New light fields based on integration theory within the Weyl ordering product of operators

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The development of quantum optics theory based on the method of integration within an ordered product of operators (IWOP) has greatly stimulated the study of quantum states in the light field, especially non-Gaussian states with various non-classical properties. In this paper, the two-mode squeezing operator is derived with integral theory within the Weyl ordering product of operators using a combinatorial field in which one mode is a chaotic field and the other mode is a vacuum field. The density operator of the new light field, its entanglement property and photon number distribution are analyzed. We also note that tracing a three-mode pure state can yield this new light field. These methods represent a theoretical approach to investigating new density operators of light fields.

Keywords: integration within an ordered product of operators, two-mode squeezing operator, Weyl ordering, entanglement property

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1. Introduction

Quantum light sources are the foundation of quantum information technology. The demand for new quantum light sources has increased in recent years. The generation of a non-classical light field has become an advanced topic in quantum optics to better complete the quantum information processing of continuous variables. Study of the properties of new quantum light fields has numerous applications in weak signal detection and quantum computing,[1,2] quantum communication[3–5] and high-precision measurement. The relevant characteristics of the light field are primarily reflected in terms of quantum states,[6,7] such as the density matrix,[8,9] P-representation,[10,11] and the Wigner function,[12,13] which partly carry the quantum state information. In this paper, the two-mode squeezing operator is applied to a combinatorial field in which one mode is chaotic. Another mode is a vacuum field, and the properties of the new light field are discussed using integration within an ordered product of operators (IWOP)[14–16] and Weyl ordering[17–22] theory. It is found that the density operators of the final state can be presented analytically after being arranged in a specific operator ordering, thereby forming a new light field worthy of attention. This method is effective because the entropy of the light field is minimized through the ordered arrangement of density operators, increasing the likelihood of forming a new light field.

As a Gaussian and entangled state, the two-mode squeezed vacuum state is one of the most popular tools for quantum-enhanced optical interferometers or continuous-variable quantum information processing. The two-mode squeezed state is an example of such a state, and it plays a vital role because it can be easily produced from reliable sources and controlled experimentally using accessible operations such as beam splitters, phase shifters and squeezers, as well as efficient detection systems.

In this paper we propose the obtaining of new light fields based on IWOP. The density operator of the new light field after squeezing the a-chaotic mode field and the b-vacuum mode field are calculated in Section 2. The new light field is derived from integration theory within the Weyl ordering product of operators and two-mode Wigner operator in Section 3. Using the squeezing mechanism, we explore a new density operator ρ and discuss the distributional characteristics of photon numbers in two different modes of the mixed state density operator ρ in Section 4. By tracing a single mode, it is shown that the resulting state is mixed. Furthermore, it is shown in Section 5 that the state can also be obtained from a three-mode state after tracing one mode.

2. Squeezing a combinatorial state of a chaotic mode and another vacuum mode to obtain the new two-mode light field

The squeezing mechanism is an integral part of quantum field theory and quantum optics. It is found that converting a degenerate parameter can result in squeezed light, which is widely used in quantum encryption, quantum computing, gravitational wave detection and precision measurement. The main purpose of this section is to calculate the density operator of the new light field after squeezing the a-chaotic mode field and the b-vacuum mode field. The two-mode squeezing

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operator[23] S_2 can be written as

$$\begin{aligned} S_2 &= \exp\left[\sigma\left(a^\dagger b^\dagger - ab\right)\right] \\ &= \operatorname{sech}\sigma \exp\left[a^\dagger b^\dagger \tanh\sigma\right] \exp\left[\left(a^\dagger a + b^\dagger b + 1\right)\ln\operatorname{sech}\sigma\right] \\ &\quad \times \exp\left[-ab\tanh\sigma\right], \end{aligned} \tag{1}$$

where σ is the squeezed parameter, and noting that $[a,a^\dagger] = [b,b^\dagger] = 1$.

