

Quantum correlation and entropic uncertainty in a quantum-dot system

Ying-Yue Yang(杨颖玥), Li-Juan Li(李丽娟), Liu Ye(叶柳), and Dong Wang(王栋)

Citation: Chin. Phys. B, 2022, 31 (**10**): 100303. DOI: 10.1088/1674-1056/ac7295
Journal homepage: http://cpb.iphy.ac.cn; http://iopscience.iop.org/cpb

What follows is a list of articles you may be interested in

Coherence migration in high-dimensional bipartite systems

Zhi-Yong Ding(丁智勇), Pan-Feng Zhou(周攀峰), Xiao-Gang Fan(范小刚),Cheng-Cheng Liu(刘程程), Juan He(何娟), and Liu Ye(叶柳)

Chin. Phys. B, 2022, 31 (**6**): 060308. DOI: 10.1088/1674-1056/ac48f8

Protecting geometric quantum discord via partially collapsing measurements of two qubits in multiple bosonic reservoirs

Xue-Yun Bai(白雪云) and Su-Ying Zhang(张素英)

Chin. Phys. B, 2022, 31 (**4**): 040308. DOI: 10.1088/1674-1056/ac4f54

Entropy squeezing for three-level atom interacting with a single-mode field

Fei-Fan Liu(刘非凡), Mao-Fa Fang(方卯发), Xiong Xu(许雄)

Chin. Phys. B, 2019, 28 (**6**): 060304. DOI: 10.1088/1674-1056/28/6/060304

Time-energy high-dimensional one-side device-independent quantum key distribution

Hai-Ze Bao(包海泽), Wan-Su Bao(鲍皖苏), Yang Wang(汪洋), Rui-Ke Chen(陈瑞柯), Hong-Xin Ma(马鸿鑫), Chun Zhou(周淳), Hong-Wei Li(李宏伟)

Chin. Phys. B, 2017, 26 (**5**): 050302. DOI: 10.1088/1674-1056/26/5/050302

Reduction of entropic uncertainty in entangled qubits system by local *JJ*-symmetric operation

Zhang Shi-Yang, Fang Mao-Fa, Zhang Yan-Liang, Guo You-Neng, Zhao Yan-Jun, Tang Wu-Wei

Chin. Phys. B, 2015, 24 (**9**): 090304. DOI: 10.1088/1674-1056/24/9/090304

Quantum correlation and entropic uncertainty in a quantum-dot system

Ying-Yue Yang(杨颖玥), Li-Juan Li(李丽娟), Liu Ye(叶柳), and Dong Wang(王栋)†

School of Physics and Optoelectronic Engineering, Anhui University, Hefei 230601, *China*

(Received 15 March 2022; revised manuscript received 23 May 2022; accepted manuscript online 24 May 2022)

We explore the dynamical behaviors of the measurement uncertainty and quantum correlation for a vertical quantum-dot system in the presence of magnetic field, including electron-electron interaction and Coulomb-blocked systems. Stemming from the quantum-memory-assisted entropic uncertainty relation, the uncertainty of interest is associated with temperature and parameters related to the magnetic field. Interestingly, the temperature has two kinds of influences on the variation of measurement uncertainty with respect to the magnetic-field-related parameters. We also discuss the relation between the lower bound of Berta *et al.* and the quantum discord. It is found that there is a natural competition between the quantum discord and the entropy $\min_{\Pi_i^{\rm B}} S_{\Pi_i^{\rm B}}(\rho_{\rm A|B})$. Finally, we bring in two improved bounds to offer a more precise limit to the entropic uncertainty.

Keywords: entropic uncertainty relation, quantum correlation, quantum dot

PACS: 03.67.–a, 03.65.Ta, 03.65.Ud **DOI:** 10.1088/1674-1056/ac7295

1. Introduction

The uncertainty principle proposed by Heisenberg[1] is famous because it is one of fundamental characteristics in quantum mechanics, which differs from those in classical counterpart. It reveals that there exists inevitable uncertainty in measurement of any two non-commutative mechanical quantities within a quantum system. Inspired by this, a universal inequality with standard deviation form is presented by Kennard and Robertson,[2,3] which can be expressed as[4,5]

$$\Delta R\Delta Z \geq \frac{1}{2}\left|\langle [R,Z]\rangle\right|, \tag{1}$$

where $[R,Z]=RZ-ZR$ is denoted as the commutator associated with a pair of incompatible observables R and Z, and $\Delta X=\sqrt{\langle X^2\rangle-\langle X\rangle^2}$ with the X's expectation value $\langle X\rangle$. Because of the state-dependent bound in Robertson's relation, it limits the accuracy to quantify the measurement uncertainty when special states are chosen. In order to remove the defect, Deutsch and Kraus[6,7] came up with a new uncertainty relation via adopting the Shannon entropy. Then, Maassen and Uffink[8] optimized the entropic uncertainty relation (EUR) as

