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Coherent interaction and action–counteraction theory in small polaron systems, and ground state properties

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Based on the coherent interaction and action–counteraction principles, we investigate the ground state properties for small polaron systems, the coherent-squeezed fluctuation correction, and the anomalous lattice quantum fluctuation, with the new variational generator containing correlated squeezed-coherent coupling and quantum entanglement. Noting that $-2t$ is the T.B.A. energy, for the coherent interaction effect, we find the ground-state energy E_0 to be $-2.428t$, in which the coherent squeezed fluctuation correction $-A_0t$ is $-0.463t$ (where t is the hopping integral, ω is the phonon frequency), with the electron–one-phonon coupling constant $g=1$ and the electron–two-phonon coupling constant $g_1=-0.1$. However, as a result of the action–counteraction effect, $\tilde{E}_0$ is $-2.788t$, but $-\tilde{A}_0t$ is $-0.735t$. As to the polaron binding energy ($E_{\rm P}$), for the coherent interaction effect, $E_{\rm P}$ is -1.38ω, but for the action–counteraction effect, $\tilde{E}_{\rm P}$ is -1.88ω. In particular, the electron–two-phonon interaction noticeably enlarges the coherent interaction and the coherent squeezed quantum fluctuation correction. By intervening with the quantum entanglement, the evolutions of the squeezed coherent state and the lattice quantum fluctuation begin to take control. At that time, we encounter a new quantum phase coherence phenomenon — the collapse and revival of inversion repeatedly for the coherent state in the entangled evolution.

Keywords: coherent interaction and action–counteraction, correlated representation variational approach, entangled-squeezed coherent state, interaction of electron with the coherent squeezed fluctuation

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1. Introduction

The investigation of the properties of coupled electron–phonon (el–ph) systems and the related polaron behavior has played a central role in condensed matter physics for a long time. Because the coupling of electrons with the lattice vibration noticeably changes the characteristics of both electrons and phonons, it leads to a number of interesting effects and phenomena.[1–8] Over the last few years, effects due to the strong el–ph interaction and polaronic signatures have been evident in several compounds, such as high-temperature cuprate superconductors,[9,12] colossal magnetoresistance manganites,[8,11,13] fullerenes,[10] and DNA.[14]

This amount of experimental results has stimulated the study of different coupled el–ph systems. Among the most studied models, there is the Holstein lattice model[1,2] (small polaron), and the Fröhlich model (large polaron).[15,16] Recently, for the Fröhlich polaron, the weak electron–phonon coupling region is fairly well described. For the arbitrary electron–phonon coupling strength, the variational approach, the variational path-integral,[22–24] the kernel polynomial,[25] and the polaron clusters method have investigated[18,19] further.

The Holstein model has motivated many studies since 1959.[1] The variational ansatz is expected to give reliable results only for ground-state properties.[26–28] In recent years, various methods, such as the variational path-integral,[25] density matrix renormalization group analysis,[29] and the exact diagonalization of small clusters,[30,31] have been used for studies of the Holstein model with special attention to the polaron problem. As we know, the local electron–phonon interaction leads to lattice localized distortion. For strongly coupled el–ph systems, the polaron is strongly localized, therefore the theory of strong-coupling el–ph systems has had some unsolved problems from the very time of its creation.[8,10,13] That is, the complex physics of the coupled el–ph systems in polar crystals and related polaronic behavior are far from a simple thing.[18,19,24,25] Recently, the variational wave function of a coherent two-phonon state for the lattice ground state was proposed by Zheng.[42] Feinberg *et al.*[43,44] generalized this idea and showed how the squeezed state of the phonon is deeply related to the anomalous lattice fluctuation. At the same time, instead of the two-phonon coherent state expansion, we have proposed the variational approximation based on the squeezed coherent state expansion. In line with this new idea, for the arbitrary electron–phonon coupling strength, the ground-state energy of the Fröhlich's polaron systems is not only lower than the squeezed phonon state approach, but also goes beyond the results given by the Feynman path integral and the existing theoretical approximation.[22–24] Equally, for the system of Holstein's polaron, the ground-state energy and the polaron binding energy also greatly exceed the results given by the two-phonon coherent state approach.[46] Furthermore, this new idea has generalized to the solution of quantum noise caused by the one-phonon coherent state field; for

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example, the persistent current in the mesoscopic ring and the quantum coherence dissipation in the quantum tunneling system with the spin–lattice phonon coupling.[47,48]

In this paper, on the basis of coherent interaction and action–counteraction between the squeezed state of the phonon $S(z)|0\rangle_b$ and the one-phonon coherent state $D(\alpha)|0\rangle_b$, we aim to solve the coupled electron (c)–phonon (b) systems (α is the coherent parameter, z the squeezed parameter). Since the development of the local squeezed action of the two-phonon state leads to the anomalous lattice quantum fluctuation, the coherent state and the ground state will move toward instability. For this reason, we intervene with the non-local quantum entangled action to restrict the local squeezed action of the phonon. As a result of the representation correlation between $S(Z)$ and $D(\alpha)$, we use the joint generator $\tilde{U}=\exp\left\{\sum_i\left[(zb_i^{+2}-z^*b_i^2)+(\alpha b_i^+-\alpha^* b_i)c_i^+c_i\right]\right\}$ instead of the decoupled generator $U=S(z)D(\alpha)$. For the intermediate and strongly coupled el–ph systems, the polaron state has developed into a polaron–soliton state, we should use the variational wave function with the polaron–soliton state $\Psi=\sum_i \psi_i c_i^+\tilde{U}|0\rangle_{b_i,c_i}$.

2. Coherent interaction theory, correlated representation variational method with quantum entanglement

2.1. One-phonon coherent state variational approximation, the nonlinear solitary wave

Taking into account the electron–two-phonon interaction, the Holstein's Hamiltonian for coupled el–ph systems in one-dimensional molecular crystals reads

$$\begin{aligned}H=&-t\sum_i(c_i^+c_{i+1}+c_i^+c_{i-1})+\omega\sum_i b_i^+b_i\\&+g\sum_i(b_i^++b_i)c_i^+c_i+g_1\sum_i(b_i^+b_i^++b_ib_i)c_i^+c_i,\end{aligned}\quad(1)$$

where c_i^+ (c_i) is the electron creation (annihilation) operator, b_i^+ (b_i) is the phonon creation (annihilation) operator, while t denotes the hopping integral parameter, g the electron–one-phonon coupling constant, and g_1 the electron–two-phonon coupling constant.