The two-mode squeezing operator S_2 is used in the two-mode vacuum state $|00\rangle = |0\rangle_1 |0\rangle_2$ and can obtained as follows:

$$S_2 |0\rangle_1 |0\rangle_2 = \operatorname{sech}\sigma\, \mathrm{e}^{a^\dagger b^\dagger \tanh\sigma} |00\rangle. \tag{2}$$

The corresponding pure density operator is

$$\begin{aligned} & S_2\left(|0\rangle_{11}\langle 0| \cdot |0\rangle_{22}\langle 0|\right) S_2^{-1} \\ &= \operatorname{sech}^2\sigma\, \mathrm{e}^{a^\dagger b^\dagger \tanh\sigma} |00\rangle\langle 00|\, \mathrm{e}^{ab\tanh\sigma}. \end{aligned} \tag{3}$$

Here S_2 plays the role of entanglement between two modes. A long-neglected but important question remains: when a single-mode vacuum field $|0\rangle_{22}\langle 0|$ and another chaotic mode field ρ_{c} are squeezed, what is the effect of two-mode squeezed chaotic and vacuum modes and can a new quantum light field appear?

$$\rho_{\mathrm{c}} = \left(1 - \mathrm{e}^{\lambda}\right) \mathrm{e}^{\lambda a^\dagger a}, \quad \operatorname{tr}\rho_{\mathrm{c}} = 1, \quad \left\langle a^\dagger a\right\rangle_{\rho_{\mathrm{c}}} = \frac{1}{\mathrm{e}^{-\lambda} - 1}. \tag{4}$$

The answers to these questions will be explained as follows. Let

$$\begin{aligned} \rho &= S_2\left(\rho_{\mathrm{c}} |0\rangle_{22}\langle 0|\right) S_2^{-1} \\ &= \left(1 - \mathrm{e}^{\lambda}\right) S_2 \left(\mathrm{e}^{\lambda a^\dagger a} \cdot |0\rangle_{22}\langle 0|\right) S_2^{-1}, \end{aligned} \tag{5}$$

according to the similarity transformation relationship of the squeezing operator

$$\begin{aligned} S_2 a S_2^{-1} &= a\cosh\sigma - b^\dagger \sinh\sigma, \\ S_2 b S_2^{-1} &= b\cosh\sigma - a^\dagger \sinh\sigma. \end{aligned} \tag{6}$$

Equation (5) requires operator reordering, and its corresponding calculation is very complex.

3. The new light field is derived from integration theory within the Weyl ordering product of operators

In order to solve the above problems, integration theory within the Weyl ordering product of operators will be discussed in this section. Let Weyl ordering of operator F be $\vdots F\left(a^\dagger, a; b^\dagger, b\right) \vdots$, where the symbol $\vdots\ \vdots$ denotes Weyl ordering. By using the invariant properties of the Weyl ordering operator under similarity transformation, we can obtain

$$S_2 \vdots F\left(a^\dagger, a; b^\dagger, b\right) \vdots S_2^{-1} = \vdots S_2 F\left(a^\dagger, a; b^\dagger, b\right) S_2^{-1} \vdots. \tag{7}$$

According to the classical Weyl correspondence, a Weyl ordering operator $\vdots h\left(a^\dagger, a; b^\dagger, b\right) \vdots$ in phase space $(\alpha, \alpha^*; \beta, \beta^*)$ is exactly a function $h(\alpha, \alpha^*; \beta, \beta^*)$, and we can derive it as follows:

$$\begin{aligned} & \vdots h\left(a^\dagger, a; b^\dagger, b\right) \vdots \\ &= 4 \int \mathrm{d}^2\alpha \int \mathrm{d}^2\beta \Delta\left(\alpha, \alpha^*; \beta, \beta^*\right) h\left(\alpha, \alpha^*; \beta, \beta^*\right), \end{aligned} \tag{8}$$

where $\Delta\left(\alpha, \alpha^*; \beta, \beta^*\right)$ is a two-mode Wigner operator,[24,25] and noting

$$\begin{aligned} \Delta\left(\alpha, \alpha^*; \beta, \beta^*\right) &= \frac{1}{\pi^2} : \exp\left[-2\left(a^\dagger - \alpha^*\right)(a - \alpha)\right. \\ &\quad \left. -2\left(b^\dagger - \beta^*\right)(b - \beta)\right] :. \end{aligned} \tag{9}$$

IWOP and the two-mode Wigner operator $\Delta\left(\alpha, \alpha^*; \beta, \beta^*\right)$ can be used to derive the normal ordering[26,27] of the operator $\vdots h\left(a^\dagger, a; b^\dagger, b\right) \vdots$. Then we can understand the corresponding physical meaning by seeking part of its trace.

