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Citation: Chin. Phys. B, 2022, 31 (**7**): 070304. DOI: 10.1088/1674-1056/ac4cbc
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Photon blockade in a cavity–atom optomechanical system

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(Received 9 December 2021; revised manuscript received 17 January 2022; accepted manuscript online 19 January 2022)

We study the single-photon blockade (1PB), two-photon blockade (2PB), and photon-induced tunneling (PIT) effects in a cavity–atom optomechanical system in which a two-level atom is coupled to a single-model cavity field via a two-photon interaction. By analyzing the eigenenergy spectrum of the system, we obtain a perfect 1PB with a high occupancy probability of single-photon excitation, which means that a high-quality and efficient single-photon source can be generated. However, PIT often occurs in many cases when we consider 2PB in analogy to 1PB. In addition, we find that a 2PB region will present in the optomechanical system, which can be proved by calculating the correlation function of the model analytically.

Keywords: photon blockade, photon-induced tunneling, cavity–atom optomechanical system

PACS: 03.67.Hk, 14.70.Bh, 42.50.–p, 42.50.Pq **DOI:** 10.1088/1674-1056/ac4cbc

1. Introduction

Single-photon sources and single-photon detectors have great potential applications in the quantum information field, such as precision measurement, quantum cryptography, and quantum key distribution.[1,2] In recent years, it has been proposed that photon blockade (PB) effects, which describe a phenomenon that a single photon could block the transmission of other photons in some systems, can be exploited to prepare single-photon sources.[3–5] There are two general ideas regarding PB effects: (i) conventional PB (CPB), which originates in the anharmonicity of the energy spectrum,[6] and (ii) unconventional PB (UPB), which results from the destructive quantum interference between the different possible transition paths.[7] On the one hand, the CPB effect, which requires some kind of large nonlinearity, has been researched in various quantum systems, such as Kerr-type nonlinear cavities[8] and optomechanical systems.[9,10] The UPB effect, on the other hand, usually requires a large coupling strength between the two components that structure the different transition paths. Based on this theory, the UPB effect has also been observed in a coupled quantum-dot-cavity system,[11] double-cavity optomechanical systems,[12] and other systems.[13–16] Previous studies have been focused mainly on the single-photon blockade (1PB). Recently, the two-photon blockade (2PB) has been experimentally[17] and theoretically[18,19] investigated in various configurations, which could be used for two-photon sources. Furthermore, the photon-bunching effect, namely, photon-induced tunneling (PIT), which means the probability of admitting subsequent photons is increased when there is one photon inside the cavity,[19] has also been explored in many systems.[20–23]

Great success has been achieved in cavity optomechanics in the quantum information field over the past decades. Cavity optomechanics is widely used in optomechanical oscillators, displacement sensing, Newtonian gravity detection, and other fields of quantum physics.[24] In addition, research attention has been given to the reaction effect of mechanical motion on the optical field, such as laser noise cooling,[25] optomechanical entanglement,[26] optomechanically induced transparency,[27,28] and PB.[9,10,29,30] Herein, we focus on studying the PB effect in a cavity–atom optomechanical system. By analyzing the resonance condition, we analytically optimize the parameter condition of the respective 1PB case. In addition, PIT usually appears more easily than a 2PB. Furthermore, by calculating the correlation functions, we find that a 2PB region would occur in the optomechanical system.

The rest of this paper is organized as follows. In Section 2, we discuss the physical model and investigate the Hamiltonian of the system. In Section 3, we analyze the eigenstates to obtain 1PB and optimize the parameters to obtain 2PB. The result can be confirmed by analytically solving the Schrödinger equation or numerically simulating the master equation. The atom-driving case is also discussed briefly. Conclusions are given in Section 4.

2. System and Hamiltonian

We consider a cavity–atom optomechanical system composed of a single-mode cavity field coupled to a two-level atom via a two-photon physical interaction,[31,32] as depicted in Fig. 1(a). The cavity is pumped by a classical laser with frequency $\omega_{\rm d}$ and amplitude ε. The interaction between an optical cavity and mechanical film in a cavity optomechanical coupling system is realized by photon radiation pressure.[33,34] The Hamiltonian of the model is ($\hbar = 1$)[34–39]

$$H_1 = \omega_a a^+ a + \omega_0 \sigma^+ \sigma + g\left(a^{+2}\sigma + a^2\sigma^+\right) + \omega_m b^+ b$$

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$$-\lambda a^{+}a\left(b^{+}+b\right)+\varepsilon\left(a^{+}\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{d}}t}+a\mathrm{e}^{\mathrm{i}\omega_{\mathrm{d}}t}\right), \quad (1)$$

where a (a^{+}) denotes the annihilation (creation) operator for the cavity with resonance frequency ω_a. The lowering (up) operator of the two-level atom is denoted σ (σ^{+}); the transition frequency of the atom ω_0; g is the coupling strength between the cavity field and two-level atom via the two-photon interaction; b (b^{+}) represents the annihilation (creation) operators for the optical (mechanical) mode with frequency ω_m; λ is the single-photon opto-mechanical coupling strength due to the radiation pressure,[40] which describes one optical cavity coupled to the mechanical resonator[41–43] ($\lambda=\frac{\omega_a}{L}\sqrt{\frac{1}{m\omega_m}}$, with L being the cavity length and m representing effective mass of the movable mirror[34,39]). In the rotating frame defined by $U_1=\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{d}}\left(a^{+}a+\sigma_z\right)t}$, the Hamiltonian in Eq. (1) can be written as

$$\begin{aligned}H &= \Delta_a a^{+}a+\Delta_0\sigma^{+}\sigma+g\left(a^{+2}\sigma+a^{2}\sigma^{+}\right)\\ &\quad+\omega_m b^{+}b-\lambda a^{+}a\left(b^{+}+b\right)+\varepsilon\left(a^{+}+a\right). \quad (2)\end{aligned}$$

