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Non-universal Fermi polaron in quasi two-dimensional quantum gases

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We consider an impurity problem in a quasi-two-dimensional Fermi gas, where a spin-down impurity is immersed in a Fermi sea of N spin-up atoms. Using a variational approach and an effective two-channel model, we obtain the energy for a wide range of interaction strength and for various different mass ratios between the impurity and the background fermion in the context of heteronuclear mixture. We demonstrate that in a quasi-two-dimensional Fermi gas there exists a transition of the ground state from polaron in the weakly interacting region to molecule in the strongly interacting region. The critical interaction strength of the polaron–molecule transition is non-universal and depends on the particle density of the background Fermi sea. We also investigate the excited repulsive polaron state, and find similar non-universal behavior.

Keywords: Fermi gas, polaron problem, two-channel model

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1. Introduction

Ultracold atomic gases have been widely used as an excellent platform for the study of quantum many-body phenomena owing to its high tunability in the past several decades.[1–3] In particular, it is of great interest to investigate low-dimensional systems of quantum gases and explore rich physics of many exotic phases of quantum matters, such as Tonks–Girardeau gas,[4,5] Fulde–Farrell–Larkin–Ovchinnikov (FFLO) state,[6] Berezinskii–Kosterlitz–Thouless (BKT) phase,[7,8] and pseudogap phase.[9,10] Low-dimensional systems of quantum gases can be realized by trapping the laser cooled atoms in various types of potentials, including atom chips,[11] light sheet,[12] TEM01 trap,[13] surface trap[14] and optical lattices.[4–10,15–17] For fermions, Feshbach resonance[18–25] further provides exciting possibilities to create low-dimensional ultracold Fermi gases with a tunable inter-particle interaction, therefore allows for exploration of strongly correlated fermionic matters with the interplay of interaction and quantum fluctuation.[26,27] It is natural to consider a Fermi gas trapped in a deep one-dimensional optical lattice, where most of the atoms are assumed to be in the lowest level of the trapping potential. As a result, we can achieve a quasi-two-dimensional (quasi-2D) Fermi system,[9,12,28–31] in which the effect of the strong trapping potential can be modeled by a one-dimensional harmonic potential $V(z) = \frac{1}{2}m\omega_z^2 z^2$. The temperature T and the Fermi energy $E_F = \hbar^2 k_F^2/2m$ also satisfy the conditions for quasi-two-dimensionality as $k_B T$ and $E_F \ll \hbar\omega_z$,[32] where k_F is the Fermi vector and m the atomic mass. However, when the interaction between fermions increases, the degrees of freedom in the third dimension become significant even when the aforementioned condition is fulfilled.[12]

The advent of two-dimensional Fermi gas has greatly renewed interest in the so-called polaron problem, which can be considered as the limiting case of a spin-imbalanced Fermi gas. It is of great importance to understand the fate of impurities as the interaction varies by tuning the Feshbach resonances. A natural expectation is that the system would undergo a transition and change its statistics as the impurity binds with background fermions.[33,34] Indeed, if the interaction is sufficiently weak, the impurity is dressed by the density fluctuation of the Fermi sea to form a polaron state.[35–37] With increasing attractive interaction, the impurity binds a background particle and forms a molecule.[38–41] Some previous studies suggest a transition between polaron and molecule states both theoretically[42–47] and experimentally,[48] while the critical interaction strength is obtained differently in different scenarios. Considering the transition takes place in the strongly interacting regime, the significant population of excited transversal modes of the harmonic potential[49–51] may lead to non-universal situation of this phase transition, and can provide the key to understand the discrepancy of the transition points between theoretical prediction and experimental outcome in this quasi-2D systems.

