Correction of intense laser–plasma interactions by QED vacuum polarization in collision of laser beams

Wen-Bo Chen(陈文博)[1,†] and Zhi-Gang Bu(步志刚)[2]

[1] *Department of Physics, Shanghai University, Shanghai* 200444, *China*

[2] *State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai* 201800, *China*

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The influence of vacuum polarization effects on the interactions of multiple ultra-intense lasers with plasmas is discussed. The nonlinear paraxial monochromatic model of the interactions has been improved by considering the Heisenberg–Euler Lagrangian density of two laser processes. Comparing the corrections of vacuum polarization effects in the collision of laser beams with one generated by a single intense laser, we find that the former has a higher order of magnitude correction. The laser collision also produces variations in the propagation direction and polarization direction of the lasers propagating in the plasma. In addition, the strong-field quantum electrodynamic (QED) effects can be enhanced by increasing the laser intensity or frequency difference, or by adjusting the incident angles of the two laser beams.

Keywords: vacuum-polarization effects, extreme intensity laser, laser–plasma interactions

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1. Introduction

The intensity of the laser source has been one of the most important issues in laser–plasma interactions. When the intensity of a focused laser reaches or exceeds $10^{18}\ (1\ \mu\mathrm{m}/\lambda)^2\mathrm{W/cm}^2$, the perturbation method becomes inapplicable and nonlinear theory of laser–plasma interactions is necessary.[1] Further, when the intensity of the laser exceeds $10^{23}\ (1\ \mu\mathrm{m}/\lambda)^2\mathrm{W/cm}^2$, plasmas in the interactions are ultra-relativistic and the physics of relativistic plasmas are strongly affected by strong-field quantum electrodynamic (QED) processes.[2] This coupling of quantum processes and relativistic collective particle dynamics can result in significantly new physics phenomena.[3,4] In this situation, the behavior of lasers with extreme intensity also becomes notably different from that of lasers with lower intensity in plasma. In particular, vacuum polarization effects cause the laser to propagate in a vacuum as if it is in nonlinear media. Thus, vacuum polarization effects must be considered when studying the relevant interaction processes.

A large number of laser–plasma interaction processes, such as inertial confinement fusion (ICF), star surface plasma layer, etc., usually are multi-light processes. An ideal single laser source propagating in plasma is almost non-existent. This makes it necessary to study the QED vacuum polarization effects induced by multiple lasers and their laser–plasma interaction processes. When there are multiple extreme intensity lasers propagating through the vacuum, the multiphoton processes of strong-field QED such as photon–photon scattering effects become important.[5–7] Most of the QED vacuum polarization effects can be attributed to the results of these processes.[8] We know there is no direct interaction between light and light propagating in a vacuum in classical electrodynamics. However, in the overlapping area of two light beams, interference appears due to the wave character of light; once they leave the overlapping area, the two light beams are no longer affected by each other and their propagation properties are unchanged. The two beams of light do not scatter in the overlapping region, and no energy or momentum are exchanged. However, the existence of QED vacuum polarization effects changes this classical description. In quantum theory, the multiphoton processes are a nonlinear interaction with the electron–positron pairs of vacuum fluctuations acting as an intermediate "media". In contrast with the well-known Faraday or Cotton–Mouton effect, which is essentially the (circular) birefringence phenomenon of light under different polarization states in a material, induced by the projection of a magnetic field along or perpendicular the propagation direction of the light and caused by the motion of the charged particles in this material under the Lorentz forces of the external magnetic field, the process of QED vacuum polarization is a photon transferring a portion of its energy and momentum to another by interacting with a virtual particle pair in the quantum vacuum, causing scattering effects.[9] This effect of extreme intensity light does not necessarily require an additional field and is typically nonlinear and dissipative. Research on multiphoton processes in strong-field QED has led to some progress in theory and experiments.[10–14] In current experimental conditions, the observation of these QED processes has not yet been implemented directly.[15] However, with the completion of infrastructures such as Extreme Light Infrastructure (ELI), Appollon and Shanghai Superintense Ultrafast Laser Facility

†Corresponding author. E-mail: d_cwb@shu.edu.cn

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(SULF), it is believed that there will be a breakthrough in the near future.

On the other hand, special media such as plasmas can effectively enhance QED vacuum polarization effects. When strong intensity lasers are injected into the plasma, plasma fields due to the interactions between the lasers and collective effects of electrons in plasma are generated. If the intensity of the incident laser is ultra-high and the number density of the plasma is large enough, the coupling effect of vacuum polarization and plasma fields becomes unable to be neglected. Considering the effects of vacuum polarization in an ultra-intense laser and plasma interaction is necessary. Such research includes Lundin *et al.* who studied the plasma dispersion relation,[16,17] Bu and Ji who studied the correction of the laser pulse frequency shift,[18] and our previous paper which investigated the correction of the nonlinear interaction.[19] These works show that plasmas are useful for studying QED vacuum polarization effects. Specific to the work related to multiple extreme intensity laser QED vacuum effects in plasmas, there are also Roberts's research on x-ray free electron laser,[20] Shen *et al.*'s study of vacuum birefringence,[21] and Koga *et al.*'s work on the use of relativistic flying mirrors to study photon–photon scattering effect observability,[22] etc.

In this paper, the laser field dynamics equation including the QED vacuum polarization in collision of laser beams is discussed in detail. Combined with the hydrodynamic equation of a plasma, nonlinear coupled equations with QED vacuum correction are obtained. The corrections of vacuum polarization from colliding lasers has a larger correction magnitude and is more varied than that from the single laser source condition. It also causes the laser propagation direction and the polarization direction to produce components that are different from the original direction. In laser–plasma interactions, the collision process not only changes the properties of the laser's direction of propagation, polarization, etc. after the overlapping region, but also causes a corresponding change in the laser-driven plasma wave (the laser wakefield). Detecting various changes of lasers in the plasma and the plasma waves may provide a good solution for observing the vacuum polarization effects in experiments.

