Evolution of polarization singularities accompanied by avoided crossing in plasmonic system

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The evolution of polarization singularities supported in a one-dimensional periodic plasmonic system is studied. The lateral inversion symmetry of the system, which breaks the in-plane inversion symmetry and up-down mirror symmetry simultaneously, yields abundant polarization states. A complete evolution process with geometry for the polarization states is traced. In the evolution, circularly polarized points (C points) can stem from 3 different processes. In addition to the previously reported processes occurring in an isolated band, a new type of C point appearing in two bands simultaneously due to the avoided band crossing, is observed. Unlike the dielectric system with a similar structure which only supports at-Γ bound states in the continuum (BICs), accidental BICs off the Γ point are realized in this plasmonic system. This work provides a new scheme of polarization manipulation for the plasmonic systems.

Keywords: polarization singularities, topological charge, avoided crossing, inversion symmetry

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1. Introduction

Polarization manipulation plays an important role in modern electromagnetic applications. For a periodic system such as photonic crystal slabs (PCSs), polarization of the far-field radiation can be characterized topologically in the momentum space. As the typical singularities of polarization, vortex polarization points (V points) and circular polarization points (C points) have attracted much attention.[1,2] The V points are vortex centers in the polarization directions of far-field radiation and the radiation intensity vanishes at the V point. The V points are closely related to the interesting phenomena of bound states in the continuum (BICs). In electromagnetics, the BIC is a special type of spatially localized eigenmode lies in the band of continuum while no radiation happens.[3–10] In PCSs the BICs can be realized when both the upward and downward radiations correspond to a V point since no radiation occurs. The unique properties of BICs have aroused much interest for the potential applications in lasing,[11,12] sensing,[13] and antennas.[14] The C points are points of circular polarization and surrounded by elliptical polarization states. As the orientation of the polarization ellipse is undefined, C points are regarded as polarization singularities.

The polarization states of the radiation from PCSs are highly dependent on the symmetry of system. For the PCSs with in-plane inversion (C_2) and up-down mirror (σ_z) symmetries the BICs can be readily observed. When the symmetry of the system is broken, the new polarization phenomena will be brought forth.[2,15–24] For example, breaking the in-plane C_2 symmetry will eliminate the original BICs but induce C points. With other polarization states accompanied, full coverage on the Poincare sphere can be realized.[17] Breaking the in-plane C_6 symmetry for at-Γ BICs with high-order topological charge can generate off-Γ BICs from the at-Γ BIC and create or annihilate C points pairwise.[2] Breaking the in-plane inversion symmetry enables metasurfaces to support sharp high-Q resonances arising from a distortion of symmetry-protected BICs.[15] The BICs with Q factor beyond typical inverse-square law are achieved by breaking the symmetry in an C_{4v} structure.[16]

Breaking the up-down mirror symmetry can also bring new features to the far-field radiation. Unidirectionally guided resonances (or unidirectional BICs) that only radiate to one side of the structure are realized in PCSs with slanted wall[21] or with asymmetric substrate.[19] The broken mirror-flip symmetry in up-down direction can be used to enhance TE–TM coupling and give rise to BIC with unique spatial characteristics.[16] Chiral quasi-BICs can be realized by introducing chiral perturbation to break the up-down mirror symmetry.[24]

The C_2 symmetry and the up-down mirror symmetry can

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be broken simultaneously in a system with lateral inversion symmetry. Recently, Zeng *et al.* investigated the evolution of polarization singularities in a PCS with lateral inversion symmetry, proposing a flexible mechanism to generate and tune C points.[25] A similar structure has also been proposed to realize the perfect single-sided radiation and absorption without mirrors.[26] In the reported researches, the evolution of the singularities happens in an isolated band and only a small range of geometry variation is involved.