When the electron undergoes a one-phonon interaction, it will induce the one-phonon coherent action $H_c^{(1)}=\sum_i \mathrm{i}\hbar\left(\lambda b_i^+-\lambda^* b_i\right)$, with coupling parameter λ. Hence the state of the field generated by the coherent action $H_c^{(1)}$ is the one-phonon coherent state, the displaced phonon state $|\alpha\rangle=D(\alpha)|0\rangle_b$ with $D(\alpha)=\exp[\sum_i(\alpha_i b_i^+-\alpha_i^* b_i)]$, where α is the coherent parameter.

Based on the one-phonon coherent state approximation, the variational wave function $|\Phi_0\rangle$ of the coupled el–ph systems takes the form with the polaron–soliton state

$$|\Phi_0\rangle=\sum_l \psi_l c_l^+ D|0\rangle,\ \ (|0\rangle=|0\rangle_b|0\rangle_c),\quad(2)$$

$$D=\mathrm{e}^{\sum_i \alpha_i^{(1)}\left(b_i^+-b_i\right)},\ \ \left(|\psi_1\rangle=\sum_l\psi_l c_l^+|0\rangle\right),$$

$$\left(\alpha_i^{(1)}=\alpha_i^{(1)*}\right),\quad(3)$$

where ψ_i denotes the polaron–soliton state at the lattice i, $\alpha_i^{(1)}$ the variational parameters to be determined, and $|0\rangle$ the vacuum state of the coupled el–ph systems.

In this manner, the ground state energy for the coupling systems will be given by

$$\begin{aligned}E_{(0)}&=\langle\psi_1|D^{-1}HD|\psi_1\rangle/\langle\psi_1|\psi_1\rangle\\&=\frac{1}{\sum_l\psi_l^2}\left\{-t\sum_i(\psi_i\psi_{i+1}+\psi_i\psi_{i-1})+\omega\sum_{i_1}\psi_{i_1}^2\sum_i{\alpha_i^{(1)}}^2\right.\\&\left.+2g\sum_i\alpha_i^{(1)}\psi_i^2+2g_1\sum_i{\alpha_i^{(1)}}^2\psi_i^2\right\}.\end{aligned}\quad(4)$$

Using the variational extremum equation

$$\frac{\partial E_{(0)}}{\partial\alpha_k^{(1)}}=\frac{2g}{\sum_l\psi_l^2}\psi_k^2+\frac{4g_1\alpha_k^{(1)}}{\sum_l\psi_l^2}\psi_k^2+2\omega\alpha_k^{(1)}=0,\quad(5)$$

we can find for the nonlinear coherent parameter $\alpha_i^{(1)}$ as

$$\alpha_k^{(1)}=-\frac{g\psi_k^2}{\omega+2g_1\psi_k^2}=\frac{\psi_k^2}{1+A_1\psi_k^2}\alpha^{(0)},$$

$$\left(A_1=\frac{2g_1}{\omega},\ \ \alpha^{(0)}=-\frac{g}{\omega}\right).\quad(6)$$

Further, the electron state $|i\rangle$ at site i and the b_i-phonon displaced state $\mathrm{e}^{\left(\alpha_i^{(1)}b_i^+-\alpha_i^{*(1)}b_i\right)}|0\rangle_b$ are not on their own, but are interrelated with each other. At that time, the displaced phonon state should be corrected as (with correlated parameter ξ_0)

$$|\alpha\rangle=D(\alpha)|0\rangle_b=\mathrm{e}^{\sum_i\xi_0\alpha_i^{(1)}(b_i^+-b_i)c_i^+c_i}|0\rangle_b.\quad(7)$$

2.2. Correlated representation variational method based on the coherent interaction theory with quantum entanglement

The physical origin of the quantum effect in solid state physics is the interaction of electrons with the squeezed quantum fluctuation of the coherent state. The electron–two-phonon interaction will induce the nonlinear two-phonon coherent action $H_c^{(2)}=\sum_i \mathrm{i}\hbar\left(\lambda_1 b_i^{+2}-\lambda_1^* b_i^2\right)$ with coupling constant λ_1, the state of the field generated by the coherent action $H_c^{(2)}$ is the two-phonon coherent state, the phonon squeezed

state $|Z\rangle = S(z)|0\rangle_b$, with $S(z) = \exp[\sum_i (zb_i^{+2} - z^* b_i^2)]$. Because of the two-order squeezed fluctuation effect created by $H_c^{(2)}$, the state of the field generated by the coherent interaction between $H_c^{(1)}$ and $H_c^{(2)}$ is the squeezed coherent state $S(z)D(\alpha)|0\rangle_b$, but not $D(\alpha)S(z)|0\rangle_b$. For this reason, the phonon field must undergo canonical transformation with the coherent interaction ($z = \gamma e^{i\theta_0}$, $\theta_0 = 0$ with the real squeezed parameter γ)

$$\begin{aligned} D^+(\alpha)S^+(z)b_i S(z)D(\alpha) &= \left(b_i \mathrm{ch}2\gamma + b_i^+ \mathrm{sh}2\gamma\right) \\ &\quad + (\alpha \mathrm{ch}2\gamma + \alpha^* \mathrm{sh}2\gamma) \end{aligned} \tag{8}$$

to evolve into the nonlinear squeezed-coherent state (s.c state) $|[z,\alpha]\rangle = S(z)D(\alpha)|0\rangle_b$. By considering that the operators $L_1 = (\alpha b^+ - \alpha^* b)c^+ c$ and $L_2 = \left(zb^{+2} - z^* b^2\right)$ do not commute, it is beneficial to use the joint generator $\tilde{U} = \exp\left\{\sum_i \left[\left(zb_i^{+2} - z^* b_i^2\right) + \left(\alpha b_i^+ - \alpha^* b_i\right)c_i^+ c_i\right]\right\}$ to yield a lower ground state than $U = S(z)D(\alpha)$.

In accordance with the principles of natural philosophy, the local action must be accompanied with non-local action in order to occur simultaneously. As a result, both the phonons in the two-phonon coherent state are in non-local correlation (each phonon does not have its own state), i.e., quantum entanglement. In this manner, γ corrects to $f_1 f_2$, α to $(f_1 + f_2)\alpha$, with $f_1^2 + f_2^2 = 1$, where f_1, f_2 are the entanglement parameters. In this way, the joint generator is corrected as

$$\begin{aligned} \tilde{U} &= \exp\left\{\sum_i \left[f_1 f_2 \left(b_i^{+2} - b_i^2\right)\right.\right. \\ &\quad \left.\left. + \xi(\gamma)(f_1 + f_2)\alpha_i^{(1)}\left(b_i^+ - b_i\right)c_i^+ c_i\right]\right\}. \end{aligned} \tag{9}$$

