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Citation: Chin. Phys. B, 2022, 31 (**9**): 094206. DOI: 10.1088/1674-1056/ac685e
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Numerical investigation of the nonlinear spectral broadening aiming at a few-cycle regime for 10 ps level Nd-doped lasers

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(Received 12 January 2022; revised manuscript received 13 April 2022; accepted manuscript online 20 April 2022)

We present a cascaded nonlinear spectral broadening scheme for Nd-doped lasers, featuring with long pulse duration and high average power. This scheme is based on two multi-pass cells (MPCs) and one multiple-plate supercontinuum generation (MPSG), and the numerical investigation is driven by a home-made Nd-doped fiber laser with 12 ps pulse duration, 50 kHz repetition rate and 100 W average power. The MPC-based first two stages allow us to broaden the pulse spectrum to 4 nm and 43 nm respectively, and subsequently, the MPSG-based third stage allows us to reach 235 nm spectral bandwidth. This broadened spectrum can support a Fourier-transfer-limited pulse duration of 9.8 fs, which is shorter than three optical cycles. To the best of our knowledge, it is the first time to demonstrate the possibility of few-cycle pulses generation based on the 10 ps level Nd-doped lasers. Such few-cycle and high average power laser sources should be attractive and prospective, benefiting from the characteristics of structure compact, low-cost and flexibility.

Keywords: nonlinear spectral broadening, picosecond lasers, high average power

PACS: 42.60.By, 42.65.–k, 42.65.Re **DOI:** 10.1088/1674-1056/ac685e

1. Introduction

Thanks to the characteristics of short pulse duration, high peak power and temporal coherence, few-cycle laser pulses have been widely applied in a variety of scientific and technological applications. For example, attosecond science, time-resolved research, particle acceleration, and the generation of ultrashort terahertz radiation.[1–8] Generally, there are two approaches to achieve a few-cycle regime. One is optical parametric chirped pulse amplification (OPCPA) technique,[9] and the other is the nonlinear compression of multi-cycle laser pulses.[10] However, limited by the gain bandwidth ($<$ 250 nm) of available nonlinear crystal, e.g., BBO, LBO and DKDP crystals, OPCPA systems can only achieve $\sim$ 10 fs level pulses.[11,12] In addition, the average powers of OPCPA systems are limited to only a few Watt (W) level, and the facilities are generally complicated and large-cost. Therefore, the nonlinear pulse compression should be the mainstream approach for few-cycle pulses generation, which is based on the external spectral broadening and subsequent chirp removing.

In addition to short pulse duration, the property of high average power has also attracted more and more attention in plenty of applications nowadays, but they are difficult to achieve simultaneously. The current applications are generally based on the Ti:sapphire and Yb-doped lasers. The Ti:sapphire lasers can easily realize the pulse duration below 100 fs, however they are typically limited to a few W in average power. In addition, Yb-doped lasers have reached a quite high level of technological maturity and are commercially available, but their price is relatively high. Moreover, the chirped pulse amplification (CPA) technology is generally necessary for these lasers to enhance their pulse energy to hundreds of μJ or higher level. In CPA lasers, the dispersion elements, such as diffraction gratings, chirped mirrors or chirped Bragg gratings, are normally employed, which will increase the complexity and cost of the lasers. However, the mode-locked Nd-doped oscillators can directly output $\sim$ 10 ps laser pulses, which can be amplified to hundreds of μJ or even several mJ without CPA technique. In other word, Nd-doped lasers exhibit the advantages of low-cost and compact configuration. Though the pulse duration of Nd-doped lasers is very long and at 10 ps level, their pulse energy and repetition rate can reach hundreds of mJ and 1–100 kHz respectively, corresponding to an average power of hundreds of W. Hence, Nd-doped laser can be a promising device for generating compact and economic laser source with few-cycle pulse duration and high average power simultaneously, based on the combination of nonlinear pulse compression technique.

