Diffusive field coupling-induced synchronization between neural circuits under energy balance

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When charged bodies come up close to each other, the field energy is diffused and their states are regulated under bidirectional field coupling. For biological neurons, the diversity in intrinsic electric and magnetic field energy can create synaptic connection for fast energy balance and synaptic current is passed across the synapse channel; as a result, energy is pumped and exchanged to induce synchronous firing modes. In this paper, a capacitor is used to connect two neural circuits and energy propagation is activated along the coupling channel. The intrinsic field energy in the two neural circuits is exchanged and the coupling intensity is controlled adaptively using the Heaviside function. Some field energy is saved in the coupling channel and is then sent back to the coupled neural circuits to reach energy balance. Therefore the circuits can reach possible energy balance and complete synchronization. It is possible that the diffusive energy of the coupled neurons inspires the synaptic connections to grow stronger for possible energy balance.

Keywords: field coupling, synchronization, neural circuits, Hamilton energy, synapse connection

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1. Introduction

From biophysical and dynamical viewpoints, neural circuits can be tamed to reproduce the main biophysical properties of some biological neurons. For example, the output voltage from nonlinear circuits can be used to approach the firing modes in neural activities, and the inner field energy can be described effectively by the equivalent Hamilton energy. Most neural circuits[1–5] can be excited by an external voltage source for the generation of similar firing patterns in biological neurons driven by noise and periodic stimuli. Synaptic plasticity[6] enables neurons to show certain self-adaption to external stimuli and the synaptic connection becomes adaptive to synchronize neurons. Each neuron is exposed to the electromagnetic field emitted by other neurons when two or more neurons are clustered closely; this energy diversity is helpful for creating synapse connection for energy balance and synchronization can be enhanced when the coupling synapses are controlled by energy diversity. From a dynamical aspect, both electric and chemical synapses can connect neurons effectively, and synchronization can be stabilized under an appropriate coupling intensity. On the other hand, it is a challenge to both realize and explore the physical evidence[7] for building hybrid synapses connecting to neurons because synaptic current can induce an electromagnetic field effect. For neural circuits, most of the electric components such as resistor, capacitor,[8] induction coil,[9] phototube,[10] Josephson junction,[11] thermistor,[12] memristor,[13,14] and combinations of these components can be used to couple neural circuits and nonlinear circuits. The greatest advantage of these coupling channels lies their controllability, because the coupling intensity can be controlled by external physical schemes. As a result, the coupling channels show some self-adaption and external energy can be captured and encoded to control the energy balance between nonlinear circuits.

2. Scheme and results

The output voltage of a capacitor and its intrinsic field energy can represent the capacitance property and electric potential energy of the intracellular ions in biological neurons. The channel current across the induction coil and reverse voltage from connection to a constant voltage source can denote the biophysical properties of ion channels embedded in the cell membrane. Therefore, a generic neural circuit[15] can be built using a capacitor, induction coil, constant voltage source, resistors and nonlinear resistor; if necessary, an additive external voltage source can be applied as an external stimulus. When a neural circuit is excited for firing, field energy can be saved in these electric components and time-varying channel currents along the branch circuits are helpful for exchanging the field energy between different components. For two or more biological neurons, the energy diversity enables the possible creation of synaptic connection and thus they can reach fast energy balance by propagating the synaptic current with changeable intensity along the synapse connection/channels. From a physical viewpoint, the triggered electric fields interact and field energy is exchanged between neurons. Therefore, it would be interesting to explore the physical mechanism for

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energy diffusion and balance by connecting two neural circuits via a capacitor, as shown in Fig. 1.

