Realizing reliable XOR logic operation via logical chaotic resonance in a triple-well potential system

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There exists an optimal range of intensity of a chaotic force in which the behavior of a chaos-driven bistable system with two weak inputs can be consistently mapped to a specific logic output. This phenomenon is called logical chaotic resonance (LCR). However, realization of a reliable exclusive disjunction (XOR) through LCR has not been reported. Here, we explore the possibility of using chaos to enhance the reliability of XOR logic operation in a triple-well potential system via LCR. The success probability P of obtaining XOR logic operation can take the maximum value of 1 in an optimal window of intensity D of a chaotic force. Namely, success probability P displays characteristic bell-shaped behavior by altering the intensity of the chaotic driving force, indicating the occurrence of LCR. Further, the effects of periodic force on LCR have been investigated. For a subthreshold chaotic force, a periodic force with appropriate amplitude and frequency can help enhance the reliability of XOR logic operation. Thus, LCR can be effectively regulated by changing the amplitude and frequency of the periodic force.

Keywords: stochastic resonance, logical stochastic resonance, logical chaotic resonance, XOR logic, triple-well potential system

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1. Introduction

The interplay between stochastic fluctuations (i.e., noise) and nonlinearity of a system can produce extraordinary dynamic phenomena. For example, a weak signal can be effectively amplified with the help of noise in the nonlinear system. This phenomenon is called stochastic resonance (SR).[1] SR can be widely observed in various nonlinear systems[2–8] and gives a potential mechanism for utilizing omnipresent noise to obtain constructive outcomes, such as noise-induced enhancement of regularity (i.e., coherence resonance)[9,10] and noise-induced synchronization.[11]

With increasing miniaturization of computational devices it has become increasingly difficult to suppress or eliminate thermal fluctuation (thermal noise) in computational devices. Noise in computational devices has become one of the main obstacles to further improvement in computational speed and performance.[12,13] Against such a background, Murali and coworkers investigated the intriguing possibility of whether noise can be used to enhance the reliability of logic operation. They confirmed that, as expected, the probability of obtaining the desired logic operation can indeed be obviously enhanced under the assistance of noise in a noise-driven bistable system. They termed this phenomenon logic stochastic resonance (LSR) due to its similarity to SR.[14]

LSR was first proposed in 2009, and has received much attention recently due to its potential applications.[15–25] Some researchers have concentrated on designing new types of LSR-based logic devices. Noise in a LSR-based logic devices can be considered as a positive force and used to obtain or enhance the reliability of logic operation. For instance, Murali *et al.* used noisy nonlinear circuits consisting of a linear capacitor, a linear resistor and four CMOS transistors with a battery, to realize reliable and flexible logic gates (AND/OR and NAND/NOR).[26] Guerra *et al.* realized a noise-assisted reprogrammable nanomechanical logic gate which can execute fundamental logic functions (AND/OR and NAND/NOR), and in which logic functions can be programmed dynamically (such as from AND to OR) by adjusting the underlying nonlinearity of the resonator.[27] In addition, noise-assisted logic devices can be implemented in a Coulomb-coupled quantum-dot rectifier,[28] resonant tunneling diodes[29,30] and Schmitt triggers.[31] LSR-based devices can also be used for the detection of underwater objects.[32,33] These new types of LSR-based noise-assisted logic devices have obvious advantages in terms of energy consumption and size, and thus may be applied in a new generation of energy-saving mechanical computers.[27] On the other hand, some researchers have carried out LSR-related studies from a theoretical perspective, and confirmed that stochastic noise is not necessary for logical resonance. Many deterministic factors, such as periodic force,[34–38] time delay,[39–43] parameters,[44,45] coupling,[46–48] and temperature,[49] can also induce LSR-like phenomena.