2. Laser field equations with QED vacuum polarization in collision of laser beams

The nonlinear effects of a QED vacuum can be described by the Heisenberg–Euler (HE) Lagrangian density. Thus, the Lagrangian density of electromagnetic fields with nonlinear QED vacuum polarization effects can be written as[23]

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_0 + \delta\mathcal{L}_{\rm HE} - \frac{1}{c}j^\mu A_\mu,\\ &= -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{128\pi}\left[4(F_{\mu\nu}F^{\mu\nu})^2 + 7(F_{\mu\nu}\hat{F}^{\mu\nu})^2\right]\\ &\quad - \frac{1}{c}j^\mu A_\mu, \end{aligned} \tag{1}$$

where $\mathcal{L}_0$ is the classical electromagnetic field Lagrangian density, and $\delta\mathcal{L}_{\rm HE}$, the HE Lagrangian density, is composed of two Lorentz invariants, $F_{\mu\nu}F^{\mu\nu} = -2(\boldsymbol{E}^2 - \boldsymbol{B}^2) \equiv -4\mathcal{F}$, $F_{\mu\nu}\hat{F}^{\mu\nu} = -4\boldsymbol{E}\cdot\boldsymbol{B} \equiv -4\mathcal{G}$. Here, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the four-dimensional electromagnetic field tensor, $\hat{F}^{\mu\nu} = \varepsilon^{\mu\nu\lambda\tau}F_{\lambda\tau}/2$, j^μ is the four-dimensional current density, and A^μ is the four-dimensional electric vector potential. The parameter $\kappa = \alpha^2\hbar^3/45\pi m_{\rm e}^4 c^5$ gives the nonlinear HE coupling, where $\alpha = e^2/\hbar c \approx 1/137$ is the fine-structure constant.

Using the Euler–Lagrange equation to obtain the Maxwell equations with the quantum vacuum effects,

$$\partial_\nu F^{\nu\mu} + \kappa\partial_\nu(4\mathcal{F}F^{\nu\mu} + 7\mathcal{G}\hat{F}^{\nu\mu}) = \frac{4\pi}{c}j^\mu, \tag{2}$$

which is

$$\begin{aligned}&\nabla\cdot\boldsymbol{E} = -4\pi e\delta n - 4\pi\rho_{\rm vac},\\ &\left(-\frac{1}{c}\partial_t\boldsymbol{E} + \nabla\times\boldsymbol{B}\right) = -\frac{4\pi}{c}en\boldsymbol{v} + \frac{4\pi}{c}\boldsymbol{j}_{\rm vac},\end{aligned} \tag{3}$$

where $\delta n = n - n_0$ is the density perturbation of electrons, n_0 is the ambient plasma density, and $\boldsymbol{v}$ is the electron fluid velocity. In our model, the ions are considered as stationary, so the effect of the mass and charge of the ions is not accounted for. Equation (3) demonstrates the vacuum polarization effects with effective vacuum "sources", vacuum "charge" $\rho_{\rm vac}$ and vacuum "current" $\boldsymbol{j}_{\rm vac}$,

$$\begin{aligned}\rho_{\rm vac} &= \nabla\cdot\frac{\kappa}{4\pi}(4\mathcal{F}\boldsymbol{E} + 7\mathcal{G}\boldsymbol{B}),\\ \boldsymbol{j}_{\rm vac} &= \partial_t\frac{\kappa}{4\pi}(4\mathcal{F}\boldsymbol{E} + 7\mathcal{G}\boldsymbol{B}) - c\nabla\times\frac{\kappa}{4\pi}(4\mathcal{F}\boldsymbol{B} - 7\mathcal{G}\boldsymbol{E}).\end{aligned} \tag{4}$$

The electromagnetic field takes the Coulomb gauge $\nabla\cdot\boldsymbol{A} = 0$, where $\boldsymbol{A}$ is the vector electromagnetic potential. Considering that the electromagnetic field is composed of two lasers, we obtain the relations $\nabla = \boldsymbol{e}_{z_1}\partial_{z_1} + \nabla_{\perp_1} + \boldsymbol{e}_{z_2}\partial_{z_2} + \nabla_{\perp_2}$; $\boldsymbol{A} = \boldsymbol{A}_{z_1} + \boldsymbol{A}_{\perp_1} + \boldsymbol{A}_{z_2} + \boldsymbol{A}_{\perp_2}$ and $\Phi = \Phi_1 + \Phi_2$, where Φ is the scalar electromagnetic potential, laser 1 propagates along the z_1 axis direction and laser 2 along the z_2 axis direction. In the case of neglecting the effects of self-focusing and diffraction of each laser beam, the propagation of these two lasers can be described in a paraxial monochromatic model, $\nabla \to \boldsymbol{e}_{z_1}\partial_{z_1} + \boldsymbol{e}_{z_2}\partial_{z_2} = \boldsymbol{e}_{z_1}\partial/\partial z_1 + \boldsymbol{e}_{z_2}\partial/\partial z_2$. Considering intense lasers with large amplitude, the transverse component of vector potentials, perpendicular to the z_1 axis and z_2 axis direction respectively, is much larger than its longitudinal component, $|\boldsymbol{A}_{z_1}| \ll |\boldsymbol{A}_{\perp_1}|$, $\boldsymbol{A}_1 \approx \boldsymbol{A}_{\perp_1} = A\boldsymbol{e}_{\perp_1}$; $|\boldsymbol{A}_{z_2}| \ll |\boldsymbol{A}_{\perp_2}|$, $\boldsymbol{A}_2 \approx \boldsymbol{A}_{\perp_2} = A\boldsymbol{e}_{\perp_2}$.