Finally, the correlation representation variational wave function with the polaron–soliton state $\left(c_l^+ \psi_l |0\rangle\right)$ expansion will correct as

$$|\Phi\rangle = \sum_l c_l^+ \psi_l \tilde{U}|0\rangle. \tag{10}$$

Moreover, because of the representation correlation between $D(\alpha)$ and $S(z)$, the nonlinear entangled coherent parameter reads

$$\tilde{\alpha}_i^{(1)} = (f_1 + f_2)\frac{e^{2f_1 f_2} - 1}{2f_1 f_2}\frac{\psi_i^2}{1 + A_1 \psi_i^2}\alpha^{(0)}, \quad (\gamma = f_1 f_2). \tag{11}$$

Including the above effects, the entangled squeezed-coherent state is corrected as follows:

$$|[z,\alpha_i]\rangle = \tilde{U}|0\rangle, \quad \tilde{U} = U_S(f)U_D\left(\tilde{\alpha}_i^{(1)}, \xi\right), \tag{12}$$

with

$$\begin{aligned} U_S(f) &= e^{\sum_i f_1 f_2 (b_i^{+2} - b_i^2)}, \\ U_D(\tilde{\alpha}_i^{(1)}, \xi) &= e^{\sum_i \xi \tilde{\alpha}_i^{(1)}(b_i^+ - b_i)c_i^+ c_i}. \end{aligned} \tag{13}$$

Owing to the coherent coupling, $\tilde{\alpha}_i^{(1)}$ will evolve into

$$\tilde{\alpha}_i = \tilde{\alpha}_i^{(1)} \mathrm{ch}(2f_1 f_2) + \tilde{\alpha}_i^{(1)*} \mathrm{sh}(2f_1 f_2), \tag{14}$$

and the correlated parameter ξ_0 is corrected to ξ. With these results, we get

$$\begin{aligned} \tilde{U}^+ b_i \tilde{U} &= U_D^+\left(\tilde{\alpha}_i^{(1)}\right)U_S^+(f)b_i U_S(f)U_D\left(\tilde{\alpha}_i^{(1)}\right) \\ &= \left(b_i \mathrm{ch}2f_1 f_2 + b_i^+ \mathrm{sh}2f_1 f_2\right) \\ &\quad + \xi(f)(f_1 + f_2)\left(\alpha_i^{(1)} \mathrm{ch}2f_1 f_2 + \alpha_i^{*(1)} \mathrm{sh}2f_1 f_2\right) \\ &\quad \times \frac{e^{2f_1 f_2} - 1}{2f_1 f_2} c_i^+ c_i. \end{aligned} \tag{15}$$

From Eq. (15), we see the attractive interaction of electrons with the coherent squeezed quantum fluctuation with quantum entanglement

$$\begin{aligned} H_{\mathrm{el-s\cdot cfl}} &= \xi(f_1 + f_2)\left(\alpha_i^{(1)} \mathrm{ch}2f_1 f_2 + \alpha_i^{(1)*} \mathrm{sh}2f_1 f_2\right) \\ &\quad \times \frac{e^{2f_1 f_2} - 1}{2f_1 f_2} c_i^+ c_i. \end{aligned} \tag{16}$$

Noting that

$$\tilde{U}^+ c_i^+ \tilde{U} = c_i^+ e^{-\xi \tilde{\alpha}_i^{(1)} e^{-2f_1 f_2}\left(b_i^+ - b_i\right)}, \tag{17}$$

$$\begin{aligned} &{}_b\langle 0|\tilde{U}^+ c_i^+ c_{i+1}\tilde{U}|0\rangle_b \\ &= {}_b\langle 0|c_i^+ c_{i+1} e^{-\left[\xi \tilde{\alpha}_i^{(1)}(\tilde{b}_i^+ - \tilde{b}_i) - \xi \tilde{\alpha}_i^{(1)}(\tilde{b}_{i+1}^+ - \tilde{b}_{i+1})\right]}|0\rangle_b \\ &= c_i^+ c_{i+1} e^{-(1/2)\xi^2 \tilde{\alpha}_i^{(1)2} e^{-4f_1 f_2}}, \end{aligned} \tag{18}$$

the non-classical ground-state energy of coupled el–ph systems takes the form

$$\begin{aligned} E_0 &= \langle \psi_1|\tilde{U}^+ H \tilde{U}|\psi_1\rangle / \langle \psi_1 \psi_1\rangle \\ &= \frac{1}{\sum_l \psi_l^2}\left\{-t\sum_l e^{-(1/2)\xi^2 \tilde{\alpha}_i^{(1)2} e^{-4f_1 f_2}}(\psi_i \psi_{i+1} + \psi_i \psi_{i-1})\right. \\ &\quad + \omega \sum_l \left[2\xi^2 \tilde{\alpha}_i^2 \psi_i^2 + \sum_l \psi_l^2 \sinh^2 2f_1 f_2\right] + 2g\sum_i \xi \tilde{\alpha}_i \psi_i^2 \\ &\quad \left. + 2g_1 \sum_i \left(2\xi^2 \tilde{\alpha}_i^2 \psi_i^2 + \sum_l \psi_l^2 \sinh 2f_1 f \cosh 2f_1 f_2\right)\right\}. \end{aligned} \tag{19}$$

Using the following approximation with the random-phase:

$$\begin{aligned} &-t\sum_i e^{-(1/2)\xi^2 \tilde{\alpha}_i^{(1)2} e^{-4f_1 f_2}}(\psi_i \psi_{i+1} + \psi_i \psi_{i-1}) \\ &\approx -t\langle \psi_1|e^{-(1/2)\xi^2 \tilde{\alpha}_i^{(1)2} e^{-4f_1 f_2}}|\psi_1\rangle \sum_i (\psi_i \psi_{i+1} + \psi_i \psi_{i-1}), \end{aligned} \tag{20}$$

the quantum tunnel effect of polarons in continuous approximation is ($\eta_0 = e^{2f_1 f_2 - 1}/(2f_1 f_2)$)

$$\tilde{t} = t\langle \psi_1|e^{-(1/2)\xi^2 \tilde{\alpha}_i^{(1)2} e^{-4f_1 f_2}}|\psi_1\rangle$$

$$\approx t\left[1-\frac{1}{2}\xi^2 \mathrm{e}^{-4f_1f_2}\int_{-\infty}^{+\infty}\frac{(f_1+f_2)^2\eta_0^2 g^2\psi^6}{\omega^2(1+A_1\psi^2)^2}\mathrm{d}x\right]\approx t. \quad (21)$$