Here we report a complete evolution process of the polarization singularities accompanied by avoided crossing in a plasmonic system which is composed of a thin metallic film sandwiched by two dielectric gratings. When the system has up-down mirror symmetry, the band crossing happens and a symmetry-protected BIC is supported in the antisymmetric mode band. When the gratings on two sides take a relative shift along the periodic direction, the up-down mirror symmetry will be broken and the avoided band crossing happens. In this process, the original symmetry-protected BIC (V point) splits into two C points; meanwhile, other C point pairs appear in a certain single band, or emerge in both two bands simultaneously due to the avoided crossing. With the continuous tuning of the misalignment, C point pairs can merge into BICs or annihilate, and *vice versa*. The topological charges are conserved in the whole process. With the relative shift between the two grating varying from 0 to half the period, a complete involution process is traced. In the end, the asymmetry of radiation of the system is briefly discussed.

2. Overview of system

The system under study, illustrated in Figs. 1(a) and 1(b), has a thin metallic film sandwiched between two identical one-dimensional dielectric gratings. The system is periodic in the x direction and uniform in the y direction. The main parameters are the period a of the system, the thickness t_m of the metal film, the width w and the thickness t_d of the dielectric ridges of the grating, and the relative shift s between the gratings on the two sides. Specifically, here $a = 600$ nm, $t_\mathrm{m} = 40$ nm, $t_\mathrm{d} = 100$ nm, and $w = 300$ nm. The permittivity of the metal film is described by the Drude-type model: $\varepsilon(\omega) = 1 - \omega_\mathrm{p}^2/(\omega^2 + \mathrm{i}\gamma\omega)$, where ω_p is the plasma frequency and γ is the collision frequency. As we focus on the radiation loss of a resonant mode, the intrinsic loss of the nanoparticles is ignored, that is, $\gamma = 0$, and the plasma frequency is set to be $\omega_\mathrm{p} = 1.37 \times 10^{16}$ rad/s. The refractive index of the dielectric layer and the background are set to be $n_\mathrm{d} = 1.515$ and $n_\mathrm{b} = 1$, respectively.

The band structure of the system along the x direction, which has been studied before,[27] is shown in Fig. 1(c). In the figure the error bars represent the imaginary parts of the resonance frequencies, the black dots indicate the position where the error bar decreases to infinitesimal, and a BIC appears since the quality factor diverges here. When the relative shift between the two dielectric gratings $s = 0$, the structure has up-down mirror symmetry and in-plane C_2 symmetry simultaneously. The modes in A band and B band are orthogonal to each other, with A band corresponding to the odd mode and B band the even mode. Owing to the orthogonality, the two dispersion curves cross straightly through each other. Furthermore, for the mode in A band at the Γ point, the profile (H_y) along the x direction is odd, which is symmetrically incompatible with the external radiation, hence a symmetry-protected BIC[4] appears here, as indicated by the black dot shown in the first panel in Fig. 1(c). When $s \neq 0$ or $a/2$, both the in-plane C_2 symmetry and up-down mirror symmetry of the structure are broken. As a result, the symmetry protected BIC at Γ point disappears. Meanwhile, avoided band crossing happens and a band gap appears due to the coupling between the two originally orthogonal modes. When s increases to $0.2432a$, an accidental BIC appears at $k_x = 0.054$ in the upper band. This BIC will vanish again when the relative shift further increases. When s increases to $a/2$, the structure possesses in-plane C_2 symmetry again. As a result, the avoided band crossing disappears and a symmetric protected BIC forms in the A band at the Γ point. Note that the structure with shift s is mirror symmetric (σ_x) with that of a–s, then the band diagram corresponding to s for $s > 0.5a$ can be obtained by that of a–s by symmetry.

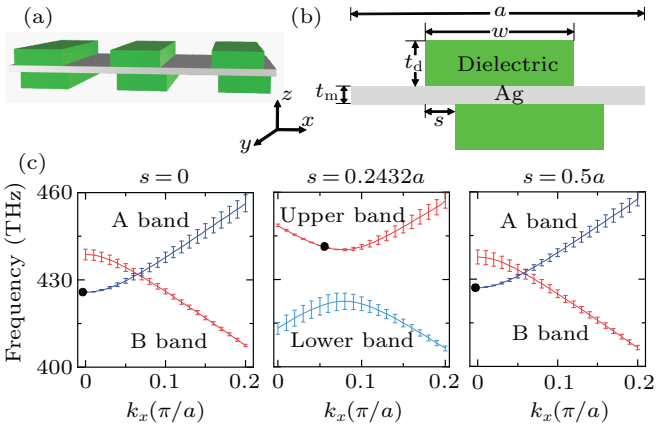


Fig. 1. (a) Schematic diagram of plasmonic system consisting of a metallic film sandwiched between two identical dielectric gratings with relative shift, (b) x–z plane view of the structure, (c) band diagrams of the structure under different shifts, with error bars denoting the imaginary parts of resonance frequencies and the black dots representing the corresponding zeros.