In our electromagnetic field model, the initial electromagnetic potentials of laser beams can be described as $\boldsymbol{A}_l(z_l,t) = \boldsymbol{A}_{l_0}(z_l,t)\,{\rm e}^{{\rm i}\Psi_l}$ and $\Phi_l(z_l,t) = \Phi_{l_0}(z_l,t)\,{\rm e}^{{\rm i}\Psi_l}$, where $\mathcal{A}_{l_0}(\boldsymbol{A}_{l_0},\Phi_{l_0})$ and $\Psi_l = -\omega_l t + k_l z_l + \psi_l$ are the amplitude and phase of the electromagnetic potential of the two lasers

($l=1,2$). During the process of the laser beams propagating in media, the beams' amplitude and phase will be perturbed. In the calculation of vacuum "sources" $\rho_{\rm vac}$ and $\boldsymbol{j}_{\rm vac}$ from Eq. (4), only the perturbations of their amplitude $\delta\mathcal{A}_l(\delta A_l,\delta\Phi_l)$ and phase $\delta\psi_l$ induced by the vacuum polarization effects need to be considered, and they are small-varying, i.e., $|\delta\mathcal{A}_l|\ll|\mathcal{A}_{l_0}|$, $|\delta\psi_l|\ll|\psi_l|$, ($l=1,2$). The electromagnetic potentials in Eq. (4) can be written as $\mathcal{A}_l[\boldsymbol{A}_l(z_l,t),\Phi_l(z_l,t)]=[\mathcal{A}_{l_0}+\delta\mathcal{A}_l(z_l,t)]\,{\rm e}^{{\rm i}[\Psi_l(z_l,t)+\delta\psi_l(z_l,t)]}$. Therefore, in the process of calculating the corrections of the vacuum polarization effects in Eq. (4), $\partial_i\mathcal{A}_l={\rm i}\partial_i(-\omega_l t+k_l z_l)\mathcal{A}_l+\mathcal{O}(z_l,t)$, where $\mathcal{O}(z_l,t)={\rm i}\partial_i(\delta\psi_l)\mathcal{A}_l+\partial_i(\delta\mathcal{A}_l)\,{\rm e}^{{\rm i}(\Psi_l+\delta\psi_l)}$ is the higher-order derivative. Only the first-order derivative is needed to obtain approximate relations $(\partial_iA_l)(\partial_jA_l)\approx(\partial_i\partial_jA_l)A_l$, $(\partial_i\Phi_l)(\partial_j\Phi_l)\approx(\partial_i\partial_j\Phi_l)\Phi_l$, $(\partial_iA_l)(\partial_j\Phi_l)\approx(\partial_i\partial_jA_l)\Phi_l\approx(\partial_i\partial_j\Phi_l)A_l\approx(\partial_i\Phi_l)(\partial_jA_l)$, where $i,j=t,z_l$, $l=1,2$; and $\partial_{i_1}A_1/\partial_{i_2}A_2\sim\Delta\varpi\Delta A$, $\partial^2_{i_1}A_1/\partial^2_{i_2}A_2\sim\Delta\varpi^2\Delta A$, $\partial_{i_1}\Phi_1/\partial_{i_2}\Phi_2\sim\Delta\varpi\Delta A$, $\partial^2_{i_1}\Phi_1/\partial^2_{i_2}\Phi_2\sim\Delta\varpi^2\Delta A$, where $\Delta\varpi\approx\omega_1/\omega_2$, $\Delta A\approx A_{1_0}/A_{2_0}$, $i=t,z$. For laser 1, the base vector representing the direction of its wave vector can be written as $\boldsymbol{e}_{z_1}=o_1\boldsymbol{e}_{z_2}+p_1\boldsymbol{e}_{\perp_2}+q_1\boldsymbol{e}_{z_2}\times\boldsymbol{e}_{\perp_2}$, and the base vector representing the direction of its vector potential can also be written as $\boldsymbol{e}_{\perp_1}=o_2\boldsymbol{e}_{z_2}+p_2\boldsymbol{e}_{\perp_2}+q_2\boldsymbol{e}_{z_2}\times\boldsymbol{e}_{\perp_2}$, $o_i^2+p_i^2+q_i^2=1$, $o_io_j+p_ip_j+q_iq_j=0$, $i,j=1,2$. The relations $o_1=\boldsymbol{e}_{z_1}\cdot\boldsymbol{e}_{z_2}$, $p_1=\boldsymbol{e}_{z_1}\cdot\boldsymbol{e}_{\perp_2}$, $o_2=\boldsymbol{e}_{\perp_1}\cdot\boldsymbol{e}_{z_2}$, $p_2=\boldsymbol{e}_{\perp_1}\cdot\boldsymbol{e}_{\perp_2}$ are the angle cosines between wave vectors and the vector potential of the two lasers. It is assumed that the frequency of laser 1 is much higher than that of laser 2 to obey the following relations, $\Delta\varpi\gg1$, $\Delta\varpi\Delta A\gg1$, so that the effects of laser 2 on laser 1 can be neglected. Therefore, we only need to calculate the wave equation of laser 2 during the propagation process. In the following calculation, the lower corner "2" can be omitted, $A_2\sim A$, $\Phi_2\sim\Phi$, $z_2\sim z$, $\perp_2\sim\perp$, $n_2\sim n$. Considering that the HE coupling coefficient κ is very small (κ about 10^{-32} cm·s^2·g^{-1}), in the case of the laser field being much smaller than the critical field $E_{\rm cr}$, the correction of vacuum polarization expressed in the form of vacuum "sources" can be written as the perturbation in terms of κ, and the high-order terms of κ can be omitted. The vacuum "sources" of laser 2 from Eq. (4) can be expanded into

$$\begin{aligned}\hat{\rho}_{\rm vac}&\approx\hat{\rho}_{{\rm vac}_0}+\delta\hat{\rho}_{\rm vac}+\delta\hat{\rho}_{{\rm vac}_{\rm G}},\\ \hat{\boldsymbol{j}}_{\rm vac}&\approx\hat{\boldsymbol{j}}_{{\rm vac}_0}+\delta\hat{\boldsymbol{j}}_{\rm vac}+\delta\hat{\boldsymbol{j}}_{{\rm vac}_{\rm G}},\end{aligned}\tag{5}$$

where $\hat{\rho}_{\rm vac}\equiv\left(e/m_{\rm e}c^2\right)\rho_{\rm vac}$ is the normalized vacuum charge and $\hat{\boldsymbol{j}}_{\rm vac}\equiv\left(e/m_{\rm e}c^2\right)\boldsymbol{j}_{\rm vac}$ is the normalized vacuum current,