The driving de-tunings $\Delta_a=\omega_a-\omega_{\mathrm{d}}$ and $\Delta_0=\omega_0-2\omega_{\mathrm{d}}$ are usually controlled by the observed PB effect. To deal with the fourth and fifth terms, in the rotating frame defined by $U_2=\mathrm{e}^{\frac{\lambda}{\omega_m}a^{+}a\left(b^{+}-b\right)}$ for Eq. (2), the transformed Hamiltonian is

$$\begin{aligned}H_2 &= \Delta_a a^{+}a+\Delta_0\sigma^{+}\sigma+g\left(a^{+2}\sigma+a^{2}\sigma^{+}\right)+\omega_m b^{+}b\\ &\quad-\frac{\lambda^2}{\omega_m}\left(a^{+}a\right)^2+\varepsilon\left(a^{+}+a\right). \quad (3)\end{aligned}$$

The Hamiltonian without a driving field in Eq. (3) can be written as

$$H_0=\Delta_a a^{+}a+\Delta_0\sigma^{+}\sigma+g\left(a^{+2}\sigma+a^{2}\sigma^{+}\right)-\frac{\lambda^2}{\omega_m}\left(a^{+}a\right)^2. \quad (4)$$

The part of the mechanical resonator can be ignored under the weak optomechanical coupling condition ($\lambda\ll\omega_m$) when we only care about the optical properties of the system.

Therefore, the separable state could be described as $|0,g\rangle$, $|1,g\rangle$, $|2,g\rangle$, $|3,g\rangle$, $|0,e\rangle$, $|1,e\rangle$ in the n-excitation ($n\leq 3$) subspace.[44,45] The eigenstates, as described in Fig. 1(b), can be calculated by the Hamiltonian in Eq. (4) and the eigenequation can be obtained as $H_0|\varepsilon_0\rangle=\varepsilon_0|\varepsilon_0\rangle$, $|\varepsilon_0\rangle=|0,g\rangle$, $\varepsilon_0=0$; $H_0|\varepsilon_1\rangle=\varepsilon_1|\varepsilon_1\rangle$, $|\varepsilon_1\rangle=|1,g\rangle$, $\varepsilon_1=\Delta_a-\frac{\lambda^2}{\omega_m}$; $H_0|\varepsilon_m\rangle=\varepsilon_m|\varepsilon_m\rangle$ ($m=2\pm,3\pm$) there. The eigenstates can help to obtain the best parameters of CPB because of the underlying principles.

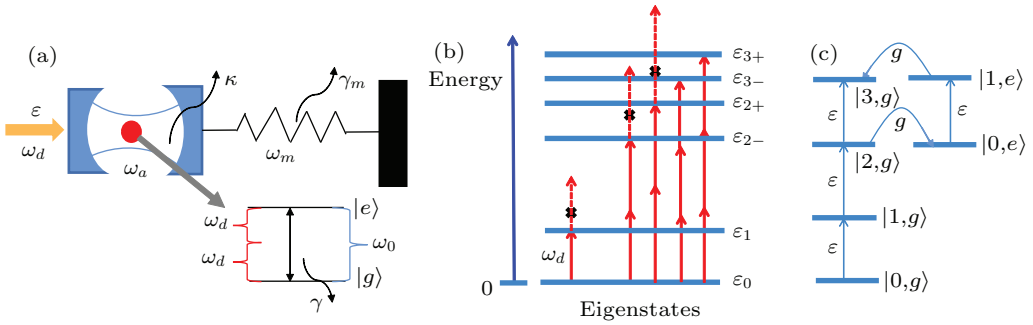


Fig. 1. (a) Sketch of the cavity–atom optomechanical system (cavity-driven), with κ, γ, and γ_m being the cavity, atomic decay rates, and damping rate of the mechanical resonator, respectively. (b) Schematic energy-level structure of the system in the subspace with zero, one, two, and three excitations. (c) Transition pathways in the cavity-driven case.

3. Photon blockade in the cavity–atom optomechanical system

3.1. Analytical solution

Considering the influence of the dissipations of the cavity field and the atom, we add the imaginary decay and gain terms into the original Hamiltonian [Eq. (4)] to obtain the non-Hermitian Hamiltonian.[46,47] The dynamical evolution of the photon state is calculated by the non-Hermitian Schrödinger equation $\mathrm{i}\hbar\frac{\partial}{\partial t}|\psi(t)\rangle=H_{\mathrm{eff}}|\psi(t)\rangle$, where

$$\begin{aligned}H_{\mathrm{eff}} &= \left(\Delta_a-\mathrm{i}\frac{\kappa}{2}\right)a^{+}a+\left(\Delta_0-\mathrm{i}\frac{\gamma}{2}\right)\sigma^{+}\sigma\\ &\quad+g\left(a^{+2}\sigma+a^{2}\sigma^{+}\right)-\frac{\lambda^2}{\omega_m}\left(a^{+}a\right)^2+\varepsilon\left(a^{+}+a\right). \quad (5)\end{aligned}$$