The mainstream technique of nonlinear pulse compression is based on the pulse propagation in inert-gas-filled hollow-core fibers (HCFs), plus the chirped mirrors which can deliver pulses with few-cycle duration.[13,14] However, such a scheme possesses a relatively low energy transmission efficiency of $\sim$ 50% and the limitation in long-term stability. Moreover, the HCF-based setups will extend to large scale

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for the compression of multi-mJ pulses or high compression ratio.[15] A more recent scheme named MPC allows compact setups and compression ratio > 30.[16] This scheme is mostly ideal for the spectral broadening of laser pulses with a peak power of tens of megawatts, and provides great prospects for peak power scaling. In addition, the beam profile and spectral homogeneity of the output pulses from MPC can also be ensured.[17,18] Meanwhile, the MPC can be constructed to have an ample aperture to make the device insensitive to the fluctuation and beam quality of the coupled laser beam.[19] However, different from the HCF scheme which is generally employed to reach few-cycle regime, MPC yielded sub-100 fs down to sub-20 fs pulses,[20,21] especially for the MPC with solid media. As an alternative, the spectral broadening by solid thin plates, i.e., multiple-plate supercontinuum generation (MPSG), has also been proposed and investigated.[22] This scheme has been successfully applied to a wide range of pulse energies, average powers and in various wavelength regions to realize few-cycle pulses.[23,24] Thus, MPSG should be an ideal complement of MPC to reach the few-cycle regime, featuring with higher flexibility, simplicity and cost-efficiency. The pulses with 534 fs duration have been compressed to <30 fs based on the combination of MPC and MPSG.[25] However, the nonlinear compression of $\sim$ 10 ps level pulses has not been fully explored. In previous works, the $\sim$ 10 ps pulses from Nd-doped laser are just compressed to 601 fs and 172 fs by one and two MPCs, respectively.[26,27] There is still a long way toward reaching the few-cycle regime.

In this work, we present a cascaded nonlinear spectral broadening scheme for 10 ps level Nd-doped lasers, aiming at a few-cycle regime. This scheme includes two MPCs and one MPSG, and is numerically investigated based on a home-made Nd-doped fiber laser delivering pulses with a pulse duration of 12 ps, a pulse energy of 2 mJ and a repetition rate of 50 kHz. In the first two MPCs, the spectrum is successively broadened to 4 nm and 43 nm, and the pulse duration is sequentially compressed to 1.12 ps and 73 fs. In the followed MPSG, the spectrum is further broadened to 235 nm, which can support a Fourier-transfer-limited (FTL) pulse duration of 9.8 fs, i.e., shorter than three optical cycles. The numerical results efficiently demonstrate the possibility of few-cycle pulses generation based on $\sim$ 10 ps level Nd-doped lasers, for the first time to the best of our knowledge. Hence, this work can make it possible to bring Nd-doped lasers with traditional long pulses into few-cycle regime, and open a new approach towards few-cycle laser sources featuring with high average power and high peak power.

2. Spectral broadening scheme

The layout of the three-stage nonlinear spectral broadening scheme is illustrated in Fig. 1. This device starts with a home-made Nd-doped fiber laser delivering pulses with a pulse energy of 2 mJ and a repetition rate of 50 kHz, corresponding to an average power of 100 W. The central wavelength of laser is 1064 nm, and the pulse duration is 12 ps. The first two nonlinear spectral broadening stages are both based on the MPC scheme, which consists of mode-matching unit, spectral broadening unit, pulse collimation unit and chirp removing unit. The third nonlinear spectral broadening stage is based on an MPSG scheme. The operation principle of MPC as nonlinear spectral broadening is based on the repeated propagation between two concave mirrors and the nonlinear media that induce a small nonlinear phase shift at each round trip. In this case, laser pulses with peak powers above the threshold for critical self-focusing (SF) power will be spectral broadened, and the beam quality degradation during the propagation through nonlinear media is mitigated.[18,19] The major nonlinear process contributing to MPSG is the Kerr effect, which will induce self-phase modulation (SPM) as well as SF. The laser pulses are spectral broadened by the SPM in each plate, while the SF in the same media produces a converging beam after propagating through.