3. Formation of off-Γ BIC

To find how the off-Γ BIC forms and vanishes, the topology for the polarization state of the radiation in the momentum space is examined. For a given band of a periodic slab, the polarization of the far-field radiation varies with the in-plane Bloch wave vector $k_{||}$. In the zero-order diffraction zone, the far-field radiating plane wave has the same $k_{||}$ as that of the

Bloch mode, hence the polarization vector $\boldsymbol{d}(k_{\|})$ in the s–p plane of the radiation can be projected onto the structure plane and mapped onto the Brillouin zone, with the projected vector written as $\boldsymbol{d}'(\boldsymbol{k}_{\|}) = \boldsymbol{d}'_x(\boldsymbol{k}_{\|})\hat{x} + \boldsymbol{d}'_y(\boldsymbol{k}_{\|})\hat{y}$.[2,3] For a fully polarized light, the polarization state can be completely described by the ellipticity angle χ, with $\tan\chi$ being the ratio of the polarization ellipse's minor to major axis, and the orientation angle θ the angle between the long axis of the polarization ellipse and the x axis.[28]

The topology of the radiation polarization in the momentum space can be presented by the distribution of $\boldsymbol{d}'(\boldsymbol{k}_{\|})$. The topological charge carried by the polarization singularity is defined as[3]

$$\nu = \frac{1}{2\pi}\oint_C \mathrm{d}\boldsymbol{k}_{\|}\cdot\nabla\theta(\boldsymbol{k}_{\|}), \tag{1}$$

where C is a closed path enclosing the polarization singular point in counterclockwise direction, ν describes the number of times the major axis of the polarization ellipse orbits the ring C. If path C encloses a V point, charge ν is an integer; if path C encloses a C point, ν is a half-integer.

Figure 2 shows the distributions of the orientation angle θ and ellipticity ρ_c of polarization states in the downward radiation for the system with different shifts at the vicinity of $s = 0.2432a$. Here the ellipticity is defined as $\rho_c = \sin(2\chi)$. When $s = 0.242a$, the ellipticity diagram shows that there are two points with $\rho_c = +1$ and -1, which correspond to a right circularly polarized state and left circularly polarized state, respectively. The θ diagram indicates that at each point, the orientation angle θ makes a π radiance change in the counterclockwise direction. So the topological charges carried by the two C points are both $+1/2$. With the increase of s, the two C points move toward each other and merge into a V point when s reaches $0.2432a$. As shown in the middle panel of Fig. 2, two L lines (a line along which the polarization is linear) with orthogonal polarization cross at this point, and θ makes a 2π change around this point, hence a V point with charge of $+1$ is formed. With the further increase of s, the V point splits into two C points again as shown in the right panel of Fig. 3. Note that the handedness of the upper and lower C points for the case of $s = 0.244a$ are opposite to those for the case of $s = 0.242a$. This process can be regarded as that two C points with the same topological charge move toward each other and merge into a V point, then depart away. In this process the topological charges are conserved. The upward radiation field exhibits a similar behavior, and the corresponding V point forms at the same point when $s = 0.2432a$, hence a BIC forms here.

