left(\frac{x}{2},2x_c\right) - \sqrt{2}\frac{\sqrt{\gamma^3_{ar}\gamma_{bl}}}{\gamma^2}B_{CC}\left(\frac{x}{2},2x_c\right), \tag{24a}$$

$$\rho_{24}(x_c,x) = \sqrt{2}t^b_{ap}t^a_{ak}X_{k,p}(x_c,x)+\sqrt{2}t^b_{ak}t^a_{ap}Y_{k,p}(x_c,x) - \sqrt{2}\frac{\sqrt{\gamma^3_{ar}\gamma_{br}}}{\gamma^2}B_{CC}(x_c,x). \tag{24b}$$

One photon is reflected to port 1 and the other is directed to port 3, 4 with amplitudes $\rho_{13}(x_1,x_2)$ and $\rho_{14}(x_1,x_2)$,

$$\rho_{13}(x_c,x) = \sqrt{2}r^a_{ak}r^b_{ak}S_{k,p}(-x_c,x) + \sqrt{2}\frac{\gamma_{ar}\sqrt{\gamma_{al}\gamma_{bl}}}{\gamma^2}B_{CC}(-x_c,x), \tag{25a}$$

$$\rho_{14}(x_c,x) = \sqrt{2}r^a_{ak}t^b_{ap}S_{k,p}\left(-\frac{x}{2},2x_c\right) - \sqrt{2}\frac{\gamma_{ar}\sqrt{\gamma_{al}\gamma_{br}}}{\gamma^2}B_{CC}\left(-\frac{x}{2},2x_c\right). \tag{25b}$$

Here

$$t^p_{p\varepsilon} = \frac{\varepsilon-\Omega+i(\gamma_{pl}-\gamma_{pr}+\gamma_{\bar{p}})/2}{\varepsilon-\Omega+i\gamma/2}, \tag{26a}$$

$$t^{\bar{p}}_{p\varepsilon} = \frac{-i\sqrt{\gamma_{pr}\gamma_{\bar{p}r}}}{\varepsilon-\Omega+i\gamma/2}, \tag{26b}$$

$$r^p_{p\varepsilon} = \frac{-i\sqrt{\gamma_{pr}\gamma_{pl}}}{\varepsilon-\Omega+i\gamma/2}, \tag{26c}$$

$$r^{\bar{p}}_{p\varepsilon} = \frac{-i\sqrt{\gamma_{pr}\gamma_{\bar{p}l}}}{\varepsilon-\Omega+i\gamma/2}. \tag{26d}$$

In the above formula, $p\neq\bar{p}$ and $p,\bar{p}\in\{a,b\}$. Herein, $t^p_{p\varepsilon}(\tilde{t}^p_{p\varepsilon})$ and $r^p_{p\varepsilon}(\tilde{r}^p_{p\varepsilon})$ are the transmitted and reflected amplitudes of the photon in the waveguide p.

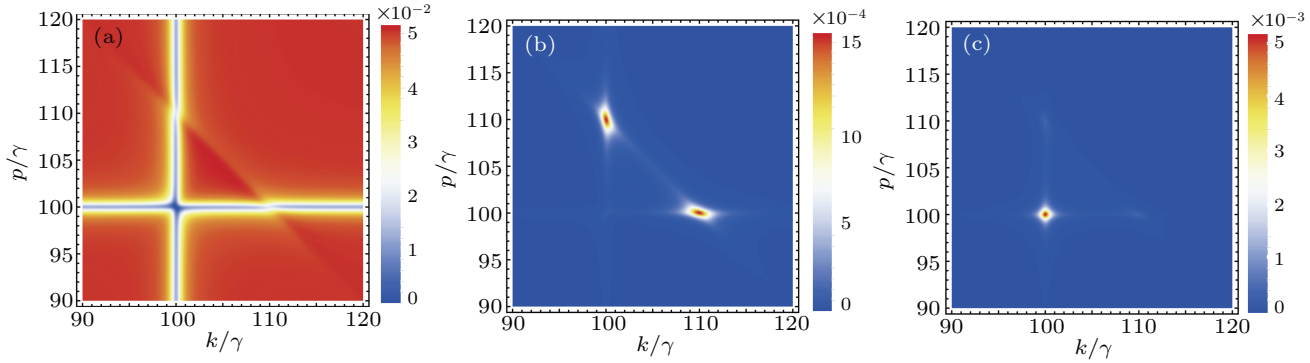


Fig. 2. Plots of dimensionless quantities: (a) T_{22}, (b) R_{11}, (c) T_{44}, and (d) R_{33}, as a function of two-photon transmission and reflection of waveguides a and b, respectively, k/γ and p/γ, when $\gamma_{ar}=0.25\gamma$, $\gamma_{al}=0.35\gamma$, $\gamma_{br}=\gamma_{bl}=0.2\gamma$ at $x=0$.