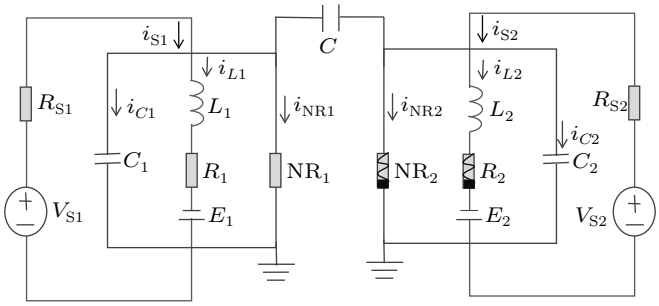


Fig. 1. Capacitive coupling between FitzHugh–Nagumo neural circuits. V_{S1} and V_{S2} denote the external voltage sources, C_1 and C_2 represent the capacitors and L_1 and L_2 are the induction coils. NR_1, NR_2, E_1, E_2, R_1, R_2, R_{S1}, and R_{S2} describe the nonlinear resistors, constant voltages and linear resistors and C defines the capacitor in the coupling channel.

As mentioned in Ref. [16], the channel current i_C across the nonlinear resistor NR can be obtained from

$$i_{NR} = -\frac{1}{\rho}\left(V - \frac{1}{3}\frac{V^3}{V_0^2}\right). \tag{1}$$

The normalized parameter ρ and V_0 describe the resistance and cut-off voltage in the linear region of the I–V curve. In addition, the channel current across the coupling channel via capacitor C' can be described by

$$i_C = C\left(\frac{dV_1}{dt} - \frac{dV_2}{dt}\right). \tag{2}$$

The variables V_1 and V_2 represent the output voltage for the two neural circuits. According to the Kirchhoff theorem, the differential equations of the circuits can be derived as follows:

$$\begin{aligned}
C_1\frac{dV_1}{dt} &= \frac{V_{S1}-V_1}{R_S} - i_{L1} + \frac{1}{\rho}\left(V_1 - \frac{1}{3}\frac{V_1^3}{V_0^2}\right) - i_C,\\
L_1\frac{di_{L1}}{dt} &= V_1 - R_1 i_{L1} + E_1,\\
C_2\frac{dV_2}{dt} &= \frac{V_{S2}-V_2}{R_S} - i_{L2} + \frac{1}{\rho}\left(V_2 - \frac{1}{3}\frac{V_2^3}{V_0^2}\right) + i_C,\\
L_2\frac{di_{L2}}{dt} &= V_2 - R_2 i_{L2} + E_2.
\end{aligned} \tag{3}$$

A generic periodic stimulus with constant B_S, amplitude B_0 and angular frequency ω_0 can be defined by

$$V_S = B_S + B_0\cos\omega_0 t. \tag{4}$$

For simplicity, we consider the case for two identical neural circuits, $C_1 = C_2$, $L_1 = L_2 = L$, $E_1 = E_2 = E$, $NR_1 = NR_2$, $R_1 = R_2 = R$, $R_{S1} = R_{S2} = R_S$. The intrinsic field energy in each neural circuit and the coupled channel can be calculated by

$$W_1 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}L_1i_{L1}^2,\ \ W_2 = \frac{1}{2}C_2V_2^2 + \frac{1}{2}L_2i_{L2}^2,$$

$$W_C = \frac{1}{2}C(V_1 - V_2)^2. \tag{5}$$

For further nonlinear analysis, the scale transformation can be applied for the variables and parameters in Eqs. (1)–(5)

$$\begin{aligned}
&x = \frac{V}{V_0},\ \ y = \frac{\rho i_L}{V_0},\ \ U = \frac{\rho V_S}{R_S V_0},\ \ \tau = \frac{t}{\rho C_1},\ \ a = \frac{E}{V_0},\\
&b = \frac{R}{\rho},\ \ c = \frac{\rho^2 C_1}{L},\ \ \xi_1 = \frac{\rho}{R_{S1}},\ \ \xi_2 = \frac{\rho}{R_{S2}},\ \ U_S = \frac{\rho B_S}{R_S V_0},\\
&U_0 = \frac{\rho B_0}{R_S V_0},\ \ \omega = \rho C\omega_0,\ \ U = U_S + U_0\cos\omega\tau.
\end{aligned} \tag{6}$$