In the weak driving limit, we can restrict the total photon number within the low-excitation subspace up to 3. Therefore, the wave function $|\psi(t)\rangle$ can be expressed by the time-separable state approximately as follows:[48]

$$\begin{aligned}|\psi(t)\rangle &= C_{0,g}(t)|0,g\rangle+C_{1,g}(t)|1,g\rangle+C_{2,g}(t)|2,g\rangle\\ &\quad+C_{0,e}(t)|0,e\rangle+C_{3,g}(t)|3,g\rangle+C_{1,e}(t)|1,e\rangle. \quad (6)\end{aligned}$$

In fact, $\left|C_{0,g}\right|\gg\left|C_{1,g}\right|\gg\left\{\left|C_{0,e}\right|,\left|C_{2,g}\right|\right\}\gg\left\{\left|C_{3,g}\right|,\left|C_{1,e}\right|\right\}$, and we can assume $C_{0,g}\simeq 1$ due to the weak-driving assumption. Then, the steady-state can be obtained by $\frac{\partial}{\partial t}C_{s,j}=0$ ($s=0,1,2,3$, $j=g,e$). Therefore, it is easy to obtain solutions of the probability amplitudes as follows:

$$C_{0,g}\simeq 1,\quad C_{1,g}=-\frac{\varepsilon}{\Delta_a-\mathrm{i}\frac{\kappa}{2}-\frac{\lambda^2}{\omega_m}},$$

$$
\begin{aligned}
C_{2,g} &= \frac{\sqrt{2}\varepsilon^2\left(\Delta_0-\mathrm{i}\frac{\gamma}{2}\right)}{\left(\Delta_a-\mathrm{i}\frac{\kappa}{2}-\frac{\lambda^2}{\omega_m}\right)\left[\left(2\Delta_a-\mathrm{i}\kappa-4\frac{\lambda^2}{\omega_m}\right)\left(\Delta_0-\mathrm{i}\frac{\gamma}{2}\right)-2g^2\right]},\\
C_{0,e} &= \frac{2g\varepsilon^2}{\left[2g^2-\left(2\Delta_a-\mathrm{i}\kappa-4\frac{\lambda^2}{\omega_m}\right)\left(\Delta_0-\mathrm{i}\frac{\gamma}{2}\right)\right]\left(\Delta_a-\mathrm{i}\frac{\kappa}{2}-\frac{\lambda^2}{\omega_m}\right)},\\
C_{3,g} &= \frac{\sqrt{6}\varepsilon^3\left(\Delta_a-\mathrm{i}\frac{\kappa}{2}+\Delta_0-\mathrm{i}\frac{\gamma}{2}-\frac{\lambda^2}{\omega_m}\right)\left(\Delta_0-\mathrm{i}\frac{\gamma}{2}\right)+2\sqrt{6}g^2\varepsilon^3}{M\left(\Delta_a-\mathrm{i}\frac{\kappa}{2}-\frac{\lambda^2}{\omega_m}\right)\left[\left(2\Delta_a-\mathrm{i}\kappa-4\frac{\lambda^2}{\omega_m}\right)\left(\Delta_0-\mathrm{i}\frac{\gamma}{2}\right)-2g^2\right]},\\
C_{1,e} &= \frac{6g\varepsilon^3\left(\Delta_0-\mathrm{i}\frac{\gamma}{2}\right)+2g\varepsilon^3\left(3\Delta_a-\mathrm{i}\frac{3}{2}\kappa-9\frac{\lambda^2}{\omega_m}\right)}{M\left(\Delta_a-\mathrm{i}\frac{\kappa}{2}-\frac{\lambda^2}{\omega_m}\right)\left[\left(2\Delta_a-\mathrm{i}\kappa-4\frac{\lambda^2}{\omega_m}\right)\left(\Delta_0-\mathrm{i}\frac{\gamma}{2}\right)-2g^2\right]},
\end{aligned}
\tag{7}
$$

with

$$
M=6g^2-\left(\Delta_a-\mathrm{i}\frac{\kappa}{2}+\Delta_0-\mathrm{i}\frac{\gamma}{2}-\frac{\lambda^2}{\omega_m}\right)\left(3\Delta_a-\mathrm{i}\frac{3}{2}\kappa-9\frac{\lambda^2}{\omega_m}\right).
$$

The PB effect can be estimated by the equal-time second-order correlation function $g^2(0)$ and the equal-time third-order correlation function $g^3(0)$. Based on Eq. (7), the steady-state correlation functions of the cavity can be analytically obtained by the steady-state solution of the Schrödinger equation:

$$
\begin{aligned}
g_a^2(0) &= \frac{\langle a^+a^+aa\rangle}{\langle a^+a\rangle^2}\\
&= \frac{2\left|C_{2,g}\right|^2+6\left|C_{3,g}\right|^2}{\left(\left|C_{1,g}\right|^2+2\left|C_{2,g}\right|^2+3\left|C_{3,g}\right|^2\right)^2}\\
&\approx \frac{2\left|C_{2,g}\right|^2}{\left|C_{1,g}\right|^4},\\
g_a^3(0) &= \frac{\langle a^+a^+a^+aaa\rangle}{\langle a^+a\rangle^3}\\
&= \frac{6\left|C_{3,g}\right|^2}{\left(\left|C_{1,g}\right|^2+2\left|C_{2,g}\right|^2+3\left|C_{3,g}\right|^2\right)^3}\\
&\approx \frac{6\left|C_{3,g}\right|^2}{\left|C_{1,g}\right|^6}.
\end{aligned}
\tag{8}
$$

Note that if $\langle\hat{n}\rangle \ll 1$ (the average photon number), when $g^2(0)<1$, it indicates 1PB; when $g^2(0)>1$ and $g^3(0)<1$, 2PB occurs, which implies two-photon bunching and three-photon anti-bunching, $g^2(0)>1$ and $g^3(0)>1$ indicate typical PIT.[19] In the weak-driving case, the photon number is very small.