In this work, we investigate the quasi-2D polaron by using a variational approach and an effective two-channel model, in which the highly excited transversal levels are taken into account by introducing a phenomenological degree of freedom

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of dressed molecule.[49–51] We consider two different kinds of background fermions, ^{6}Li and ^{40}K, which have been well investigated in experiments.[30,48,52–54] Therefore three different mass ratios $\eta = m_\downarrow/m_\uparrow = 1, 6.64$ and $1/6.64$ are investigated. The main purpose of this work is to reveal the significant effects induced by the excited transversal levels of a quasi-2D geometry, such that the value of $\hbar\omega_z/E_F$ can strongly affect the position of the polaron–molecule transition. We numerically calculate the transition points for different mass ratios, and compare our results with previous experiments. We show the results of polaron energy, the wave function for the polaron state, and the effective mass are sensitively dependent on particle density and mass ratio. Furthermore, we discuss the excited repulsive polaron state in this system and find similar non-universal behavior.

2. Formalism

We consider an atomic Fermi gas trapped in a D-dimensional harmonic potential with an isotropic trapping frequency of ω_z in the transversal directions. Although in the following discussion we will focus on the quasi-2D case with $D = 2$ only, in this section we present a general formalism which is valid for both cases of $D = 1$ and 2. Besides, while it is possible to use a single-channel approach with a pseudo-potential for a wide Feshbach resonance, for more general purposes, we adopt the conventional two-channel field theory which is valid for both wide and narrow Feshbach resonances. The Hamiltonian thus can be written as[50,55]

$$\begin{aligned} H =& \sum_{\sigma=\uparrow,\downarrow}\int \mathrm{d}^3\boldsymbol{r}\Psi_\sigma^\dagger\left(-\frac{1}{2}\nabla^2+\frac{1}{2}\sum_{i=1}^{3-D}x_i^2\right)\Psi_\sigma \\ &+\int \mathrm{d}^3\boldsymbol{r}\Phi^\dagger\left(-\frac{1}{4}\nabla^2+\sum_{i=1}^{3-D}x_i^2+\nu_\mathrm{b}\right)\Phi \\ &+g_\mathrm{b}\int\int \mathrm{d}^3\boldsymbol{r}(\psi_\uparrow^\dagger\psi_\downarrow^\dagger\Phi+\mathrm{H.c.}) \\ &+U_\mathrm{b}\int\int \mathrm{d}^3\boldsymbol{r}\int \mathrm{d}^3\boldsymbol{r}\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow, \end{aligned} \tag{1}$$

where $\Psi_\sigma(\boldsymbol{r})$ ($\sigma=\uparrow,\downarrow$) is the atomic field operator and $\Phi(\boldsymbol{r})$ is the molecular field operator. Here, we use the natural unit $\hbar = m = 1$ and scale all energies with $\hbar\omega_z$. The bare parameters (ν_b, g_b, U_b) in the Hamiltonian are defined as follows: ν_b is the detuning, g_b is the atom–molecule coupling constant, and U_b is background atomic interaction constant. They are related to the corresponding physical parameters via the renormalization relations

$$\begin{aligned} &U_\mathrm{c}^{-1} = \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{2\varepsilon_{\boldsymbol{k}}}, \quad \Gamma^{-1} = 1-U_\mathrm{p}U_\mathrm{c}^{-1}, \\ &U_\mathrm{b} = \Gamma U_\mathrm{p}, \quad g_\mathrm{b} = \Gamma g_\mathrm{p}, \quad \nu_\mathrm{b} = \nu_\mathrm{p}+\Gamma g_\mathrm{p}^2U_\mathrm{c}^{-1}, \end{aligned} \tag{2}$$