From Eqs. (18) and (21), because the coherent elastic scattering of hopping electrons with the phonon (i.e., D–W scattering) leads to the narrowing effect of the polaron bandwidth, t will be reduced to $t\mathrm{e}^{-(1/2)\xi^2\alpha^{(0)2}}$ ($\approx 0.375t$ with $g=1.0$, $\omega=1.9$, $\xi=1.4$). However, by considering the polaron–soliton effect, the D–W scattering effect is corrected to $t\mathrm{e}^{-(1/2)\xi^2\tilde{\alpha}^{(1)2}\mathrm{e}^{-4\gamma}}\approx t$ since $\alpha^{(0)}$ is corrected to $\tilde{\alpha}_i^{(1)}(\psi_i)$ with $|\mathrm{e}^{-4\gamma}\tilde{\alpha}_i^{(1)}(\psi_i)|\ll|\alpha^{(0)}|$. After the operation of the extreme-variation equation $\partial E_0/\partial\psi_k=0$, we obtain

$$\begin{aligned}&\tilde{t}(\psi_{k+1}+\psi_{k-1})-2g\xi\tilde{\alpha}_k\psi_k-2(\omega+2g_1)\xi^2\tilde{\alpha}_k^2\psi_k\\&-\psi_k\sum_i\psi_i\Big[\tilde{t}(\psi_{i+1}+\psi_{i-1})\\&-2g\xi\tilde{\alpha}_i\psi_i-2(\omega+2g_1)\xi^2\tilde{\alpha}_i^2\psi_i\Big]=0. \quad (22)\end{aligned}$$

Using the continuous approximation

$$\begin{aligned}&\psi_k\to\psi,\ \ \psi_{k+1}+\psi_{k-1}=\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}+2\psi,\\&\sum_i\psi_i^2=\int_{-\infty}^{+\infty}\psi^2(x)\,\mathrm{d}x=1,\\&\tilde{\alpha}_k\to\tilde{\alpha}(\psi)=\alpha_0\eta_0(f_1+f_2)\frac{\psi^2}{1+A_1\psi^2}, \quad (23)\end{aligned}$$

then Eq. (22) turns out to be

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}+\frac{2A}{1+A_1\psi^2}\psi^3-\frac{AB}{(1+A_1\psi^2)^2}\psi^5=A_0\psi, \quad (24)$$

where

$$\begin{aligned}&A_1=\frac{2g_1}{\omega},\ \ A=-\frac{\alpha_0\eta_0(f_1+f_2)\xi g}{\tilde{t}},\\&B=-\frac{2\alpha_0\eta_0(f_1+f_2)(\omega+2g_1)\xi}{g},\\&A_0=\int_{-\infty}^{+\infty}\psi\left[\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}+\frac{2A}{1+A_1\psi^2}\psi^3-\frac{AB}{(1+A_1\psi^2)^2}\psi^5\right]\mathrm{d}x, \quad (25)\end{aligned}$$

with

$$\begin{aligned}&\alpha_0=\alpha^{(0)}\cosh(2f_1f_2)+\alpha^{(0)*}\sinh(2f_1f_2),\\&\eta_0=\frac{\mathrm{e}^{2f_1f_2}-1}{2f_1f_2},\ \ \left(\alpha^{(0)}=-\frac{g}{\omega}\right).\end{aligned}$$

By using Eqs. (22)–(25), the non-classical ground-state energy for the polaron–soliton systems reads

$$\tilde{E}=\varepsilon_0(f_1f_2)-2t-A_0t, \quad (26)$$

where

$$\varepsilon_0(f_1,f_2)=\omega\sinh^2(2f_1f_2)+2g_1\sinh(2f_1f_2)\cosh(2f_1f_2). \quad (27)$$

Clearly, $-A_0t$ represents the fluctuation correction caused by the interaction of electrons with the squeezed quantum fluctuation of the entangled coherent state. Further, equations (24)–(27) form the self-consisted and closed systems of the equation which is different from the references.[35–39]

On the other hand, the displaced phonon–electron correlation parameter ξ can be determined by the variational equation $\partial E_0/\partial\xi=0$ as follows:

$$\begin{aligned}&\xi\Big[t\sum_i\tilde{\alpha}_i^2\mathrm{e}^{-4f_1f_2}\mathrm{e}^{-(1/2)\xi^2\tilde{\alpha}_i^{(1)2}\mathrm{e}^{-4f_1f_2}}(\psi_i\psi_{i+1}+\psi_i\psi_{i-1})\\&+4\sum_i(2g_1+\omega)\tilde{\alpha}_i^2\psi_i^2\Big]+2g\sum_i\tilde{\alpha}_i\psi_i^2=0. \quad (28)\end{aligned}$$

Since $(1/2)\xi^2\tilde{\alpha}_i^{(1)2}\mathrm{e}^{-4f_1f_2}\ll 1$ and $\mathrm{e}^{-(1/2)\xi^2\tilde{\alpha}_i^{(1)2}\mathrm{e}^{-4f_1f_2}}\approx 1$, by the continuous approximation, equation (28) can reduce to

$$a\xi^3-b\xi+c=0, \quad (29)$$

where

$$\begin{aligned}a=&\frac{1}{2}t\mathrm{e}^{-8f_1f_2}\int_{-\infty}^{+\infty}\frac{g^4(\alpha_0\eta_0)^4(f_1+f_2)^4}{\omega^4(1+A_1\psi^2)^4}\psi^9\\&\times\left\{\left[-\frac{2A}{1+A_1\psi^2}\psi^3+\frac{AB}{(1+A_1\psi^2)^2}\psi^5+A_0\psi\right]+2\psi\right\}\mathrm{d}x, \quad (30)\end{aligned}$$

$$\begin{aligned}b=&t\mathrm{e}^{-4f_1f_2}\int_{-\infty}^{+\infty}\frac{g^2(\alpha_0\eta_0)^2(f_1+f_2)^2}{\omega^2(1+A_1\psi^2)^2}\psi^5\\&\times\left\{\left[-\frac{2A}{1+A_1\psi^2}\psi^3+\frac{AB}{(1+A_1\psi^2)^2}\psi^5+A_0\psi\right]+2\psi\right\}\mathrm{d}x\\&+4(\omega+2g_1)\int_{-\infty}^{+\infty}\frac{g^2(\alpha_0\eta_0)^2(f_1+f_2)^2}{\omega^2(1+A_1\psi^2)^2}\psi^6\mathrm{d}x, \quad (31)\end{aligned}$$

$$c=2\int_{-\infty}^{+\infty}\frac{g(\alpha_0\eta_0)(f_1+f_2)}{\omega(1+A_1\psi^2)}\psi^4\mathrm{d}x. \quad (32)$$

Later, $\xi^{(1)}$, $\tilde{t}^{(1)}$, $A^{(1)}$, $B^{(1)}$, $A_0^{(1)}$, and $E_0^{(1)}$ are referred to as the electron–one-phonon interaction, while $\xi^{(2)}$, $\tilde{t}^{(2)}$, $A^{(2)}$, $B^{(2)}$, $A_0^{(2)}$, and $E_0^{(2)}$ are referred to as the electron–two-phonon interaction, respectively.