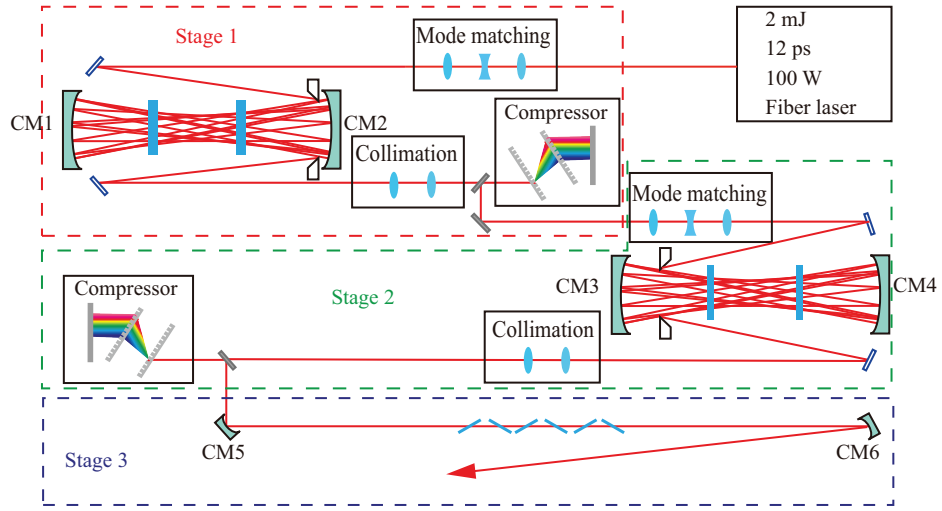


Fig. 1. Schematic of the three cascaded nonlinear spectral broadening scheme, including two MPCs and one MPSG.

In the first nonlinear spectral broadening stage, the laser pulses are firstly coupled to the MPC by a three-lens based mode-matching unit. This MPC consists of two identical 2-inch concave mirrors (CM1, CM2), with the radius of curvature of 300 mm. The separation between CM1 and CM2 is 570 mm. In addition, two pieces of fused silica with 2 mm thickness and 50 mm diameter are placed in the middle of this MPC as the nonlinear media, with a separation of 230 mm. The diameters of the pulse beam on the surfaces of the fused silica and the concave mirrors are 0.6 mm and 1.33 mm, respectively. This MPC scheme can ensure on the one hand that each step of propagation contributes efficiently to induce nonlinear phase, and on the other hand, that catastrophic SF due to the converging spot-sizes on subsequent steps is avoided. The laser pulses coupled in and out of this MPC is realized by two scraper mirrors, and 20 round trips are achieved. As a result, a total propagating distance in fused silica of 160 mm is finished in this MPC. The laser pulses output from this MPC are firstly collimated, and then injected into a Treacy compressor to remove the positive chirp. As the negative group delay dispersion (GDD) needed to compensate, the chirp of the pulse is too large to compensate with chirped mirrors, a pair of transmission gratings are applied to compress the chirped pulses at the cost of introducing extra third-order dispersion (TOD). In this compressor, two 1000 gr/mm gratings are adopted, and the laser pulses are injected on the gratings around the Littrow angel for high diffraction efficiency.

The second MPC consists of CM3 and CM4, with 3-inch diameter and 500 mm radius of curvature. The separation between CM3 and CM4 is 885 mm. In view of the pulse shortening and resultant peak power increasing in the first stage, the beam diameter on the surfaces of fused silica here is increased to 1 mm to avoid the optical damage on coatings; while the diameter of the pulse beam on the surfaces of concave mirrors is 1.37 mm. The thickness of fused silica is still 2 mm, and the laser pulses are aligned to pass through 13 round trips inside this MPC. Consequently, a total optical path of 104 mm in fused silica is achieved in this MPC. In the pulse compression stage, the Treacy compressor is designed with two 500 gr/mm gratings.

In order to achieve broader spectrum that supports few-cycle pulse duration, the second MPC is followed by an MPSG. There are two concave mirrors (CM5 and CM6) and six fused silica plates in this MPSG. The focal lengths of CM5 and CM6 are 1500 mm and 1000 mm, respectively. The first fused silica plate is placed after the focal spot, and the beam diameter on it is about 332 μm. For simplicity, the beam diameters on all six plates are regarded the same, assuming a balance between diffraction and SF is realized. Each plate is 0.1 mm-thickness, 10 mm-diameter and placed at Brewster's angle of 55.5°. Such thin plates are used to suppress the catastrophic effect of strong focusing in bulk materials, meanwhile, maintain a quasi-waveguide and confine the beam so as to achieve effective spectral broadening. After passing through all the six plates, the diverging beam are collimated with CM6.

3. Simulated results

The pulse propagation in media can be described by the nonlinear Schrödinger equation.[28–30] In our numerical model, dispersion, Kerr effect, self-steepening effect and Raman optical nonlinearities are taken into account. In the above three cascaded nonlinear spectral broadening stages, all the employed nonlinear media are fused silica. For the laser radiations with a central wavelength of 1064 nm in fused silica, the nonlinear refractive index (n_2) is $\sim 3\times10^{-16}$ cm^2/W, the GDD (β_2) and the third order dispersion TOD (β_3) is 16.473 fs^2/mm and 44.394 fs^3/mm, respectively.