where the subscript p denoting the physical parameters and $\varepsilon_{\boldsymbol{k}} = \hbar^2\boldsymbol{k}^2/2m$. The integral in U_c^{-1} is defined in three dimensions, with a momentum cut off k_c. The physical parameters are defined from the parameters of the Feshbach resonance as $U_\mathrm{p} = 4\pi a_\mathrm{bg}/a_\mathrm{t}$, $g_\mathrm{p} = \sqrt{4\pi\mu_\mathrm{co}W|a_\mathrm{bg}|/a_\mathrm{t}\hbar\omega_z}$, and $\nu_\mathrm{p} = \mu_\mathrm{co}(B-B_0)/\hbar\omega_z$, where μ_co is the differential magnetic moment between the two channels, and the s-wave scattering length near resonance is $a_\mathrm{s} = a_\mathrm{bg}[1-W/(B-B_0)]$ with the background scattering length a_bg, the resonance width W and center field of the resonance B_0. Here, the trapping length scale $a_\mathrm{t} = \sqrt{\hbar/m\omega_z}$ is set as the length unit.

The Hamiltonian Eq. (1) captures the full ingredients of the trapping potential and the Feshbach resonance, however, it can not be directly linked to the unique properties of low-dimensional physics. Hence it is desirable to obtain an effective low-dimensional Hamiltonian if the many-body energy scales related to the density are much smaller than the trapping energy $\hbar\omega_z$. In this limit, one can obtain an effective two-dimensional Hamiltonian of the following form:[38–41,50,55]

$$\begin{aligned} H_\mathrm{eff} =& \sum_{\boldsymbol{k},\sigma}\varepsilon_{\boldsymbol{k},\sigma}c_{\boldsymbol{k},\sigma}^\dagger c_{\boldsymbol{k},\sigma}+\sum_{\boldsymbol{q}}\left(\frac{1}{2}\varepsilon_{\boldsymbol{q}}+\lambda_\mathrm{b}\right)b_{\boldsymbol{q}}^\dagger b_{\boldsymbol{q}} \\ &+\frac{\alpha_\mathrm{b}}{L^{D/2}}\sum_{\boldsymbol{k}\boldsymbol{q}}\left(b_{\boldsymbol{q}}^\dagger c_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\uparrow}c_{-\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\downarrow}+b_{\boldsymbol{q}}c_{-\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\downarrow}^\dagger c_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\uparrow}^\dagger\right) \\ &+\frac{V_\mathrm{b}}{L^D}\sum_{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q}}c_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\uparrow}^\dagger c_{-\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\downarrow}^\dagger c_{\boldsymbol{k}'+\frac{\boldsymbol{q}}{2},\downarrow}c_{-\boldsymbol{k}'+\frac{\boldsymbol{q}}{2},\uparrow}, \end{aligned} \tag{3}$$

where $\boldsymbol{k}$, $\boldsymbol{k}'$ and $\boldsymbol{q}$ are now two-dimensional momenta, and the bare parameters of the relative detuning λ_b (in units of $\hbar\omega_z$), the coupling constant α_b (in units of $\hbar\omega_z a_\mathrm{t}^{D/2}$), and the background interaction in the open channel V_b (in units of $\hbar\omega_z a_\mathrm{t}^D$) are related to the physical ones by

$$\begin{aligned} &V_\mathrm{c}^{-1} = \int\frac{\mathrm{d}^D\boldsymbol{k}}{(2\pi)^D}\frac{1}{2\varepsilon_{\boldsymbol{k}}+3-D}, \quad \Omega^{-1} = 1-V_\mathrm{p}V_\mathrm{c}^{-1}, \\ &V_\mathrm{p} = \Omega^{-1}V_\mathrm{b}, \quad \alpha_\mathrm{p} = \Omega^{-1}\alpha_\mathrm{b}, \quad \lambda_\mathrm{p} = \lambda_\mathrm{b}-\Omega\alpha_\mathrm{p}^2V_\mathrm{c}^{-1}. \end{aligned} \tag{4}$$