3. Ground state based on the principles of coherent interaction and action–counteraction

3.1. Ground state based on the coherent interaction theory with quantum entanglement

When $g_1\neq 0$, noting that $2|g_1|<\omega$, $\psi^2<1$, and $|A_1|\psi^2\ll 1$, as a reasonable non-linear expansion approximation to Eq. (24), the result is likely shown to be

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}-A_0\psi+2A\psi^3-(2A_1A+AB)\psi^5$$

$$+2A_1(A_1A+AB)\psi^7-A_1^2(2A_1A+3AB)\psi^9=0. \quad (33)$$

We now consider the first-order effect to be g_1 only since $g_1\approx-0.01\sim-0.2$, then the nonlinear equation for ψ takes the form

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2}-A_0^{(2)}\psi+2A^{(2)}\psi^3-(2A^{(2)}A_1+A^{(2)}B^{(2)})\psi^5=0. \quad (34)$$

Using the boundary condition

$$\psi|_{x\to\infty}=0,\quad \psi_x|_{x\to\infty}=0, \quad (35)$$

then Eq. (34) has the solitary solution as follows:

$$\psi^{(2)}(x)=\frac{1}{[(\alpha_2+\beta_2)+2\beta_2\sinh^2\sqrt{b_0}(x-x_0)]^{1/2}}, \quad (36)$$

where

$$\alpha_2=-\frac{b_1}{2b_0},\quad \beta_2=\frac{1}{2b_0}(b_1^2-4b_0b_2)^{1/2}, \quad (37)$$

with $b_0=A_0^{(2)}$, $b_1=-A^{(2)}$, $b_2=\frac{1}{3}\left(2A^{(2)}A_1+A_0^{(2)}B^{(2)}\right)$. By the use of the normalization of $\psi^{(2)}$ we get

$$\alpha_2=\frac{\eta_2}{\sqrt{b_0}\tanh\eta_2},\quad \eta_2^2=b_2>0,\quad \left(\eta_2=\frac{\beta_2}{\alpha_2}\right), \quad (38)$$

with $b_0=A_0^{(2)}=(b_1^2/4b_2)\tanh^2\sqrt{b_2}$. As a result, we find for the ground state energy

$$E_0^{(2)}=\varepsilon_0^{(2)}(f_1,f_2)-2t-\frac{b_1^2\tilde{t}^{(2)}}{4b_2}\tanh^2\sqrt{b_2}. \quad (39)$$

In the meantime, since $b_0>0$, $b_1<0$, $b_2>0$, and $\tanh^2\sqrt{b_2}<1$, the existence condition of the stable solitary wave (36), $\alpha_2+\beta_2>0$, $\beta_2>0$, is also satisfied fully.

3.2. Action–counteraction law correction

From Fig. 1, when $\gamma\approx0.16$, $f_1\approx0.158$, we find the ground state energy $E_0^{(2)}=-2.428t$ with the one-phonon coherent state $|\alpha_0(\gamma_0=0.16,f_1=0.158)\rangle$. At the same time, owing to the coherent interaction, the phonon already has become a displaced squeezed phonon $\tilde{b}(v_0)=[b+\alpha_{(0)}(f_1=0.158)]\mathrm{ch}\,\gamma_0+[b^+\alpha^*_{(0)}(f_1=0.158)]\mathrm{sh}\,\gamma_0$. With this result, we will see the coherent state as

$$\left|\tilde{\alpha}_{(0)}(f_1=0.158,\gamma_0=0.16)\right\rangle=\mathrm{e}^{\sum\limits_i(\mathrm{ch}2\gamma_0+\mathrm{sh}2\gamma_0)|\alpha_{(0)}|(b_i^+-b_i)}|0\rangle_b$$

but not

$$\left|\alpha_{(0)}(f_1=0.158,\gamma_0=0.16)\right\rangle=\mathrm{e}^{\sum\limits_i|\alpha_{(0)}|(b_i^+-b_i)}|0\rangle_b.$$

In the meantime, the squeezed state of phonon $\left|Z_{(0)}(\gamma=0.16)\right\rangle$ (later, $S(z)|0\rangle_b=|Z(\gamma)\rangle$) also developed to a new squeezed state $\left|\tilde{Z}_{(0)}(\tilde{\gamma}_0)\right\rangle$ caused by the action–counteraction effect from $\left|\tilde{\alpha}_{(0)}(\gamma_0=0.16)\right\rangle$. That is, we now see a new ground state energy $\tilde{E}_0^{(2)}$ but not $E_0^{(2)}=-2.428t$.

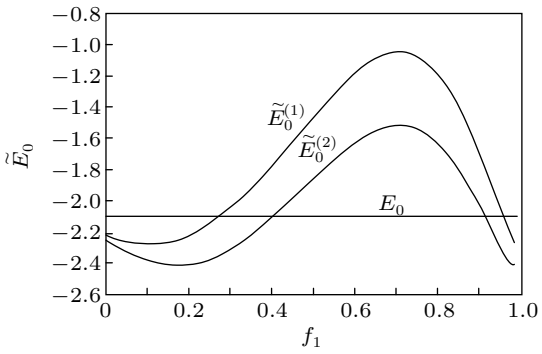


Fig. 1. Variation of ground state energy $\tilde{E}_0$ with the quantum entanglement versus the entangled parameter f_1 at $\omega=1.0$, $t=1.0$, $g=1.0$, $g_1=-0.1$, where $\tilde{E}^{(1)},\tilde{E}_0^{(2)}$ stand for the electron–one/two-phonon interaction, respectively, $E_{(0)}$ shown in Ref. [38].

As is well known, considering the representation correlation between $D(\alpha)$ and $S(z)$, the generator will be described by the joint generator with electron-displaced phonon correlation

$$U_{\mathrm{corr}}=\mathrm{e}^{\sum\limits_i\left[\left(z_ib_i^{+2}-z^*b_i^2\right)+\xi\left(\alpha_ib_i^+-\alpha_i^*b_i\right)c_i^+c_i\right]}. \quad (40)$$