In the first nonlinear spectral broadening stage, considering the energy loss comes from the 160 passes through the end faces of fused silica media and the 78 times reflection on the concave mirrors, the pulse energy after the MPC is estimated to be 1.74 mJ, corresponding to a transmission of $\sim 87\%$. With the pulse energy reduces in MPC, the per-pass nonlinear phase (i.e., B-integral) is also gradually decreasing. The average per-pass and total B-integral in this MPC are estimated to be 0.124π and 4.96π (beam averaged values), respectively.

During the process of spectral broadening in MPC, the pulse temporal profile is almost unchanged with the accumulation of B-integral due to the long pulse duration of 12 ps, as shown by Fig. 2(a). The spectrum is gradually broadened to 4 nm from 0.25 nm, corresponding to a spectral broadening ratio of ~ 16, as shown in Fig. 2(b). The temporal evolution of pulses with GDD compensation is also simulated, and the shortest pulse duration occurs at the GDD compensated with an amount of -2.3×10^6 fs^2. In this case, the highest pulses compression ratio of ~ 10.7 can be achieved, as shown in Figs. 2(c)–2(d). Thus, the incident angle and the grating pair separation of this Treacy compressor are optimized to 32.14° and 326.7 mm, respectively, to provide corresponding amount of GDD. As a result, the spectral broadened pulses are compressed to 1.12 ps, as shown in Fig. 2(e). In addition, the transmission of Treacy compressor is regarded as 90%, here referring to the reported efficiency.[26]

In the second nonlinear spectral broadening stage, the pulse energies before and after MPC are 1.56 mJ and 1.43 mJ, respectively, corresponding to a transmission efficiency of $\sim 92\%$. This transmission is higher than that of the first MPC, due to the fewer round trips. The averaged B-integral of each pass in this MPC is calculated to be about $\sim 0.38\pi$, which contributes to a total B-integral of $\sim 9.89\pi$ (beam averaged values). Compared with the first MPC, the single-pass and total B-integral in this MPC are both larger because of the shortening of pulse duration and then the improvement of laser intensity.