$$H(R)+H(Z)\geq \log_2\frac{1}{c}, \tag{2}$$

with $H(X)=-\sum_m p_m(X)\log_2 p_m(X)$ represents for the Shannon entropy. Note that the probability distribution of the m-th outcome is described by $p_m(X)=\langle\varphi_i|\rho|\varphi_i\rangle$ with the eigenstates $|\varphi_i\rangle$ of X when measuring ρ in a quantum system. In addition, $c=\max_{i,j}\{|\langle r_i|z_j\rangle|^2\}$ is the complementarity of R and Z, where $|r_i\rangle$ and $|z_j\rangle$ denote the eigenvectors of R and Z, respectively.

Later, a quantum-memory-assisted entropic uncertainty relation (QMA-EUR), which has been put forward by Refs. [9,10] and experimentally demonstrated by Refs. [11,12], modifies the outcomes of the measurement uncertainty. Remarkably, the relation can be illustrated by a well-known uncertainty game. In the game, we assume that there are two legitimate participants, Alice and Bob. First, Bob prepares two entangled particles A and B. A is subsequently sent to Alice while particle B is serving as a memory. Then, Alice chooses one of two measurements R and Z, and it is agreed in advance to measure her particle and send her choice to Bob. According to Alice's measurement choice, Bob can predict the outcomes measured by Alice as accurate as possible. The uncertainty can be mathematically expressed as[10]

$$S(R|B)+S(Z|B)\geq \log_2\frac{1}{c}+S(A|B). \tag{3}$$

Here, the conditional von Neumman entropy is described as $S(X|B)=S(\rho_{XB})-S(\rho_{\rm B})$ with $S(\rho)=-\sum_i\lambda_i\log_2\lambda_i$ and $\rho_{XB}=\sum_i(|\varphi_i\rangle\langle\varphi_i|\otimes\mathbb{1})\rho_{\rm AB}(|\varphi_i\rangle\langle\varphi_i|\otimes\mathbb{1})$. Note that λ_i and $\mathbb{1}$ represent the eigenvalues of ρ and an identity operator, respectively. Interestingly, one can obtain the following conclusions: (i) If A and B are maximally entangled, Bob can predict perfectly the measurement outcomes, attributed to $S(A|B)=-\log_2\frac{1}{c}$. (ii) When quantum memory B vanishes, we can obtain the inequality $H(R)+H(Z)\geq\log_2\frac{1}{c}+H(A)$, whose bound is stronger than that of Eq. (2), on account of $H(A)\geq 0$. In addition, several groups have contributed to exploring entropic uncertainty relations and made some progress.[13–16]

As a matter of fact, the QMA-EUR can be applied to achieve many kinds of quantum tasks in the realm of

†Corresponding author. E-mail: dwang@ahu.edu.cn

© 2022 Chinese Physical Society and IOP Publishing Ltd

quantum information and computation, such as quantum cryptography,[17,18] quantum teleportation,[19] quantum key distribution,[20–23] quantum metrology[24–26] and entanglement witness.[11,12,27] Moreover, the relation has been experimentally demonstrated based on all-optics platforms, promoting its applications in real-life quantum information processing.[28–33]

Recently, the quantum coherence in a vertical quantum-dot system (QDS) was investigated in Ref. [34], which demonstrated the coherence for electrons in the structure of a solid states. Remarkably, these investigations make for a further understanding of the coherent control of spins in diamond and manipulations electron spin qubits in QDSs. Nevertheless, the measurement uncertainty has never been revealed in any QDS, which play a fundamental and nontrivial role in quantum information processing. With this in mind, the aim of the paper is to address this issue.

The remainder of this paper is organized as follows. We first review the Hamiltonian of the quantum systems in Section 2. In Section 3, we focus on the dynamical behavior of the QMA-EUR. Meanwhile, we analyze the physical interpretation of the measurement uncertainty in terms of the quantum correlation. Next, several improved bounds in QDSs are investigated in Section 4. Finally, a concise conclusion is rendered in Section 5.