As the first step, we have to find the Weyl ordering form $\mathrm{e}^{\lambda a^\dagger a} \cdot |0\rangle_{22}\langle 0|$.

Considering the Weyl ordering of $|0\rangle_{22}\langle 0|$ as

$$|0\rangle_{22}\langle 0| = 2 \vdots \exp\left[-2b^\dagger b\right] \vdots, \tag{10}$$

combined with the corresponding Weyl ordering formula of the operator $H\left(a^\dagger, a\right)$

$$\begin{aligned} H\left(a^\dagger, a\right) &= 2 \int \frac{\mathrm{d}^2\gamma}{\pi} \langle -\gamma| H\left(a^\dagger, a\right) |\gamma\rangle \\ &\quad \times \vdots \exp\left[2\left(\gamma^* a - \gamma a^\dagger + a^\dagger a\right)\right] \vdots, \end{aligned} \tag{11}$$

where $|\gamma\rangle$ is a coherent state,[28,29] the Weyl ordering of $\mathrm{e}^{\lambda a^\dagger a}$ can be derived as

$$\begin{aligned} \mathrm{e}^{\lambda a^\dagger a} &= 2 \int \frac{\mathrm{d}^2\gamma}{\pi}\, {}_a\langle -\gamma| : \exp\left[\left(\mathrm{e}^{\lambda} - 1\right) a^\dagger a\right] : |\gamma\rangle_a \\ &\quad \times \exp\left[2\left(\gamma^* a - \gamma a^\dagger + a^\dagger a\right)\right] \vdots \\ &= 2 \int \frac{\mathrm{d}^2\gamma}{\pi} \vdots \exp\left[-\left(\mathrm{e}^{\lambda} + 1\right)\right] |\gamma|^2 \\ &\quad +2\left(\gamma^* a - \gamma a^\dagger + a^\dagger a\right) \vdots \\ &= \frac{2}{\mathrm{e}^{\lambda} + 1} \vdots \exp\left[2\frac{\mathrm{e}^{\lambda} - 1}{\mathrm{e}^{\lambda} + 1} a^\dagger a\right] \vdots. \end{aligned} \tag{12}$$

Thus

$$\mathrm{e}^{\lambda a^\dagger a} \cdot |0\rangle_{22}\langle 0| = \frac{4}{\mathrm{e}^{\lambda} + 1} \vdots \exp\left[2\frac{\mathrm{e}^{\lambda} - 1}{\mathrm{e}^{\lambda} + 1} a^\dagger a - 2b^\dagger b\right] \vdots. \tag{13}$$

Substituting Eq. (13) into Eq. (5), we can obtain

$$\begin{aligned} & \left(1 - \mathrm{e}^{\lambda}\right) S_2 \left(\mathrm{e}^{\lambda a^\dagger a} \cdot |\rangle_{22}\langle 0|\right) S_2^{-1} \\ &= \frac{4\left(1 - \mathrm{e}^{\lambda}\right)}{\mathrm{e}^{\lambda} + 1} S_2 \vdots \exp\left[\frac{2\left(\mathrm{e}^{\lambda} - 1\right)}{\mathrm{e}^{\lambda} + 1} a^\dagger a - 2b^\dagger b\right] \vdots S_2^{-1} \end{aligned}$$

$$= \frac{4\left(1-\mathrm{e}^{\lambda}\right)}{\mathrm{e}^{\lambda}+1} \vdots \exp\left[\frac{2\left(\mathrm{e}^{\lambda}-1\right)}{\mathrm{e}^{\lambda}+1}\left(a^{\dagger}\cosh\sigma - b\sinh\sigma\right)\right.$$
$$\times\left(a\cosh\sigma - b^{\dagger}\sinh\sigma\right)\Big] - 2\left(b^{\dagger}\cosh\sigma - a\sinh\sigma\right)$$
$$\times\left(b\cosh\sigma - a^{\dagger}\sinh\sigma\right)\vdots, \tag{14}$$