As a consequence, the dynamics of the coupled neural circuits can be presented in dimensionless form as follows:[16]

$$\begin{aligned}
\frac{dx_1}{d\tau} &= x_1(1-\xi_1) - \frac{1}{3}x_1^3 - y_1 + \xi_1 U_1\\
&\quad + \frac{C}{C_1+2C}\left[\left(x_2(1-\xi_2) - \frac{1}{3}x_2^3 - y_2 + \xi_2 U_2\right)\right.\\
&\quad \left. - \left(x_1(1-\xi_1) - \frac{1}{3}x_1^3 - y_1 + \xi_1 U_1\right)\right],\\
\frac{dy_1}{d\tau} &= c(x_1 + a - by_1),\\
\frac{dx_2}{d\tau} &= x_2(1-\xi_2) - \frac{1}{3}x_2^3 - y_2 + \xi_2 U_2\\
&\quad - \frac{C}{C_1+2C}\left[\left(x_2(1-\xi_2) - \frac{1}{3}x_2^3 - y_2 + \xi_2 U_2\right)\right.\\
&\quad \left. - \left(x_1(1-\xi_1) - \frac{1}{3}x_1^3 - y_1 + \xi_1 U_1\right)\right],\\
\frac{dy_2}{d\tau} &= c(x_2 + a - by_2).
\end{aligned} \tag{7}$$

The coupling intensity $\lambda_C = C/(2C+C_1)$ can be tamed to regulate synchronous firing in the two neurons by continuously pumping field energy along the coupling channel. When the coupling intensity λ_C is selected as zero, equation (7) is reduced to describe the dynamics of two isolated FitzHugh–Nagumo neurons. The two neurons become non-identical when one parameter shows diversity, $\xi_1 \neq \xi_2$. Note that the dimensionless energy function for a neuron and coupling channel can also be approached by

$$\begin{aligned}
&H_1 = \frac{W_1}{C_1V_0^2} = \frac{1}{2}x_1^2 + \frac{1}{2c}y_1^2,\ \ H_2 = \frac{W_2}{C_1V_0^2} = \frac{1}{2}x_2^2 + \frac{1}{2c}y_2^2,\\
&H_C = \frac{W_C}{C_1V_0^2} = \frac{1}{2}\frac{C}{C_1}(x_1-x_2)^2 = \frac{1}{2}\frac{\lambda_C}{1-2\lambda_C}(x_1-x_2)^2.
\end{aligned} \tag{8}$$

From Eq. (7), the coupling channel can be continuously controlled by changing the capacitance and then the coupling intensity is further increased to reach complete synchronization between two identical neurons. As a result, the Hamilton energy of the neuron becomes close to that of another one, and the field energy in the coupling channel becomes zero within a finite transient period. As confirmed in Ref. [17], the saturation gain method can be used to adjust the coupling channel

to reach complete synchronization between neurons. In fact, it is important to discern the self-adaptive property of synaptic connection, which indicates that the coupling channel should be dependent on the energy diversity between neurons. Therefore, we suggest that the coupling channel should be guided by

$$\Delta H = |H_1 - H_2| = \left| \left(\frac{1}{2}x_1^2 + \frac{1}{2c}y_1^2 \right) - \left(\frac{1}{2}x_2^2 + \frac{1}{2c}y_2^2 \right) \right|,$$
$$\frac{\mathrm{d}\lambda_C}{\mathrm{d}\tau} = \sigma\lambda_C K(\Delta H - \varepsilon),$$
$$K(p) = 1, \quad p \geq 0, \quad K(p) = 0, \quad p < 0. \tag{9}$$

That is, the Heaviside function $K(\cdot)$ and threshold ε control the working state of the coupling intensity. When the parameter ε is selected to have a tiny value, the coupling intensity is increased exponentially with constant gain σ as $\lambda_C \sim \lambda_0 \exp(\sigma\tau)$ before reaching energy balance between two neurons. When the threshold ε is increased, it means that two neurons can reach phase lock because the coupling intensity can be below the threshold and there is a large energy diversity and continuous energy pumping along the coupling channel. To detect the occurrence of complete synchronization, the error function is estimated from

$$\theta(e_x, e_y) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}. \tag{10}$$