3.2. Numerical solution

However, we utilize the quantum master equation with the initial Hamiltonian H to correctly account for the driven-dissipative character of the system, which is also a method of exact solution:[49–51]

$$
\begin{aligned}
\frac{\partial\rho}{\partial t} &= \mathrm{i}[\rho,H]+\frac{1}{2}\kappa(\bar{n}_{tha}+1)L[a]\\
&\quad+\frac{1}{2}\kappa(\bar{n}_{tha})L\left[a^+\right]+\frac{1}{2}\gamma(\bar{n}_{th1}+1)L[\sigma]\\
&\quad+\frac{1}{2}\gamma(\bar{n}_{th1})L\left[\sigma^+\right]+\frac{1}{2}\gamma_m(\bar{n}_{thb}+1)L[b]\\
&\quad+\frac{1}{2}\gamma_m(\bar{n}_{th1})L\left[b^+\right],
\end{aligned}
\tag{9}
$$

where the Hamiltonian H is expressed in Eq. (2), $L[o]=2o\rho o^+-o^+o\rho-\rho o^+o$ are Lindblad terms accounting for the arbitrary system operator o; κ, γ, and γ_m are the cavity, atomic decay rates, and damping rate of the mechanical resonator, respectively; $\bar{n}_{thx}=\left[\exp\left(\hbar\omega_x/k_{\mathrm{B}}T_x\right)-1\right]^{-1}$, $x=\{a,1,b\}$ are the average thermal excitation numbers of the bath at temperatures T_a and $T_1\,(x=\{a,1\})$, and the mechanical thermal excitation number at the temperature $T_b\,(x=b)$, with k_{B} the Boltzmann constant.

In the system, we are concerned about $g^2(0)$ and $g^3(0)$ in the steady state, and must thus solve the master equation for the steady-state density operator by setting $\partial\rho/\partial t=0$. The exact numerical result is given by

$$
\begin{aligned}
g_a^2(0) &= \mathrm{Tr}\left(a^+a^+aa\rho_s\right)/\left[\mathrm{Tr}\left(a^+a\rho_s\right)\right]^2,\\
g_a^3(0) &= \mathrm{Tr}\left(a^+a^+a^+aaa\rho_s\right)/\left[\mathrm{Tr}\left(a^+a\rho_s\right)\right]^3.
\end{aligned}
\tag{10}
$$

However, there is an obvious deviation between the two methods in the higher-order correlation functions. This is because we neglect the influence of quantum jumps in solving the effective Hamiltonian.[18]

3.2.1. Single-photon blockade

The PB effect is usually divided into CPB and UPB. The CPB effect results from the anharmonicity of the eigenenergy spectrum, as shown in Fig. 1(b). This means that the driving frequency ω_{d} matches the energy separation ω_a between the zero-excitation state ε_0 and the single-excitation state ε_1. In addition, the single-photon transition $|\varepsilon_0\rangle\rightarrow|\varepsilon_1\rangle$ becomes resonant, but the subsequent transitions $|\varepsilon_1\rangle\rightarrow|\varepsilon_{2\pm}\rangle$ induced by the second photon are blockaded. As in Fig. 1(c), there are two transition pathways from ε_0 to ε_3, and the UPB effect usually occurs due to the destructive quantum-interference phenomenon between them. It is always expressed as $\left|C_{2,g}\right|=0$, which means that there are no two-photon probabilities in the

cavity. In our system, the parameter condition for the perfect 1PB effect ($|C_{2,g}| = 0$) can be obtained as $\Delta_0 = 0, \gamma = 0$, which can be explained based on the destructive quantum interference between the two paths $|\varepsilon_0\rangle \to |\varepsilon_{2\pm}\rangle$.[31] If we analogize 1PB in the UPB effect to consider 2PB, the requirement is not satisfied. The result, $|C_{3,g}| = 0$, only ensures that one of the factors, $g^3(0) < 1$ [the other factor is $g^2(0) > 1$], could not be satisfied.

In studying the 1PB effect in this system, the dips shown in Fig. 2 can be obtained by the optimal relations $\varepsilon_0 - \varepsilon_1 = 0$ in the resonant transitions between the ground and first-excited states [Fig. 1(b)].[52,53] We can obtain the best parameter $\Delta_a = \frac{\lambda^2}{\omega_m}$ for the 1PB effect from previous calculations (Section 2). In Fig. 2, we plot the equal-time second-order correlation function $g^2(0)$ and the equal-time third-order correlation function $g^3(0)$. It is clear that the dips are in $\Delta_a = 0$ [Fig. 2(a)], 0 [Fig. 2(b)], and 1 [Fig. 2(c)]. Owing to the constraint of approximate calculation conditions ($\lambda \ll \omega_m$), the parameter conditions of the dips are basically distributed around zero. In Fig. 2(a), it can be seen that the correlation function $g^3(0)$ exhibits some fluctuations near -2. This is because of the quantum interference between the two transitions $|\varepsilon_0\rangle \to |\varepsilon_{2\pm}\rangle$. We calculate the logarithm of the correlation function to observe the extreme point in Fig. 2(c).