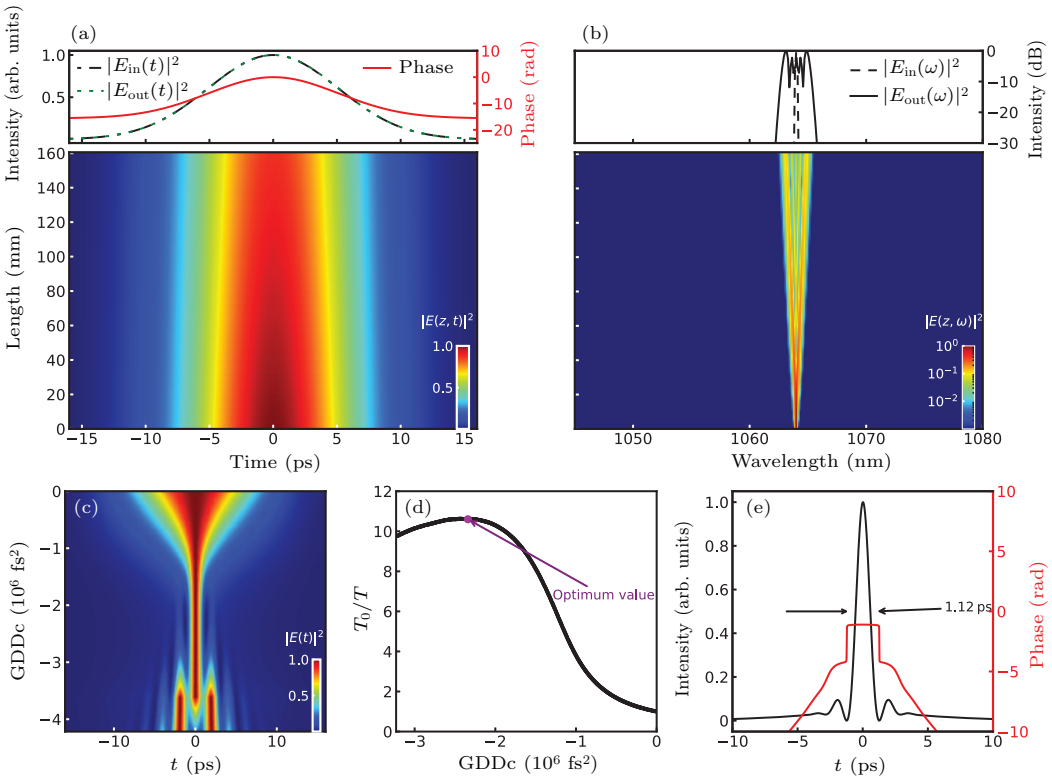


Fig. 2. The first nonlinear spectral broadening stage. The evolution of (a) temporal intensity and (b) spectrum with the accumulation of fused silica thickness; (c) the temporal intensity evolution of spectral broadened pulses with the compensation amount of GDD; (d) the pulse compression ratio (T_0/T) evolution with GDD compensation; (e) the temporal profile and phase of compressed pulses.

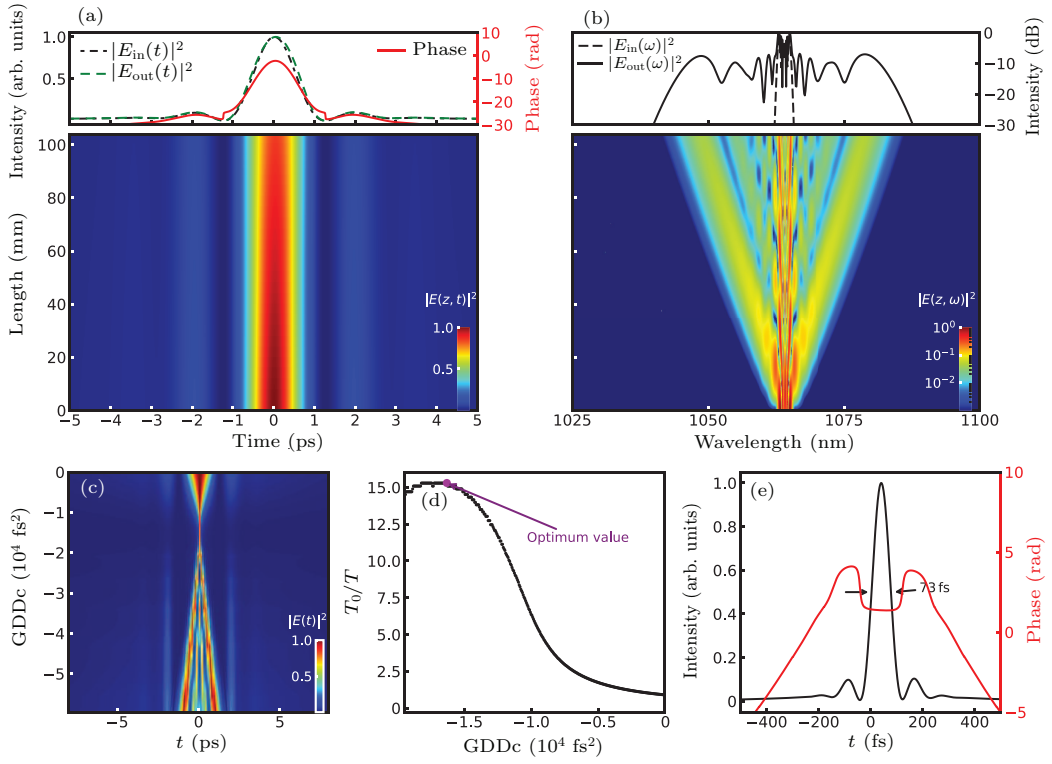


Fig. 3. The second nonlinear spectral broadening stage. The evolution of (a) temporal intensity and (b) spectrum with the accumulation of fused silica thickness; (c) the temporal intensity evolution of spectral broadened pulses with the compensation amount of GDD; (d) the pulse compression ratio (T_0/T) evolution with GDD compensation; (e) the temporal profile and phase of compressed pulses.

After 26 passes in the second MPC, there is still no obvious change of the temporal profile for the 1.12 ps pulses, as shown in Fig. 3(a). The spectral broadening caused by this MPC generates the components from 1042 nm to 1085 nm, and the corresponding spectral broadening ratio is ~ 10.7, as shown in Fig. 3(b). Based on the numerical calculation, the optimal compensation amount of GDD is about -1.63×10^4 fs^2, as shown in Figs. 3(c)–3(d). After optimizing the compressor,