All these parameters in the effective two-channel model can be determined phenomenologically by matching the two-body bound state solved exactly from the original Hamiltonian Eq. (1).[50] To compare with experiments, we focus on the setup of two component Fermi gases of ^{6}Li and ^{40}K atoms, as well as their heteronuclear mixtures. The scattering parameters for ^{6}Li (^{40}K) are $W \simeq 300$G (8G), $a_\mathrm{bg} \simeq -1405a_\mathrm{B}$ ($174a_\mathrm{B}$), $\mu_\mathrm{co} \simeq 2\mu_\mathrm{B}$ ($1.68\mu_\mathrm{B}$) with Bohr radius a_B and Bohr magneton μ_B.[19–22,56] With a strong trapping frequency of $\omega_z \simeq 2\pi\times 40$ kHz, the physical parameters for the interaction are given by $g_\mathrm{p} = 310.2141$ (26.1660) and $U_\mathrm{p} = -4.5531$ (1.4459). In the following calculation, one can use a common form of the renormalization relation for a quasi-2D model

$$\frac{1}{V_\mathrm{b}-\frac{\alpha_\mathrm{b}^2}{\lambda_\mathrm{b}-x}} = \frac{1}{V_\mathrm{p}-\frac{\alpha_\mathrm{p}^2}{\lambda_\mathrm{p}-x}}-\sum_{\boldsymbol{k}}\frac{1}{2\varepsilon_{\boldsymbol{k}}+\hbar\omega_z}, \tag{5}$$

where $\varepsilon_{\boldsymbol{k}}$ represents the two-dimensional dispersion.

This system has two possible ground states. For weak interaction, the ground state is expected to be a polaron with an approximate wave function[35]

$$|P(\boldsymbol{Q})\rangle = \gamma c^{\dagger}_{\boldsymbol{Q}\downarrow}|N\rangle_{\uparrow} + \sum_{\boldsymbol{q}} \alpha_{\boldsymbol{q}} b^{\dagger}_{\boldsymbol{Q}+\boldsymbol{q}} c_{\boldsymbol{q}\uparrow}|N\rangle_{\uparrow} + \sum_{\boldsymbol{k},\boldsymbol{q}} \beta_{\boldsymbol{k}\boldsymbol{q}} c^{\dagger}_{\boldsymbol{Q}+\boldsymbol{q}-\boldsymbol{k}\downarrow} c^{\dagger}_{\boldsymbol{k}\uparrow} c_{\boldsymbol{q}\uparrow}|N\rangle_{\uparrow}, \tag{6}$$

where the first term is a bare impurity on top of the non-interacting majority Fermi sea $|N\rangle_{\uparrow}$, and the second term represents a dressed molecule couples to the background Fermi sea to create a hole therein. The last term describes excitations containing one particle–hole pair. Here, $\boldsymbol{Q}$ represents the center-of-mass momentum, the summation of $\boldsymbol{k}$ is always over $|\boldsymbol{k}| > k_{\mathrm{F}}$, and the summation of $\boldsymbol{q}$ is restricted below the Fermi level.

The eigenenergy E of the polaron state can be determined by minimizing $\langle P(\boldsymbol{Q})|H - E|P(\boldsymbol{Q})\rangle$, leading to the following equation:

$$E_{\mathrm{P}} - \varepsilon_{\boldsymbol{Q}\downarrow} = \sum_{\boldsymbol{q}} \left[\frac{1}{V_{\mathrm{p}} - \frac{\alpha_{\mathrm{p}}^2}{(\frac{1}{2}\varepsilon_{\boldsymbol{Q}+\boldsymbol{q}} + \lambda_{\mathrm{p}}) + \varepsilon_{\boldsymbol{q}\uparrow} - E_{\mathrm{P}}}} + \sum_{\boldsymbol{k}} \frac{1}{E_{\boldsymbol{Q}\boldsymbol{k}\boldsymbol{q}}} - \sum_{\boldsymbol{k}} \frac{1}{2\varepsilon_{\boldsymbol{k}} + \hbar\omega_z} \right]^{-1}, \tag{7}$$

where $E_{\boldsymbol{Q}\boldsymbol{k}\boldsymbol{q}} = -E_{\mathrm{P}} + \varepsilon_{\boldsymbol{Q}+\boldsymbol{q}-\boldsymbol{k}\downarrow} + \varepsilon_{\boldsymbol{k}\uparrow} - \varepsilon_{\boldsymbol{q}\uparrow}$ and $E_{\mathrm{P}} = E - \sum_{|\boldsymbol{k}_0|<k_{\mathrm{F}}} \varepsilon_{\boldsymbol{k}_0\uparrow}$.