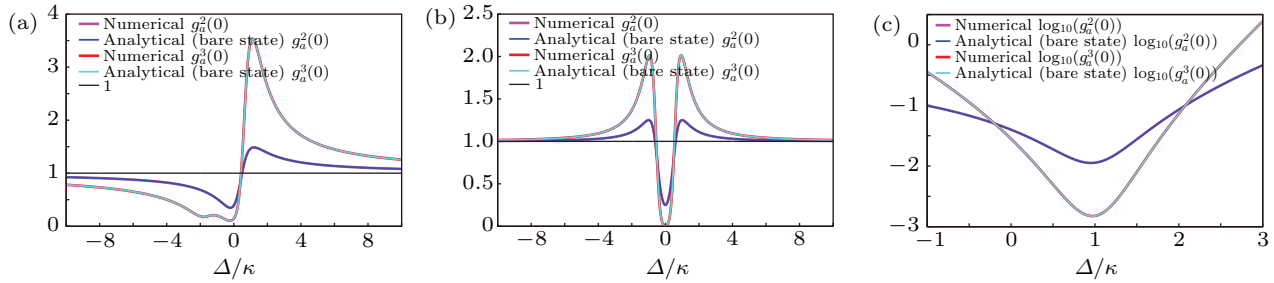


Fig. 2. Correlation functions of the cavity. The pink ($g^2(0)$) and red ($g^3(0)$) lines are plotted using the numerical solution of Eq. (10), while the blue ($g^2(0)$) and light cyan ($g^3(0)$) curves correspond to the exact analytical result in Eq. (8). (a) $\varepsilon = 0.01\kappa$, $g = 0.7\kappa$, $\gamma = \kappa$, $\Delta_a = \Delta$, $\Delta_0 = \kappa$, $\lambda = 0$, $\omega_m = 100\kappa$, $\gamma_m = \frac{\omega_m}{10^6}$. (b) $\varepsilon = 0.01\kappa$, $g = 0.5\kappa$, $\gamma = \kappa$, $\lambda = 0$, $\omega_m = 100\kappa$, $\gamma_m = \frac{\omega_m}{10^6}$, $\Delta_a = \Delta_0 = \Delta$. (c) $\varepsilon = 0.01\kappa$, $g = 2\kappa$, $\gamma = \kappa$, $\Delta_a = \Delta$, $\Delta_0 = \kappa$, $\lambda = 100\kappa$, $\omega_m = 10000\kappa$, $\gamma_m = \frac{\omega_m}{10^6}$.

3.2.2. Two-photon blockade

We analyzed the 2PB effect as follows. When the two-photon transitions ($|\varepsilon_0\rangle \to |\varepsilon_{2\pm}\rangle$) become resonant, the parameter condition can be obtained by $\varepsilon_0 - \varepsilon_{2\pm} = 0$ [in the resonant case, $\omega_0 = 2\omega_a, \Delta_a = \frac{1}{2}\Delta_0 = \frac{\lambda^2}{\omega_m} \pm \sqrt{\left(\frac{\lambda^2}{\omega_m}\right)^2 + \frac{1}{2}g^2}$]. Figure 3(a) clearly shows $g^2(0) > 1$ and $g^3(0) > 1$ at the point of $\Delta_a = -1.4042$ and $\Delta_a = 1.4242$ (we calculate the logarithm of the function), which represents PIT, not 2PB. However, the result of calculating the resonance could obtain the peak of photon number probability. In Fig. 3(b), the probability of a single photon reaching the maximum at the point of $\Delta_a = 0.0100$ corresponds to the dip in Fig. 3(a) ($\Delta_a = \frac{\lambda^2}{\omega_m}$). The same thing can be observed in Fig. 3(c) for the probability of two-photon blockade ($\Delta_a = -1.4042$ and $\Delta_a = 1.4242$). Therefore, it is not determined that the 2PB effect will occur in two-photon transitions becoming resonant.

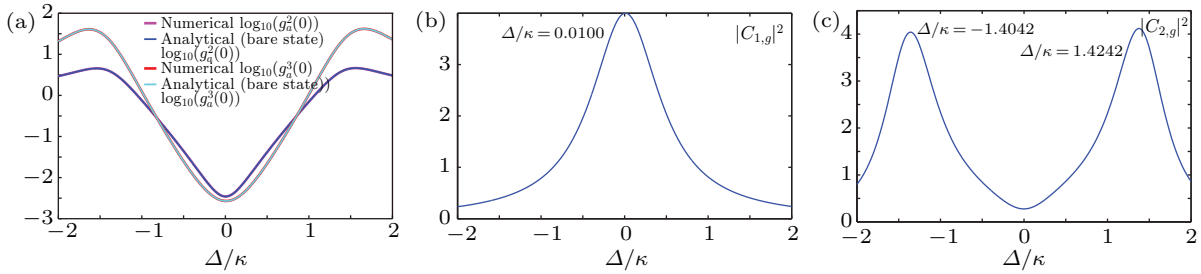


Fig. 3. Photon transition resonance, $\frac{1}{2}\Delta_0 = \Delta_a = \Delta$, $\varepsilon = 0.01\kappa$, $g = 2\kappa$, $\gamma = \kappa$, $\lambda = 10\kappa$, $\omega_m = 10000\kappa$, $\gamma_m = \frac{\omega_m}{10^6}$. (a) Correlation function: the pink ($g^2(0)$) and red ($g^3(0)$) lines are plotted using the numerical solution of Eq. (10), while the blue ($g^2(0)$) and light cyan ($g^3(0)$) curves correspond to the exact analytical result in Eq. (8). (b) and (c) Probabilities of (b) single-photon and (c) two-photon blockades.