2. Physical mode

In quantum mechanics, optical and electronic properties of quantum dots differ highly from those in larger-scale particles, which can stimulate more and more interests to explore deeper phenomena and mechanics in quantum information theory.[35,36] Typically, Hamiltonian of a quantum-dot system can be written as[34,37]

$$\mathcal{H} = \frac{\mu}{4}\hat{S}_1 \cdot \hat{S}_2 - \nu B_0 \hat{S}_z, \tag{4}$$

where μ is the difference of energy between the single and triplet states, and the first term can be regarded as the exchange coupling. Here we consider about $\boldsymbol{B} = B_0 z$ so that the Zeeman effect is incorporated. Note that ν is the dot gyromagnetic ratio. Additionally, $\hat{S}_1$ and $\hat{S}_2$ are two fictitious 1/2-spins. $\hat{S}_z$ is the z-projection of the total spin quantum number S.

Hence, we obtain the eigenvectors and eigenvalues of Hamiltonian (4) as follows:

$$\begin{aligned} E_1 &= \frac{\mu}{16}, \\ |\psi_1\rangle &= \frac{1}{\sqrt{2}}\left(\chi_{\frac{1}{2}}(\hat{S}_{1z})\chi_{-\frac{1}{2}}(\hat{S}_{2z}) + \chi_{-\frac{1}{2}}(\hat{S}_{1z})\chi_{\frac{1}{2}}(\hat{S}_{2z})\right), \quad (5) \\ E_2 &= \frac{-3\mu}{16}, \\ |\psi_2\rangle &= \frac{1}{\sqrt{2}}\left(\chi_{\frac{1}{2}}(\hat{S}_{1z})\chi_{-\frac{1}{2}}(\hat{S}_{2z}) - \chi_{-\frac{1}{2}}(\hat{S}_{1z})\chi_{\frac{1}{2}}(\hat{S}_{2z})\right), \quad (6) \end{aligned}$$

$$E_3 = \frac{\mu}{16} + \nu B_0, \quad |\psi_3\rangle = \chi_{-\frac{1}{2}}(\hat{S}_{1z})\chi_{-\frac{1}{2}}(\hat{S}_{2z}), \tag{7}$$

$$E_4 = \frac{\mu}{16} - \nu B_0, \quad |\psi_4\rangle = \chi_{\frac{1}{2}}(\hat{S}_{1z})\chi_{\frac{1}{2}}(\hat{S}_{2z}). \tag{8}$$

For simplicity, Plank's constant $\hbar = 1$ and Boltzmann's constant $K = 1$ are set. Herein, $|\psi_3\rangle$ and $|\psi_4\rangle$ are factorizable states without entanglement. On the other hand, $|\psi_1\rangle$ and $|\psi_2\rangle$ are two of maximally entangled states, with the spin function $\chi_\alpha(s_z)\left(\alpha = \pm\frac{1}{2}\right)$ for each electron. The spin function can be transferred to

$$\begin{aligned} \chi_{\frac{1}{2}}\chi_{\frac{1}{2}} &= |\uparrow\uparrow\rangle, \\ \chi_{\frac{1}{2}}\chi_{-\frac{1}{2}} &= |\uparrow\downarrow\rangle, \\ \chi_{-\frac{1}{2}}\chi_{\frac{1}{2}} &= |\downarrow\uparrow\rangle, \\ \chi_{-\frac{1}{2}}\chi_{-\frac{1}{2}} &= |\downarrow\downarrow\rangle, \end{aligned} \tag{9}$$

where $\uparrow$ and $\downarrow$ stand for two different directions of the electronic spin.

For a solid state system in thermal equilibrium, the state can be expressed as $\rho = \exp\left(-\frac{\mathcal{H}}{KT}\right)/Z$ with the partition function $Z = \mathrm{Tr}\left(\exp\left(-\frac{\mathcal{H}}{KT}\right)\right)$ at a finite temperature T, which can be re-expressed by the probability of the eigenvectors of the Hamiltonian as follows:

$$\rho = \sum_i a_i |\psi_i\rangle\langle\psi_i|, \tag{10}$$

where $a_i = \exp\left(-\frac{E_i}{KT}\right)$ represents the probability distribution.