After some straightforward algebra and integration, if lengthy, we can show that

$$U_{\mathrm{corr}}=\mathrm{e}^{\sum\limits_i\left(z_ib_i^{+2}-z^*b_i^2\right)}\cdot\mathrm{e}^{\sum\limits_i\left(Rb_i^+-R^*b_i\right)}\cdot\mathrm{e}^{\frac{w}{4\gamma^2}\left(\mathrm{sh}2\gamma-2\gamma\mathrm{ch}2\gamma-2\gamma^2\right)}, \quad (41)$$

where $R=\frac{1}{2\gamma}\xi|\alpha|\left[\mathrm{e}^{\mathrm{i}\theta_1}\mathrm{sh}2\gamma+\mathrm{e}^{\mathrm{i}(\theta-\theta_1)}(\mathrm{ch}2\gamma-1)\right]c_i^+c_i$ and $w=\alpha^{*2}\mathrm{e}^{-\mathrm{i}\theta}-\alpha^2\mathrm{e}^{\mathrm{i}\theta}$. We now take $\theta_1=0$, $\theta=0$, then $R=\xi\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}|\alpha|c_i^+c_i$. With these results,

$$U_{\mathrm{corr}}=\mathrm{e}^{\sum\limits_i|z|\left(b_i^{+2}-b_i^2\right)}\cdot\mathrm{e}^{\sum\limits_i\xi\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}|\alpha|\left(b_i^+-b_i\right)c_i^+c_i}, \quad (42)$$

$$U_{\mathrm{corr}}^+b_iU_{\mathrm{corr}}=(b_i\mathrm{ch}2\gamma+b_i^+\mathrm{sh}2\gamma)+\xi\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}|\alpha|(\mathrm{ch}2\gamma+\mathrm{sh}2\gamma)c_i^+c_i. \quad (43)$$

That is, in line with the coherent interaction of the coherent state $|\alpha\rangle$ with the phonon squeezed state $|Z(\gamma)\rangle$, the coherent state $|\alpha\rangle$ evolves to the new squeezed one-phonon coherent state

$$\left|\tilde{\alpha}_\xi(\gamma)\right\rangle=\mathrm{e}^{\sum\limits_i\xi(\gamma)\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}(\mathrm{ch}2\gamma+\mathrm{sh}2\gamma)|\alpha|\left(b_i^+-b_i\right)}|0\rangle_b$$

with

$$\tilde{\alpha}_\xi(\gamma)=\xi(\gamma)\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}(\mathrm{ch}2\gamma+\mathrm{sh}2\gamma)\alpha.$$

In accordance with the action–counteraction law, state $\left|\tilde{\alpha}_\xi(\gamma)\right\rangle$ will give the phonon squeezed state $|Z(\gamma)\rangle$ will obtain a counteraction from state $\left|\tilde{\alpha}_\xi(\gamma)\right\rangle$ in a same amount to develop into the new squeezed state

$$\left|\tilde{Z}(\gamma)\right\rangle=\mathrm{e}^{\sum\limits_i\xi(\gamma)\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}(\mathrm{ch}2\gamma+\mathrm{sh}2\gamma)|z|\left(b_i^{+2}-b_i^2\right)}|0\rangle_b.$$

On the other hand, the phonon squeezed state $|Z(\gamma)\rangle$ originates from the time evolution of the two-phonon coherent action $\mathrm{e}^{-\mathrm{i}H_{\mathrm{c}}^{(2)}(\lambda)t/\hbar}|\Psi^{(2)}\rangle(0)$. That is to say, $H_{\mathrm{c}}^{(2)}(\lambda)$ should also obtain the same counteraction amount originating from state $\left|\tilde{\alpha}_{\xi}(\gamma)\right\rangle$ to develop into the new two-phonon coherent action $\tilde{H}_{\mathrm{c}}^{(2)}\left(\tilde{\lambda}_1\right)$ with $\tilde{\lambda}_1=\xi(\gamma)\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}(\mathrm{ch}\,2\gamma+\mathrm{sh}\,2\gamma)\lambda_1$. As stated above, the electron–two-phonon interaction induces the two-phonon coherent action $H_{\mathrm{c}}^{(2)}$. In the meantime, we see the electron–two-phonon interaction develop into a new $\tilde{H}_{\mathrm{el-ph}}^{(2)}(\tilde{g}_1)$ but not $H_{\mathrm{el-ph}}^{(2)}(g_1)$, with

$$\tilde{g}_1=g_{1\mathrm{eff}}=\xi(\gamma)\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}(\mathrm{ch}\,2\gamma+\mathrm{sh}\,2\gamma)g_1. \tag{44}$$

From Figs. 1 and 4, the minimum for $E_0^{(2)}$ occurs in $f_1=0.16$, $\gamma=f_1f_2=0.158$, and $\xi(\gamma)=1.4$, and we find $g_{1\mathrm{eff}}=-0.225$. With this result, $-2g_{1\mathrm{eff}}\mathrm{ch}\,2\gamma\mathrm{sh}\,2\gamma=-0.156t$, the coherent squeezed quantum fluctuation correction $-\tilde{A}_0t$ changes from $-0.463t$ to $-0.735t$, but $E_0^{(2)}$ from $-2.428t$ to $\tilde{E}_0^{(2)}=-2.788t$.

4. Lattice phonon quantum fluctuation and binding energy of polaron based on the quantum entangled effect

In the light of the entangled state representation (9), we now investigate the new quantum entanglement behavior for the lattice quantum fluctuation and the binding energy of polarons in the polaron–soliton systems. Noting that

$$\begin{aligned}X_i&=(\hbar/2M\omega)^{1/2}\left(b_i^{+}+b_i\right),\\ P_i&=\mathrm{i}\,(M\hbar\omega/2)^{1/2}\left(b_i^{+}-b_i\right),\end{aligned} \tag{45}$$

we have

$$\tilde{U}^{+}X_i\tilde{U}=\sqrt{\frac{\hbar}{2M\omega}}\left[\left(b_i^{+}+b_i\right)\mathrm{e}^{2f_1f_2}+2\tilde{\alpha}_ic_i^{+}c_i\right], \tag{46}$$

$$\tilde{U}^{+}P_i\tilde{U}=\mathrm{i}\sqrt{\frac{M\hbar\omega}{2}}\left(b_i^{+}-b_i\right)\mathrm{e}^{2f_2f_2}. \tag{47}$$

With the above results, we find for the mean square fluctuations of P_i and X_i as

$$\langle\Phi|\Delta^2\tilde{X}|\rangle=\frac{\hbar}{2M\omega}\mathrm{e}^{4f_1f_2}\left[1+4\mathrm{e}^{-4f_1f_2}\left\langle\Delta^2\tilde{n}\right\rangle\right], \tag{48}$$

$$\langle\Phi|\Delta^2\tilde{P}|\rangle=\frac{\hbar M\omega}{2}\mathrm{e}^{-4f_1f_2}, \tag{49}$$

and the uncertainty between the phonon coordinate and momentum is

$$\left\langle\Delta^2\tilde{X}\right\rangle\left\langle\Delta^2\tilde{P}\right\rangle=\frac{\hbar^2}{4}\left[1+4\mathrm{e}^{-4f_1f_2}\left\langle\Delta^2\tilde{n}\right\rangle\right], \tag{50}$$

where $\left\langle\Delta^2\tilde{n}\right\rangle$ is the mean square fluctuation for the electron density. Under the continuity approximation, $\langle\Delta^2\tilde{n}\rangle$ can be written in the form

$$\langle\Delta^2\tilde{n}\rangle=2\int_{-\infty}^{+\infty}\frac{g^2(\alpha_0\eta_0)^2(f_1+f_2)^2\xi^2}{\omega^2(1+A_1\psi^2)^2}\psi^6\mathrm{d}x$$

$$-\int_{-\infty}^{+\infty}\frac{g^2(\alpha_0\eta_0)^2(f_1+f_2)^2\xi^2}{\omega^2(1+A_1\psi^2)^2}\psi^8\mathrm{d}x. \tag{51}$$