The probability of two photons appearing at the same port is defined as $T_{jj}(x)=\left|\rho_{jj}(x_1,x_2)\right|^2$ and $R_{jj}(x)=\left|\rho_{jj}(x_1,x_2)\right|^2$. Obviously, those amplitude wave functions contain the contributions of plane wave and bound wave, and R_{11}, R_{33}, T_{22}, T_{44} depends only upon x ($x=x_1-x_2$). If without the kerr medium in the cavity (i.e., $U=0$), the scattering state is only affected by the plane wave. At the point $x=0$, we can show that there is an optimal point for the transmitted photons exhibiting strongly repulsive interaction ($T_{22}\approx0$) at single photon resonance ($vk=vp=\Omega$) [Fig. 2(a)]. Therefore, we magnify the interaction of the transmitted photons through Fig. 3(a), and the result shows that only when $\gamma_{ar}/\gamma=1/4$, the transmitted

photon exhibits strongly repulsive interaction. In addition, the anti-bunching effect of reflected two-photons strongly depends on the energies of the two incident photons, in Fig. 2(b). At point $vk = vp = \Omega$, the reflected two photons always exhibit an anti-bunching effect, for all parameter γ_{ar}/γ and γ_{al}/γ in Fig. 3(b); and perpetually exude bunching attraction effect for two-photon resonance ($\omega = 2\Omega + U$) in Fig. 4(b). In addition, the two photons transferred into waveguide b always exhibit a bundling effect due to the influence of the bound state. Naturally, in the case of $vk + vp = 2\Omega + U$ [see Figs. 4(a)–4(d)], the optical cavity can absorb and release two photons at the same time, not to mention the influence of the bound state of the two photons, which directly makes all the outgoing possibilities show bunching effect.

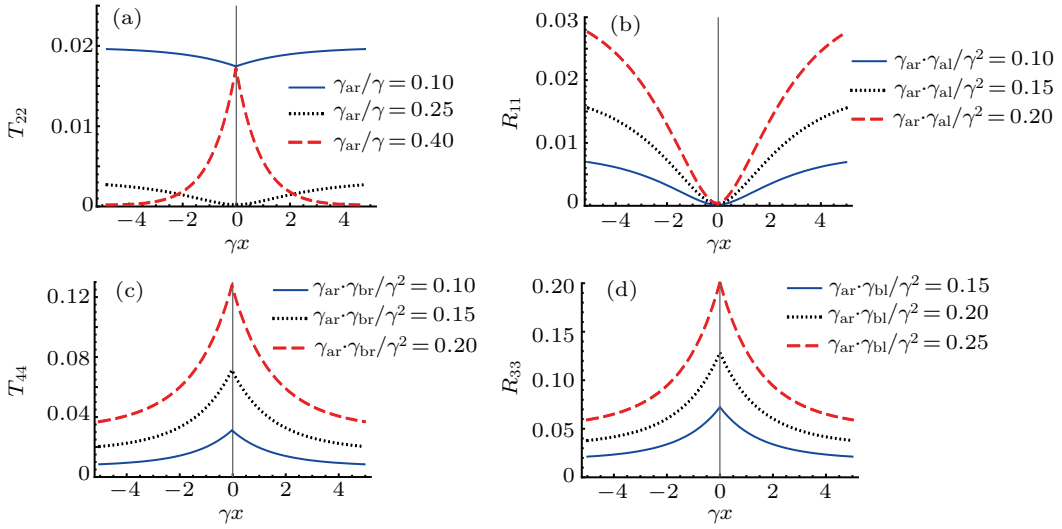


Fig. 3. The (a) T_{22}, (b) R_{11}, (c) T_{44}, and (d) R_{33}, as functions of relative coordinate γx. The other parameters are $vk = vp = \Omega = 100\gamma$, $U = 10\gamma$.