This means that complete synchronization is reached when the error function shown in Eq. (10) is decreased to zero within the transient period. Furthermore, Hilbert transformation can be applied to the sampled time series for the variable (x_1, x_2) and phase series are obtained to calculate the phase error; as a result, phase locking can be explored for two neurons with parameter diversity (mismatch). To be consistent with the parameter setting in previous works, the parameters are selected as $a = 0.7$, $b = 0.8$, $c = 0.1$, and $\xi_1 = \xi_2 = 0.175$, the external stimuli U_1 and U_2 are equal to maintain the same excitability and thus the two neurons can be excited to present different firing modes. The fourth Runge–Kutta algorithm is used to find numerical solutions for the dynamical equations and Hamilton energy function with time step $h = 0.01$. For simplicity, we consider two identical neurons $\xi_1 = \xi_2$, and the same time-varying current $U_1 = U_2 = 1.0 + 2\cos\omega\tau$ is imposed on the two neurons synchronously. As is known, the excitability of the isolated neuron is dependent on the external stimulus, and an appropriate external stimulus or adjacent coupling can change the excitability and its firing modes can be regulated effectively. From a physical viewpoint, a larger amplitude in the external forcing can inject a larger energy and the oscillation states in the neural activities can be more easily changed. Here, we just consider the case that frequency in the external stimulus is changed, and the evolution of the Hamilton in an isolated neuron is also estimated in Fig. 2.

Figure 2 compares the interspike interval (ISI) and $\langle H \rangle$ of the neuronal circuit, where ISI describes the firing pattern of the neuronal circuit and $\langle H \rangle$ describes the average level of the saved energy. It is obvious that both parameters vary nonlinearly with the frequency of the external forcing and correspond to each other. By changing the frequency of the external forcing, distinct mode transition is completely induced, such as spiking, bursting and chaotic states. As defined in Eq. (9), the growth of coupling intensity, which depicts the synapse connection, is dependent on the parameter σ. Moreover, the intensity is adaptively adjusted by the Heaviside function $K(\cdot)$. The error function and energy propagation along the coupling channel are estimated during the adaptive growth of coupling intensity, and the results are shown in Fig. 3.

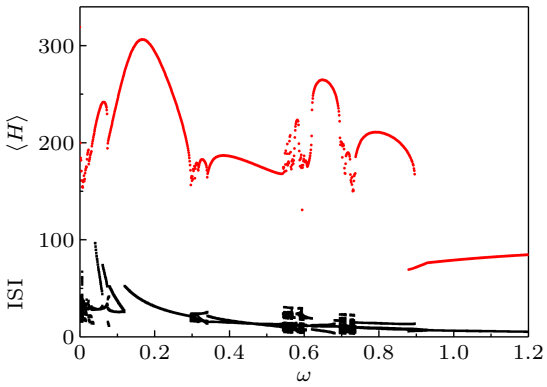


Fig. 2. Bifurcation diagrams for the ISI and average value for the Hamilton energy function (H) are calculated by changing the frequency in the external stimulus ($a = 0.7$, $b = 0.8$, $c = 0.1$, $\xi_1 = \xi_2 = 0.175$). The symbol $\langle H \rangle$ indicates the average of the Hamilton energy in time and the external stimulus is applied as $U = 1.0 + 2\cos\omega\tau$.