Next, we discuss the 2PB effect in our system in detail. The previous studies on cavity–atom systems generally achieve the 1PB effect. Even if the 2PB effect is considered, it mostly ends with the result of the PIT effect (under cavity driving). We also calculate the results in a cavity–atom system. Figure 4 shows the difference between the cavity–atom system and the cavity–atom optomechanical system. It is clear that a region with 2PB [within the red wireframe area in Figs. 4(b) and 4(e)] appeared by comparing Figs. 4(a) [Fig. 4(d)] and 4(b) [Fig. 4(e)], which can be explained based on the optomechanical coupling strength. Mathematically, it is difficult to calculate the equation $g^2(0) = g^3(0)$ to make

the intersection of $g^2(0)$ and $g^3(0)$ be below 1 in the cavity–atom system [Figs. 4(a) and 4(d)]. However, we can build a 2PB region by adding a mechanical system to increase the value of the intersection of correlation functions. To check the correctness, we introduced the relative deviation of a given photon-number distribution from the corresponding Poisson distribution that is defined as $[P(n)-\mathcal{P}(n)]/\mathcal{P}(n)$,[54,55] where $\mathcal{P}(n)=\mathrm{e}^{-\langle a^+a\rangle}\frac{\langle a^+a\rangle^n}{n!}$ denotes the probability of an n-photon state under the Poisson distribution, and $P(n)$ is the photon-number distribution probability. Figures 4(c) and 4(f) show the probability of the two-photon case exceeding the Poisson distribution, which can be concluded to be the 2PB case at the points of $\Delta_a=0.57$ and $\Delta_a=0.7$, respectively. The 2PB effect has been observed in Ref. [17], which uses the parameters $\frac{\kappa}{2\pi}=2.0$ MHz, $\frac{\gamma}{2\pi}=3.0$ MHz, $\frac{\Delta}{2\pi}=18.0$ MHz, $\frac{g}{2\pi}=16.38$ MHz, $\frac{\varepsilon}{2\pi}=0.55$ MHz. These ratios between the parameters and the cavity decay rate κ are slightly different in order of magnitudes from the parameters we give. The mechanical system could be controlled by $\lambda=\frac{\omega_a}{L}\sqrt{\frac{1}{m\omega_m}}$. Therefore, this scheme is feasible in real applications.