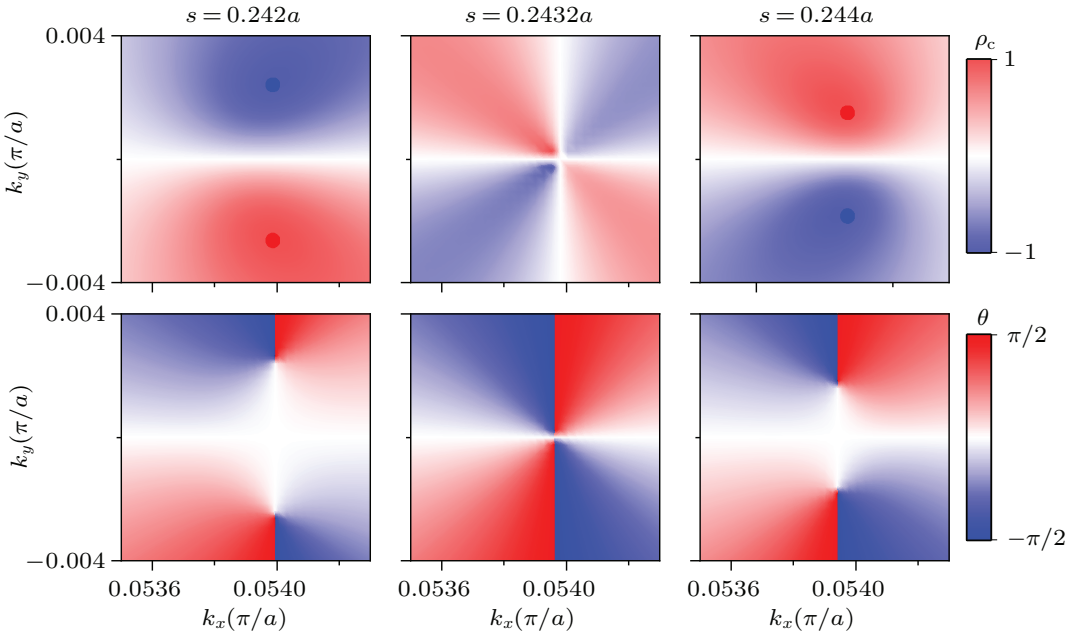


Fig. 2. Evolutions of polarization states near $s = 0.2432a$, with upper panels and lower panels showing the distributions of ellipticity and orientation angle of the polarization ellipses, respectively.

4. Global evolution of polarization singularities

To further trace the whole evolution of these singularities in the momentum space with respect to the parameter s, the projected polarization states in the downward direction with s varying from 0 to $a/2$ are calculated, and the results are shown in Fig. 3. Here the polarization states are represented by the corresponding ellipses. Note that when $|k_y|$ is larger than 0.9, the band diagram extends outside the light cone and the corresponding states are evanescent.

When $s = 0$, due to the up-down mirror symmetry and the in-plane inversion symmetry of the structure, there is a symmetry protected BIC at the Γ point in A band, as mentioned above.

When the two dielectric gratings slightly shift, the mirror

symmetry is broken and avoided crossing happens. The symmetry protected BIC splits into two C points, marked as C_1 and C_2 in the lower band. When s increases to $0.02a$, other two pairs of C points (C_3 and C_4, C_5 and C_6) appear. The pair in the lower band (C_3 and C_4) carries $-1/2$ charges while the pair in the upper band (C_5 and C_6) has $+1/2$ charges. With the further increase of s, the two C points (C_5 and C_6) in the upper band move around the right side, while the two pairs of C points in the lower band (C_1 and C_3, C_2 and C_4) will collide in pair and annihilate when s reaches $0.07a$. When s increases to $0.14a$, other 4 C points (C_7 and C_9, C_8 and C_{10}) emerge in pairs. The C points in each pair have the same handedness while opposite charges. With the further increase of s, C_9 and C_{10} move outwards and eventually go out of the light cone when s reaches $0.18a$, becoming regular bound states. Meanwhile, C_7 and C_8, C_5 and C_6, move toward each other. When s approaches to $0.2432a$, they merge into two BICs located symmetrically on the k_x axis with charges of $+1$. With further increase of s, the two BICs break into two pairs of C points again. When s exceeds about $0.28a$, two C points, C'_9 and C'_{10}, will move into the light cone from outside. However, the handedness is different from that of C_9 and C_{10}. With further increase of s, C_8 and C'_9, C_7 and C'_{10} will collide and annihilate when $s$approaches to $0.40a$, and C_5 and C_6 will move outwards. When s increases to about $0.41a$, two pairs of C points (C'_1 and C'_3, C'_2 and C'_4,) appear in the lower band. When s exceeds $0.48a$, the 4 C points (C'_3 and C'_4 in the lower band, C_5 and C_6 in the upper band) vanish simultaneously. When s approaches to $0.5a$, the voided crossing disappears, and C'_1 and C'_2 merge into a BIC.