As a result, one can obtain the density matrix of the system's state:

$$\rho = \begin{pmatrix} \upsilon & 0 & 0 & 0 \\ 0 & \gamma & \lambda & 0 \\ 0 & \lambda & \gamma & 0 \\ 0 & 0 & 0 & \omega \end{pmatrix}, \tag{11}$$

with

$$\upsilon = \frac{1}{z}\exp\left(-\frac{\mu - 16\beta}{16T}\right), \tag{12}$$

$$\gamma = \frac{1}{2z}\left[\exp\left(-\frac{\mu}{16T}\right) + \exp\left(\frac{3\mu}{16T}\right)\right], \tag{13}$$

$$\omega = \frac{1}{z}\exp\left(-\frac{\mu + 16\beta}{16T}\right), \tag{14}$$

$$\lambda = \frac{1}{2z}\left[\exp\left(-\frac{\mu}{16T}\right) - \exp\left(\frac{3\mu}{16T}\right)\right]. \tag{15}$$

Herein, we take $\beta = \nu B_0$. The partition function z is given by

$$z = \exp\left(-\frac{\mu}{16T}\right)\left[2\cosh\left(\frac{\beta}{T}\right) + 1\right] + \exp\left(\frac{3\mu}{16T}\right). \tag{16}$$

3. The entropic uncertainty relation in quantum-dot system

First of all, we will focus on exploiting the dynamical evolution of the measurement uncertainty in the quantum-dot systems. Herein, we take two Pauli observables, $\hat{\sigma}_x$ and $\hat{\sigma}_z$,

as the incompatibility. Therefore, the post-measurement states can be written as

$$\begin{aligned}\rho_{\hat{\sigma}_x\mathrm{B}} &= \frac{\upsilon+\gamma}{2}\left(|00\rangle\langle 00|+|10\rangle\langle 10|\right)\\ &+\frac{\gamma+\omega}{2}\left(|01\rangle\langle 01|+|11\rangle\langle 11|\right)\\ &+\frac{\lambda}{2}\big(|00\rangle\langle 11|+|01\rangle\langle 10|\\ &+|10\rangle\langle 01|+|11\rangle\langle 00|\big), \qquad (17)\\ \rho_{\hat{\sigma}_z\mathrm{B}} &= \upsilon|00\rangle\langle 00|+\gamma\left(|01\rangle\langle 01|+|10\rangle\langle 10|\right)\\ &+\omega|11\rangle\langle 11|. \qquad (18)\end{aligned}$$

In addition, the reduced density matrix ρ_{B} can be given by $\rho_{\mathrm{B}}=\mathrm{Tr}_{\mathrm{A}}(\rho_{\mathrm{AB}})$. After that, one can easily obtain the eigenvalues, and the von Neumann entropy can be expressed as

$$S(\rho_{\hat{\sigma}_x\mathrm{B}}) = -2\eta^{+}\log_2\eta^{+}-2\eta^{-}\log_2\eta^{-}, \qquad (19)$$

$$S\left(\rho_{\hat{\sigma}_z\mathrm{B}}\right) = -\upsilon\log_2\upsilon-2\gamma\log_2\gamma-\omega\log_2\omega, \qquad (20)$$

$$S(\rho_{\mathrm{B}}) = -(\upsilon+\gamma)\log_2(\upsilon+\gamma)-(\omega+\gamma)\log_2(\omega+\gamma), \qquad (21)$$

with $\eta^{\pm}=\frac{1}{4}(\upsilon+2\gamma+\omega\pm\sqrt{(\upsilon-\omega)^2+4\lambda^2})$.

Hence, the left-hand side of Eq. (3) can be rewritten as

$$U_{\mathrm{L}} = S(\rho_{\hat{\sigma}_x\mathrm{B}})+S\left(\rho_{\hat{\sigma}_z\mathrm{B}}\right)-2S(\rho_{\mathrm{B}}). \qquad (22)$$

Additionally, due to the fact that the overlap always equals 1/2 for $\hat{\sigma}_x$ and $\hat{\sigma}_z$, the right-hand side of Eq. (3) is equal to

$$\begin{aligned}U_{\mathrm{R}} &= 1-\upsilon\log_2\upsilon-(\gamma+\lambda)\log_2(\gamma+\lambda)\\ &-(\gamma-\lambda)\log_2(\gamma-\lambda)-\omega\log_2\omega-S(\rho_{\mathrm{B}}). \qquad (23)\end{aligned}$$

To reveal the nature of the measurement uncertainty of interest in quantum-dot systems, we depict the entropy uncertainty and the lower bound of entropy uncertainty as a function of temperature T in Fig. 1. From Fig. 1, it is clear that the uncertainty increases with the growth of temperature. Intuitively, the higher the temperature is, the more disordered the system is, and hence the weaker the systemic quantum correlation will be, which results in the inaccuracy of the prediction to the outcomes of Alice's measurement. In addition, one can see that the lower bound also increases with the growth of T and it always satisfies $U_{\mathrm{L}}\geq U_{\mathrm{R}}$. When the temperature is enough high, it is easy to find that the measurement uncertainty shrinks to a fixed value and $U_{\mathrm{R}}\to U_{\mathrm{L}}$.