It is worth noting that for realization of LSR-based logic devices, most studies have focused on the basic logic gates,

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such as AND/OR and NAND/NOR, and only a few consider complex logic devices. Compared with bistable systems, multi-stable systems have more potential wells and thus can distinguish between logic input sets (1,0) and (0,1). Because of this, besides the basic logic gates, complex logic operation can also be realized in multi-stable systems. Storni *et al.* first examined the ability to directly obtain complex XOR logic in multi-stable dynamical systems without the need for the traditional concatenation of NOR/NAND gates.[50] Zhang *et al.* discussed the effects of Gaussian colored noise[51] and non-Gaussian noise[52] on LSR in a triple-well potential system. We have also investigated the possibility of obtaining logic and set–reset latch operations in a triple-well potential system.[53]

Intrinsic or extrinsic chaotic activities play the role of stochastic noise, and can cause extraordinary dynamic phenomena.[54–58] Chaotic activities in deterministic nonlinear systems can induce chaotic resonance (CR), which is very similar to SR. CR can also be used to amplify or detect weak signals as for SR. Interestingly, chaos can also induce a logic resonance phenomenon (LCR).[59,60] To be specific, if the intensity of chaotic activities is within an optimal window, the reliability of logic operation can be obviously enhanced and the desired logic operation is always achieved. In addition, LCR can be dramatically enhanced via periodic force and coupling between different sub-systems.[61] We have confirmed that LCR can be used to realize a chaos-assisted memory device via the implementation of reliable set–reset latch operation in chaos-driven bistable system.[62] However, realization of a reliable exclusive disjunction (XOR) through LCR has not been reported. In this study, we explore the possibility of obtaining reliable XOR logic operation in a triple-well potential system via LCR.

2. Model and scheme

Considering an overdamped Brownian particle in the following triple-well potential, the Langevin-type dynamical system is described by[52,53]

$$\gamma\dot{x} = F(x) + I_1(t) + I_2(t) + DX(t). \tag{1}$$

The parameter γ represents a dissipation constant. With no loss of generality, $\gamma = 1$ is considered in this study. The nonlinear function $F(x) = -0.06x^5 + 0.8x^3 - 2(1+a)x$ is obtained as the gradient of a triple-well potential $U(x) = x^2(0.1x^2-1)^2 + ax^2$. The parameter a controls the number and depth of potential wells. As shown in Fig. 1, the system can be tristable or monostable by altering the parameter a. Unless otherwise specified, a is set to 0.25 through this study. For $a = 0.25$, the bilateral and middle potential wells are defined by the threshold values of the unstable points $x_\pm = \pm 2.23607$ (Fig. 1). Two unrelated inputs denoted by $I_1(t)$ and $I_2(t)$ in Eq. (1) are two-level aperiodic square waves. They can be interpreted as two independent binary logic inputs according to the following rule: $I_1(t)$ and $I_2(t)$ take the value 0.8 for logic input 1 and -0.8 for logic input 0 (Table 1). The parameter D governs the intensity of the chaotic force $X(t)$, and $X(t)$ is generated from the standard Lorenz system given by[59,61,62]

$$\begin{aligned} \dot{X} &= \sigma Y - \sigma X, \\ \dot{Y} &= \rho X - Y - XZ, \\ \dot{Z} &= XY - \beta Z. \end{aligned} \tag{2}$$

The parameters σ, ρ, and β take values of 10, 28, and 8/3, respectively, so that the generated $X(t)$ is indeed chaotic (Fig. 2). The dynamics of the system can be obtained by integrating Eqs. (1) and (2) numerically using Euler's method with a fixed time step $\Delta t = 0.001$ time units. Note that control runs with smaller time steps disclosed no significant difference in the final results.

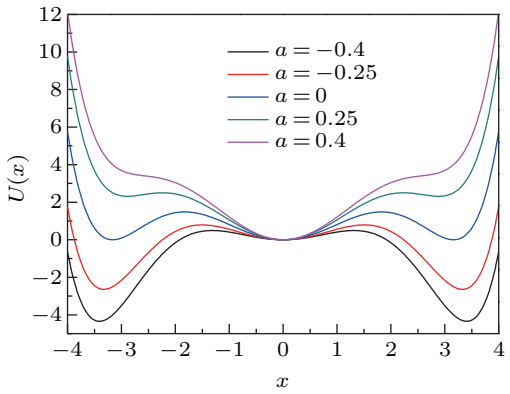


Fig. 1. Potential $U(x)$ for different values of a. The system can be tri-stable or mono-stable by changing the parameter a.