Noting that the electron–two-phonon interaction will enlarge the phonon squeeze action and will affect both the squeezed coherent state and the representation correlation, γ will develop to $\tilde{\gamma}\gg\gamma$, $\alpha_0\eta_0$ to $|\tilde{\alpha}_0\tilde{\eta}_0|\gg|\alpha_0\eta_0|$, so $\left\langle\Delta^2\tilde{n}\right\rangle=(f_1+f_2)^2(\tilde{\alpha}_0\tilde{\eta}_0)^2\xi^2\langle\Delta^2n\rangle\gg\langle\Delta^2n\rangle$, thus $\left\langle\Delta^2\tilde{X}\right\rangle\gg\left\langle\Delta^2X\right\rangle$, as shown in Figs. 2 and 3.

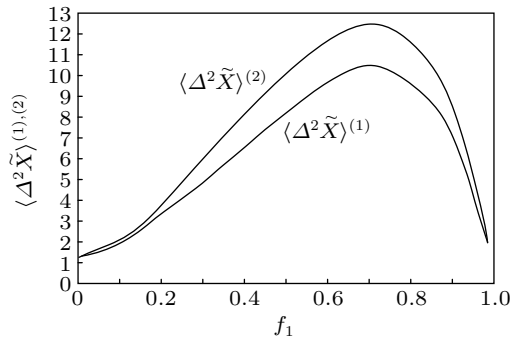


Fig. 2. .Variation of lattice quantum fluctuation $\left\langle\Delta^2\tilde{X}\right\rangle$ (unit $\hbar/2M\omega$) versus entangled parameter f_1 at $\omega=1.0$, $t=1.0$, $g=1.0$, $g_1=-0.1$, where $\left\langle\Delta^2\tilde{X}\right\rangle^{(1)}$, $\left\langle\Delta^2\tilde{X}\right\rangle^{(2)}$ stand for the electron–one-phonon/two-phonon interaction, respectively.

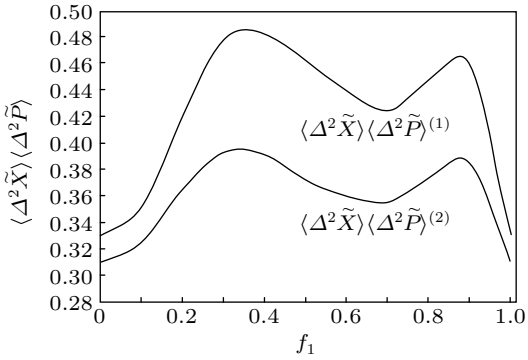


Fig. 3. Variation of quantum fluctuation in between the phonon coordinate and the phonon momentum versus the entangled parameter f_1 at $\omega=1.0$, $t=1.0$, $g=1.0$, $g_1=-0.1$, where $\left\langle\Delta^2\tilde{X}\right\rangle\left\langle\Delta^2\tilde{P}\right\rangle^{(1)}$, $\left\langle\Delta^2\tilde{X}\right\rangle\left\langle\Delta^2\tilde{P}\right\rangle^{(2)}$ stand for the electron-one/two-phonon interaction, respectively.

On the other hand, in terms of the coherent interaction between $D(\alpha)|0\rangle_b$ and $S(z)|0\rangle_b$, the binding energy of the polaron also leads to the important correction,

$$\begin{aligned}E_{\mathrm{P}}^{(2)}&=-\sum_i\langle\psi_1|\left(\tilde{\alpha}_ic_i^{+}c_i\right)^2|\psi_1\rangle\hbar\omega\\&=-2\hbar\omega\int_{-\infty}^{+\infty}\frac{g^2(\alpha_0\eta_0)^2(f_1+f_2)^2\xi^2}{\omega^2(1+A_1\psi^2)^2}\psi^6\mathrm{d}x.\end{aligned} \tag{52}$$

On the basis of the action–counteraction law, from Fig. 5 we see the minimum for $E_{\mathrm{P}}^{(1)}=-0.80\omega$ and $E_{\mathrm{P}}^{(2)}=-1.38\omega$ to occur in $f_1=0.7$, $\xi=0.4$ ($\gamma=f_1f_2\approx0.5$), at that time $g_{1\mathrm{eff}}=\xi\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}(\mathrm{ch}\,2\gamma+\mathrm{sh}\,2\gamma)g_1=-0.187$. By noting $\frac{1}{(1+A_1\psi^2)^2}\approx1-2A_1\psi^2$,

$$E_{\mathrm{P}}^{(2)}=E_{\mathrm{P}}^{(1)}+2A_1(2\hbar\omega)\int_{-\infty}^{+\infty}g^2(\alpha_0\eta_0)^2(f_1+f_2)^2$$

$$\times\xi^2\psi^6\cdot\left(\psi^2\right)\mathrm{d}x$$
$$= E_{\mathrm{P}}^{(1)} + 2A_1 I_1. \tag{53}$$

With this result, $I_1 = \frac{E_{\mathrm{P}}^{(2)} - E_{\mathrm{P}}^{(1)}}{4g_1} = 1.45$, we then have

$$\tilde{E}_{\mathrm{P}}^{(2)} = E_{\mathrm{P}}^{(1)} + 2\times 2g_{1\mathrm{eff}} I_1 \approx -1.883\omega. \tag{54}$$

Up to now, the best result was obtained using the correlated representation variational approach, $E_{\mathrm{P}} = -0.729\omega$,[49,50] but a better result can be obtained based on the variational path-integral approach, $E_{\mathrm{P}} = -1.00\omega$[25] (with $g = 1.0$, $g_1 = -0.1$, $t = 1.0$, $\gamma = 1.0$). Clearly, not only is the coherent interaction theory approach better than the existing E_{P}, the action–counteraction effect correction ($\tilde{E}_{\mathrm{P}}^{(2)}$) is also far beyond the existing one.

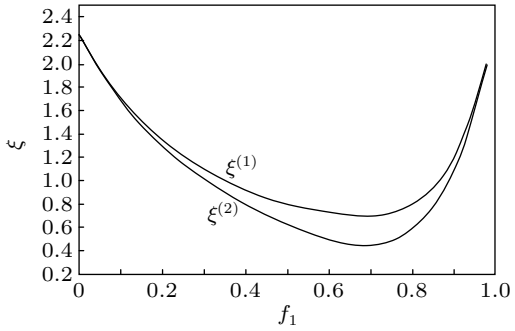


Fig. 4. Variation of phonon–electron correlation parameter $\xi^{(1)}$ and $\xi^{(2)}$ versus f_1 at $\omega = 1.0$, $t = 1.0$, $g = 1.0$, $g_1 = -0.1$, respectively.