In the following, we focus on the effect of the parameters μ and β on the entropic uncertainty in Fig. 2. Interestingly, the behaviors of the measurement uncertainty are divided into two regions, i.e., $T=0.1$ region and $T=1$ region. The evolutions in the two regions are different. The entropic uncertainty will firstly increase and subsequently decrease with the growth of μ and β when temperature $T=0.1$, and the entropic uncertainty has a maximum value in the whole evolution. On the other hand, the uncertainty always decreases with increasing μ and β in the $T=1$ regime, which is contrast to the non-monotonic variation of entropic uncertainty with increasing μ and β in the $T=0.1$ region.

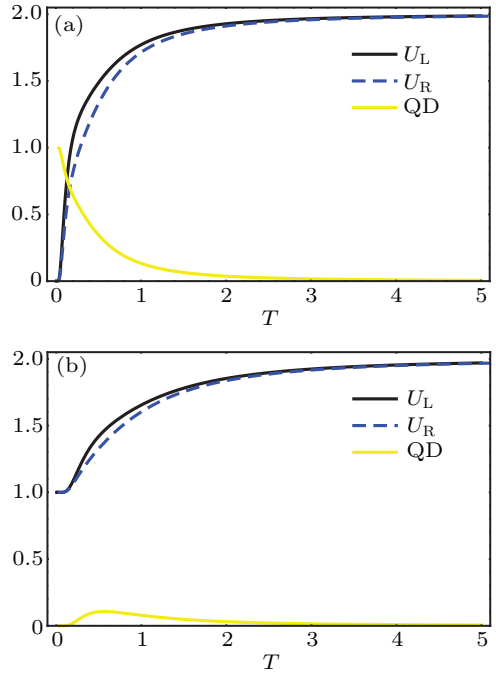


Fig. 1. Measurement uncertainty and the quantum discord depicted as functions of temperature T for different values of μ and β: (a) $\mu=5$, $\beta=1$, (b) $\mu=5$, $\beta=2$.

Next, we turn to probe the systemic quantum correlation. Generally, the quantum discord (QD) is usually deemed as one of effective quantification of quantum correlation,[38–40] which is defined by

$$\mathrm{QD}(\rho_{\mathrm{AB}}) = I(\rho_{\mathrm{AB}})-\mathrm{CC}(\rho_{\mathrm{AB}}), \qquad (24)$$

where $I(\rho_{\mathrm{AB}})$ is the total correlation, which is denoted as

$$I(\rho_{\mathrm{AB}}) = S(\rho_{\mathrm{A}})+S(\rho_{\mathrm{B}})-S(\rho_{\mathrm{AB}}), \qquad (25)$$

and $\mathrm{CC}(\rho_{\mathrm{AB}})$ is the classical correlation between subsystems A and B, which is given by

$$\mathrm{CC}(\rho_{\mathrm{AB}}) = \max_{\Pi_i^{\mathrm{B}}}\left[S(\rho_{\mathrm{A}})-S_{\Pi_i^{\mathrm{B}}}\left(\rho_{\mathrm{A|B}}\right)\right], \qquad (26)$$

where $\rho_{\mathrm{A(B)}}$ is the reduced density matrix of ρ_{AB} tracing over B (A). Π_i^{B} represents a set of possible positive-operator-valued measurements on particle B, the conditional entropy of A is given by $S_{\Pi_i^{\mathrm{B}}}\left(\rho_{\mathrm{A|B}}\right)=\sum_i q_i S\left(\rho_i^{\mathrm{A}}\right)$ with the corresponding probability $q_i=\mathrm{Tr}_{\mathrm{AB}}\left(\rho_{\mathrm{AB}}\Pi_i^{\mathrm{B}}\right)$, where ρ_i^{A} is the density operator defined by $\rho_i^{\mathrm{A}}=\frac{1}{q_i}\mathrm{Tr}_{\mathrm{B}}\left(\Pi_i^{\mathrm{B}}\rho_{\mathrm{AB}}\Pi_i^{\mathrm{B}}\right)$.