and the classical correspondence is

$$h(\alpha,\alpha^{*};\beta,\beta^{*})$$
$$= \frac{1-\mathrm{e}^{\lambda}}{\mathrm{e}^{\lambda}+1}\exp\left\{\frac{2\left(\mathrm{e}^{\lambda}-1\right)}{\mathrm{e}^{\lambda}+1}\left[|\alpha|^{2}\cosh^{2}\sigma + |\beta|^{2}\sinh^{2}\sigma\right]\right.$$
$$\left.-2|\beta|^{2}\cosh^{2}\sigma - 2|\alpha|^{2}\sinh\sigma + \frac{2\sinh 2\sigma}{\mathrm{e}^{\lambda}+1}\left(\alpha\beta+\alpha^{*}\beta^{*}\right)\right\}. \tag{15}$$

Substituting Eq. (15) into Eq. (8), combining Eq. (9) and the IWOP technique, we have

$$\begin{aligned}\rho &= \left(1-\mathrm{e}^{\lambda}\right)S_{2}\left(\mathrm{e}^{\lambda a^{\dagger}a}\cdot|0\rangle_{22}\langle 0|\right)S_{2}^{-1}\\ &= \int\frac{4\mathrm{d}^{2}\alpha\mathrm{d}^{2}\beta}{\pi^{2}}:\exp\left[-2\left(a^{\dagger}-\alpha^{*}\right)\left(a-\alpha\right)\right.\\ &\quad\left.-2\left(b^{\dagger}-\beta^{*}\right)\left(b-\beta\right)\right]:h(\alpha,\alpha^{*};\beta,\beta^{*})\\ &= \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma:\exp\left[a^{\dagger}a\left(\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma-1\right)\right.\\ &\quad\left.-b^{\dagger}b+ab\tanh\sigma+a^{\dagger}b^{\dagger}\tanh\sigma\right]:.\end{aligned} \tag{16}$$

Using the normally ordered form

$$|0\rangle_{22}\langle 0| =: \mathrm{e}^{-b^{\dagger}b}:, \tag{17}$$
$$\mathrm{e}^{fa^{\dagger}a} =: \exp\left[\left(\mathrm{e}^{f}-1\right)a^{\dagger}a\right]:, \tag{18}$$

the display form corresponding to the new light field density operator ρ can be calculated as follows:

$$\begin{aligned}\rho &= S_{2}\left(\rho_{\mathrm{c}}|0\rangle_{22}\langle 0|\right)S_{2}^{-1}\\ &= \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma\,\mathrm{e}^{a^{\dagger}b^{\dagger}\tanh\sigma}\\ &\quad\times\exp\left[a^{\dagger}a\ln\left(\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma\right)\right]|0\rangle_{22}\langle 0|\,\mathrm{e}^{ab\tanh\sigma}.\end{aligned} \tag{19}$$

This is a new density operator, and we have found a new quantum light field in theory.

4. Properties of the new light field

In order to understand the physical meaning of the density operator and explore the properties of the new light field, we sought the trace of the a-mode by using a coherent state $|\alpha\rangle_{a}$ of the a-mode and Eq. (19).

$$\begin{aligned}&\mathrm{tr}_{a}\rho\\ &= \mathrm{tr}_{a}\left[\int\frac{\mathrm{d}^{2}\alpha}{\pi}|\alpha\rangle_{aa}\langle\alpha|\,\rho\right]\\ &= \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma\int\frac{\mathrm{d}^{2}\alpha}{\pi}\langle\alpha|\,\mathrm{e}^{a^{\dagger}b^{\dagger}\tanh\sigma}\\ &\quad\times\exp\left[a^{\dagger}a\ln\left(\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma\right)\right]|0\rangle_{22}\langle 0|\,\mathrm{e}^{ab\tanh\sigma}|\alpha\rangle\\ &= \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma\int\frac{\mathrm{d}^{2}\alpha}{\pi}:\exp\left[-|\alpha|^{2}\right.\\ &\quad\left.\times\left(1-\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma\right)+ab\tanh\sigma+a^{*}b^{\dagger}\tanh\sigma-b^{\dagger}b\right]:\\ &= \frac{\left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma}{1-\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma}:\exp\left[\left(\frac{\tanh^{2}\sigma}{1-\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma}-1\right)b^{\dagger}b\right]:\\ &= \frac{\left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma}{1-\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma}\exp\left[b^{\dagger}b\ln\frac{\tanh^{2}\sigma}{1-\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma}\right]\\ &= \left(1-\mathrm{e}^{\lambda'}\right)\mathrm{e}^{\lambda'a^{\dagger}a}.\end{aligned} \tag{20}$$