As mentioned before, the structure with shift s is mirror symmetric (σ_x) with that of a–s, then the polarization vector $\boldsymbol{d}'(\boldsymbol{k}_\parallel)$ corresponding to s is also mirror symmetric with that for a–s. Hence the polarization topology for structure with $s > a/2$ can be obtained from that of a–s. It worth noting that the radiations on the two sides of the structure are different since the up-down mirror symmetry is broken. However, $\boldsymbol{d}'(\boldsymbol{k}_\parallel)$ in upward radiation can be obtained by inversing that in downward radiation due to the lateral inversion symmetry of the system.[25]

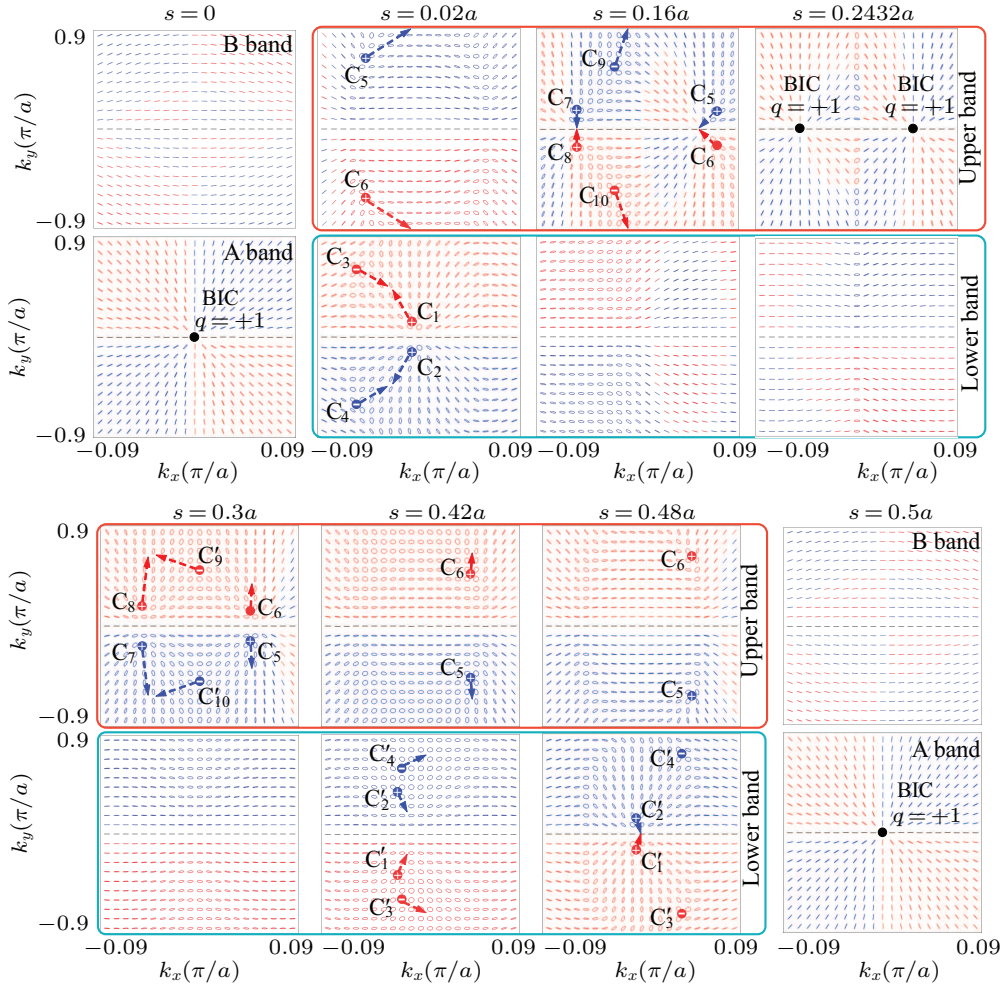


Fig. 3. Evolutions of polarization singularities with geometry variation, with ellipses (dots) in red and blue corresponding to the right-handed and left-handed elliptical (circular) polarizations, respectively, symbols "+" and "−" denoting the sign of the topological charges for C points, and arrows indicating the direction of motion for C points with the increase of s.

