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Steering quantum nonlocalities of quantum dot system suffering from decoherence

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The important applications of quantum dot system are to implement logic operations and achieve universal quantum computing based on different quantum nonlocalities. Here, we characterize the quantum steering, Bell nonlocality, and nonlocal advantage of quantum coherence (NAQC) of quantum dot system suffering nonunital and unital channels. The results reveal that quantum steering, Bell nonlocality, and NAQC can display the traits of dissipation, enhancement, and freezing. One can achieve the detections of quantum steering, Bell nonlocality, and NAQC of quantum dot system in different situations. Among these quantum nonlocalities, NAQC is the most fragile, and it is most easily influenced by different system parameters. Furthermore, considering quantum dot system coupling with amplitude damping channel and phase damping channel, these quantum nonlocalities degenerate with the enlargement of the channel parameters t and Γ. Remarkably, measurement reversal can effectively control and enhance quantum steering, Bell nonlocality, and NAQC of quantum dot system suffering from decoherence, especially in the scenarios of the amplitude damping channel and strong operation strength.

Keywords: quantum nonlocalities, quantum dot system, decoherence, steering

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1. Introduction

Based on the background of the Einstein–Podolsky–Rosen (EPR) paradox,[1] quantum steering, as one of significant quantum nonlocalities different from classical world, was first illuminated for two-qubit system in Schrödinger's efforts.[2,3] This quantum nonlocality describes a nonclassical phenomenon that the state of one side for a bipartite state can be steered via implementing measurement on the other side.[2–4] Subsequently, various quantum steering criteria embody seminal part in witnessing quantum steering and performing a variety of quantum information tasks.[5–15] These criteria mainly include steering criteria based on uncertainty relations,[16–21] fine-grained steering criteria,[22] linear and nonlinear steering criteria,[23–25] *etc*. The relevant investigations have been widely carried out in recent years.[26–29] Bell inequalities, including Clauser–Horne–Shimony–Holt (CHSH) inequality,[30] can quantify the strength of another kind of quantum nonlocality that the information can be encoded in the quantum correlations of the measurement outcomes between two parties shared by a two-qubit state.[31,32] The violations of Bell inequalities mean that the Bell nonlocality exists in the two-qubit state. A series of applications in quantum information processing depend on the Bell nonlocality,[33] such as multiparty conference key agreement,[34] communication complexity,[35] and self-testing.[36]

As another promising quantum nonlocality, quantum coherence is used to measure the coherence of quantum states.[37–39] Different quantitative means of quantum coherence were introduced, for instance, the l_1 norm of coherence,[37] the relative entropy of coherence,[37] the robustness of coherence,[40–42] the skew information of coherence.[43,44] The quantum coherence plays a crucial role in quantum biology, quantum phase transitions, quantum thermodynamics, quantum algorithms, quantum metrology, and quantum channel discrimination.[38,39] Of particular note is that considering a complete set of mutually unbiased bases as reference bases and any single-qubit state, the sum of quantum coherence with respect to the three mutually unbiased bases can puncture none of the corresponding upper bounds. If one chooses the l_1 norm of coherence, the relative entropy of coherence, and the skew information of coherence to measure the coherence of any single-qubit state, the corresponding upper bounds are equal to $\sqrt{6}$, 2.23, and 2, respectively. Later on, Mondal *et al*. revealed that the bound can be punctured by the average coherence of subsystem for a bipartite system through local measurements, and considered this quantum nonlocality as the nonlocal advantage of quantum coherence (NAQC).[45] The NAQC can be detected via various co-

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herence measures.[45] For instance, if the average l_1 norm of coherence of subsystem for a bipartite system can puncture corresponding upper bound $\sqrt{6}$, in the system there exists the NAQC. For this reason, the l_1 norm of coherence can be used to capture the NAQC. Several representative efforts have been made in the field NAQC,[46–52] such as the hierarchy between the NAQC and deferent quantum nonlocalities,[47] the detection of NAQC in high-dimensional system,[48] the NAQC under sequential observers,[49] the experimental demonstration of NAQC,[50] and the achievements of NAQC in a variety of quantum systems.[51–54]

It is worth emphasizing that the coupling between the system and environment can accelerate the dissipation of quantum nonlocalities, which brings us great challenges to realize quantum information tasks. As a consequence, it is of essential importance in finding an avenue to battle against the degenerations of quantum nonlocalities. In addition, quantum dot systems have aroused wide attention due to their applications in quantum technologies.[55–64] For this reason, it is necessary to characterize and steer quantum nonlocalities of quantum dot system suffering decoherence. However, that is still lacking. In this work, the quantum steering, Bell nonlocality, and NAQC of quantum dot system coupled with nonunital and unital channels are investigated. The results reveal how the various parameters influence the quantum steering, Bell nonlocality, and NAQC of system. Also, in order to more effectively resist the dissipation of quantum nonlocalities of quantum dot system suffering decoherence so as to be more conducive to the realizations of various quantum information tasks via quantum dot system. We use local measurement reversal to explore the quantum nonlocalities and achieve the increases of quantum steering, Bell nonlocality, and NAQC of quantum dot system under the environmental influences.

The rest of the paper is organized as follows. Quantum dot system is briefly introduced in Section 2. The quantum steering, Bell nonlocality, and NAQC of quantum dot system are examined in Section 3. Considering nonunital and unital channels, the influences of environments on quantum nonlocalities of quantum dot system are investigated in Section 4. In Section 5, the enlargements of quantum steering, Bell nonlocality, and NAQC are realized for quantum dot system suffers decoherence. Finally, some conclusions are drawn in Section 6.

2. Quantum dot system

Considering the Hamiltonian of quantum dot in Ref. [65], which can be given by

$$H_{\rm d}=\sum_{n,s}\varepsilon_n d^{\dagger}_{ns}d_{ns}-E_{\rm S}S^2_{\rm tot}-E_zS^z+E_{\rm C}(N-N_0). \tag{1}$$

Here, $N=\sum_{n,s}d^{\dagger}_{ns}d_{ns}$ represents the total number of electrons in a quantum dot; $N_0=2$ is even integer number of the electron for a Coulomb-blockade valley and can be controlled by the voltage of nearby gate.[66] $E_{\rm C}$ is the charge energy considered here to be on a largest energy scale (for empirically convenient situation), On the average, $\delta E\ll E_{\rm C}$ and δE is the mean level spacing; ε_n is the energy level of the single-particle in lateral quantum dot system (namely, dots have completely broken spatial symmetry), and their distribution obeys Wigner–Dyson ensemble and random matrix theory. Also, assuming electron number is even, we consider the intradot exchange interactions that are in favor of ferromagnetic configurations. Among $E_{\rm C}$, δE, and the interaction strength $E_{\rm S}$ ($E_{\rm S}>0$), there exists an inequality $E_{\rm S}\ll\delta E\ll E_{\rm C}$. We assume the system to have a Coulomb-blockade valley,[67–69] and the number of electrons to be $N_0=2$. The two active orbital levels are denoted by $n=-1$ and $n=+1$. The total spin of dots (for $E_{\rm S}\ll\delta E$) $S=(\sum_{n,s,s'}d^{\dagger}_{ns}\sigma_{ss'}d^{\dagger}_{ns'})/2$ is equal to 0.5 (0) for N is odd (even). Using total spin quantum number S and z-projection S_z, the lowest energy states can be written as ($|S,S^z\rangle$)