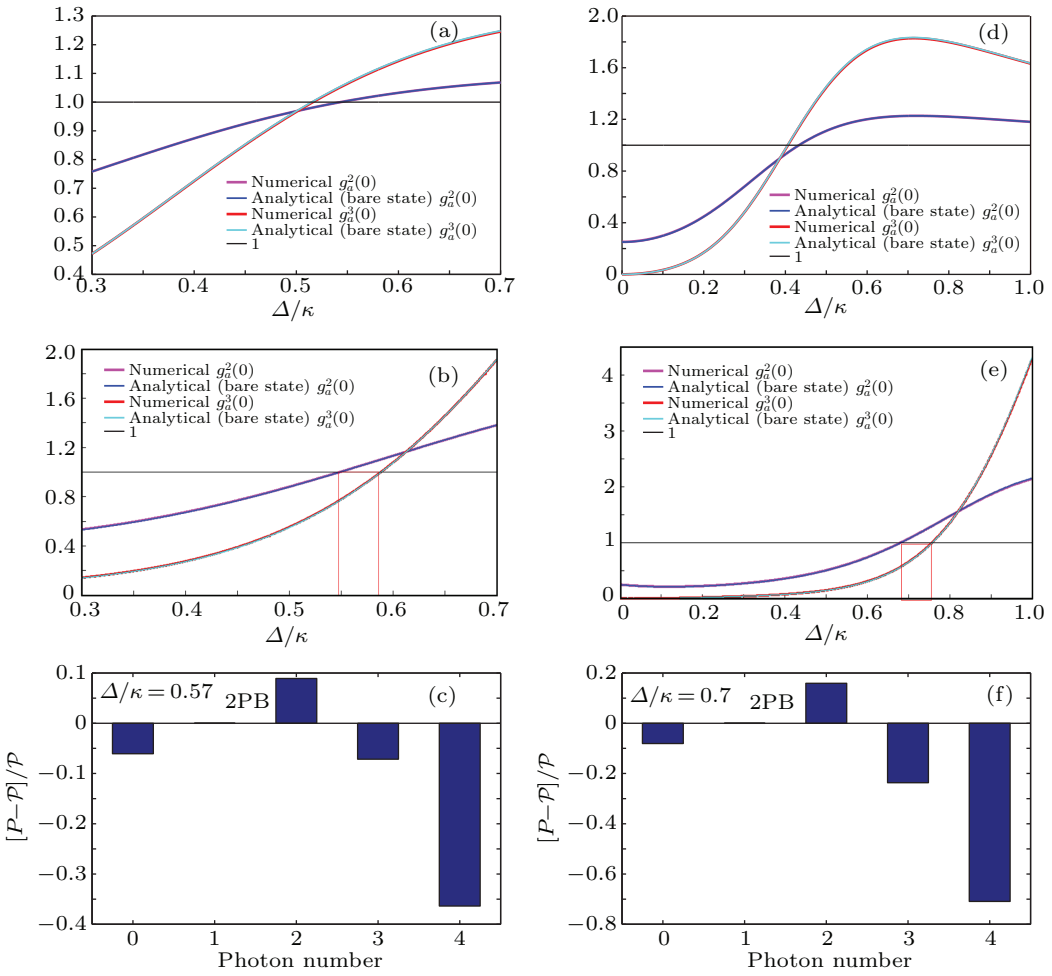


Fig. 4. Correlation function and relative deviation of the cavity. The pink ($g^2(0)$) and red ($g^3(0)$) lines are plotted using the numerical solution of Eq. (10), while the blue ($g^2(0)$) and light cyan ($g^3(0)$) curves correspond to the exact analytical result in Eq. (8). (a) The cavity–atom system: $\varepsilon=0.03\kappa$, $g=0.3\kappa$, $\gamma=\kappa$; (b) cavity–atom optomechanical system: $\lambda=5\kappa$, $\omega_m=100\kappa$, $\gamma_m=\frac{\omega_m}{10^6}$; (c) $\Delta_a=\Delta_0=\Delta=0.57$; (d) cavity–atom system: $\varepsilon=0.03\kappa$, $g=0.5\kappa$, $\gamma=\kappa$; (e) cavity–atom optomechanical system: $\lambda=6\kappa$, $\omega_m=100\kappa$, $\gamma_m=\frac{\omega_m}{10^6}$; (f) $\Delta_a=\Delta=0.7$, $\Delta_0=2\Delta_a$.

3.3. Discussion

We next consider the case of atom driving, which seems a good project by which to obtain the 2PB effect.[31] However, we calculate $C_{1,g}=0$ in an analytical solution, which represents that $g^2(0)$ and $g^3(0)$ are relatively large and that PIT appeared. Owing to the two-photon physical transition process, the judgment of the 2PB effect can rely on the criterion $g^4(0)$, which is calculated by the master equation. In the cavity–atom system, 2PB may emerge because of the transition resonance ($\omega_0=2\omega_a, |\varepsilon_0\rangle\rightarrow|\varepsilon_{2\pm}\rangle, |\varepsilon_0\rangle\rightarrow|\varepsilon_{4\pm}\rangle$). However, our scheme could build a region of the 2PB effect under cavity driving.

Moreover, cavity optomechanics could also achieve the 2PB effect, but $|C_2|\neq 0$ in this system, which means that the perfect 1PB effect could not be obtained. In other words, cavity optomechanics could not use interference to enhance the PB effect. Our system could use this phenomenon, which is briefly discussed in Subsection 3.2.1.

4. Conclusions

In summary, we have investigated the 1PB, 2PB, and PIT effects in a cavity–atom optomechanical system. By analyzing the transition resonance, we find that the 1PB and PIT actions can be observed easily in the cavity–atom system, while 2PB is difficult to realize. Therefore, adding an optomechanical system could help us build a 2PB region. Moreover, we briefly discuss the PIT effect that may occur under atom driving. Our analysis results are obtained by solving the equations of motion for these probability amplitudes and are confirmed by solving the quantum master equation in the truncated Hilbert space.