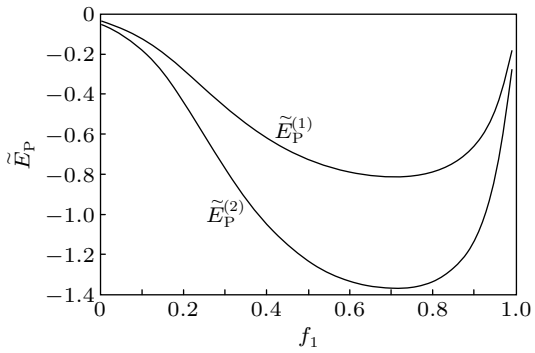


Fig. 5. Polaron binding energy $\tilde{E}_{\mathrm{P}}$ versus the entangled parameter f_1 at $\omega = 1.0$, $t = 1.0$, $g = 1.0$, $g_1 = -0.1$, where $\tilde{E}_{\mathrm{P}}^{(1)}$, $\tilde{E}_{\mathrm{P}}^{(2)}$ stand for the electron-one/two-phonon interaction, respectively.

5. Summary, discussion and remarks

In synthesizing Section 1 and the results in Sections 2–4, some key physics and principal comments are obtained.

(i) Because of the coherent interaction between $S(z)|0\rangle_b$ and $D(\alpha)|0\rangle_b$, the coupled el–ph systems certainly evolve into the squeezed coherent state $S(z)D(\alpha)|0\rangle_b$ with quantum entanglement but not $D(\alpha)S(z)|0\rangle_b$. The main quantum effects originate from the interaction of electrons with the coherent squeezed quantum fluctuation with quantum entanglement. As a result of the coherent interaction, the coherent parameter $\alpha_i^{(1)}$ is enlarged to $\tilde{\alpha}_i$ with $|\tilde{\alpha}_i| \gg |\alpha_i^{(1)}|$. In this way, the electron-one-phonon interaction energy $2g\sum_i \alpha_i^{(1)}\psi_i^2$ lowers to $2g\sum_i \xi\tilde{\alpha}_i\psi_i^2$; in particular, the electron–two-phonon interaction energy $2g_1\sum_i \alpha_i^{(1)^2}\psi_i^2$ decreases even more to $4g_1\sum_i \xi^2\tilde{\alpha}_i^2\psi_i^2$. On the other hand, the phonon energy $\omega\sum_i \alpha_i^{(1)^2}$ greatly weakens to $2\omega\sum_i \xi^2\tilde{\alpha}_i^2\psi_i^2$ since $\psi_i^2 \ll 1$. With these results, the coherent squeezed fluctuation correction $-A_0^{(2)}t$ changes from $-0.159t$ to $-0.463t$; meanwhile $E_0^{(1)}$ from $-2.159t$ to $-2.428t$, ($E_0^{(2)}$ from $-2.083t$ to $-2.251t$), while for the existing theory[47,48] $E_{(0)}^{(1)} = -2.083t$, $E_{(0)}^{(2)} = -2.131t$ (with $g = 1.0$, $g_1 = -0.1$, $\omega = 1.0$, $t = 1.0$).

(ii) Besides the T.B.A. energy $-2t$, the essential negative energy contributions are from the electron–phonon interaction $H_{\mathrm{el-ph}}^{(1)}$ and $H_{\mathrm{el-ph}}^{(2)}$. The intervention of $H_{\mathrm{el-ph}}^{(2)}$ will not only noticeably enlarge the coherent interaction and the interaction of elections with the coherent squeezed fluctuation, but also the representation correlation between $S(z)$ with $D(\alpha)$. More importantly, based on the principles of natural philosophy, we can arrive at the action–counteraction corrections for the ground state energy and the polaron binding energy, in which $-A_0^{(2)}(t)$ corrects from $-0.463t$ to $-0.735t$, $E_0^{(2)}$ corrects from $-2.428t$ to $-2.788t$, $E_{\mathrm{P}}^{(2)}$ from -1.381ω to -1.883ω, respectively. Looking back on the existing theory,[25–31] on the basis of the correlated representation variational approach,[35–41] $E_0^{(2)} = -2.131t$, $E_{\mathrm{P}} = -0.729\omega$; while, with the variational path-integral approach,[25] $E_{\mathrm{P}} = -1.00\omega$ with $g = 1.0$, $g_1 = -0.1$, $\omega = 1.0$, $t = 1.0$, respectively.

(iii) The local squeezed action and the non-local entangled action are not a result of their own action, but coexist with each other. That is, the local squeezed action parameter $\gamma = f_1 f_2$ and the non-local entangled action parameter $(f_1 + f_2)$ are interrelated with $f_1^2 + f_2^2 = 1$. Because the local squeezed action and the non-local entangled action restrict each other's evolution, the evolutions of the lattice quantum fluctuation and the coherent state cannot move forward to collapse and the stability of the ground state is assured. At that time, we will encounter a new phenomenon of the quantum phase coherence, in which the coherent state shows the revival of the collapsed inversion repeatedly.

(iv) The correlation between the hopping electron state and the displaced state of the b_i-phonon remarkably weakens the phonon energy from $\omega\sum_i \tilde{\alpha}_i^2$ to $\omega\sum_i 2\xi\tilde{\alpha}_i^2\psi_i^2$ since $\psi_i^2 \ll 1$. On the other hand, the joint generator variational ansatz $\tilde{U} = \exp\left\{\sum_i\left[\left(zb_i^{+2} - z^* b_i^2\right) + \xi\left(\alpha b_i^+ - \alpha^* b_i\right)c_i^+ c_i\right]\right\}$ not only leads to the renormalized coherent parameter $\alpha_R = \xi\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}\alpha$, but also to the renormalized two-phonon coherent action $H_{\mathrm{CR}}^{(2)} = \xi\frac{\mathrm{e}^{2\gamma}-1}{2\gamma}H_{\mathrm{c}}^{(2)}(\lambda_1)$. With this effect, we yield a more negative ground state and polaron binding

energy. By use of the polaron–soliton state wave function, the Debye–Waller coherent scattering effect is reduced from $te^{-(1/2)\xi^2\alpha(0)^2} \approx 0.375t$ $(g = 1.0, \omega = 1.0, \xi = 1.4)$ to $te^{-(1/2)\xi^2\tilde{\alpha}^{(1)^2}e^{-4\gamma}} \approx t$.

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