$$\begin{aligned}
&|1,1\rangle=d^{\dagger}_{+1\uparrow}d^{\dagger}_{-1\uparrow}|0\rangle\,,|1,-1\rangle=d^{\dagger}_{+1\downarrow}d^{\dagger}_{-1\downarrow}|0\rangle\,,\\
&|1,0\rangle=\frac{1}{\sqrt{2}}(d^{\dagger}_{+1\uparrow}d^{\dagger}_{-1\downarrow}+d^{\dagger}_{+1\downarrow}d^{\dagger}_{-1\uparrow})|0\rangle\,,\\
&|0,0\rangle=d^{\dagger}_{-1\uparrow}d^{\dagger}_{-1\downarrow}|0\rangle\,.
\end{aligned} \tag{2}$$

Here, $|0\rangle$ indicates the ground state for dot with N_0-2 electrons. If a two-level system is close to the singlet-triplet transition, the two-electron quantum dot can be considered as a two-level system. The exact correspondence can be structured after introducing fictitious 1/2-spins (S_1, S_2), namely,

$$\begin{aligned}
&|1,1\rangle\Leftrightarrow|\uparrow_1,\uparrow_2\rangle\,,\quad |1,-1\rangle\Leftrightarrow|\downarrow_1,\downarrow_2\rangle\,,\\
&|1,0\rangle\Leftrightarrow\frac{1}{\sqrt{2}}(|\uparrow_1,\downarrow_2\rangle+|\downarrow_1,\uparrow_2\rangle),\\
&|0,0\rangle\Leftrightarrow\frac{1}{\sqrt{2}}(|\uparrow_1,\downarrow_2\rangle-|\downarrow_1,\uparrow_2\rangle).
\end{aligned} \tag{3}$$

According to these spins, the low-energy Hamiltonian[66] corresponding to Eq. (1) is

$$H=\frac{k_0}{4}S_1\cdot S_2-\gamma B_0S_z. \tag{4}$$

Here, the Zeeman effect is represented by the magnetic field $\boldsymbol{B}=\boldsymbol{B}_0\hat{S}_z$, γ is the dot gyromagnetic ratio. The first term on the right-hand of Eq. (4) indicates an exchange coupling with $k_0=\delta-2E_{\rm S}$ (the energy difference between the single state and the triplet state). If $k_0>0$ and $B_0=0$, the ground state (energy is $E_{0,0}=0$) is a singlet state and the triplet state is three-fold degenerate. If $B_0>0$, the triplet state is split by the magnetic field. As a consequence, the energy levels $E_{0,0}$ and $E_{1,1}$ are degenerative at $B_0=k_0/4\gamma$. The eigenvalues and eigenvectors of the Hamiltonian in Eq. (4) are

$$H|\phi_1\rangle=E_1|\phi_1\rangle=\left(\frac{k_0}{16}+\gamma B_0\right)|\downarrow_1\downarrow_2\rangle\,,$$

$$H|\phi_2\rangle = E_2|\phi_2\rangle = \left(\frac{k_0}{16} - \gamma B_0\right)|\uparrow_1\uparrow_2\rangle,$$
$$H|\phi_3\rangle = E_3|\phi_3\rangle = \frac{k_0}{16\sqrt{2}}(|\uparrow_1\downarrow_2\rangle + |\downarrow_1\uparrow_2\rangle),$$
$$H|\phi_4\rangle = E_4|\phi_4\rangle = \frac{-3k_0}{16\sqrt{2}}(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle). \quad (5)$$

Here, $|\phi_1\rangle$ and $|\phi_2\rangle$ are separable states, $|\phi_3\rangle$ and $|\phi_4\rangle$ are maximally entangled states. Then, we choose $\hbar = 1$ and the Boltzmann constant $K = 1$. At thermal equilibrium and on the standard orthonormal basis, the state of the quantum dot system is given by[70]

$$\rho = \frac{1}{z}\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & c & 0 \\ 0 & c & b & 0 \\ 0 & 0 & 0 & d \end{pmatrix}, \quad (6)$$

with

$$a = \exp\left(-\frac{k_0 - 16\gamma B_0}{16T}\right),$$
$$b = \frac{1}{2}\left(\exp\left(-\frac{k_0}{16T}\right) + \exp\left(\frac{3k_0}{16T}\right)\right),$$
$$c = \frac{1}{2}\left(\exp\left(-\frac{k_0}{16T}\right) - \exp\left(\frac{3k_0}{16T}\right)\right),$$
$$d = \exp\left(-\frac{k_0 + 16\gamma B_0}{16T}\right), \quad r = \gamma B_0. \quad (7)$$

3. Characterizing quantum nonlocalities of quantum dot system

In this work, we use quantum steering criterion based on general entropic uncertainty relation to detect quantum steering. The CHSH inequality and l_1 norm of coherence are used to capture Bell nonlocality and NAQC, respectively. To begin with, we calculate the quantum steering of quantum dot system. $\rho_{AB} = (I_2 \otimes I_2 + \boldsymbol{a}\cdot\boldsymbol{\sigma} \otimes I_2 + I_2 \otimes \boldsymbol{b}\cdot\boldsymbol{\sigma} + \sum_{m,n} c_{mn}\sigma_m \otimes \sigma_n)/4$ $(m,n \in \{x,y,z\})$ is the state shared by Alice and Bob, a set of measurements $A_k \otimes B_k$ is implemented on Alice and Bob, respectively. The quantum steering can be witnessed through the violation of steering criterion based on the general entropic uncertainty relation,[18] viz.

$$\sum_k [S_q(B_k|A_k) + (1-q)C(A_k,B_k)] \geq C_B^{(q)}, \quad (8)$$

with

$$C(A,B) = \sum_i p_i^q [\ln_q(p_i)]^2 - \sum_{i,j} p_{ij}^q \ln_q(p_i)\ln_q(p_{ij}), \quad (9)$$
$$C_B^{(q)} = m\ln_q\left(\frac{md}{d+m-1}\right). \quad (10)$$

Here, $S_q(B|A) = S_q(A,B) - S_q(A)$ is the conditional entropy, $S_q(P) = -\sum_i p_i^q \ln_q(p_i)$ is the Tsallis entropy with $\ln_q(x) = (x^{1-q}-1)/(1-q)$, parameter $q > 1d$ is the dimension of system, m is the number of mutually unbiased bases, and $C_B^{(q)}$ is the bound of steering criterion based on general entropic uncertainty relation for $q \in (0;2]$. Also, steering criterion based on general entropic uncertainty relation can be represented by the form of measurement probability,[18] namely,

$$\frac{1}{q-1}\left[\sum_k\left(1 - \sum_{ij}\frac{(p_{ij}^{(k)})^q}{(p_i^{(k)})^{q-1}}\right)\right] \geq C_B^{(q)}. \quad (11)$$