From Fig. 3, it can be seen that the C points can be categorized into three types according to their generation mechanisms. One is those generated from the V points (BICs) due to the breaking of the in-plane C_2 symmetry, like C_1 and C_2. The two C points carry the same quantity of charge but opposite handedness, which is reported in other researches.[1,17] The second type, like C_7 and C_9, C_8 and C_{10}, are generated from originally 0-charged polarization states. At the beginning, the paired two C points are very close to each other, and have the same handedness but opposite charges. Such C point pairs have also been reported in Zeng's work.[25] The third type, like C_3, C_4, C_5, and C_6 (or C_3', C_4', C_5, and C_6), which appear or vanish simultaneously, is different from the first two types. If the two C points in the same band are regarded as a pair, they are far apart from each other, opposite to the first two types in which they two C points are located closely when they first appear. If the two C points in different bands but similar in-plane positions are regarded as a pair, like C_3 and C_5 (or C_4 and C_6), both the charge and handedness are different, which is also different from the first two types in which either the charge or the handedness is identical with each other.

The generation of the third type of C points is related to the avoided band crossing. Figure 4(a) shows the band diagram for $s = 0$, where the two bands cross each other. Figures 4(b) and 4(c) are the projected polarization states of the upper band and lower band of the system with $s = 0.02a$, respectively. The black lines are the projected intersection lines of A band and B band in Fig. 4(a). It can be seen that the two C point pairs are both located at the vicinity of the intersection line. From Fig. 3 it can be seen that the radiation field of the A band and the B band are both nearly linearly polarized. Moreover, at the vicinity where C_3, C_4, C_5, and C_6 appear when $s = 0.02a$, the polarizations in bands A and B are nearly perpendicular to each other. These phenomena suggest that the formation of the C points can be understood as the superposition between the polarization states in A band and B band. When avoided crossing happens, the states in A band and B band will mix to form a new band structure. At the vicinity of the crossing, the new formed states can be expressed as[29]

$$|\varphi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|\varphi_{\mathrm{A}}\rangle \pm \mathrm{e}^{\mathrm{i}\phi}|\varphi_{\mathrm{B}}\rangle), \tag{2}$$

with "+" and "−" corresponding to one of the new formed states, and φ is a phase angle dependent on the system parameters. At certain locations φ_{A} and φ_{B} have nearly orthogonal polarizations and equal amplitudes, and if φ approaches to $\pm\pi/2$ for a certain s (for example, $s = 0.02a$ in our system), circular polarization states will form there. According to Eq. (2), if one state is left circularly polarized, the other one must be right circularly polarized. This explains why C_3 and C_5 (C_4 and C_6) have opposite handedness. On the other hand, the conservation of topological charge requires C_3 and C_5 (C_4 and C_6) to carry opposite charges since the total charge is 0 at the vicinity of C_3 and C_5 at the beginning. Meanwhile, the mirror symmetry in the y direction (σ_y) guarantees that C_3 and C_4 (C_5 and C_6) have opposite handedness and same charges. Similarly, the generation of C_3', C_4', C_5, and C_6 when s decreases from $0.5a$ to $0.48a$, as shown in Figs. 4(d)–4(f), can also be explained in this way.