Furthermore, we plot the quantum discord as shown in Figs. 1 and 2. In contrast to the dynamics of the entropic uncertainty, the QD decreases with increasing T. Based on this

phenomenon, we here offer a physical explanation. By connecting Eqs. (3) and (24)–(26), the lower bound can be derived as

$$\begin{aligned} U_{\mathrm{R}} &= 1+S(\rho_{\mathrm{AB}})-S(\rho_{\mathrm{B}}) \\ &= -\mathrm{QD}(\rho_{\mathrm{AB}})+\min_{\Pi_i^{\mathrm{B}}} S_{\Pi_i^{\mathrm{B}}}\left(\rho_{\mathrm{A|B}}\right)+1. \end{aligned} \tag{27}$$

From Figs. 1 and 2, one can see that the lower bound is mainly anti-correlated with the quantum discord. For example, the lower bound is correlation with the quantum discord with the growth of β in the $T=1$ regime as shown in Fig. 2(b). It looks like that uncertainty is anti-correlated with the quantum discord in Fig. 1(a), uncertainty increases as temperature increases, while QD decreases until it reaches 0. However, in Fig. 2(c), QD gradually decreases from 1 and remains at 0, but the uncertainty is not a synchronous process with QD, it is a process of first increasing, then decreasing and finally maintaining. This means that the dynamics of the uncertainty is not always synchronous with the tendency of the quantum correlation as shown in Fig. 2(c). The reason is that the lower bound does not only rely on the systemic QD, but also depend on the conditional entropy $\min_{\Pi_i^{\mathrm{B}}} S_{\Pi_i^{\mathrm{B}}}\left(\rho_{\mathrm{A|B}}\right)$. In fact, the quantum discord have a competitive relation with the conditional entropy as the derivation given above. In the current scenario, we notice that the QD is a predominant factor for the evolution of the measurement uncertainty from the present results.

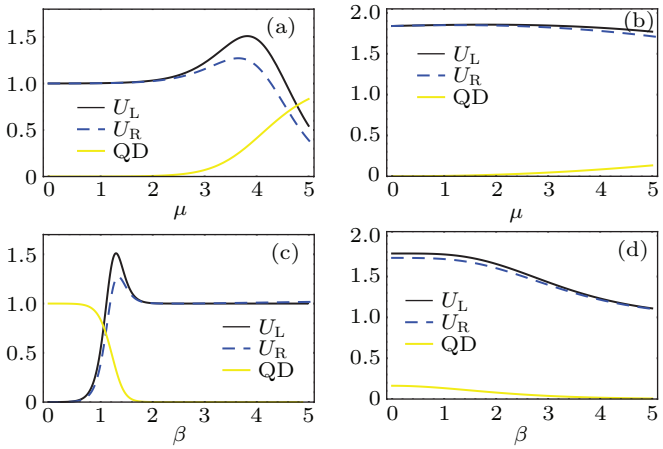


Fig. 2. Measurement uncertainty and the quantum discord depicted as functions of μ and β in different regimes of temperature T: (a) $T=0.1$, $\beta=1$, (b) $T=1$, $\beta=1$, (c) $\mu=5$, $T=0.1$, (d) $\mu=5$, $T=1$.

4. The improved bounds in quantum-dot system

The uncertainty principle is regarded as as an important tool to quantify the precision of the measurement outcomes concerning two incompatible observables, the uncertainty bound essentially plays a prominent role in quantum information theory.[41] Recently, more and more attention has been paid to optimizing the lower bounds. Typically, Pati *et al.*[42] proposed an improved bound via quantum correlation between the subsystems, which has the form of

$$\begin{aligned} &S(\mathrm{R}|\mathrm{B})+S(\mathrm{Z}|\mathrm{B}) \\ &\geq \log_2\frac{1}{c}+S(\mathrm{A}|\mathrm{B})+\max\{0,\mathrm{QD}-\mathrm{CC}\}. \end{aligned} \tag{28}$$

In addition, Adabi *et al.*[43] optimized the lower bound by considering the Holevo quantity and mutual information, which can be expressed as

$$S(\mathrm{R}|\mathrm{B})+S(\mathrm{Z}|\mathrm{B})\geq\log_2\frac{1}{c}+S(\mathrm{A}|\mathrm{B})+\max\{0,\delta'\}, \tag{29}$$

with $\delta'=I(\rho_{\mathrm{AB}})-I(\rho_{\mathrm{RB}})-I(\rho_{\mathrm{ZB}})$.