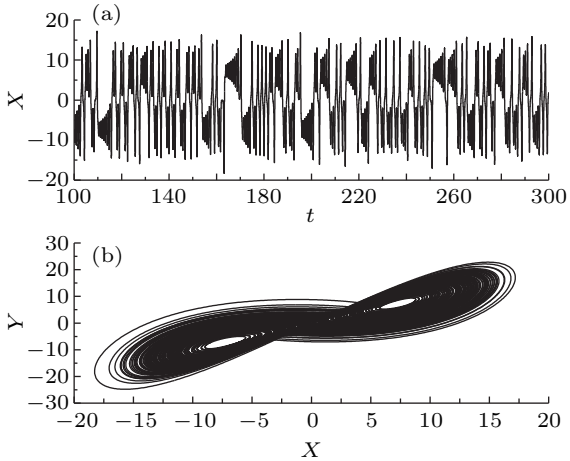


Fig. 2. (a) Sampled time series of $X(t)$ generated from a chaotic Lorenz system. (b) A chaotic attractor.

The response of the system to two inputs $I_1(t)$ and $I_2(t)$ can be interpreted as binary logic outputs according to the following encoding scheme: The response of the system is mapped to logic output 1 if the Brownian particle lies in the central potential well (Fig. 1, Table 1) or logic output 0 if the

Brownian particle lies in the bilateral potential wells (Fig. 1, Table 1). Then, we use the success probability $P = n/N$ to quantitatively characterize the reliability of XOR logic operation. Here n indicates the number of successful runs, while $N = 100$ denotes the total number of runs. A successful run means that the logic output of the system is always correct for any possible combination of logic inputs (Table 1). Note that there is delayed effect between the inputs and the corresponding response of system. Therefore, we must tolerate the undesired logic results during the transient processes when calculating the success probability P. Without loss of generality, the transient processes are defined as the first 10% of the minimum time interval (1000 time units) during which inputs $I_1(t)$ and $I_2(t)$ remain unchanged.

Table 1. Truth table for a XOR logic gate.

Values of I_1 and I_2	Logic input set	Logic output of XOR gate
$(-0.8, -0.8)$	(0, 0)	(bilateral wells)
$(-0.8, 0.8)/(0.8, -0.8)$	(0, 1)/(1, 0)	1 (central well)
(0.8, 0.8)	(1, 1)	(bilateral wells)