$$\begin{aligned}\hat{\rho}_{{\rm vac}_0}&=-6\kappa e^2n_0^2(4\pi)F_0\delta\hat{n},\\ \delta\hat{\rho}_{\rm vac}&=-2\kappa e^2n_0^2(4\pi)(F_1+F_2o_1)\delta\hat{n}\\&\quad+2\kappa e^2n_0^2(4\pi)F_2p_1\hat{n}\beta_z\frac{a}{\phi},\\ \delta\hat{\rho}_{{\rm vac}_{\rm G}}&=7\kappa e^2n_0^2(4\pi)Gq_1\delta\hat{n}\frac{a}{\phi},\\ \hat{\boldsymbol{j}}_{{\rm vac}_0}&=6\kappa ce^2n_0^2(4\pi)F_0\hat{n}(\beta_z\boldsymbol{e}_z+\beta_\perp\boldsymbol{e}_\perp),\\ \delta\hat{\boldsymbol{j}}_{\rm vac}&=2\kappa ce^2n_0^2(4\pi)(F_1+F_2)\hat{n}(\beta_z\boldsymbol{e}_z+\beta_\perp\boldsymbol{e}_\perp)\\&\quad-2\kappa ce^2n_0^2(4\pi)F_2\delta\hat{n}\frac{a}{\phi}\left[p_1\boldsymbol{e}_z+(1-o_1)\boldsymbol{e}_\perp\right],\\ \delta\hat{\boldsymbol{j}}_{{\rm vac}_{\rm G}}&=-7\kappa ce^2n_0^2(4\pi)G\Big\{\delta\hat{n}(q_1\boldsymbol{e}_\perp-p_1\boldsymbol{e}_z\times\boldsymbol{e}_\perp)\\&\quad+\hat{n}\beta_z\frac{a}{\phi}[q_1\boldsymbol{e}_z+(1-o_1)\boldsymbol{e}_z\times\boldsymbol{e}_\perp]\Big\},\end{aligned}\tag{6}$$

and the coefficients F_1, F_2 in Eq. (6) are

$$\begin{aligned}F_1&\approx\Delta\varpi^2\Delta A^2F_0+2\delta F,\\ F_2&\approx\Delta\varpi(2\Delta\varpi^2\Delta A^2F_0+\delta F).\end{aligned}\tag{7}$$

Here,

$$\begin{aligned}F_0&\approx\delta\hat{n}\phi-\hat{n}\beta_\perp a,\\ \delta F&\approx2\Delta\varpi\Delta A\Big\{\delta\hat{n}\left[o_1\phi+(p_2-p_2o_1+p_1o_2)\frac{a^2}{\phi}\right]\\&\quad-\hat{n}\left[(p_1+o_2)\beta_z+p_2\beta_\perp\right]a\Big\},\\ G&\approx-\Delta\varpi\Delta A\Big\{\delta\hat{n}\left[q_1+(p_1q_2-p_2q_1)\right]\\&\quad-\hat{n}\left[q_2+(q_1o_2-q_2o_1)\right]\beta_z\frac{a}{\phi}\Big\}a,\end{aligned}\tag{8}$$

where $\phi\equiv(e/m_{\rm e}c^2)\Phi$ and $a\equiv(e/m_{\rm e}c^2)A$ are the normalized scalar and vector electromagnetic potentials, $\delta\hat{n}=\delta n/n_0$, $\hat{n}=n/n_0$, $\beta_\perp=v_\perp/c$, $\beta_z=v_z/c$. New components of the wave vector $\delta\boldsymbol{k}_{\rm G}$, along the direction of $q_1\boldsymbol{e}_\perp-p_1\boldsymbol{e}_z\times\boldsymbol{e}_\perp$, and new components of vector potential $\delta\boldsymbol{A}_{\rm G}$, along $q_1\boldsymbol{e}_z+(1-o_1)\boldsymbol{e}_z\times\boldsymbol{e}_\perp$, produced by one of the vacuum "sources" $\delta\hat{\rho}_{{\rm vac}_{\rm G}}$ and $\delta\hat{\boldsymbol{j}}_{{\rm vac}_{\rm G}}$ are in a different direction from their original $\boldsymbol{k}$, along the z axis, and $\boldsymbol{A}$, along the $\perp$ axis. If we only focus on the variations of field equations along the original $\boldsymbol{k}$ and $\boldsymbol{A}$ direction of laser 2, we can neglect the influence of $\delta\hat{\rho}_{{\rm vac}_{\rm G}}$ and $\delta\hat{\boldsymbol{j}}_{{\rm vac}_{\rm G}}$, i.e., assume $G\to0$, $\delta\hat{\rho}_{{\rm vac}_{\rm G}}\approx0$ and $\delta\hat{\boldsymbol{j}}_{{\rm vac}_{\rm G}}\approx0$. Ignoring the new components $\delta\boldsymbol{k}_{\rm G}$ and $\delta\boldsymbol{A}_{\rm G}$ in the wave equations, Eq. (3) can approximate the wave equations with the vacuum polarization effects describing laser 2. Its expressions are

$$\begin{aligned}\partial_z^2\phi&=k_{\rm p}^2\delta\hat{n}-2\kappa e^2n_0^2(4\pi)^2\\&\quad\times\left[(3F_0+F_1+F_2o_1)\delta\hat{n}-F_2p_1\hat{n}\beta_z\frac{a}{\phi}\right],\\ \frac{1}{c}\partial_t\partial_z\phi&=-k_{\rm p}^2\hat{n}\beta_z+2\kappa e^2n_0^2(4\pi)^2\\&\quad\times\left[(3F_0+F_1+F_2)\hat{n}\beta_z-F_2p_1\delta\hat{n}\frac{a}{\phi}\right],\\ (\partial_z^2-\frac{1}{c^2}\partial_t^2)a&=k_{\rm p}^2\hat{n}\beta_\perp-2\kappa e^2n_0^2(4\pi)^2\\&\quad\times\left[(3F_0+F_1+F_2)\hat{n}\beta_\perp-F_2(1-o_1)\delta\hat{n}\frac{a}{\phi}\right],\end{aligned}\tag{9}$$

where $k_{\rm p}^2=\omega_{\rm p}^2/c^2=4\pi e^2n_0/m_{\rm e}c^2$ is the plasma wave number.