In order to discuss the bounds of Eq. (3) and Eqs. (27) and (28), we plot the evolution of U_{L}, U_{R}, U_{b_1} and U_{b_2} in Fig. 3 versus all parameters. Notice that $U_{\mathrm{b}_1}=\log_2\frac{1}{c}+S(\mathrm{A}|\mathrm{B})+\max\{0,\mathrm{QD}-\mathrm{CC}\}$ is denoted as the bound of Pati *et al.* expressed by Eq. (27), $U_{\mathrm{b}_2}=\log_2\frac{1}{c}+S(\mathrm{A}|\mathrm{B})+\max\{0,\delta'\}$ is the bound of Adabi *et al.* expressed by Eq. (28). Following Fig. 3, the bounds of interest are more and more tight, i.e., $U_{\mathrm{R}}\leq U_{\mathrm{b}_1}\leq U_{\mathrm{b}_2}\leq U_{\mathrm{L}}$. The result of Adabi *et al.* limits the uncertainty of the outcomes predicted by Bob superior to other bounds, and the tightness between the entropic uncertainty and the lower bound of Adabi *et al.* is too small to distinguish from Fig. 3, which verifies that the bound of Adabi *et al.* is optimal.

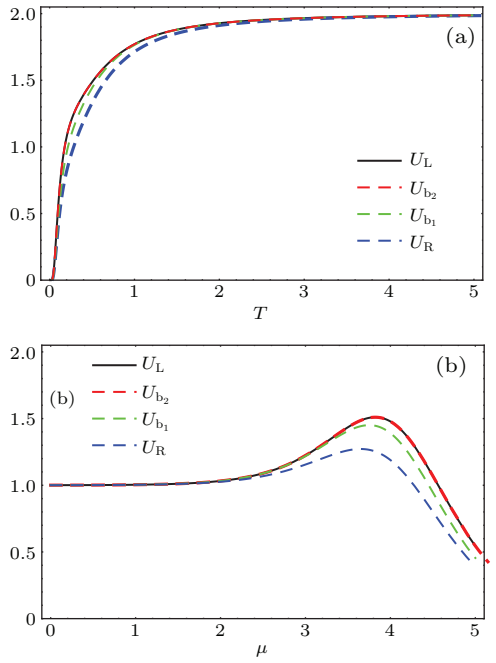


Fig. 3. Measurement uncertainty, the lower bound and two improved bound depicted as functions of T, μ and β: (a) $\mu=5$, $\beta=1$, (b) $T=0.1$, $\beta=1$.

5. Conclusion

In summary, we explore the uncertainty of interest and quantum correlation in a quantum-dot system with the magnetic field. As a consequence, we find that the higher temperature can induce the higher measurement uncertainty, which can undermine the entanglement between particles A and B. It is worth noting that the different-temperature regimes have

different influences on the behaviors of measurement uncertainty as functions of μ and β. In the region of high temperature, the uncertainty decreases as μ or β increases. However, the measurement uncertainty does not monotonously vary with the growth of μ and β in the region of low temperature. In fact, the uncertainty firstly increases and subsequently decreases as μ or β inflates. That is to say, the entropic uncertainty has a maximum value during the evolution. Interestingly, we find that the lower bound does not only always depend on the quantum discord but also rely on the conditional entropy $\min_{\Pi_i^{\rm B}} S_{\Pi_i^{\rm B}}\left(\rho_{\rm A|B}\right)$. Moreover, we compare two improved bounds, i.e., the lower bounds of Pati *et al.* and Adabi *et al.* It is proved that the improved bound can effectively enhance the accuracy of the limitation to the measurement uncertainty. Moreover, any quantum system is essentially disclosed, and inevitably suffers from decoherence.[44–46] Thus, to clarify how the noises in QDS affect the QMA-EUR is required in practical quantum information processing, which may be considered in the near future. With these in mind, we argue that our results may be beneficial to further understanding the entanglement dynamics of the solid-state systems and quantum measurement precision in practical quantum information processing.

Acknowledgements

Project supported by the National Natural Science Foundation of China (Grant Nos. 12075001, 61601002, and 12175001), the Anhui Provincial Key Research and Development Plan (Grant No. 2022b13020004), the Anhui Provincial Natural Science Foundation (Grant No. 1508085QF139), and the Fund of CAS Key Laboratory of Quantum Information (Grant No. KQI201701).