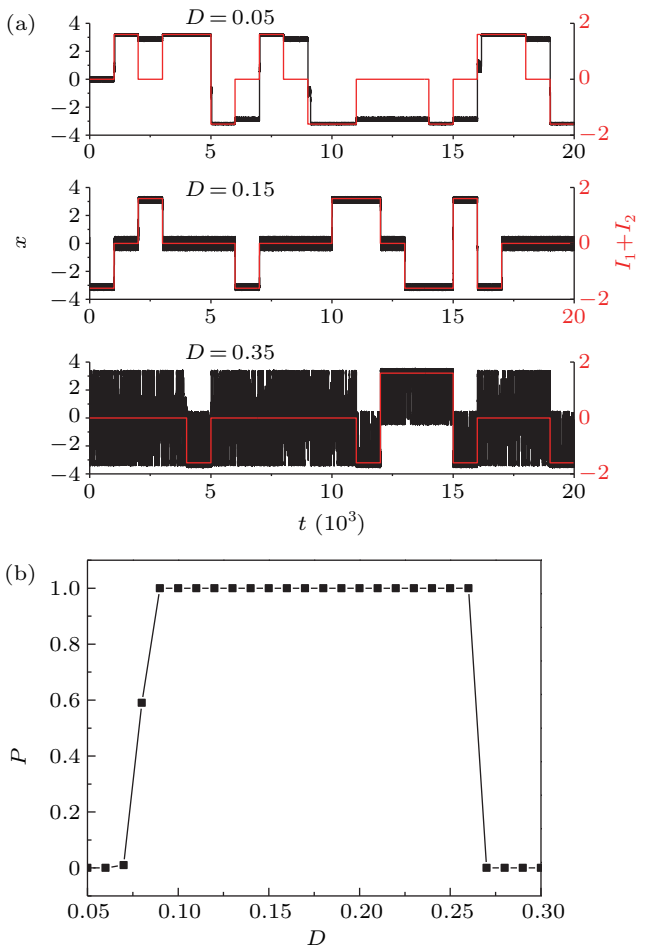


Fig. 3. (a) Sampled time series of system output x (black line) and the sum of inputs I_1 and I_2 (red line) under different intensities D of the chaotic driving force. I_1 and I_2 are two-level aperiodic square waves. Thus, the sum of I_1 and I_2 is a three-level aperiodic square wave. Clearly, reliable XOR logic operation can be obtained for a moderate intensity D of the chaotic force. (b) Success probability P as a function of intensity D of the chaotic force. The characteristic resonant curve can be clearly observed, indicating the occurrence of logical chaotic resonance for XOR logic.

3. Results

3.1. Chaos-induced logical resonance for XOR logic

As shown in Fig. 3(a), when the intensity D of the chaotic force is relatively small (such as $D = 0.05$), the logic output of the triple-well potential system is not always correct according to the truth table of the XOR gate. Increasing the intensity D of the chaotic force (such as $D = 0.15$) can maximize the success probability P of obtaining XOR logic operation, and XOR logic operation is always achieved. However, when the intensity D of chaotic forcing is beyond a certain threshold, the desired logic operation is unreliable. In this case, the Brownian particle is mainly controlled by the chaotic force and thus jumps back and forth between bilateral and middle potential wells. The above results indicate that the LCR phenomenon exists in the tri-stable system when the system carries out XOR logic operation.

The success probability P is used to quantitatively describe the reliability of XOR logic operation. P is plotted in Fig. 3(b) as a function of intensity D of the chaotic force. It is obvious that P evolves non-monotonously with increasing intensity D of the chaotic driving force. The curve of P versus D displays resonant characteristics (Fig. 3(b)). This result further confirms that LCR can occur in a tri-stable system. More precisely, there is an optimal window of intensity D of the chaotic force in which reliable XOR logic operation can be implemented in a chaos-driven tri-stable system.

3.2. Effects of periodic driving force on logical chaotic resonance for XOR logic

In this section, the effects of periodic driving force on LCR are investigated. To do this, the dynamical Eq. (1) is rewritten as

$$\gamma\dot{x} = F(x) + I_1(t) + I_2(t) + DX(t) + B\cos(\omega t). \tag{3}$$

The last term in Eq. (3) signifies a periodic driving force with amplitude B and frequency ω. Contour plots of success probability P are drawn in the B–D plane for different frequencies ω (Fig. 4). Increasing amplitude B allows the optimal window (where $P = 1$) of intensity D of the periodic driving force to shrink in size and move from right to left (Figs. 4(a)–4(f)). However, when the frequency is above certain a threshold, the size and location of the optimal window of intensity D of the chaotic driving force is hardly influenced by increasing amplitude B and increasing frequency ω. This result can also be observed from contour plots of success probability P in the $\log_{10}(\omega)$–D plane (Fig. 5). When frequency is relatively low, as frequency increases the optimal window of intensity D becomes wider, and moves from left to right (Fig. 5). As amplitude B is increased, the change in the optimal window of intensity D with increasing frequency becomes more and more obvious (Fig. 5). In addition, with the increase in amplitude B, the red region of reliability moves upwards (Fig. 5),

which means that for relatively large amplitude B, a relatively high-frequency periodic force is necessary if we want to obtain reliable XOR logic operation (Fig. 5). In other words, the optimal frequency has a dependence on amplitude. The dependence between frequency $\log_{10}(\omega)$ and amplitude B can be further revealed by contour plots of success probability P in the $\log_{10}(\omega)$–B plane for different intensities D (Fig. 6). It is worth noting that when the intensity D of the chaotic force is small, the chaotic force is so weak that it cannot induce LCR alone (Fig. 3). Namely, the chaotic force is subthreshold. Even for a subthreshold chaotic force, when the amplitude (or frequency) is fixed LCR can still be obtained by adjusting the frequency (or amplitude) (Figs. 6(a)–6(c)). However, for large intensity D, LCR cannot be always obtained, regardless of whether or not there is a periodic force (Fig. 6(i)). Taken together, a periodic force has a great impact on LCR, and thus LCR can be effectively regulated by changing the amplitude and frequency of the periodic force.