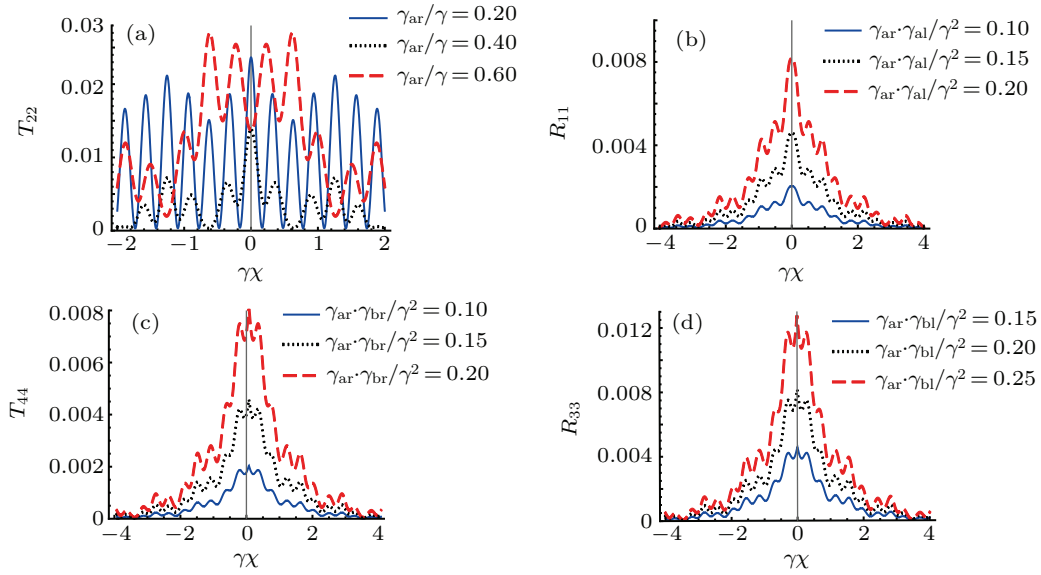


Fig. 4. The (a) T_{22}, (b) R_{11}, (c) T_{44}, and (d) R_{33}, as functions of relative coordinate γx. The other parameters are $\Omega = vp = 100\gamma$, $vk = 110\gamma$, $U = 10\gamma$.

In order to discuss the transfer rate of photon from waveguide a into waveguide b, we introduce the definition of routing probability as follows:

$$P_b = \frac{\int dx \langle \Phi | \psi_b(x) \psi_b^\dagger(x) | \Phi \rangle}{\int dx \langle \Phi | \psi_b(x) \psi_b^\dagger(x) + \psi_a(x) \psi_a^\dagger(x) | \Phi \rangle}. \quad (27)$$

where $\psi_b(x) = \hat{R}_b^\dagger(x) + \hat{L}_b^\dagger(x)$ is the field operator of waveguide b, and the $|\Phi\rangle$ is the scattering state of two photons. Due to the fact that the integral interval in real space is from negative infinity to positive infinity, the effect of bound states can be ignored. Therefore, the contribution of routing rate only comes from plane wave. Interestingly, due to the existence of bound states, in the case of two-photon resonance point, both photons can appear at the same port, but the bound states do not play any role in the routing rate. The result is shown in Fig. 5

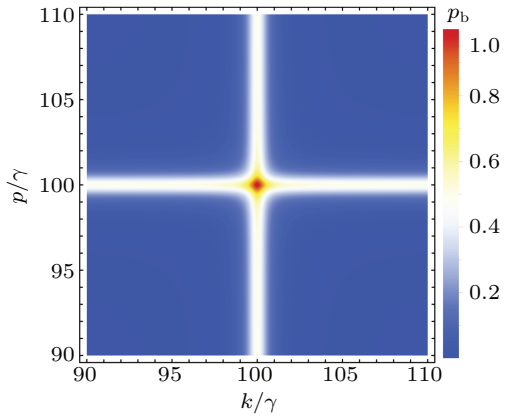


Fig. 5. The probability P_b of the input photons from port 1 routed to port 3 as a function of energy vk and vp at the maximally chiral couplings $\gamma_{ar} = \gamma_{br} = \gamma/2$. Other parameters are setting as follows: $\Omega = vp = 100\gamma$, $vk = 110\gamma$, $U = 10\gamma$.