Here, $p_{ij}^{(k)}$ is the probability of outcome (i,j) for a set of measurements $A_k \otimes B_k$, and $p_i^{(k)}$ is the probability of marginal outcome for measurement A_k. If $q \to 1$, $C_B^{(q)}$ is not optimal for even dimensions, one needs to replace $C_B^{(q)}$ with the bound C_B,[71] and the form of C_B is

$$C_B = \begin{cases} (d+1)\ln\left(\frac{d+1}{2}\right), & d \text{ odd}, \\ \frac{d}{2}\ln\left(\frac{d}{2}\right) + \left(\frac{d}{2}+1\right)\ln\left(\frac{d}{2}+1\right), & d \text{ even}. \end{cases} \quad (12)$$

For brevity, we use the

$$\mathrm{EPR}(\rho_{AB}) = C_B^{(q)} - \sum_k [S_q(B_k|A_k) + (1-q)C(A_k,B_k)] \quad (13)$$

to represent the detectable quantum steering of ρ_{AB} under the steering criterion based on general entropic uncertainty relation. The $\mathrm{EPR}(\rho_{AB}) > 0$ manifests that in the system there must exist the quantum steering. By calculation, one can attain the quantum steering of quantum dot system, that is

$$\begin{aligned}\mathrm{EPR}(\rho) = {} & \frac{2}{(2+b)(a+b)(2+a+2b)z} \\ & \times\{a^3(2+b) + a[6b^3 + 2b(10+c^2-4z) \\ & + b^2(16-3z) + 4(2+c^2-z)] \\ & + 2b[2b^3 + b(8+c^2-3z) \\ & + 2(2+c^2-z) - b^2(z-6)] \\ & + a^2[8+4b^2 - b(z-10) - 2z]\}. \end{aligned} \quad (14)$$

The maximal violation of CHSH inequality can be used to detect Bell nonlocality. The Bell operator is

$$B_{\mathrm{CHSH}} = \boldsymbol{a}\cdot\boldsymbol{\sigma} \otimes (\boldsymbol{b}+\boldsymbol{b}')\cdot\boldsymbol{\sigma} + \boldsymbol{a}'\cdot\boldsymbol{\sigma} \otimes (\boldsymbol{b}-\boldsymbol{b}')\cdot\boldsymbol{\sigma}. \quad (15)$$

Here $\boldsymbol{a}$, $\boldsymbol{a}'$, $\boldsymbol{b}$, and $\boldsymbol{b}'$ are the unit vectors. Hence, the CHSH inequality[30] is

$$\left|\langle B_{\mathrm{CHSH}}\rangle_{\rho_{AB}}\right| = |\mathrm{tr}(\rho_{AB}B_{\mathrm{CHSH}})| \leq 2. \quad (16)$$

For two-qubit X state ρ_{AB}^X, the quantification of Bell nonlocality can be given by[72,73]

$$BN(\rho_{AB}^X) = \mathrm{Max}\left\{0, B_{\mathrm{CHSH}}^{\mathrm{Max}}(\rho_{AB}^X) - 2\right\}, \quad (17)$$

with

$$B_{\mathrm{CHSH}}^{\mathrm{Max}}(\rho_{AB}^X) = 2\sqrt{\sum_{i=x,y,z} t_i^2 - \lambda_{\min}}, \quad (18)$$

where $\lambda_{\min} = \min\{t_x^2, t_y^2, t_z^2\}$ and $t_i = \mathrm{tr}[\rho_{AB}^X \sigma_i \otimes \sigma_i]$. Based on Eqs. (6) and (17), one can obtain Bell nonlocality of quantum dot system

$$\mathrm{BN}(\rho)=\mathrm{Max}\left\{0,\mathrm{Max}\left\{4\sqrt{2}\left|\frac{c}{z}\right|,\frac{2}{|z|}\sqrt{4c^2+(a-2b+d)^2}\right\}-2\right\}. \tag{19}$$

Considering $\{|k_i\rangle\}$ as reference base, the l_1 norm of coherence[37] can be characterized by

$$C_{l_1}(\rho_{AB})=\sum_{i\neq j}\left|\langle k_i|\rho_{AB}|k_j\rangle\right|. \tag{20}$$

After Alice performs measurement on particle A of ρ_{AB}, she announces her choice base σ_i and measurement outcome $a\in\{0,1\}$ (with probability $p(a|\sigma_i)=\mathrm{tr}[(\Pi_i^a\otimes I)\rho_{AB}]$, and $\Pi_i^a=[I+(-1)^a\sigma_i]/2$) to Bob. After that, Bob randomly measures the coherence of particle B in the eigenbasis of σ_j $(j\neq i)$. The average coherence of Bob's conditional state $\rho_{B|\sigma_i^a}$ is[45]

$$\bar{C}_{l_1}^{\sigma_j}(\{p(a|\sigma_i),\rho_{B|\sigma_i^a}\})=\sum_a p(a|\sigma_i)C_{l_1}^{\sigma_j}(\rho_{B|\sigma_i^a}), \tag{21}$$

with $\rho_{B|\sigma_i^a}=\mathrm{tr}_A[(\Pi_i^a\otimes I\rho_{AB})/p(a|\sigma_i)]$, and $C_{l_1}^{\sigma_j}$ is the l_1 norm of coherence quantified in the eigenbasis of σ_j. If one considers all measurements σ_i $(i=x,y,x)$ implemented by Alice, NAQC can be achieved by using l_1 norm of coherence.[45] That is to say, the violation of inequality

$$C_{l_1}^{na}(\rho_{AB})=\frac{1}{2}\sum_{i,j,a,i\neq j}p(a|\sigma_i)C_{l_1}^{\sigma_j}(\rho_{B|\sigma_i^a})\leq C_{l_1}^{m}, \tag{22}$$

signifies the existence of NAQC in the state ρ_{AB}. Here, the bound $C_{l_1}^{m}=\sqrt{6}$. Thereby, the NAQC of quantum dot system can be given by

$$\mathrm{NAQC}(\rho)=C_x+C_y+C_z-\sqrt{6}, \tag{23}$$

with

$$C_x=C_y=\left|\frac{c}{z}\right|+\frac{1}{2|z|}\sqrt{4c^2+(a-d)^2}), \tag{24}$$

$$C_z=\frac{a}{z}\left|\frac{a-b}{a+b}\right|+\frac{d}{z}\left|\frac{b-d}{b+d}\right|+\frac{b}{z}\left(\left|\frac{a-b}{a+b}\right|+\left|\frac{b-d}{b+d}\right|\right). \tag{25}$$