3. Response of multi-light vacuum polarization in nonlinear intense laser field–plasma interactions

A new nonlinear coupling model of colliding extreme intensity lasers is given via combining the intense laser–plasma interaction theory with the wave equations including the quantum vacuum effects. The plasma wave (laser wakefield) generated by the laser in the plasma is described by the multicomponent fluid plasma model. The hydrodynamic equations consist of a continuity equation and a momentum equation,

$$\partial_t n + \nabla\cdot(n\boldsymbol{v}) = 0,$$
$$\partial_t \boldsymbol{p} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{p} = -e\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right),$$

where $\boldsymbol{p} = m\boldsymbol{v}\gamma$ is the electron momentum, $\gamma = (1-\boldsymbol{\beta}^2)^{-1/2}$ and $\boldsymbol{\beta} = \boldsymbol{v}/c$ is the relativistic factor. The laser-induced plasma waves described by the wave equation (9) are also valid under the near-axis approximation. The movement of free electrons in the plasma wave can be decomposed into transverse rapid quiver motion and longitudinal drift motion, $\boldsymbol{\beta} = \beta_\perp \boldsymbol{e}_\perp + \beta_z \boldsymbol{e}_z$. Hence, the momentum equation is divided into two components, transverse and longitudinal. The transverse part of the momentum equation, $\frac{1}{c}\partial_t(\gamma\beta_\perp) + \beta_z\partial_z(\gamma\beta_\perp) = \frac{1}{c}\partial_t a + \beta_z(\partial_z a)$, can easily be obtained in terms of the normalized vector potential $a = \gamma\beta_\perp$ and the relativistic factor $\gamma^2 = (1-\beta_z^2-\beta_\perp^2)^{-1} = (1+a^2)/(1-\beta_z^2)$. When the intensity of the injected laser is far beyond a certain value (for a 1 μm wavelength laser this value is 10^{18} W/cm^2), i.e., $a \gg 1$, the electrons in the laser wakefield become relativistic. The relation between the longitudinal drift velocity and the transverse quiver velocity of electrons in the laser wakefield is $\beta_\perp \ll \beta_z \sim 1$. With conversion to speed of light coordinates, $\xi = z - ct$, $\tau = t$, the hydrodynamic equation describing the plasma wave can be written as

$$\frac{1}{c}\partial_\tau n = \partial_\xi[(n(1-\beta_z)],$$
$$\partial_\xi[\gamma(1-\beta_z)-\phi] = -\frac{1}{c}\partial_\tau(\gamma\beta_z). \tag{10}$$

The dynamic equations of the laser, Eq. (9), are rewritten as

$$\begin{aligned}
\partial_\xi^2\phi &= k_\mathrm{p}^2\delta\hat{n} - 2\kappa e^2 n_0^2(4\pi)^2 \\
&\quad\times\left[(3F_0+F_1+F_2o_1)\delta\hat{n} - F_2p_1\hat{n}\beta_z\frac{a}{\phi}\right], \\
\left(\frac{1}{c}\partial_\xi\partial_\tau - \partial_\xi^2\right)\phi &= -k_\mathrm{p}^2\hat{n}\beta_z + 2\kappa e^2 n_0^2(4\pi)^2 \\
&\quad\times\left[(3F_0+F_1+F_2)\hat{n}\beta_z - F_2p_1\delta\hat{n}\frac{a}{\phi}\right], \\
\left(\frac{2}{c}\partial_\xi\partial_\tau - \frac{1}{c^2}\partial_\tau^2\right)a &= k_\mathrm{p}^2\frac{\hat{n}}{\gamma}a - 2\kappa e^2 n_0^2(4\pi)^2 \\
&\quad\times\left[(3F_0+F_1+F_2)\hat{n}\frac{\hat{n}}{\gamma}a - F_2(1-o_1)\delta\hat{n}\frac{a}{\phi}\right].
\end{aligned} \tag{11}$$

Assuming the laser frequency is much higher than the plasma frequency, $\omega \gg \omega_\mathrm{p}$, one has a quasistatic approximation, $|\frac{1}{c}\partial_\tau\int_\xi^0 n\,\mathrm{d}\xi'| \ll n_0$, $|\frac{1}{c}\partial_\tau\int_\xi^0(\gamma\beta_z)\,\mathrm{d}\xi'| \ll 1$, so that the hydrodynamic equation (10) can be further written as

$$\begin{aligned}
\delta\hat{n} &= \frac{1}{2}\left[\frac{1+a^2}{(1+\phi)^2} - 1\right], \\
\beta_z &= \frac{1+a^2-(1+\phi)^2}{1+a^2+(1+\phi)^2}, \\
\gamma &= \frac{1+a^2+(1+\phi)^2}{2(1+\phi)}.
\end{aligned} \tag{12}$$

Thus, the coupled field equation (11) can be simplified as

$$\partial_\xi^2\phi = \frac{k_\mathrm{p}^2}{2}(1-\eta_{\mathrm{vac}_0}-\eta_{\mathrm{vac}_1})\left[\frac{1+a^2}{(1+\phi)^2}-1\right], \tag{13a}$$

$$\frac{1}{c}\partial_\xi\partial_\tau\phi = \frac{k_\mathrm{p}^2}{2}\eta_{\mathrm{vac}_2}\left[\frac{1+a^2}{(1+\phi)^2}-1\right], \tag{13b}$$

$$\left(\frac{2}{c}\partial_\xi\partial_\tau - \frac{1}{c^2}\partial_\tau^2\right)a = k_\mathrm{p}^2(1-\eta_{\mathrm{vac}_0}-\eta_{\mathrm{vac}_3})\frac{a}{1+\phi}, \tag{13c}$$

with

$$\begin{aligned}
\eta_{\mathrm{vac}_0} &= \Lambda^3 n_0 3F_0, \\
\eta_{\mathrm{vac}_1} &= \Lambda^3 n_0\left[F_1 + F_2(o_1 - p_1\frac{a}{\phi})\right], \\
\eta_{\mathrm{vac}_2} &= \Lambda^3 n_0 F_2(1-o_1), \\
\eta_{\mathrm{vac}_3} &= \Lambda^3 n_0\left\{F_1 + F_2\left[1-(1-o_1)\frac{a^2-(2+\phi)\phi}{2(1+\phi)\phi}\right]\right\},
\end{aligned} \tag{14}$$

where the coefficients F_1, F_2 are still represented by Eq. (7). Note that

$$F_0 = -\frac{(a^2+\phi^2)(2+\phi)}{2(1+\phi)^2},$$
$$\begin{aligned}
\delta F = \Delta\varpi\Delta A\Bigg\{&\left[o_1\phi - (p_1+o_2)a + (p_2 - p_2o_1 + p_1o_2)\frac{a^2}{\phi}\right] \\
&\times\left[\frac{1+a^2}{(1+\phi)^2}-1\right] - p_2\frac{2a^2}{1+\phi}\Bigg\},
\end{aligned}$$