4. Discussion and conclusion

In summary, we have presented the exact analytic solution of few photons in two one-dimensional waveguides chirally coupled by a nonlinear cavity. Since the Kerr-type nonlinearity has no effect at the single-photon level, the single-photon routing is not different from that of a two-level atom in two one-dimensional waveguides.[59] Interestingly, at the two-photon level, through some appropriate special inputs, we have measured the transmission, reflection and transfer characteristics of two photons. When two identical resonant photons $(vk = vp = \Omega)$ are incident, with the change of the parameter γ_{ar}/γ, the transmitted photons thus change from bunching to antibunching, and back to bunching. Hence the nonlinear cavity can induce either an effective repulsion or attraction between two transmitted photons. In addition, at $x = 0$, the antibunching in R_{11} for all γ_{ar} and γ_{al} in fact has analogous physical source as the antibunching experimentally.[60] At the same time, T_{44} and R_{33} show an anti-bunching effect different from the previous two-level quantum emitter,[59] that is, it is always in the bunching effect. Surprisingly, when the total frequency of the two incident photons resonates with the two-photon cavity field $(vk + vp = 2 + U)$, R_{11}, T_{22}, R_{33}, and T_{44} have bunching effects with any γ_{ar}, γ_{al}, γ_{br} and γ_{bl}. This shows that the role of bound states will be greater in two-photon resonance. However, the bound state has no contribution to the routing rate.

Acknowledgements

Project supported by the National Natural Science Foundation of China (Grant Nos. 11975095, 12075082, and 11935006) and the Science and Technology Innovation Program of Hunan Province, China (Grant No. 2020RC4047)