In order to investigate the influences of temperature T on quantum steering, Bell nonlocality, and NAQC, we characterize the dependence of quantum steering, Bell nonlocality, and NAQC on temperature T in Fig. 1. As demonstrated from Fig. 1(a), the quantum steering, Bell nonlocality, and NAQC sharply degenerate withthe increase of temperature after a short freezing process. In contrast, the NAQC of system is most vulnerable, and it is easily affected by enhancing temperature. The sudden death of NAQC takes place at lower temperature. The reasons for the results can be explained as follows. The state of quantum dot system can be described by $\rho=\sum_{i=1}^4 p_i|\phi_i\rangle\langle\phi_i|$.[70] Here, $p_i=\mathrm{e}^{-E_i/KT}/\mathrm{tr}(\mathrm{e}^{-H/T})$ is the probability distribution. For the settings of parameters in Fig. 1(a), the state of quantum dot system lies in $|\phi_4\rangle$ (*i.e.* maximally entangled states) at $T=0$. For this reason, different nonlocalities reach the corresponding maximum values at $T=0$ in Fig. 1. After a short process of increasing temperature, state $|\phi_4\rangle$ of quantum dot system gradually mixes with the higher energy levels $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$. The degree of mixing is enhanced with temperature rising. The results are responsible for the situations that quantum steering. The Bell nonlocality and NAQC sharply dissipate with the increase of temperature after a short freezing process. If parameter k_0 is increased to 7, as displayed in Fig. 1(b), quantum steering, Bell nonlocality, and NAQC can effectively battle against the effect of temperature. To be clarity, the three quantum nonlocalities experience a relatively long freezing process, and these traits are different form the ones in the scenario of $k_0=3$. The reason of freezing these quantum nonlocalities is that the increase of k_0 effectively restrains the mixing of the state $|\phi_4\rangle$ and the higher energy levels ($|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$) at low temperature. In addition, the degenerative tendencies of these quantum nonlocalities in the case of $k_0=7$ are slower than the ones in the case of $k_0=3$. That is to say, the enlargement of k_0 can effectively resist the effects of temperature on the quantum steering, Bell nonlocality, and NAQC.

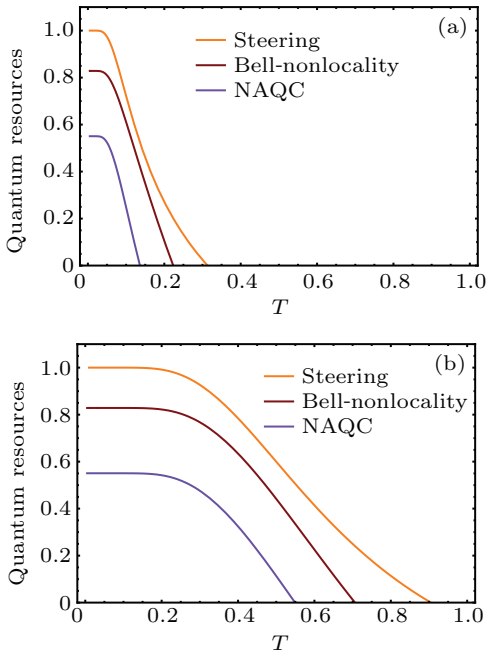


Fig. 1. Curves of quantum steering, Bell nonlocality, and NAQC with respect to temperature T for (a) $k_0=3$ and $r=0.5$, and (b) $k_0=7$ and $r=0.5$.

Now, we come to focus our attention on quantum steering, Bell nonlocality, and NAQC of quantum dot system under different parameter r, as depicted in Fig. 2. As described in Fig. 2(a), the increase of r cannot affect quantum steering nor Bell nonlocality nor NAQC in the initial stage. These quantum nonlocalities are frozen at different fixed values, respectively. However, quantum steering, Bell nonlocality, and NAQC sharply decrease if their r values reach corresponding

critical values, and disappear eventually. The results can be understood as follows. The quantum dot system is in almost maximally entangled state $|\phi_4\rangle$ in the case of ($k_0 = 7$, $T = 0.1$, $r = 0$), and the increase of r cannot break the situation. Consequently, the quantum steering, Bell nonlocality, and NAQC are frozen in the initial stage. The mixing ratios of $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$ (especially separable states $|\phi_2\rangle$) in quantum dot system are conspicuously enhanced when their r values reach corresponding critical values. Hence these quantum nonlocalities sharply dissipate and disappear eventually. The NAQC, as the most fragile quantum nonlocality, dies at weaker r. The influences of r on quantum nonlocalities under the condition of $T = 0.5$ (Fig. 2(b)) are different from those under the condition of $T = 0.1$ (Fig. 2(a)) The increase of temperature from 0.1 to 0.5 leads to the fact that the state $|\phi_4\rangle$ of quantum dot system mixes with $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$ at $r = 0$. The mixing ratios of $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$ (especially $|\phi_2\rangle$) increase with the growing r. The mechanisms are responsible for the facts that the freezing phenomena of quantum nonlocalities are destroyed by increasing the temperature. The quantum steering, Bell nonlocality, and NAQC gradually decay with the enlargement of r in Fig. 2(b).

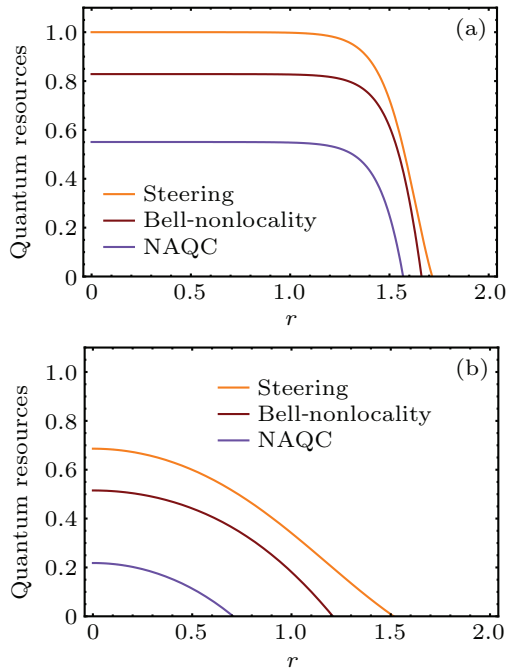


Fig. 2. Curves of quantum steering, Bell nonlocality, and NAQC with respect to parameter r for (a) $k_0 = 7$ and $T = 0.1$, and (b) $k_0 = 7$ and $T = 0.5$.

At the last stage, the dependence of quantum steering, Bell nonlocality, and NAQC on parameter k_0 are characterized in Fig. 3. It is demonstrated that the quantum dot system cannot achieve quantum steering nor Bell nonlocality nor NAQC at weak k_0. The system can only achieve these quantum nonlocalities if k_0 reaches the critical values, respectively. After that, the quantum steering, Bell nonlocality, and NAQC are enhanced with k_0 growing. Finally, the quantum steering, Bell nonlocality, and NAQC are frozen due to the strong parameter k_0. The mechanisms of the above phenomena can be formulated as follows. If the k_0 is weak, the state of quantum dot system is dominated by the separable state $|\phi_2\rangle$, therefore, the three quantum nonlocalities cannot be detected in this situation. Of particular note is that the mixing ratios of $|\phi_2\rangle$ and $|\phi_4\rangle$ in the state of quantum dot system will be swapped by increasing k_0. The state of quantum dot system is gradually dominated by the maximally entangled state $|\phi_4\rangle$ with k_0 increasing. Thus, quantum steering, Bell nonlocality, and NAQC can be captured at critical values of k_0. Subsequently, these quantum nonlocalities strengthen as k_0 goes up. Finally, these quantum nonlocalities are frozen at strong k_0. Also, one can find from Figs. 3(a) and 3(b) that the increase of r cannot affect the tendencies nor characteristics of different quantum nonlocalities, but can affect only the critical value k_0 of the quantum nonlocality.