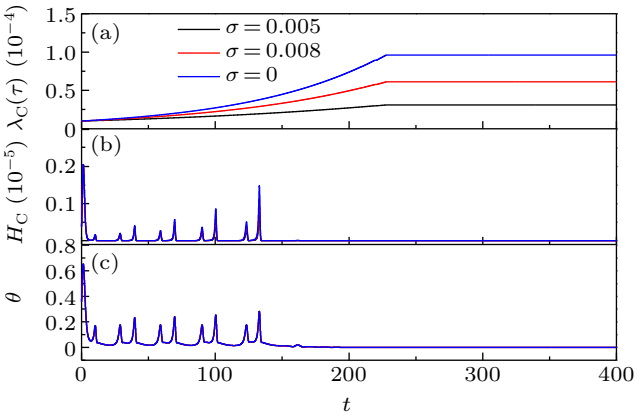


Fig. 3. Estimation of the growth of coupling intensity, energy pumping and error function for two coupled neurons presenting bursting states. The parameters are fixed as $a = 0.7$, $b = 0.8$, $c = 0.1$, $\omega = 0.005$, $\xi_1 = \xi_2 = 0.175$, and $\sigma = 0.005$, 0.008, 0.01: (a) coupling intensity, (b) channel energy, (c) error function. The external stimulus is applied as $U = 1.0 + 2\cos\omega\tau$ and the initial values are selected as (0.32, 0.59, 0.56, 0.33, 0.00001).

Within a finite transient period, two identical bursting neurons reach complete synchronization and the error function for two neurons is also decreased to zero. Meanwhile, the coupling intensity is stabilized to a saturation value and the

energy in the coupling channel also decreases to zero. It is interesting to discuss a similar case for two spiking neurons, and the results are plotted in Fig. 4.

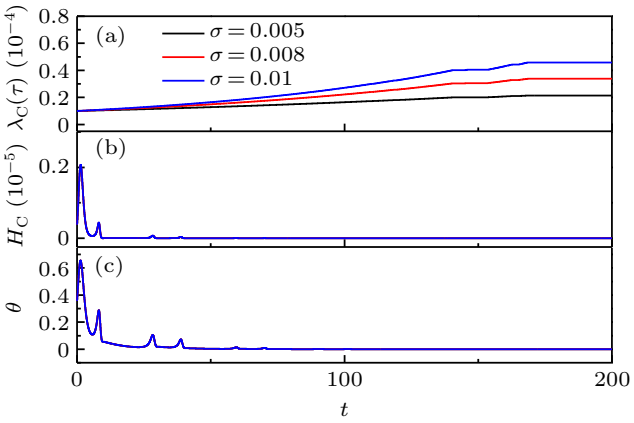


Fig. 4. Estimation of the growth of coupling intensity, energy pumping and error function for two coupled neurons presenting spiking modes. The parameters are fixed as $a = 0.7$, $b = 0.8$, $c = 0.1$, $\omega = 0.2$, $\xi_1 = \xi_2 = 0.175$, and $\sigma = 0.005$, 0.008, 0.01: (a) coupling intensity, (b) channel energy, (c) error function. The external stimulus is applied as $U = 1.0 + 2\cos\omega\tau$ and the initial values are selected as (0.32, 0.59, 0.56, 0.33, 0.00001).

As presented in Fig. 4, further increase in the coupling intensity is terminated and it reaches a saturation value and the energy in the coupling channel is pumped to zero, which means that two spiking neurons reach possible energy balance and complete synchronization. Surely, the error function for two spiking neurons also decreases to zero quickly and the energy balance is maintained for complete synchronization. Finally, it is worth looking at the synchronization stability between two identical neurons within chaotic states; the results are shown in Fig. 5.

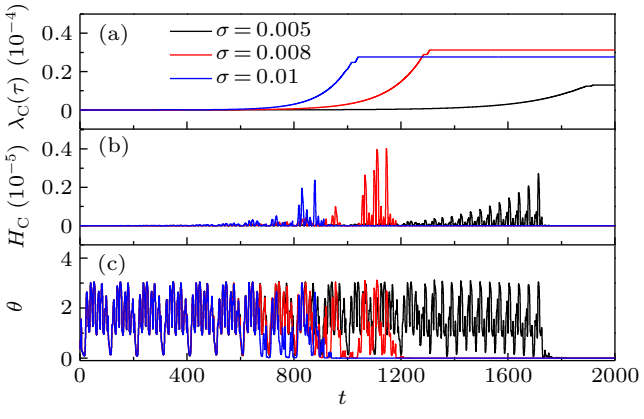


Fig. 5. Estimation of the growth of coupling intensity, energy pumping and error function for two coupled neurons presenting chaotic modes. The parameters are fixed as $a = 0.7$, $b = 0.8$, $c = 0.1$, $\omega = 0.7$, $\xi_1 = \xi_2 = 0.175$ and $\sigma = 0.005$, 0.008, 0.01: (a) coupling intensity, (b) channel energy, (c) error function. The external stimulus is applied as $U = 1.0 + 2\cos\omega\tau$ and the initial values are selected as (0.32, 0.59, 0.56, 0.33, 0.00001).