References

[1] Tao X H and Yang Y 2017 *Chin. Phys. Lett.* **34** 038501
[2] Tang G Z, Sun S H and Chen H 2016 *Chin. Phys. Lett.* **33** 120301
[3] Liu C Q, Zhu C H and Wang L H 2016 *Chin. Phys. Lett.* **33** 100301
[4] Tan Y G and Liu Q 2016 *Chin. Phys. Lett.* **33** 090303
[5] Gu W Y, Zhao S H and Dong C 2019 *Acta Phys. Sin.* **68** 240301 (in Chinese)
[6] Chen Y H, Wang J D and Du C 2019 *Acta Phys. Sin.* **68** 130301 (in Chinese)
[7] Yang L, Ma H Y and Zheng C 2017 *Acta Phys. Sin.* **66** 230303
[8] Zhang S J, Chen X and Bao W S 2020 *Chin. Phys. B* **29** 020301
[9] Chang L W, Zhang Y Q and Tian X X 2020 *Chin. Phys. B* **29** 010304
[10] Shi J J, Li B P and Huang D 2020 *Chin. Phys. B* **29** 040301
[11] Shen J T, Povinelli M L, Sandhu S and Fan S H 2007 *Phys. Rev. B* **75** 035320
[12] Shen J T and Fan S H 2009 *Phys. Rev. A* **79** 023838
[13] Pan J, Sandhu S, Huo Y, Stuhrmann N, Povinelli M L, Harris J S, Fejer M M and Fan S 2010 *Phys. Rev. B* **81** 041101(R)
[14] Sandhu S, Povinelli M L and Fan S H 2010 *Appl. Phys. Lett.* **96** 231108
[15] Tan H T, Zhang W M and Li G X 2011 *Phys. Rev. A* **83** 062310
[16] Englund D, Majumdar A, Faraon A, Toishi M, Stoltz N, Petroff P and Vukovi J 2010 *Phys. Rev. Lett.* **104** 073904
[17] Cheng M T and Song Y Y 2012 *Opt. Lett.* **37** 978
[18] Huang J F, Shi T, Sun C P and Nori F 2013 *Phys. Rev. A* **88** 013836
[19] Shen J T and Fan S H 2005 *Opt. Lett.* **30** 2001
[20] Shen J T and Fan S H 2007 *Phys. Rev. Lett.* **98** 153003
[21] Shen J T and Fan S H 2007 *Phys. Rev. A* **76** 062709
[22] Zhou L, Gong Z R, Liu Y X, Sun C P and Nori F 2008 *Phys. Rev. Lett.* **101** 100501
[23] Zang X and Jiang C 2010 *J. Phys. B* **43** 215501
[24] Lu J, Zhou L, Fu H C and Kuang L M 2010 *Phys. Rev. A* **81** 062111
[25] Gonzalez T A, Cano D M, Moreno E, Moreno M L, Tejedor C and Garcia Vidal F J 2011 *Phys. Rev. Lett.* **106** 020501
[26] Li T Y, Huang J F and Law C K 2015 *Phys. Rev. A* **91** 043834
[27] Li T, Miranowicz A, Hu X D, Xia K Y and Nori F 2018 *Phys. Rev. A* **97** 062318
[28] Yan C H, Li M, Xu X B, Zhang Y L, Yuan H and Zou C L 2020 *Phys. Rev. A* **102** 053719
[29] Shen J T and Fan S H 2009 *Phys. Rev. A* **79** 023837
[30] Roy D 2010 *Phys. Rev. B* **81** 155117
[31] Liao J Q and Law C K 2013 *Phys. Rev. A* **87** 043809
[32] Duan L M and Kimble H J 2004 *Phys. Rev. Lett.* **92** 127902
[33] Koshino K, Ishizaka S and Nakamura Y 2010 *Phys. Rev. A* **82** 010301
[34] Shi T, Fan S H and Sun C P 2011 *Phys. Rev. A* **84** 063803
[35] Ji Z and Gao S 2012 *Opt. Commun.* **285** 1302
[36] Dong J H, Jiang Q, Hu Q M, Zou B S and Zhang Y Y 2019 *Phys. Rev. A* **100** 013840
[37] Liao J Q and Law C K 2010 *Phys. Rev. A* **82** 053836
[38] Xu X W and Li Y 2014 *Phys. Rev. A* **90** 033832
[39] Xu S S and Fan S H 2019 *Phys. Rev. A* **99** 063806
[40] Kekatpure R D, Barnard E S, Cai W and Brongersma M L 2010 *Phys. Rev. Lett.* **104** 243902
[41] Han Z and Bozhevolnyi S I 2011 *Opt. Express* **19** 3251
[42] Roy D 2011 *Phys. Rev. Lett.* **106** 053601
[43] Xiao Y F, Li M, Liu Y C, Li Y, Sun X and Gong Q 2010 *Phys. Rev. A* **82** 065804
[44] Lu H, Liu X, Mao D and Wang G X 2012 *Opt. Lett.* **37** 3780
[45] Joanesarson B, Jake I S, Heuck M and Mork J 2020 *Phys. Rev. A* **101** 063809
[46] O'Shea D, Junge C, Volz J and Rauschenbeutel A 2013 *Phys. Rev. Lett.* **111** 193601
[47] Dong G, Zhang Y, Kamran M A and Zou B 2013 *J. Appl. Phys.* **113** 143105
[48] Shi T and Fan S H 2013 *Phys. Rev. A* **87** 063818
[49] Xu S, Rephaeli E and Fan S H 2013 *Phys. Rev. Lett.* **111** 223602
[50] Lodahl P, Mahmoodian S, Stobbe S, Rauschenbeutel A, Schneeweiss P, Volz J, Pichler H and Zoller P 2017 *Nature* **541** 473
[51] Berman P R 2020 *Phys. Rev. A* **101** 013830
[52] Volz J P and Rauschenbeutel A 2014 *Science* **346** 67
[53] Volz M S H W and Rauschenbeutel A 2016 *Science* **354** 1577
[54] Coles R J, Price D M, Dixon J E, Royall B, Clarke E, Kok P, Skolnick M S, Fox A M and Makhonin M N 2016 *Nat. Commun.* **7** 11183
[55] Gong S H, Alpeggiani F, Sciacca B, Garnett E C and Kuipers L 2018 *Science* **359** 443
[56] Mahmoodian S, Calajo G, Chang D E, Hammerer K and Sorensen A S 2020 *Phys. Rev. X* **10** 031011
[57] Cheng M T, Ma X S, Zhang J Y and Wang B 2016 *Opt. Express* **24** 19988
[58] Carlos G B, Esteban M, Francisco G V J and Alejandro G T 2016 *Phys. Rev. A* **94** 063817
[59] Yang Y, Lu J and Zhou L 2022 *Commun. Theor. Phys.* **74** 025101
[60] Hoffges J T, Baldauf H W and Eichler T 1997 *Opt. Commun.* **133** 170