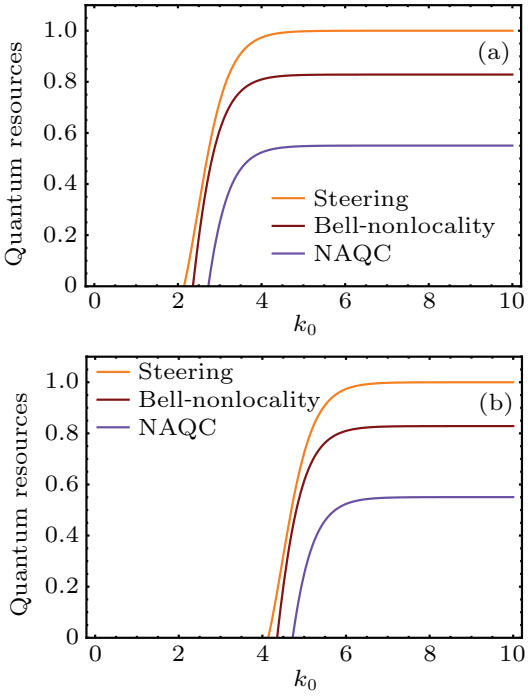


Fig. 3. Curves of quantum steering, Bell nonlocality, and NAQC with respect to parameter k_0 for (a) $T = 0.1$ and $r = 0.5$, and (b) $T = 0.1$ and $r = 1$.

4. Investigating quantum nonlocalities of quantum dot system under decoherence

In this section, we observe the quantum steering, Bell nonlocality, and NAQC of system influenced by different decoherence channels. Considering that particle A and particle B of the quantum dot system are subjected to the effect of amplitude damping (AD) channel (a nonunital noise). The Kraus operators of AD channel can be written as[74]

$$K_1^{\mathrm{AD}} = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, \quad K_2^{\mathrm{AD}} = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}, \tag{26}$$

where $p = 1 - \mathrm{e}^{-\Gamma t}$. The output state can be obtained as follows:

$$\rho_{\rm AD}=\frac{1}{z}\begin{pmatrix} a+2bp+dp^2 & 0 & 0 & 0\\ 0 & (1-p)(b+dp) & c(1-p) & 0\\ 0 & c(1-p) & (1-p)(b+dp) & 0\\ 0 & 0 & 0 & d(p-1)^2 \end{pmatrix}. \tag{27}$$

The detectable quantum steering of $\rho_{\rm AD}$ can be calculated and indicated below:

$$\begin{aligned}{\rm EPR}(\rho_{\rm AD}) =& \frac{2(p-1)}{(2+b)(2+a+2b)^2(a+b+2p+bp)}\\ &\times\{4b^2(1+b)-2bc^2(2+b)\\ &+4p[b(1+b)(4+2b+b^2)-(2+b)c^2]\\ &+2p^2[4(1+b)(2+2b+b^2)+(2+b)^2c^2]\\ &+a^2(4+b-2p)(b+2p)\\ &+2a[2c^2(p-1)+8p+b^3(1+p)\\ &+b(4-c^2+pc^2+12p)\\ &+2b^2(3+p+p^2)]\}. \end{aligned}\tag{28}$$

The Bell nonlocality of $\rho_{\rm AD}$ is

$${\rm BN}(\rho_{\rm AD})={\rm Max}\{B^1_{\rm AD},B^2_{\rm AD}\}-2, \tag{29}$$

with

$$B^1_{\rm AD}=4\sqrt{2}(1-p)\left|\frac{c}{z}\right|, \tag{30}$$

$$B^2_{\rm AD}=2\left|\frac{1}{z}\right|\sqrt{\begin{aligned}&a^2+d^2+4b^2(1-2p)^2+4bd(2p-1)^3+2a(2p-1)[2b+d(2p-1)]\\ &+4(p-1)[c^2(p-1)+2d^2p(1-2p+2p^2)]\end{aligned}}. \tag{31}$$

The NAQC of system can be represented by

$${\rm NAQC}(\rho_{\rm AD})=C^x_{\rm AD}+C^y_{\rm AD}+C^z_{\rm AD}-\sqrt{6}, \tag{32}$$

with

$$C^x_{\rm AD}=C^y_{\rm AD}=(1-p)\left|\frac{c}{z}\right|+\frac{1}{2|z|}\sqrt{a^2-2ad+4c^2(p-1)^2+4ap(b+d)+[d-2p(b+d)]^2}, \tag{33}$$

$$\begin{aligned}C^z_{\rm AD} =& \frac{1}{z}\left\{a\left|\frac{a+dp(2p-1)+b(3p-1)}{a+b+p(b+d)}\right|+d\left[(1-p)\left|\frac{b-d+2dp}{b+d}\right|+p\left|\frac{a+dp(2p-1)+b(3p-1)}{a+b+p(b+d)}\right|\right]\right.\\ &\left.+b\left[(1-p)\left|\frac{b-d+2dp}{b+d}\right|+(1+p)\left|\frac{a+dp(2p-1)+b(3p-1)}{a+b+p(b+d)}\right|\right\}.\end{aligned} \tag{34}$$

To examine the influences of AD channel on quantum steering, Bell nonlocality, and NAQC of quantum dot system, we herein provide quantum steering, Bell nonlocality, and NAQC each as a function of channel parameter t as indicated in Fig. 4. Our results reveal that the quantum steering, Bell nonlocality, and NAQC decrease as t goes up. The effect of AD channel gives rise to the fact that the freezing phenomena of quantum steering, Bell nonlocality, and NAQC cannot take place in quantum dot system. Besides this, the increase of Γ from 0.5 to 2 as indicated in Figs. 4(a) and 4(b) enhances the influences of AD channel on quantum nonlocalities, and induces the results that the sudden death of quantum steering, Bell nonlocality, and NAQC occur more rapidly. By investigating, we reveal that the eigenvector of $\rho_{\rm AD}$ is consistent with that of ρ. That is to say, the $\rho_{\rm AD}$ can also be characterized by the probabilistic combination of $|\phi_i\rangle$ $(i=1,2,3,4)$. The above phenomenon can be attributed to the mechanism that the coupling between noise channel and quantum dot system reduces (increases) the mixing ratio of $|\phi_4\rangle$ ($|\phi_2\rangle$) in the state of quantum dot system, and enhances the dissipation of information stored in quantum dot system. The stronger the channel parameters t and Γ, the more obvious the coupling is.