with an incident angle and a grating pair separation of 15.4° and 13.71 mm, the spectral broadened pulses are compressed to 73 fs, as shown in Fig. 3(e). Taking into account the original pulse duration of 1.12 ps at the output of the first nonlinear spectral broadening stage, a pulse compression factor of ~ 15 is achieved in this stage. Combining 90% transmission of Treacy compressor, the pulses output from the second nonlinear spectral broadening stage are potential to have the peak power of ~ 17.6 GW and the average power of ~ 64 W. In the above-mentioned two spectral broadening stages, the additional TODs introduced by grating compressors are about 6.9×10^6 fs^3 and 3.2×10^4 fs^3, respectively. The influence on pulse compression induced by the above TODs has also been considered in our simulation, and the influence is slight and ignorable. This is mainly due to fact that the pulse spectra are still relatively narrow here, i.e., the pulse durations are relatively long. In order to reach the few-cycle regime, the pulse spectrum should be further broadened. Thereby, a third nonlinear spectral broadening stage based on MPSG is followed.

In this MPSG, the laser intensity on fused silica plates is $\sim 2.03\times10^{13}$ W/cm^2, assuming a constant pulse duration of 73 fs. The total B-integral is calculated to be $\sim 6.8\pi$ (beam averaged values). The spectrum after each fused silica plate is simulated. As shown in Fig. 4(a), the spectral broadening remains nearly symmetrical in the first two plates, which indicates that SPM plays the dominant role in spectral broadening. The self-steepening (SS) effect is occurring and strengthened in each subsequent plates, and the spectral broadening starts to blue shift and asymmetric. As a result, a broader spectrum spanning from 955 nm to 1190 nm is achieved, corresponding to a spectral broadening ratio of ~ 5.5. This spectrum can support an FTL pulse duration of ~ 9.8 fs, which is shorter than three optical cycles. Thus, few-cycle pulses are potential to be realized by proper chirp removing, based on this three cascaded nonlinear spectral broadening scheme.

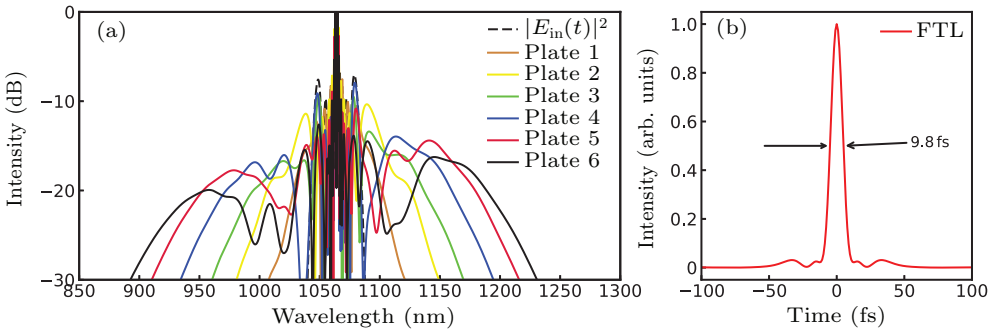


Fig. 4. The third nonlinear spectral broadening stage. (a) The spectrum evolution after each fused silica plate in a log scale, (b) and the corresponding FTL pulse profile.

4. Conclusion and outlook

In conclusion, a three-stage nonlinear spectral broadening scheme is numerically investigated for 10 ps level Nd-doped lasers, based on the combination of two MPCs and one MPSG. The numerical results show that the spectrum of the 12 ps pulses output from a home-made Nd-doped fiber laser can be broadened to 235 nm, which can support an FTL pulse duration of 9.8 fs. Thus, it is a promising scheme to realize the laser sources with ultra-broadband spectrum and high average power, and can make it possible to bring Nd-doped lasers with 10 ps level pulses into a few-cycle regime. In the following work, some other dispersion compensation methods will be studied and introduced into the MPC based first two stages for realizing a better dispersion compensation, and hence, few-cycle pulses can be directly achieved by using chirped mirrors after the MPSG based third stage.

Acknowledgements

Project supported by the National Natural Science Foundation of China (Grant No. 61925507), the National Key R&D Program of China (Grant No. 2017YFE0123700), the Strategic Priority Research Program of Chinese Academic Sciences (Grant No. XDB1603), the Shanghai Municipal Science and Technology Major Project (Grant No. 2017SHZDZX02), the Shanghai Natural Science Foundation (Grant No. 20ZR1464600), the Program of Shanghai Academic/Technology Research Leader (Grant No. 18XD1404200), the Shanghai Sailing Program (Grant No. 21YF1453800), and Youth Innovation Promotion Association of Chinese Academic Sciences (Grant No. Y202059).

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