Equation (20) represents the chaotic field, where

$$\ln\frac{\tanh^{2}\sigma}{1-\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma} = \lambda'. \tag{21}$$

We obtain

$$\frac{\left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma}{1-\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma} = 1-\mathrm{e}^{\lambda'}. \tag{22}$$

Conversely, we sought the trace of the b-mode by using a coherent state $|\beta\rangle_{b}$ of the b-mode as follows:

$$\begin{aligned}&\mathrm{tr}_{b}\rho\\ &= \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma\int\frac{\mathrm{d}^{2}\beta}{\pi}{}_{b}\langle\beta|\,\mathrm{e}^{a^{\dagger}b^{\dagger}\tanh\sigma}\\ &\quad\times\exp\left[a^{\dagger}a\ln\left(\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma\right)\right]|0\rangle_{22}\langle 0|\,\mathrm{e}^{ab\tanh\sigma}|\beta\rangle_{b}\\ &= \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma\int\frac{\mathrm{d}^{2}\beta}{\pi}\,\mathrm{e}^{-|\beta|^{2}+\alpha^{\dagger}\beta^{*}\tanh\sigma}\\ &\quad\times\exp\left[a^{\dagger}a\ln\left(\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma\right)\right]\mathrm{e}^{\alpha\beta\tanh\sigma}\\ &= \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma\int\frac{\mathrm{d}^{2}\beta}{\pi}\\ &\quad\times:\mathrm{e}^{-|\beta|^{2}+\alpha^{\dagger}\beta^{*}\tanh\sigma+\alpha\beta\tanh\sigma+a^{\dagger}a\left(\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma-1\right)}:\\ &= \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma:\exp\left[a^{\dagger}a\left(\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma+\tanh^{2}\sigma-1\right)\right]:\\ &= \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma\exp\left[a^{\dagger}a\ln\left(\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma+\tanh^{2}\sigma\right)\right]\\ &= \left(1-\mathrm{e}^{\lambda''}\right)\mathrm{e}^{\lambda''a^{\dagger}a}.\end{aligned} \tag{23}$$

Equation (23) also represents the chaotic field, where we have

$$\ln\left(\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma+\tanh^{2}\sigma\right) = \lambda''\neq\lambda'. \tag{24}$$

Thus, we obtain

$$1-\mathrm{e}^{\lambda''} = 1-\mathrm{e}^{\lambda}\,\mathrm{sech}^{2}\sigma-\tanh^{2}\sigma = \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^{2}\sigma. \tag{25}$$

Clearly, two different kinds of chaotic fields are produced by two types of traces, which shows that the two-mode squeezing has the function of entanglement.

The distributional characteristics of photon numbers in two different modes of the mixed state density operator ρ will be discussed. When $\mathrm{Tr} = \mathrm{tr}_a \mathrm{tr}_b$, using Eqs. (21) and (25), the following equations result:

$$\begin{aligned}\left\langle a^{\dagger}a\right\rangle_{\rho} &= \mathrm{Tr}\left[\rho a^{\dagger}a\right] = \mathrm{tr}_a\left(a^{\dagger}a\mathrm{tr}_b\rho\right)\\ &= \left(1-\mathrm{e}^{\lambda''}\right)\mathrm{tr}_a\left(a^{\dagger}a\mathrm{e}^{\lambda'' a^{\dagger}a}\right)\\ &= \left(1-\mathrm{e}^{\lambda''}\right)\mathrm{tr}_a\left(\sum_{n=0}\mathrm{e}^{\lambda'' n}|n\rangle\langle n|n\right)\\ &= \left(\mathrm{e}^{-\lambda''}-1\right)^{-1} = \frac{\mathrm{e}^{\lambda}+\sinh^2\sigma}{1-\mathrm{e}^{\lambda}}. \end{aligned} \tag{26}$$