Now, we turn our attention to investigating the quantum steering, Bell nonlocality, and NAQC of quantum dot system suffering unital channel, namely, phase damping (PD) channel. The Kraus operators of PD channel are[74]

$$K^{\rm PD}_1=\begin{pmatrix}1 & 0\\ 0 & \sqrt{1-p}\end{pmatrix},\quad K^{\rm PD}_2=\begin{pmatrix}0 & 0\\ 0 & \sqrt{p}\end{pmatrix}, \tag{35}$$

with $p=1-{\rm e}^{-\Gamma t}$. After the A particle and B particle of ρ are subjected to the effect of PD channel, the final state is

$$\rho_{\rm PD}=\frac{1}{z}\begin{pmatrix}a & 0 & 0 & 0\\ 0 & b & c(1-p) & 0\\ 0 & c(1-p) & b & 0\\ 0 & 0 & 0 & d\end{pmatrix}. \tag{36}$$

The $\rho_{\rm PD}$ can also be represented by the probabilistic combination of $|\phi_i\rangle$ $(i=1,2,3,4)$. One can obtain the detectable

quantum steering of $\rho_{\rm PD}$,

$$\begin{aligned}\mathrm{EPR}(\rho_{\rm PD}) = & \frac{1}{(2+b)(a+b)(2+a+2b)^2} \\ & \times[-2b(2+a+2b)(2b+4a+4b) \\ & +4c^2(1-p)^2(2+b)(a+b)]. \end{aligned} \tag{37}$$

Bell nonlocality of $\rho_{\rm AD}$ in the form

$$\mathrm{BN}(\rho_{\rm PD}) = \mathrm{Max}\{B^1_{\rm PD}, B^2_{\rm PD}\} - 2, \tag{38}$$

with

$$B^1_{\rm PD} = 4\sqrt{2}(1-p)\left|\frac{c}{z}\right|, \tag{39}$$

$$B^2_{\rm PD} = \frac{2}{|z|}\sqrt{(a-2b+d)^2+4c^2(p-1)^2}. \tag{40}$$

The NAQC can be written as

$$\mathrm{NAQC}(\rho_{\rm PD}) = C^x_{\rm PD} + C^y_{\rm PD} + C^z_{\rm PD} - \sqrt{6}, \tag{41}$$

$$\begin{aligned} C^x_{\rm PD} &= C^y_{\rm PD} \\ &= \frac{1}{2|z|}\left[\sqrt{(a-d)^2+4c^2(p-1)^2}+(1-p)|c|\right], \end{aligned} \tag{42}$$

$$C^z_{\rm PD} = \frac{a}{z}\left|\frac{a-b}{a+b}\right| + \frac{d}{z}\left|\frac{b-d}{b+d}\right| + \frac{b}{z}\left(\left|\frac{a-b}{a+b}\right| + \left|\frac{b-d}{b+d}\right|\right). \tag{43}$$

Figure 5 shows the curves of quantum steering, Bell nonlocality, and NAQC with respect to parameter t of PD channel. The dynamics and mechanisms of these quantum nonlocalities under the influence of PD channel are similar to those under the influence of AD channel in Fig. 4. Quantum steering, Bell nonlocality, and NAQC degrade with increasingincreasing. The enlargement of Γ suppresses the amplitude of quantum steering, Bell nonlocality, and NAQC.

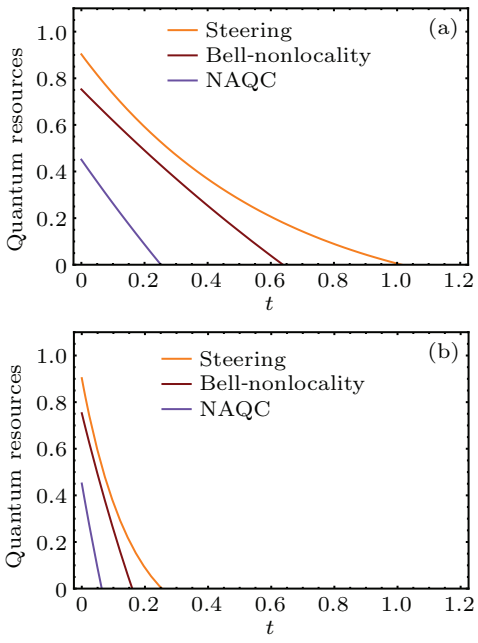


Fig. 4. Curves of quantum steering, Bell nonlocality, and NAQC with respect to channel parameter t at $k_0 = 5$, $r = 0.5$, $T = 0.2$: (a) $\Gamma = 0.5$ and (b) $\Gamma = 2$.

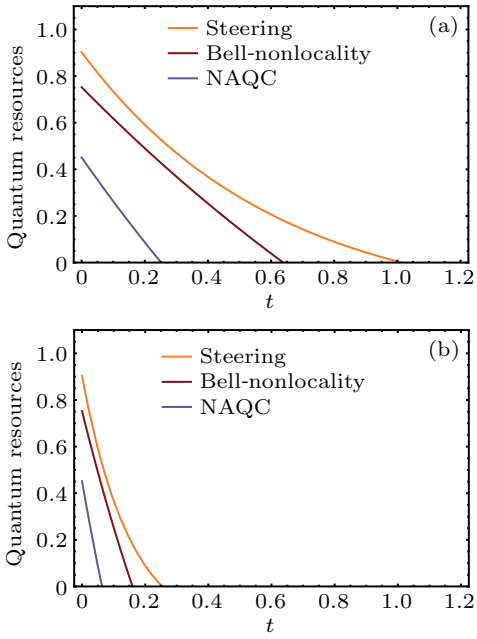


Fig. 5. Couves of quantum steering, Bell nonlocality, and NAQC with respect to channel parameter t at $k_0 = 5$, $r = 0.5$, $T = 0.2$, and $\Gamma = 0.5$ (a) and $\Gamma = 1$ (b).

5. Controlling quantum nonlocalities for quantum dot system suffering decoherence

The results obtained above reveal that the AD and PD noise channels strongly influence quantum steering, Bell nonlocality, and NAQC of the quantum dot system. In this scenario, we investigate the lifting quantum steering, Bell nonlocality, and NAQC for the quantum dot system coupling with AD channel and PD channel by implementing measurement reversal. The measurement reversal is one of uncollapsed partial measurements, and the operator can be expressed as[75]

$$E = \begin{pmatrix} \sqrt{1-k} & 0 \\ 0 & 1 \end{pmatrix}, \tag{44}$$

where k ($0 \le k \le 1$) is the operator's strength of the measurement. To begin with, we pay attention to controlling quantum nonlocalities of quantum dot system suffering the AD noise channel. For this purpose, we implement the measurement reversal on particles A and B of $\rho_{\rm AD}$ (as shown in Eq. (27)). The final state is given by $\rho_{\rm AD-r}$, and the nonzero matrix elements of $\rho_{\rm AD-r}$ are

$$\rho^{11}_{\rm AD-r} = \frac{(k-1)^2[a+p(2b+dp)]}{a(k-1)^2+(kp-1)[2b(k-1)+d(kp-1)]}, \tag{45}$$

$$\begin{aligned}\rho^{22}_{\rm AD-r} &= \rho^{33}_{\rm AD-r} \\ &= \frac{(k-1)(p-1)(b+dp)}{a(k-1)^2+(kp-1)[2b(k-1)+d(kp-1)]}, \end{aligned} \tag{46}$$

$$\rho^{44}_{\rm AD-r} = \frac{d(p-1)^2}{a(k-1)^2+(kp-1)[2b(k-1)+d(kp-1)]}, \tag{47}$$

$$\begin{aligned}\rho^{23}_{\rm AD-r} &= \rho^{32}_{\rm AD-r} \\ &= \frac{c(k-1)(p-1)}{a(k-1)^2+(kp-1)[2b(k-1)+d(kp-1)]}. \end{aligned} \tag{48}$$