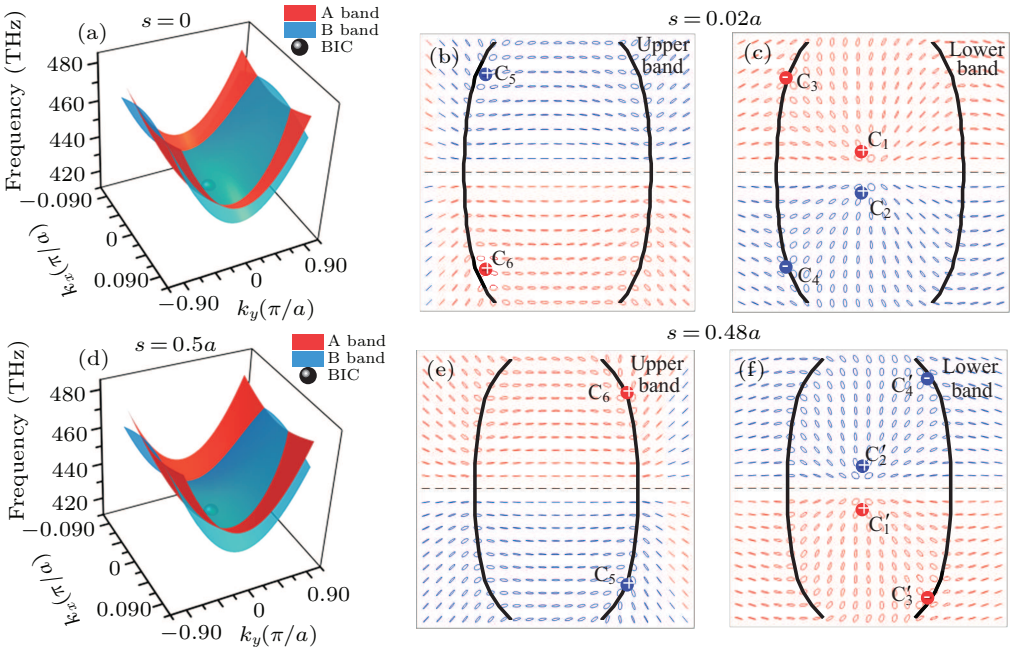


Fig. 4. Generation of C points due to avoided band crossing. (a) and (d) Band diagrams with band crossing at $s = 0$ and $s = 0.5a$, respectively. ((b), (c) and (e), (f)) Projected polarization states of upper and lower bands of the system with $s = 0.02a$ and $s = 0.48a$, respectively. The black lines are the projected intersection lines of A band and B band shown in panels (a) and (d).

5. Discussion

In the end, we make a comparison between the plasmonic system here and the reported dielectric system with same symmetry. According to the reported results,[25,26] the dielectric system exhibits high up-down asymmetry in radiation. However, for the plasmonic system here, the ratio of the Q factor for the upward radiation to downward radiation is less than 8 according to our calculation, which is far less than the reported results. The low asymmetric ratio comes from the low light transmission of the metal film in our system. According to previous work,[25] the bound of the asymmetric ratio f^2 (the ratio of the upward radiation power to the downward radiation power) is given by $(1-t)/(1+t) \leq f^2 \leq (1+t/(1-t)$, where t is the direct transmission through the structure (the Fabry–Perot background). For the dielectric slab, the full transmission ($t = 1$) can be reached and thus extremely high asymmetric ratio can be realized. While for the plasmonic system here, the full transmission cannot be attained and high asymmetric ratio is not exhibited either. Another difference is that accidental BICs at off-Γ points are supported in the plasmonic system while in the dielectric system only the at-Γ BIC is supported. Off-Γ BICs are propagating states without loss and can be applied to on-chip beam guiding[30] and directional vector beam generation.[31]

6. Conclusion and perspectives

In conclusion, we have studied the evolution of the polarization singularities in a periodic plasmonic system with lateral inversion symmetry. In the evolution, BICs can split into C point pairs and conversely, C points can merge into BICs. Meanwhile, two C points with opposite charges can also collide and annihilate. C points are generated in three ways in this system. In addition to the known mechanism reported in other researches, avoided band crossing can also induce C point pairs. The whole process is governed by the law of charge conservation. Unlike the reported dielectric system in which only at-Γ BIC is supported, off-Γ BICs are observed in our plasmonic system. This work proposes a new scheme for generating the polarization singularities and may find its applications in polarization manipulation.