If the interaction strength is sufficiently strong, the impurity atom and one background particle prefer to form a bound state, i.e., a dimer molecule. We can also write a trial wave function for such state with one pair of particle–hole excitation as

$$|M(\boldsymbol{Q})\rangle = \gamma b^{\dagger}_{\boldsymbol{Q}}|N-1\rangle_{\uparrow} + \sum_{\boldsymbol{k}} \alpha_{\boldsymbol{k}} c^{\dagger}_{\boldsymbol{Q}-\boldsymbol{k}\downarrow} c^{\dagger}_{\boldsymbol{k}\uparrow}|N-1\rangle_{\uparrow} + \sum_{\boldsymbol{k},\boldsymbol{q}} \beta_{\boldsymbol{k}\boldsymbol{q}} b^{\dagger}_{\boldsymbol{Q}+\boldsymbol{q}-\boldsymbol{k}} c^{\dagger}_{\boldsymbol{k}\uparrow} c_{\boldsymbol{q}\uparrow}|N-1\rangle_{\uparrow} + \sum_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{q}} \delta_{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q}} c^{\dagger}_{\boldsymbol{Q}+\boldsymbol{q}-\boldsymbol{k}-\boldsymbol{k}'\downarrow} c^{\dagger}_{\boldsymbol{k}\uparrow} c^{\dagger}_{\boldsymbol{k}'\uparrow} c_{\boldsymbol{q}\uparrow}|N-1\rangle_{\uparrow}.$$

Note that the Fermi sea here only contains $N-1$ spin-up fermions, thus the threshold energy would be different from the polaron state. Minimizing $\langle M(\boldsymbol{Q})|H - E|M(\boldsymbol{Q})\rangle$, we obtain the molecule energy equation

$$\left[\frac{1}{V_{\mathrm{p}} - \frac{\alpha_{\mathrm{p}}^2}{\lambda_{\mathrm{p}} - (E_{\mathrm{M}} - \frac{1}{2}\varepsilon_{\boldsymbol{Q}})}} - \sum_{\boldsymbol{k}_1} \frac{1}{2\varepsilon_{\boldsymbol{k}_1} + \hbar\omega_z} + \sum_{\boldsymbol{k}'} \frac{1}{E_{\boldsymbol{Q}\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q}}} \right] G(\boldsymbol{k},\boldsymbol{q}) - \frac{1}{E_{\boldsymbol{Q}\boldsymbol{k}}} \sum_{\boldsymbol{q}_1} G(\boldsymbol{k},\boldsymbol{q}_1) - \sum_{\boldsymbol{k}'} \frac{G(\boldsymbol{k}',\boldsymbol{q})}{E_{\boldsymbol{Q}\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q}}} + \frac{1}{E_{\boldsymbol{k}}} \cdot \frac{\sum_{\boldsymbol{k}',\boldsymbol{q}_1} \frac{G(\boldsymbol{k}',\boldsymbol{q}_1)}{E_{\boldsymbol{Q}\boldsymbol{k}'}}}{\sum_{\boldsymbol{k}'} \frac{1}{E_{\boldsymbol{Q}\boldsymbol{k}'}} - \left(\frac{1}{V_{\mathrm{p}} - \frac{\alpha_{\mathrm{p}}^2}{\lambda_{\mathrm{p}} - (E_{\mathrm{M}} - \varepsilon_{\boldsymbol{Q}}/2)}} - \sum_{\boldsymbol{k}_1} \frac{1}{2\varepsilon_{\boldsymbol{k}_1} + \hbar\omega_z} \right)} = 0, \tag{8}$$