One can derive the detectable quantum steering of $\rho_{\mathrm{AD}-r}$, viz.

$$\mathrm{EPR}(\rho_{\mathrm{AD}-r}) = -\frac{2(k-1)(p-1)}{A_1B_1C_1} \times \{D_1+2a(k-1)E_1+2F_1\}, \quad (49)$$

with

$$A_1 = 2kp-2+b(k-1), \quad (50)$$
$$B_1 = a(k-1)-2p-b(1+p)+2kp(b+p), \quad (51)$$
$$C_1 = [a(k-1)^2+2(kp-1)(-1-b+bk+kp)]^2, \quad (52)$$
$$D_1 = a^2(k-1)^3(b+2p)[b(k-1)+2(-2+p+kp)], \quad (53)$$
$$E_1 = b^3(k-1)^2(-1-p+2kp) +2(kp-1)[c^2(-1+k+p-kp)+4p(kp-1)^2] +b[-c^2(k-1)^2(p-1)+4(kp-1)^2(-1-3p+4kp)] +2b^2(k-1)[3+p(1+p-7k-3kp+5k^2p)], \quad (54)$$
$$F_1 = 2b^4p(k-1)^3(kp-1) +2b^3(k-1)(kp-1)(-1-p+2kp)(-1-2p+3kp) +4p(kp-1)^2[c^2(-1+k+p-kp)+2p(kp-1)^2] +2b(kp-1)(-1-2p+3kp) \times[c^2(-1+k+p-kp)+4p(kp-1)^2] +b^2\{-c^2(k-1)^2(p-1)(-1-p+2kp)+2(kp-1)^2 \times[1+6p+6p^2+kp(-8-18p+13kp)]\}. \quad (55)$$

The Bell nonlocality of $\rho_{\mathrm{AD}-r}$ can be given by

$$\mathrm{BN}(\rho_{\mathrm{AD}-r}) = \mathrm{Max}\{B^1_{\mathrm{AD}-r}, B^2_{\mathrm{AD}-r}\}-2, \quad (56)$$

with

$$B^1_{\mathrm{AD}-r} = 4\sqrt{2}(1-p)(1-k)\left|\frac{c}{a(k-1)^2+(kp-1)[2b(k-1)+d(kp-1)]}\right|, \quad (57)$$
$$B^2_{\mathrm{AD}-r} = 2\sqrt{\frac{4c^2(k-1)^2(p-1)^2+\{a(k-1)^2+(1-2p+kp)[d+2b(k-1)+dp(k-2)]\}^2}{\{a(k-1)^2+(kp-1)[2b(k-1)+d(kp-1)]\}^2}}. \quad (58)$$

The NAQC of $\rho_{\mathrm{AD}-r}$ can be attained by

$$\mathrm{NAQC}(\rho_{\mathrm{AD}-r}) = C^x_{\mathrm{AD}-r}+C^y_{\mathrm{AD}-r}+C^z_{\mathrm{AD}-r}-\sqrt{6}, \quad (59)$$

with

$$C^x_{\mathrm{AD}-r} = C^y_{\mathrm{AD}-r} = \frac{1}{|a(k-1)^2+(kp-1)[2b(k-1)+d(kp-1)]|}\Bigg\{(1-p)(1-k)\,|c| +\frac{1}{2}\sqrt{\begin{array}{l}a^2(k-1)^4+4c^2(k-1)^2(p-1)^2+2a(k-1)^2[2bp(k-1)^2+d(-1+2p-2kp^2+k^2p^2)]\\ +[2bp(k-1)^2+d(-1+2p-2kp^2+k^2p^2)]^2.\end{array}}\Bigg\}, \quad (60)$$

$$C^z_{\mathrm{AD}-r} = \frac{1}{a(k-1)^2+(kp-1)[2b(k-1)+d(kp-1)]}\Bigg\{(k-1)[-b+a(k-1)-(b+d)p+kp(2b+dp)] \times\left|\frac{b+a(k-1)+bp(2k-3)+db(1-2p+kp)}{a+b-ak+p(b+d)-kp(2b+dp)}\right|+(p-1)[(kb-b)+(kpd-d)]\left|\frac{d+b(k-1)+dp(k-2)}{b(k-1)+d(kp-1)}\right|\Bigg\}. \quad (61)$$

Next, we come to probe the quantum dot system suffering the PD channel. If one perform the measurement reversal on particle A and particle B of ρ_{PD}, the final state is represented by $\rho_{\mathrm{PD}-r}$, and the corresponding nonzero matrix elements are

$$\rho^{11}_{\mathrm{PD}-r} = \frac{a(k-1)^2}{d-2b(k-1)+a(k-1)^2}, \quad (62)$$
$$\rho^{22}_{\mathrm{PD}-r} = \rho^{33}_{\mathrm{PD}-r} = \frac{b(k-1)}{d-2b(k-1)+a(k-1)^2}, \quad (63)$$
$$\rho^{44}_{\mathrm{PD}-r} = \frac{d}{d-2b(k-1)+a(k-1)^2}, \quad (64)$$
$$\rho^{23}_{\mathrm{PD}-r} = \rho^{32}_{\mathrm{PD}-r} = \frac{c(k-1)(p-1)}{d-2b(k-1)+a(k-1)^2}. \quad (65)$$

One can attain the detectable quantum steering of $\rho_{\mathrm{PD}-r}$, namely,

$$\mathrm{EPR}(\rho_{\mathrm{PD}-r}) = -\frac{2(k-1)}{A_2}\{B_2-C_2\}, \quad (66)$$

$$A_2 = [-2+b(k-1)][2-2b(k-1) +a(k-1)^2]^2(a+b-ak), \quad (67)$$
$$B_2 = a^2b(-4+bk-b)(k-1)^3 -2b\{2b^2(k-1)+b[-2+c^2(k-1)^2(p-1)^2] -2c^2(k-1)(p-1)^2\}, \quad (68)$$
$$C_2 = 2a(k-1)\{-6b^2(k-1)+b^3(k-1)^2 -b[-4+c^2(k-1)^2(p-1)^2] +2c^2(k-1)(p-1)^2\}. \quad (69)$$

The Bell nonlocality of $\rho_{\mathrm{PD}-r}$ is expressed as

$$\mathrm{BN}(\rho_{\mathrm{PD}-r}) = \mathrm{Max}\{B^1_{\mathrm{PD}-r}, B^2_{\mathrm{PD}-r}\} - 2, \tag{70}$$

$$B^1_{\mathrm{PD}-r} = 4\sqrt{2}(1-p)(1-k)\left|\frac{c}{d-2b(k-1)+a(k-1)^2}\right|, \tag{71}$$

$$B^2_{\mathrm{PD}-r} = 2\sqrt{\frac{[d+2b(k-1)+a(k-1)^2]^2+4c^2(k-1)^2(p-1)^2}{[d-2b(k-1)+a(k-1)^2]^2}}. \tag{72}$$