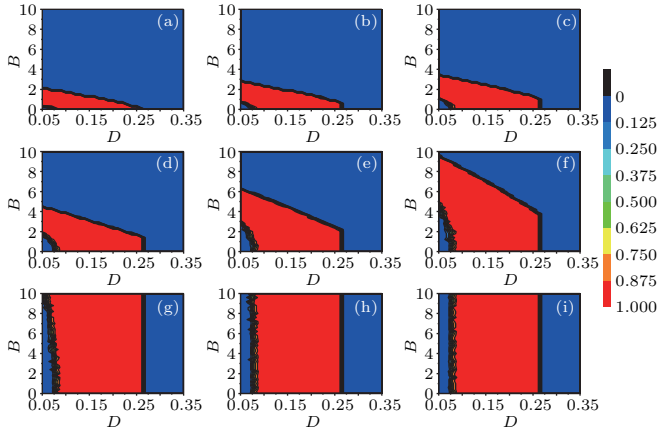


Fig. 4. Contour plots of success probability P in the B–D plane for different frequencies ω of the periodic driving force. The red reliable region (where $P = 1$) indicates the occurrence of LCR for XOR logic. All insets share the same color bar. (a) $\log_{10}(\omega) = 0$; (b) $\log_{10}(\omega) = 0.4$; (c) $\log_{10}(\omega) = 0.6$; (d) $\log_{10}(\omega) = 0.8$; (e) $\log_{10}(\omega) = 1$; (f) $\log_{10}(\omega) = 1.2$; (g) $\log_{10}(\omega) = 1.6$; (h) $\log_{10}(\omega) = 2$; (i) $\log_{10}(\omega) = 3$.

We can also observe that the area of the red reliable region in the contour plot of P changes with changing parameters (Figs. 4–6). To qualify this phenomenon, the area ratios of the red reliable region to the total area of plane are plotted in Fig. 7. As frequency $\log_{10}(\omega)$ increases, the area ratio of the red reliable region initially increases rapidly and finally reaches a plateau (Fig. 7(a)), which is in accordance with the results shown in Fig. 4. Meanwhile, the area ratio of the red reliable region decreases monotonically as amplitude B increases (Fig. 7(b)), which is consistent with the results displayed in Fig. 5. Interestingly, the area ratio of the red reliable region evolves non-monotonically as intensity D increases, displaying a resonance-like effect (Fig. 7(c)). This result corresponds to the observation in Fig. 6.

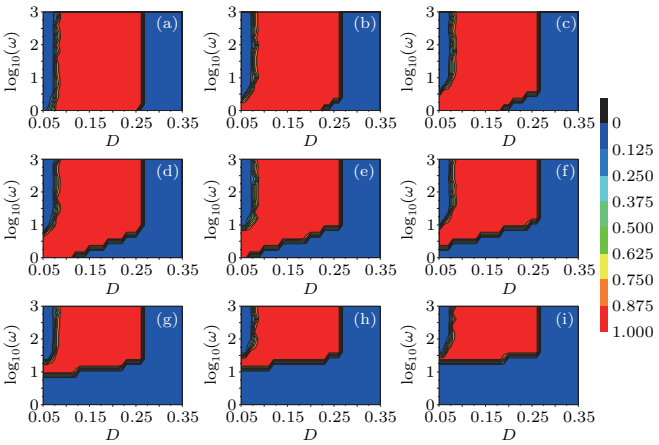


Fig. 5. Contour plots of success probability P in the $\log_{10}(\omega)$–D plane for different amplitudes B of the periodic driving force. The red reliable region (where $P = 1$) indicates the occurrence of LCR for XOR logic. All insets share the same color bar. (a) $B = 0.2$; (b) $B = 0.6$; (c) $B = 1$; (d) $B = 1.6$; (e) $B = 2$; (f) $B = 2.6$; (g) $B = 5$; (h) $B = 8$; (i) $B = 10$.