For no squeezing in the system, $\sinh^2\sigma = 0$, $\left\langle a^{\dagger}a\right\rangle_{\rho} \to \left\langle a^{\dagger}a\right\rangle_{\rho_c} = \frac{1}{\mathrm{e}^{-\lambda}-1}$. Similarly, using Eqs. (22) and (24), we obtain

$$\begin{aligned}\left\langle b^{\dagger}b\right\rangle &= \mathrm{Tr}\left[\rho b^{\dagger}b\right] = \mathrm{tr}_b\left(b^{\dagger}b\mathrm{tr}_a\rho\right)\\ &= \left(\mathrm{e}^{-\lambda'}-1\right)^{-1} = \frac{\sinh^2\sigma}{1-\mathrm{e}^{\lambda}}.\end{aligned}$$

When there is no squeezing in the system $\sinh^2\sigma = 0\left\langle b^{\dagger}b\right\rangle \to 0$, the physical meaning is reflected as $|0\rangle_{22}\langle 0|$.

In this section, therefore, using the squeezing mechanism, we have explored a new density operator

$$\begin{aligned}\rho &= \left(1-\mathrm{e}^{\lambda}\right)\mathrm{sech}^2\sigma\,\mathrm{e}^{a^{\dagger}b^{\dagger}\tanh\sigma}\\ &\quad\times\exp\left[a^{\dagger}a\ln\left(\mathrm{e}^{\lambda}\,\mathrm{sech}^2\sigma\right)\right]|0\rangle_{22}\langle 0|\,\mathrm{e}^{ab\tanh\sigma},\end{aligned}$$

which shows that two-mode squeezing plays the role of quantum entanglement.

5. Trace the pure state of the higher dimension to derive the new light field

In Section 4 we applied the two-mode squeezing operator to the a-chaotic mode field and the b-vacuum mode field and calculated the density operator ρ of the new light field, which is a mixed state. In this section we will find a three-mode pure state whose single-mode measurement will generate the corresponding mixed state ρ. For the sake of discussion, let $\mathrm{e}^{\lambda} = \tanh^2\tau$, so Eq. (19) can be written as follows:

$$\begin{aligned}&\mathrm{sech}^2\sigma\,\mathrm{sech}^2\tau\,\mathrm{e}^{a_2^{\dagger}a_3^{\dagger}\tanh\sigma}\left(\mathrm{sech}^2\sigma\tanh^2\tau\right)^{a_3^{\dagger}a_3^{\dagger}}\\ &\times|0\rangle_{22}\langle 0|\,\mathrm{e}^{a_2a_3\tanh\sigma}\equiv\rho.\end{aligned} \tag{27}$$

With the help of IWOP technology, we expand by one degree of freedom

$$\begin{aligned}\rho &= \mathrm{sech}^2\sigma\,\mathrm{sech}^2\tau : \exp\left[\left(a_2^{\dagger}a_3^{\dagger}+a_2a_3\right)\tanh\sigma\right.\\ &\quad\left.+\left(\mathrm{sech}^2\sigma\tanh^2\tau-1\right)a_3^{\dagger}a_3-a_2^{\dagger}a_2\right]:\\ &= \mathrm{sech}^2\sigma\,\mathrm{sech}^2\tau\int\frac{\mathrm{d}^2z}{\pi}\\ &\quad\times:\exp\left[-|z|^2+\left(z^*a_3^{\dagger}+za_3\right)\mathrm{sech}\sigma\tanh\tau\right.\\ &\quad\left.+\left(a_2^{\dagger}a_3^{\dagger}+a_2a_3\right)\tanh\sigma-a_2^{\dagger}a_2-a_3^{\dagger}a_3\right]:\\ &= \mathrm{sech}^2\sigma\,\mathrm{sech}^2\tau\int\frac{\mathrm{d}^2z}{\pi}{}_1\langle z|\\ &\quad\times\exp\left[a_1^{\dagger}a_3^{\dagger}\,\mathrm{sech}\sigma\tanh\tau+a_2^{\dagger}a_3^{\dagger}\tanh\sigma\right]\\ &\quad\times|000\rangle\langle 000|\exp\left[a_1a_3\,\mathrm{sech}\sigma\tanh\tau+a_2a_3\tanh\sigma\right]|z\rangle_1\\ &= \mathrm{sech}^2\sigma\,\mathrm{sech}^2\tau\,\mathrm{tr}_1\left\{\exp\left[a_1^{\dagger}a_3^{\dagger}\,\mathrm{sech}\sigma\tanh\tau+a_2^{\dagger}a_3^{\dagger}\tanh\sigma\right]\right.\\ &\quad\left.\times|000\rangle\langle 000|\exp\left[a_1a_3\,\mathrm{sech}\sigma\tanh\tau+a_2a_3\tanh\sigma\right]\right\}.\end{aligned} \tag{28}$$