References

[1] Eisaman M D, Fan J, Migdall A and Polyakov S V 2011 *Rev. Sci. Instrum.* **82** 071101
[2] Hadfield and Robert H 2009 *Nat. Photon.* **3** 696
[3] Miranowicz A, Paprzycka M, Liu Y X , Bajer J and Nori F 2012 *Phys. Rev. A* **87** 1110
[4] Zou F, Liao J Q and Liao D G 2014 *Opt. Express* **28** 16175
[5] Imamoğlu A, Schmidt H, Woods G and Deutsch M 1997 *Phys. Rev. Lett.* **79** 1467
[6] Birnbaum K M, Boca A, Miller R, Boozer A D, Northup T E and Kimble H J 2005 *Nature* **436** 87
[7] Flayac H and Savona V 2017 *Phys. Rev. A* **96** 053810
[8] Lin H Y, Wang X Q, Yao Z H and Zou D D 2020 *Opt. Express* **28** 17643
[9] Wang D Y, Bai C H, Han X, Liu S, Zhang S and Wang H F 2014 *Opt. Lett.* **45** 2604
[10] Wang D Y, Bai C H, Liu S, Zhang S and Wang H F 2014 *Phys. Rev. A* **99** 043818
[11] Snijders H J, Frey J A, Norman J, Flayac H, Savona V, Gossard A C, Bowers J E, van Exter M P, Bouwmeester D and Löffler W 2018 *Phys. Rev. Lett.* **121** 043601
[12] Stannigel K, Komar P, Habraken S J M, Bennett S D, Lukin M D, Zoller P and Rabl P 2012 *Phys. Rev. Lett.* **109** 013603
[13] Liew T C H and Savona V 2010 *Phys. Rev. Lett.* **104** 183601
[14] Lemonde M A, Didier N and Clerk A A 2014 *Phys. Rev. A* **90** 063824
[15] Gerace D and Savona V 2014 *Phys. Rev. A* **89** 031803
[16] Zhou Y H, Shen H Z and Yi X X 2015 *Phys. Rev. A* **92** 023838
[17] Hamsen C, Tolazzi K N, Wilk T and Rempe G 2017 *Phys. Rev. Lett.* **118** 133604
[18] Bin Q, Lü X Y, Bin S W and Wu Y 2018 *Phys. Rev. A* **98** 043858
[19] Kowalewska K A, Abo S I, Chimczak G, Peřina J J, Nori F and Miranowicz A 2019 *Phys. Rev. A* **100** 053857
[20] Xu X W, Li Y J and Liu Y X 2013 *Phys. Rev. A* **87** 025803
[21] Majumdar A, Bajcsy M, Rundquist A and Vučković J 2012 *Phys. Rev. Lett.* **108** 183601
[22] Rundquist A, Bajcsy M, Majumdar A, Sarmiento T, Fischer K, Lagoudakis Konstantinos G, Buckley S, Piggott Alexander Y and Vučković J 2014 *Phys. Rev. A* **90** 023846
[23] Faraon A, and Fushman I, Englund D, Stoltz N, Petroff P and Vučković J 2008 *Nat. Phys.* **4** 859
[24] Metcalfe M 2014 *Appl. Phys. Rev.* **1** 031105
[25] Safavi-Naeini, A. H, Chan J, Hill J T, Gröeblacher S, Miao H X, Chen Y B, Aspelmeyer M and Painter O 2013 *New J. Phys.* **15** 035007
[26] Vitali D, Gigan S, Ferreira A, Böhm H R, Tombesi P, Guerreiro A, Vedral V, Zeilinger A and Aspelmeyer M 2007 *Phys. Rev. Lett.* **98** 030405
[27] Agarwal G S and Huang S 2010 *Phys. Rev. A* **81** 041803
[28] Weis S, Rivière R, Deléglise S, Gavartin E, Arcizet O, Schliesser A and Kippenberg T J 2010 *Science* **330** 6010
[29] Rabl P 2011 *Phys. Rev. Lett.* **107** 063601
[30] Nunnenkamp A, Børkje K, and Girvin S M 2011 *Phys. Rev. Lett.* **107** 063602
[31] Zou F, Zhang X Y, Xu X W, Huang J F and Liao J Q 2020 *Phys. Rev. A* **102** 053710
[32] Felicetti S, Rossatto D Z, Rico E, Solano E and Forn-Díaz P 2018 *Phys. Rev. A* **97** 013851
[33] Kippenberg T J and Vahala K J 2007 *Opt. Express* **15** 17172
[34] Teng J H 2014 *Theoretical study of the quantum properties in cavity optomechanical system* (Ph.D. dissertation) (Dalian: Dalian University of Technology) (in Chinese)
[35] Law C K 1995 *Phys. Rev. A* **51** 2537
[36] Jayich A M, Sankey J C, Zwickl B M, Yang C, Thompson J D, Girvin S M, Clerk A A, Marquardt F and Harris J G E 2008 *New J. Phys.* **10** 095008
[37] Bhattacharya M, Uys H and Meystre P 2008 *Phys. Rev. A* **77** 033819
[38] Teng J H, Wang H F and Yi X X 2013 *Mod. Phys. Lett. B* **27** 50226
[39] Barzanjeh Sh, Naderi M H and Soltanolkotabi M 2011 *Phys. Rev. A* **84** 063850
[40] Xiong B, Li X, Chao S L and Zhou L 2018 *Opt. Lett.* **43** 6053
[41] Bhattacharya M and Meystre P 2007 *Phys. Rev. Lett.* **99** 073601
[42] Feng L J and Gong S Q 2021 *Phys. Rev. A* **103** 043509
[43] Yin T S, Lü X Y, Zheng L L, Wang M, Li S and Wu Y 2017 *Phys. Rev. A* **95** 053861
[44] Carreño J C López and Laussy F P 2016 *Phys. Rev. A* **94** 063825
[45] Schuster I, Kubanek A, Fuhrmanek A, Puppe T, Pinkse P W H, Murr K and Rempe G 2008 *Nat. Phys.* **4** 382
[46] Gardiner C W and Zoller P 2000 *Quantum Noise* (Berlin: Springer) pp. 81–86
[47] Plenio M B and Knight P L 1998 *Rev. Mod. Phys.* **70** 101
[48] Rosenblum S, Parkins S and Dayan B 2011 *Phys. Rev. A* **84** 033854
[49] Walls D F and Milburn G J 2008 *Quantum Optics* (Berlin: Springer) pp. 197–244
[50] Agarwal G S 1973 *Prog. Opt.* **11** 1
[51] Carmichael H 1993 *An Open Systems Approach to Quantum Optics* (Berlin: Springer) pp. 6–28
[52] Shen H Z, Xue W S and Yi X X 2020 *Opt. Lett.* **45** 4424
[53] Liang X Y, Duan Z L, Guo Q, Liu C J, Guan S G and Ren Y 2019 *Phys. Rev. A* **100** 063834
[54] Gao Y P, Liu X F, Wang T J, Cao C and Wang C 2019 *Phys. Rev. A* **100** 043831
[55] Huang R, Miranowicz A, Liao J Q, Nori F and Jing H 2018 *Phys. Rev. Lett.* **121** 153601