$\Lambda^3 = (8/45)\alpha^2\lambda_\mathrm{C}^3 \approx 5.45\times10^{-37}$ cm^3 is the coupling coefficient of vacuum polarization. In Eq. (14), η_{vac_0}, η_{vac_1}, η_{vac_2}, and η_{vac_3} are the correction rates of QED vacuum polarization effects. If there is no contribution of vacuum polarization effects, i.e., $F_0 \sim F_1 \sim F_2 \to 0$, and $\eta_{\mathrm{vac}_0} \sim \eta_{\mathrm{vac}_1} \sim \eta_{\mathrm{vac}_2} \sim \eta_{\mathrm{vac}_3} \sim 0$, Eq. (13) are reduced to the classical nonlinear equations. If there is only a single laser source, i.e., $F_1 \sim F_2 \to 0$, and $\eta_{\mathrm{vac}_1} \sim \eta_{\mathrm{vac}_2} \sim \eta_{vac_3} \sim 0$, equation (13) is reduced to the form without collision corrections which is given in Ref. [19].

4. Discussion and conclusion

In this paper, a new nonlinear paraxial monochromatic coupling model is given, which can deal with the contribution of multi-light vacuum polarization effects in a intense laser–plasma interaction. This model allows us to study the physical

conditions of multi-beam ultra-intense laser propagation in a laser–plasma environment.

The HE correction rate, $\eta_{\mathrm{vac}_0} = \Lambda^3 n_0 3F_0$, corresponds to the modification of the coupled field caused by the vacuum "sources" $\hat{\rho}_{\mathrm{vac}_0}$ and $\hat{j}_{\mathrm{vac}_0}$ in Eq. (5), which are generated from the extremely high intensity of the laser itself. Assuming the nonlinear condition, i.e., the amplitude of the normalized vector potential $a_0^2 \gg 1$, then F_0 in η_{vac_0} is about $|F_0|_{\max} \sim a_0^2$. It is easy to obtain the relation between a_0, the laser intensity I and the laser wavelength λ as $n_0 a_0^2 \sim 7.3\times10^{-11} n_0[\lambda\ (\mathrm{cm})]^2 I\ (\mathrm{W/cm^2}) = 7.3\times10^{-11}\cdot \pi(mc^2/e^2)(\omega_{\mathrm{p}}/\omega)^2 I\ (\mathrm{W/cm^2}) = 816(\omega_{\mathrm{p}}/\omega)^2 I\ (\mathrm{W/cm^2})$.[24] In the case that $\omega_{\mathrm{p}}/\omega \sim 0.1$, the incident laser intensity reaches $I \sim 10^{25}$ W/cm^2, and plasma number density $n_0 \sim 10^{24}$ cm^{-3}, then $a_0 \sim 10$, and the correction parameter $\eta_{\mathrm{vac}_{0\max}} = \Lambda^3 n_0 a_0^2$ can reach 10^{-11}.

The collision correction rates, η_{vac_1}, η_{vac_2}, and η_{vac_3}, correspond to the influence of the QED vacuum polarization effects of the two colliding lasers in the laser–plasma interaction system, due to the modifications of the coupled field caused by the vacuum "sources" $\delta\hat{\rho}_{\mathrm{vac}}$ and $\delta\hat{j}_{\mathrm{vac}}$ in Eq. (5), which add their value if the lasers collide in the quantum vacuum. These correction rates, η_{vac_1}, η_{vac_2}, and η_{vac_3}, consist of F_1, F_2. Under the nonlinear condition ($a_0 \gg 1$), from Eq. (7), the relation between these two coefficients is $F_2 \sim \Delta\varpi F_1$. F_1, F_2 is composed of F_0, δF, where δF depends on the angle between the two laser propagation directions.

When the directions of the two laser beam propagations (wave vector directions) are the same, then we have $o_1 = 1$, $p_1 = o_2 = 0$. At this point, $\delta F = \Delta\varpi\Delta A\left\{\phi\left[\frac{1+a^2}{(1+\phi)^2}-1\right]-p_2\frac{2a^2}{1+\phi}\right\}$. In particular, if the polarization directions (vector potential directions) of the two lasers are also the same, i.e., $p_2 = 1$, then $\delta F = 2\Delta\varpi\Delta A F_0$. The corresponding parameter $|\delta F|_{\max} \sim \Delta\varpi\Delta A a_0^2$. Thus there are $\eta_{\mathrm{vac}_1} \approx \eta_{\mathrm{vac}_3} = \Lambda^3 n_0(1+\Delta\varpi)F_1$, $\eta_{\mathrm{vac}_2} \approx 0$. We assume $\Delta\varpi \sim 10$, $\Delta A \sim 1$, then $\eta_{\mathrm{vac}_{1\max}} = \Lambda^3 n_0 a_0^2 \Delta\varpi^3 \Delta A^2$. In the case of $a_0 \sim 10$, $\eta_{\mathrm{vac}_{1\max}}$ can reach 10^{-8}.