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References

[1] Ye W, Gao Y and Liu J 2020 *Phys. Rev. Lett.* **124** 153904
[2] Yoda T and Notomi M 2020 *Phys. Rev. Lett.* **125** 053902
[3] Zhen B, Hsu C W, Lu L, Stone A D and Soljacic M 2014 *Phys. Rev. Lett.* **113** 257401
[4] Hsu C W, Zhen B, Stone A D, Joannopoulos J D and Soljačić M 2016 *Nat. Rev. Mater.* **1** 16048
[5] Dai S, Liu L, Han D and Zi J 2018 *Phys. Rev. B* **98** 081405
[6] Yuan L and Lu Y Y 2018 *Phys. Rev. A* **97** 043828
[7] Chen W, Chen Y and Liu W 2019 *Phys. Rev. Lett.* **122** 153907
[8] Song Q J, Hu J S, Dai S W, Zheng C X, Han D Z, Zi J, Zhang Z Q and Chan C T 2020 *Sci. Adv.* **6** eabc1160
[9] Sadreev A F 2021 *Rep. Prog. Phys.* **84** 055901
[10] Yuan L, Luo X and Lu Y Y 2021 *Phys. Rev. A* **104** 023521
[11] Kodigala A, Lepetit T, Gu Q, Bahari B, Fainman Y and Kante B 2017 *Nature* **541** 196
[12] Huang C, Zhang C, Xiao S M, Wang Y H, Fan Y B, Liu Y L, Zhang N, Qu G Y, Ji H J, Han J C, Ge L, Kivshar Y and Song Q H 2020 *Science* **367** 1018
[13] Yesilkoy F, Arvelo E R, Jahani Y, Liu M, Tittl A, Cevher V, Kivshar Y and Altug H 2019 *Nat. Photon.* **13** 390
[14] Melik-Gaykazyan E, Koshelev K, Choi J H, Kruk S S, Bogdanov A, Park H G and Kivshar Y 2021 *Nano Lett.* **21** 1765
[15] Koshelev K, Lepeshov S, Liu M, Bogdanov A and Kivshar Y 2018 *Phys. Rev. Lett.* **121** 193903
[16] Wang H F, Gupta S K, Zhu X Y, Lu M H, Liu X P and Chen Y F 2018 *Phys. Rev. B* **98** 214101
[17] Liu W, Wang B, Zhang Y, Wang J, Zhao M, Guan F, Liu X, Shi L and Zi J 2019 *Phys. Rev. Lett.* **123** 116104
[18] Mermet-Lyaudoz R, Dubois F, Hoang N V, Drouard E and Hai S N 2019 arXiv: 1905.03868 [physics.optics]
[19] Lee S G, Kim S H and Kee C S 2020 *Nanophotonics* **9** 4373
[20] Ovcharenko A I, Blanchard C, Hugonin J P and Sauvan C 2020 *Phys. Rev. B* **101** 155303
[21] Yin X, Jin J, Soljacic M, Peng C and Zhen B 2020 *Nature* **580** 467
[22] Głowadzka W, Wasiak M and Czyszanowski T 2021 *Nanophotonics* **10** 3979
[23] Han S, Pitchappa P, Wang W, Srivastava Y K, Rybin M V and Singh R 2021 *Adv. Opt. Mater.* **9** 2002001
[24] Overvig A, Yu N and Alu A 2021 *Phys. Rev. Lett.* **126** 073001
[25] Zeng Y, Hu G, Liu K, Tang Z and Qiu C W 2021 *Phys. Rev. Lett.* **127** 176101
[26] Zhou H, Zhen B, Hsu C W, Miller O D, Johnson S G, Joannopoulos J D and Soljačić M 2016 *Optica* **3** 001079
[27] Meng Y, Zhang R Y, Zhang Q, Liu Z, Wu X, Xiao J, Xiang H, Han D and Wen W 2017 *J. Phys. D: Appl. Phys.* **50** 485101
[28] Doeleman H M, Monticone F, den Hollander W, Alù A and Koenderink A F 2018 *Nat. Photon.* **12** 397
[29] Cohen-Tannaoudji C, Diu B and Laloe F 1992 *Quantum Mechanics* (New York: Wiley-VCH) pp. 410–412
[30] Lin Y, Feng T, Lan S, Liu J and Xu Y 2020 *Phys. Rev. Appl.* **13** 064032
[31] Kang M, Zhang S, Xiao M and Xu H 2021 *Phys. Rev. Lett.* **126** 117402