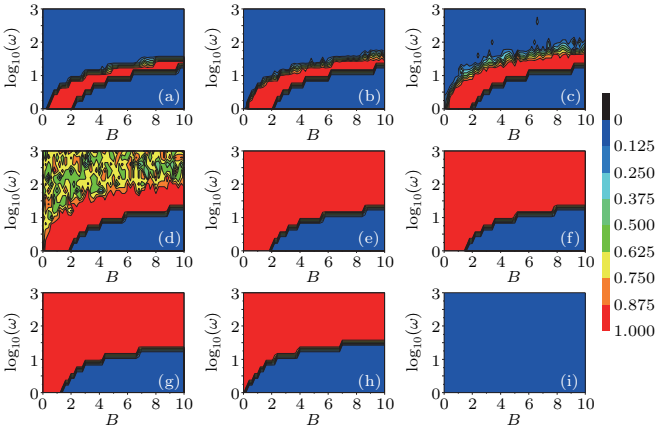


Fig. 6. Contour plots of success probability P in $\log_{10}(\omega)B$ plane for different intensities D of the chaotic driving force. The red reliable region indicates the occurrence of LCR. All insets share the same color bar. (a) $D = 0.05$; (b) $D = 0.06$; (c) $D = 0.07$; (d) $D = 0.08$; (e) $D = 0.09$; (f) $D = 0.12$; (g) $D = 0.16$; (h) $D = 0.26$; (i) $D = 0.28$.

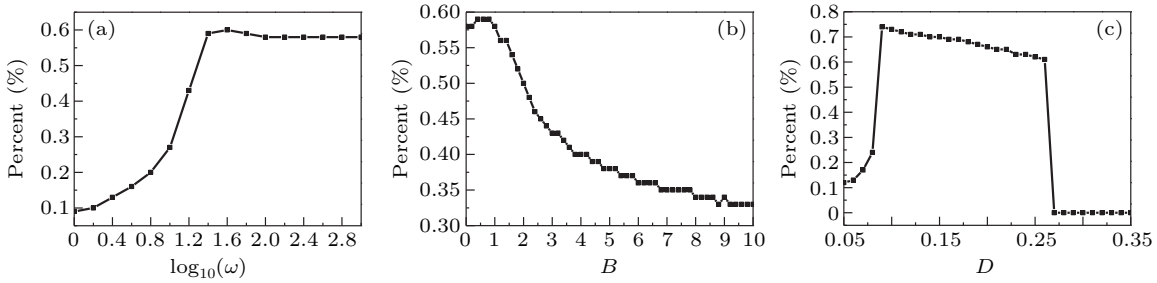


Fig. 7. Area ratio of the red reliable region to the total area of (a) the B–D plane, (b) the $\log_{10}(\omega)$–D plane and (c) the $\log_{10}(\omega)$–B plane. The ranges of B and D in (a) are the same as in Fig. 4. The ranges of $\log_{10}(\omega)$ and D in (b) are the same as in Fig. 5. The ranges of $\log_{10}(\omega)$ and B in (c) are the same as in Fig. 6.

4. Conclusion

In this study, we have investigated logical chaotic resonance (LCR) in a triple-well potential system under a chaotic force and periodic force. The probability of successfully obtaining XOR logic operation has been numerically calculated for different parameters. When the system is subjected only to chaotic force, the success probability P shows a bell-shaped behavior with varying intensity D of the chaotic force. That is to say, simulation results have revealed that is an optimal window of intensity D of the chaotic force in which the reliability of XOR logic operation can be significantly enhanced. Namely, LCR for XOR logic operation can be realized in a chaotic force-driven triple-well potential system. The effects of periodic force on LCR were explored. When the intensity D of the chaotic force is small, a periodic force with appropriate amplitude and frequency can help to enhance the reliability of XOR logic operation (i.e., LCR can occur). When the intensity D of the chaotic force is large, LCR never appear. Therefore, a periodic force has a great impact on LCR, and LCR can be effectively regulated by changing the amplitude and frequency of the periodic force. Taken together, a chaotic force and a periodic force allow the triple-well potential system to execute XOR logic operation through LCR.

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