When the directions of the two laser beam propagations are orthogonal, and the polarization direction of laser 1 is parallel to the direction of $\boldsymbol{e}_z \times \boldsymbol{e}_\perp$, then $o_1 = o_2 = p_2 = 0$, $p_1 = \pm1$. At this point, $\delta F = \Delta\varpi\Delta A \cdot \mp a\left[\frac{1+a^2}{(1+\phi)^2}-1\right]$. The corresponding parameter $|\delta F|_{\max} \sim \Delta\varpi\Delta A a_0^3$. Thus there are $\eta_{\mathrm{vac}_1} = \Lambda^3 n_0 F_1(1 \mp \Delta\varpi\frac{a}{\phi})$, $\eta_{\mathrm{vac}_2} = \Lambda^3 n_0 \Delta\varpi F_1$, $\eta_{\mathrm{vac}_3} = \Lambda^3 n_0 F_1[1+\Delta\varpi(3\phi^2+4\phi-a^2)/2(1+\phi)\phi]$. We assume $\Delta\varpi \sim 10$, $\Delta A \sim 1$, then $\eta_{\mathrm{vac}_{1\max}} = \Lambda^3 n_0(a_0^2\Delta\varpi^2\Delta A^2 + a_0^3\Delta\varpi\Delta A)\Delta\varpi a_0$, $\eta_{\mathrm{vac}_{2\max}} = \Lambda^3 n_0(a_0^2\Delta\varpi^2\Delta A^2 + a_0^3\Delta\varpi\Delta A)\Delta\varpi$, $\eta_{\mathrm{vac}_{3\max}} = \Lambda^3 n_0(a_0^2\Delta\varpi^2\Delta A^2 + a_0^3\Delta\varpi\Delta A)\Delta\varpi a_0^2$. In the case of $a_0 \sim 10$, $\eta_{\mathrm{vac}_{1\max}}$ can reach 10^{-7}, $\eta_{\mathrm{vac}_{2\max}}$ can reach 10^{-8}, and $\eta_{\mathrm{vac}_{3\max}}$ can reach 10^{-6}.

When the directions of the two laser propagations are opposite, then $o_1 = -|p_2| = -1$, $p_1 = o_2 = 0$. At this point, $\delta F = \Delta\varpi\Delta A\left\{(-\phi+2p_2\frac{a^2}{\phi})\left[\frac{1+a^2}{(1+\phi)^2}-1\right]-p_2\frac{2a^2}{1+\phi}\right\}$. The corresponding parameter $|\delta F|_{\max} \sim \Delta\varpi\Delta A a_0^4$. At this point, $\eta_{\mathrm{vac}_1} = -\Lambda^3 n_0\Delta\varpi F_1$, $\eta_{\mathrm{vac}_2} = 2\Lambda^3 n_0\Delta\varpi F_1$, $\eta_{\mathrm{vac}_3} = \Lambda^3 n_0[\Delta\varpi F_1(2\phi^2+3\phi-a^2)/(1+\phi)\phi]$. We assume $\Delta\varpi \sim 10$, $\Delta A \sim 1$, then $\eta_{\mathrm{vac}_{1\max}} \sim \eta_{\mathrm{vac}_{2\max}} = \Lambda^3 n_0(a_0^2\Delta\varpi^2\Delta A^2 + a_0^4\Delta\varpi\Delta A)\Delta\varpi$, $\eta_{\mathrm{vac}_{3\max}} = \Lambda^3 n_0(a_0^2\Delta\varpi^2\Delta A^2 + a_0^4\Delta\varpi\Delta A)\Delta\varpi a_0^2$. In the case of $a_0 \sim 10$, the parameter $\eta_{\mathrm{vac}_{1\max}} \sim \eta_{\mathrm{vac}_{2\max}}$ can reach 10^{-7} magnitude, and $\eta_{\mathrm{vac}_{3\max}}$ can reach 10^{-5}.

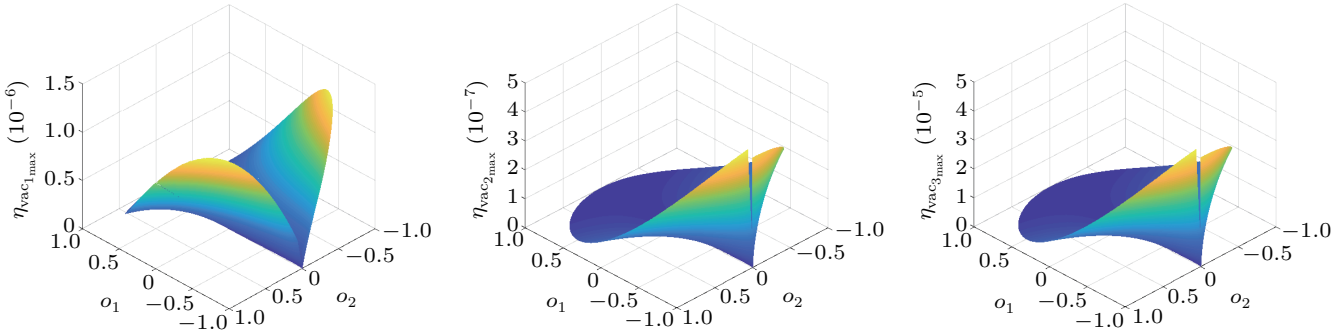


Fig. 1. Magnitudes of collision correction rates (a) $\eta_{\mathrm{vac}_{1\max}}$, (b) $\eta_{\mathrm{vac}_{2\max}}$, and (c) $\eta_{\mathrm{vac}_{3\max}}$ versus angle cosines of the two lasers o_1, o_2 for $p_1 > 0$, $q_1 = 0$, $\Delta\varpi \sim 10$, $\Delta A \sim 1$, $I = 10^{25}$ W/cm^2, $n_0 = 10^{24}$ cm^{-3} and $\omega_{\mathrm{p}}/\omega = 0.1$.

According to the above discussions, we found that compared with η_{vac_0}, the collision correction rates η_{vac_1}, η_{vac_2} and η_{vac_3} are significantly larger in magnitude. Especially noteworthy is the existence of η_{vac_2} and Eq. (13b); new effects of the quantum vacuum appear in the coupled field along the direction of the laser and its wakefield propagation. These do not exist in the classical equations or the one only considering the quantum vacuum effect of a single laser beam. Moreover, figure 1 shows that by changing the incident angle and the polarization direction of the two lasers, the corresponding magnitude of the collision correction rates also changes. It is possible to effectively enhance the magnitude of η_{vac_1}, η_{vac_2}, and η_{vac_3} in the laser–plasma interaction system by increasing the intensity of the incident lasers or the frequency difference between the two lasers, or the number density of the plasma. The results indicate that as well as a higher intensity of lasers, a higher number density of free electrons and ions in a plasma also helps the virtual particle pairs by quantum fluctuations to

transfer energy and momentum between multiple colliding intense lasers. For example, for the electron number density of plasma n_0, in future ICF experiments, $n_0 \sim 10^{25}$ cm^{-3}, and in the regime of astrophysics, $n_0 \sim 10^{30}$ cm^{-3} in compact objects such as dwarfs. In these multi-light physical environments, the effect of vacuum polarization is quite significant. These are all shown in Figs. 2–4.