The tr_1 in Eq. (28) represents the trace of the first mode and $\{\cdots\}$ a pure state. Then we prove that the right vector is the result of two two-mode squeezing operators $S_{23}(\sigma)S_{31}(\tau)$ acting on the three-mode vacuum state $|000\rangle$, that is

$$\begin{aligned}&\mathrm{sech}\sigma\tanh\tau\exp\left[a_1^{\dagger}a_3^{\dagger}\,\mathrm{sech}\sigma\tanh\tau+a_2^{\dagger}a_3^{\dagger}\tanh\sigma\right]|000\rangle\\ &= S_{23}(\sigma)S_{31}(\tau)|000\rangle.\end{aligned} \tag{29}$$

In fact

$$S_{31}(\tau)|000\rangle = \mathrm{sech}\tau\exp\left(a_1^{\dagger}a_3^{\dagger}\tanh\tau\right)|000\rangle. \tag{30}$$

Thus,

$$\begin{aligned}&S_{23}(\sigma)S_{31}(\tau)|000\rangle\\ &= \mathrm{sech}\tau S_{23}(\sigma)\exp\left(a_1^{\dagger}a_3^{\dagger}\tanh\tau\right)|000\rangle\\ &= \mathrm{sech}\tau S_{23}(\sigma)\exp\left(a_1^{\dagger}a_3^{\dagger}\tanh\tau\right)S_{23}^{-1}(\sigma)S_{23}(\sigma)|000\rangle\\ &= \mathrm{sech}\tau\exp\left[a_1^{\dagger}\left(a_3^{\dagger}\cosh\sigma+a_2\sinh\sigma\right)\tanh\tau\right]\\ &\quad\times\mathrm{sech}\sigma\exp\left(a_2^{\dagger}a_3^{\dagger}\tanh\sigma\right)|000\rangle\\ &= \mathrm{sech}\sigma\,\mathrm{sech}\tau\exp\left[a_1^{\dagger}a_3^{\dagger}\,\mathrm{sech}\sigma\tanh\tau+a_2^{\dagger}a_3^{\dagger}\tanh\sigma\right]|000\rangle.\end{aligned} \tag{31}$$

If we let

$$\rho' = S_{23}(\sigma)S_{31}(\tau)|000\rangle\langle 000|S_{31}^{\dagger}(\tau)S_{23}^{\dagger}(\sigma), \tag{32}$$

then we have

$$\mathrm{tr}_1\rho' = \rho, \tag{33}$$

where ρ' is a new density operator, a new three-mode quantum state of the light field in terms of physical meaning. It can collapse into the mixed state $S_2(\rho_c|0\rangle_{22}\langle 0|)S_2^{-1}$ by measuring its corresponding first mode. This shows that the measurement leads to the collapse of the high-dimensional pure state to the low-dimensional mixed state.

6. Conclusion

This paper studies an effective way to obtain a new light field in theory, using IWOP, coherent state representation and Weyl ordering. The density operator of the new light field, its entanglement property and its photon number distribution are analyzed. The results show that the new optical field density operator ρ can be formed by applying the two-mode squeezing operator to the a-chaotic mode field and the b-vacuum mode field. The density operator ρ is a mixed state with an entanglement function. Meanwhile, we obtain a three-mode pure state by tracing pure states of a higher dimension. By measuring its corresponding first mode, the new light field can be derived. The research results are a significant guide for preparing quantum states in experiments.

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