Similar to the cases for busting and spiking neurons, the coupling intensity is adaptively saturated and the energy in the coupling channel is pumped to zero within a certain transient period. These results indicate that two identical neurons in the same firing modes can be coupled to reach complete synchronization within a finite transient period by increasing the coupling intensity adaptively. As a result, the energy saved in the coupling channel is sent back to the neurons for further energy balance and exchange. When parameter diversity occurs, two neurons become non-identical and their firing modes become different; as a result, this scheme is effective in controlling the stability of phase synchronization and phase locking. For simplicity, the two neurons are excited with different external stimuli $U_1 = 1.0 + 2\cos\omega_1\tau$ and $U_2 = 1.0 + 2\cos\omega_2\tau$ and the same adaptive scheme is used to explore the synchronization stability between neurons with different firing modes (Fig. 6).

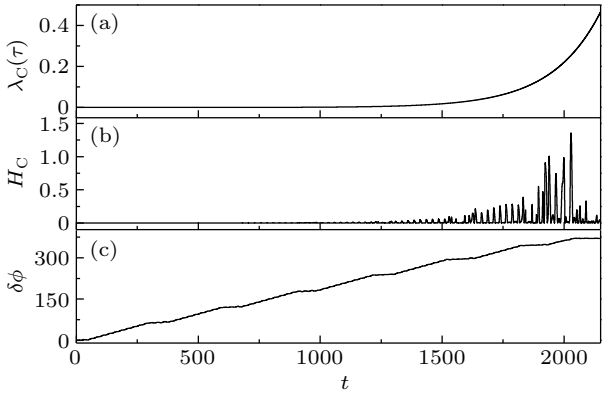


Fig. 6. Estimation of the growth of coupling intensity, energy pumping and error function for two coupled neurons presenting different modes. The parameters are fixed as $a = 0.7$, $b = 0.8$, $c = 0.1$, $\xi_1 = \xi_2 = 0.175$, and $\sigma = 0.005$: (a) coupling intensity, (b) channel energy, (c) phase error. The external stimulus is applied as $U = 1.0 + 2\cos\omega\tau$, $\omega_1 = 0.02$, $\omega_2 = 0.5$, and the initial values are selected as (0.32, 0.59, 0.56, 0.33, 0.00001). Phase series are derived by applying Hilbert transformation on the sampled time series for the membrane potential of each neuron.

The coupling intensity is further increased with time; however, energy pumping along the coupling channel is continued and complete synchronization between two different neurons becomes difficult. Nevertheless, phase synchronization is observed when the intensity is greater than 0.4 in Fig. 6.

In summary, capacitor connection facilitates fast energy exchange between neurons and neural circuits, and energy pumping is controlled in an adaptive way. Involvement of the Heaviside function can control the coupling intensity and working state of the coupling channels and the physical evidence for exploring the self-controllability and creation of synaptic connection can be confirmed. A similar scheme can be used to discern the formation and regulation of electric synapses to neurons and networks.[18,19] This work provides possible guidance for controlling the collective behaviors of neural networks and artificial neurons by injecting external energy in a gradient fashion.

3. Conclusion

In this paper, a capacitor which pumps electric field energy is used to couple two neural circuits. Moreover, the cou-

pling intensity is assumed to be controlled by the Heaviside function for the synchronization state. If the difference in Hamiltonian energy is larger than the threshold, the coupling intensity adjusts itself adaptively until the difference is less than the threshold. This work provides new insights for understanding how the intensity of synaptic connection is adaptively activated apart from the energy balance. That is, the energy transportation could be an effective way to regulate the collective behaviors of neural networks.

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