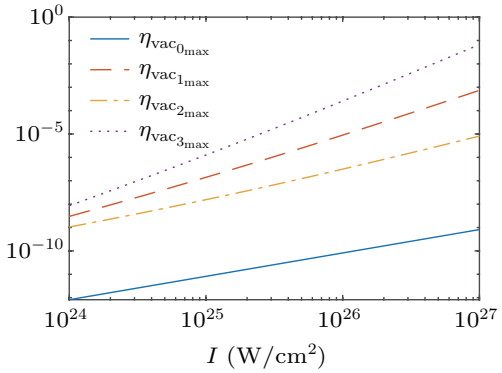


Fig. 2. Magnitudes of vacuum polarization effect correction rates $\eta_{\mathrm{vac}_{0\max}}$, $\eta_{\mathrm{vac}_{1\max}}$, $\eta_{\mathrm{vac}_{2\max}}$, and $\eta_{\mathrm{vac}_{3\max}}$ in different laser intensity I for $o_1 = o_2 = p_2 = 0$, $p_1 = 1$, $\Delta\varpi \sim 10$, $\Delta A \sim 1$, $n_0 = 10^{24}$ cm^{-3} and $\omega_{\mathrm{p}}/\omega = 0.1$.

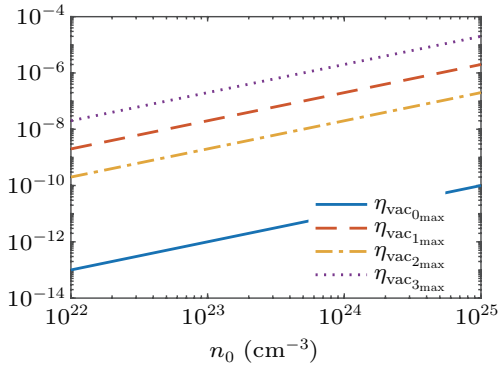


Fig. 3. Magnitudes of vacuum polarization effect correction rates $\eta_{\mathrm{vac}_{0\max}}$, $\eta_{\mathrm{vac}_{1\max}}$, $\eta_{\mathrm{vac}_{2\max}}$, and $\eta_{\mathrm{vac}_{3\max}}$ in different plasma density n_0 for $o_1 = o_2 = p_2 = 0$, $p_1 = 1$, $\Delta\varpi \sim 10$, $\Delta A \sim 1$, $a_0 \sim 10$ and $I = 10^{25}$ W/cm^2.

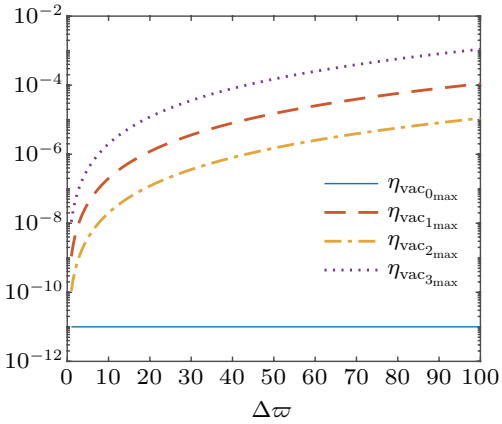


Fig. 4. Magnitudes of vacuum polarization effect correction rates $\eta_{\mathrm{vac}_{0\max}}$, $\eta_{\mathrm{vac}_{1\max}}$, $\eta_{\mathrm{vac}_{2\max}}$, and $\eta_{\mathrm{vac}_{3\max}}$ versus frequency ratios of the two laser $\Delta\varpi$ for $o_1 = o_2 = p_2 = 0$, $p_1 = 1$, $\Delta A \sim 1$, $I = 10^{25}$ W/cm^2, $n_0 = 10^{24}$ cm^{-3} and $\omega_{\mathrm{p}}/\omega = 0.1$.

On the other hand, due to the multi-light vacuum polarization effects, some scattered photons are generated with different directions from the lasers' original propagation direction and polarization direction. Some of them induce the vacuum "sources" $\delta\hat{\rho}_{\mathrm{vac_G}}$ and $\delta\hat{\boldsymbol{j}}_{\mathrm{vac_G}}$ to modify the coupled field. Comparing $\delta\hat{\rho}_{\mathrm{vac_G}}$ and $\delta\hat{\boldsymbol{j}}_{\mathrm{vac_G}}$ with other kinds of vacuum "sources" $\delta\hat{\rho}_{\mathrm{vac}}$ and $\delta\hat{\boldsymbol{j}}_{\mathrm{vac}}$ in Eq. (6), and comparing the corresponding coefficient G with F_0 and δF in Eq. (8), we can estimate the approximate magnitude range of the correction rate $\eta_{\mathrm{vac_G}}$ caused by $\delta\hat{\rho}_{\mathrm{vac_G}}$ and $\delta\hat{\boldsymbol{j}}_{\mathrm{vac_G}}$ is about $\Lambda^3 n_0 a_0^4 \Delta\varpi\Delta A < \eta_{\mathrm{vac_G}} < \Lambda^3 n_0 a_0^5 \Delta\varpi\Delta A$. Assuming the same parameters as before, $\Delta\varpi \sim 10$, $\Delta A \sim 1$, $a_0 \sim 10$, we obtain $10^{-7} < \eta_{\mathrm{vac_G}} < 10^{-6}$. The existence of these kinds of scattered photons also causes the wave vector $\boldsymbol{k}$ of the laser to produce a component along the direction of $q_1\boldsymbol{e}_\perp - p_1\boldsymbol{e}_z \times \boldsymbol{e}_\perp$, different from its original propagation direction along the z axis, and the vector potential $\boldsymbol{A}$ of the laser to produce a component along the direction of $q_1\boldsymbol{e}_z + (1-o_1)\boldsymbol{e}_z \times \boldsymbol{e}_\perp$, different from its original polarization direction along the $\perp$ axis. Further detailed analysis of this kind of correction requires three-dimensional nonlinear laser–plasma interaction theory. The propagation of the scattered photons in the plasma can further cause secondary effects. These may also be observed in state-of-the-art experiments of QED quantum vacuum polarization effects.

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