The NAQC of $\rho_{\mathrm{PD}-r}$ can be denoted as

$$\mathrm{NAQC}(\rho_{\mathrm{PD}-r}) = C^x_{\mathrm{PD}-r} + C^y_{\mathrm{PD}-r} + C^z_{\mathrm{PD}-r} - \sqrt{6}, \tag{73}$$

with

$$\begin{aligned} C^x_{\mathrm{PD}-r} &= C^y_{\mathrm{PD}-r} \\ &= \frac{1}{|d-2b(k-1)+a(k-1)^2|}\left[(k-1)(p-1)\,|c| + \frac{1}{2}\sqrt{d^2+2ad(k-1)^2+a^2(k-1)^4+4c^2(k-1)^2(p-1)^2}\right], \end{aligned} \tag{74}$$

$$\begin{aligned} C^z_{\mathrm{PD}-r} = \frac{1}{d-2b(k-1)+a(k-1)^2}\Bigg\{ & a(k-1)^2\left|\frac{b+a(k-1)}{a+b-ak}\right| + d\left|\frac{d+b(k-1)}{b+d-bk}\right| \\ & - b(k-1)\left[\left|\frac{b+a(k-1)}{a+b-ak}\right| + \left|\frac{d+b(k-1)}{b+d-bk}\right|\right]\Bigg\}. \end{aligned} \tag{75}$$

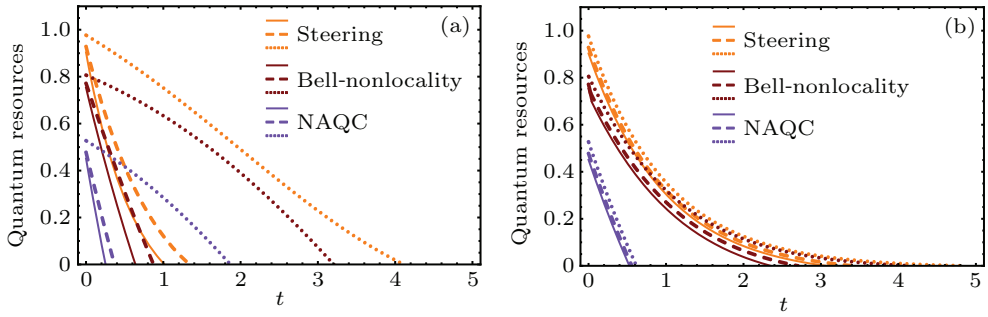


Fig. 6. Curves of quantum steering, Bell nonlocality, and NAQC with respect to channel parameter t at $k_0 = 5$, $r = 0.5$, $T = 0.2$, and $\Gamma = 0.5$, indicating the results of (a) AD channel and (b) PD channel, with solid curves denoting the results of $k = 0$, dashed curves for the results of $k = 0.3$, and dotted curves for the results of $k = 0.9$.

In order to demonstrate the effects of measurement reversal on quantum nonlocalities of quantum dot system in AD channel and PD channel, we characterize quantum steering, Bell nonlocality, and NAQC of system as a function of channel parameter t under different operation strengths ($k = 0, 0.3, 0.9$) of measurement reversal in Fig. 6. It is important to mention here that the measurement reversal does affect neither of the eigenvectors of the system, viz. the $\rho_{\mathrm{AD}-r}$ and $\rho_{\mathrm{PD}-r}$ are still the probabilistic combination of $|\phi_i\rangle$ $(i = 1,2,3,4)$, respectively. Owing to the fact that the measurement reversal can significantly increase the mixing ratio of $|\phi_4\rangle$ in the state of quantum dot system suffering the AD channel, and effectively suppress the dissipation of information stored in quantum dot system. Accordingly, the measurement reversal can effectively enhance the quantum steering, Bell nonlocality, and NAQC of quantum dot system influenced by the AD channel, which are revealed in Fig. 6(a). The stronger the operation strength, the more obvious the enhancement of quantum nonlocality is. If operation strength k increases to 0.9, the amplitude of quantum steering, Bell nonlocality, and NAQC are remarkably strengthened. The degenerating trends of these quantum nonlocalities become relatively gentle. These traits are responsible for the delay of sudden death of quantum nonlocality, and the quantum nonlocalities of quantum dot system can effectively restrain the influence of AD noise with the help of the measurement reversal. Additionally, the measurement reversal can also realize the increases of quantum steering, Bell nonlocality, and NAQC of quantum dot system suffering PD channel, as exhibited in Fig. 6(b). Note that compared with the results of AD channel in Fig. 6(a), the influences of measurement reversal on different quantum nonlocalities of quantum dot system under PD channel are not significant. Even if the operation strength k increases to 0.9, the enlargement of quantum steering, Bell nonlocality, and NAQC are not obvious. The results can be explicated as follows. Even if the operation strength k is strong, the measurement reversal cannot effectively influence the mix-

ing ratio of $|\phi_1\rangle$, $|\phi_2\rangle$, $|\phi_3\rangle$, and $|\phi_4\rangle$ in the state of quantum dot system suffering PD channel, and cannot obviously suppress the dissipation of information stored in quantum dot system either. For this reason, measurement reversal is more effective for steering quantum nonlocalities of quantum dot system suffering the AD channel.

6. Conclusions

The quantum steering, Bell nonlocality, and NAQC of quantum dot system coupling with decoherence are investigated in this work. The results reveal that the quantum steering, Bell nonlocality, and NAQC of quantum dot system can be frozen by reducing temperature. The degeneration of parameter k_0 can destroy these freezing phenomena at different temperatures. The increases of k_0 can help the quantum steering, Bell nonlocality, and NAQC to resist the influence of temperature. The enlargement of parameter r cannot affect quantum steering nor Bell nonlocality nor NAQC at low temperature. These quantum nonlocalities sharply decline at critical values of r. The freezing phenomena of quantum nonlocalities at different values of r disappear via enhancing the temperature. The quantum steering, Bell nonlocality, and NAQC gradually strengthen with k_0 increasing. And these quantum nonlocalities are frozen ultimately. One cannot capture the quantum steering nor Bell nonlocality nor NAQC of quantum dot system at high temperature, strong r, and weak k_0. The NAQC, as the most fragile quantum nonlocality, is most easily affected by different parameters of system. Also, it turns out that quantum steering, Bell nonlocality, and NAQC decrease as the parameter t of AD channel and PD channel go up. The enlargement of channel parameter Γ results in the fact that the sudden death of quantum steering, Bell nonlocality, and NAQC take place more rapidly. It is worth emphasizing that the measurement reversal can effectively control and strengthen the quantum steering, Bell nonlocality, and NAQC of quantum dot system suffering noise channel. The stronger the operation strength, the more effective the protection of quantum nonlocality is. The influences of measurement reversal on these quantum nonlocalities of quantum dot system coupling with AD channel are stronger than those of quantum dot system coupling with PD channel.

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