References

[1] Heisenberg W 1927 *Z. Phys.* **43** 172
[2] Kennard E H 1927 *Z. Phys.* **44** 326
[3] Robertson H P 1929 *Phys. Rev.* **34** 163
[4] Wang K K, Zhan X, Bian Z H, Li J, Zhang Y S and Xue P 2016 *Phys. Rev. A* **93** 052108
[5] Maccone L and Pati A K 2014 *Phys. Rev. Lett.* **113** 260401
[6] Deutsch D 1983 *Phys. Rev. Lett.* **50** 631
[7] Kraus K 1987 *Phys. Rev. D* **35** 3070
[8] Maassen H and Uffink J B M 1988 *Phys. Rev. Lett.* **60** 1103
[9] Renes J M and Boilea J C 2009 *Phys. Rev. Lett.* **103** 020402
[10] Berta M, Christandl M, Colbeck R, Renes J M and Renner R 2010 *Nat. Phys.* **6** 659
[11] Li C F, Xu J S, Xu X Y, Li K and Guo G C 2011 *Nat. Phys.* **7** 752
[12] Prevedel R, Hamel D R, Colbeck R, Fisher K and Resch K J 2011 *Nat. Phys.* **7** 757
[13] Dolatkhah H, Haseli S, Salimi S and Khorashad A S 2019 *Quantum Inf. Process.* **18** 13
[14] Zhang Z Y and Liu J M 2022 *Physica A* **589** 126639
[15] Jiang T Y, Fang Y Y, Li Y H, Xu X Y and Liu J M 2022 *Ann. Phys. (Berlin)* **534** 2100352
[16] Li L J, Ming F, Song X K, Ye L and Wang D 2022 *Acta Phys. Sin.* **71** 070302 (in Chinese)
[17] Konig R, Wehner S and Wullschleger J 2012 *IEEE Trans. Inf. Theory* **58** 1962
[18] Dupuis F, Fawzi O and Wehner S 2015 *IEEE Trans. Inf. Theory* **61** 1093
[19] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[20] N J Cerf, M Bourennane, A Karlsson and Gisin N 2002 *Phys. Rev. Lett.* **88** 127902
[21] Koashi M 2009 *New J. Phys.* **11** 045018
[22] Grosshans F and Cerf N J 2004 *Phys. Rev. Lett.* **92** 047905
[23] Ming F, Wang D, Fan X G, Shi W N, Ye L and Chen J L 2020 *Phys. Rev. A* **102** 012206
[24] Hall M J W and Wiseman H M 2012 *New J. Phys.* **14** 033040
[25] Yu C S 2017 *Phys. Rev. A* **95** 042337
[26] Jarzyna M and Demkowicz-Dobrzański R 2015 *New J. Phys.* **17** 013010
[27] Hu M L and Fan H 2013 *Phys. Rev. A* **88** 014105
[28] Rodriguez E B and Aguilar L M A 2018 *Sci. Rep.* **8** 4010
[29] Wang D, Shi W N, Hoehn R D, Ming F, Sun W Y, Kais S and Ye L 2018 *Ann. Phys. (Berlin)* **530** 1800080
[30] Yang Y Y, Sun W Y, Shi W N, Ming F, Wang D and Ye L 2019 *Front. Phys.* **14** 31601
[31] Xie B F, Ming F, Wang D, L Ye and Chen J L 2021 *Phys. Rev. A* **104** 062204
[32] Li L J, Ming F, Song X K, Ye L and Wang D 2021 *Eur. Phys. J. C* **81** 728
[33] Li L J, Ming F, Shi W N, Ye L and Wang D 2021 *Physica E* **133** 114802
[34] Berrada K 2020 *Physica E* **116** 113784
[35] Chotorlishvili L, Gudyma A, Waetzel J, Ernst A and Berakdar J 2019 *Phys. Rev. B* **100** 174413
[36] Lee B, Pursley B C, Carter S G, Economou S E, Yakes M K, Grim J Q, Bracker A S and Gammon D 2019 *Phys. Rev. B* **100** 125438
[37] M Pustilnik and Glazman L I 2000 *Phys. Rev. Lett.* **85** 2993
[38] Ollivier H and Zurek W H 2001 *Phys. Rev. Lett.* **88** 017901
[39] Luo S L 2008 *Phys. Rev. A* **77** 042303
[40] Sun K, Ye X J, Xu J S, Xu X Y, Tang J S, Wu Y C, Chen J L, Li C F and Guo G C 2016 *Phys. Rev. Lett.* **116** 160404
[41] Wang D, Ming F, Hu M L and Ye L 2019 *Ann. Phys. (Berlin)* **531** 1900124
[42] Pati A K, Wilde M M, Usha Devi A R, Rajagopal A K and Sudha 2012 *Phys. Rev. A* **86** 042105
[43] Adabi F, Salimi S and Haseli S 2016 *Phys. Rev. A* **93** 062123
[44] Hu M L, Zhang Y H and Fan H 2021 *Chin. Phys. B* **30** 030308
[45] Hu M L and Fan H 2020 *Sci. China-Phys. Mech. Astron.* **63** 230322
[46] Yu T and Eberly J H 2006 *Phys. Rev. Lett.* **97** 140403