where $E_{\boldsymbol{Q}\boldsymbol{k}} = E_{\mathrm{M}} - \varepsilon_{\boldsymbol{Q}-\boldsymbol{k}\downarrow} - \varepsilon_{\boldsymbol{k}\uparrow} - \sum_{|\boldsymbol{q}|<k_{\mathrm{F}}} V_{\mathrm{b}}$, $E_{\boldsymbol{Q}\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q}} = -E_{\mathrm{M}} + \varepsilon_{\boldsymbol{Q}+\boldsymbol{q}-\boldsymbol{k}-\boldsymbol{k}'\downarrow} + \varepsilon_{\boldsymbol{k}\uparrow} + \varepsilon_{\boldsymbol{k}'\uparrow} - \varepsilon_{\boldsymbol{q}\uparrow}$, E_{M} is defined as $E - \sum_{|\boldsymbol{k}_0|<k_{\mathrm{F}}} \varepsilon_{\boldsymbol{k}_0\uparrow}$, and the function G is defined as $G(\boldsymbol{k},\boldsymbol{q}) = \sum_{\boldsymbol{k}'} \delta_{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q}} / \sum_{\boldsymbol{k}'} \alpha_{\boldsymbol{k}'}$.

3. Results

Equations (7) and (8) can be solved numerically by discretizing the integrals to solvable matrix equations. The results for a zero center-of-mass momentum impurity are plotted in Fig. 1. We firstly consider the case of mass ratio $\eta = m_{\downarrow}/m_{\uparrow} = 1$ and $\hbar\omega_z/E_{\mathrm{F}} = 7.85$, which is relevant to an experimentally feasible configuration where the impurity and background fermions are both ^{40}K atoms. As we can see in Fig. 1, there exists a polaron–molecule transition under these conditions. The transition takes place at $\ln(k_{\mathrm{F}}a_{2\mathrm{D}}) = -0.6224$, where the two-dimensional scattering length is defined as $a_{2\mathrm{D}} = a_{\mathrm{t}}\sqrt{\pi/B}\exp(-\sqrt{\pi/2})a_{\mathrm{t}}/a_{\mathrm{s}}$ and $B \approx 0.905$.[43] Compared with the experimental measurement of $\ln(k_{\mathrm{F}}a_{2\mathrm{D}}) = -0.4$ reported in Ref. [48], our result is comparable with the pervious theoretical estimation of $\ln(k_{\mathrm{F}}a_{2\mathrm{D}}) = -0.6$ by using another version of quasi-2D model,[43] and better than the outcome of -0.8 via a strictly two-dimensional theory.[42]

The most important finding of our treatment is the non-universality of the system. The polaron–molecule transition point $\ln(k_{\mathrm{F}}a_{2\mathrm{D}})$ is not a universal constant, but depends on the density of background fermions and the trapping geometry. In Fig. 2, we show the dependence of transition point on $\hbar\omega_z/E_{\mathrm{F}}$ for various mass ratio η, where the region below each transition line represents the molecule state regime for the corresponding case. For all mass ratios, the polaron–molecule transition is always present, and the transition point moves monotonically towards the strongly interacting limit by increasing $\hbar\omega_z/E_{\mathrm{F}}$, leading to a narrower regime of molecule state. This trend can be understood by noticing that with increasing transversal confinement, the ground level population

of the trapping potential is enhanced. This effect will lead to stronger fluctuation and tend to break the molecule state formed by the impurity and one background atom. As a result, the molecule state can only be stabilized with stronger interaction. We also notice that the transition line with heavier impurity, e.g., mass ratio $\eta = 6.64$, lies above the ones with lighter impurity, i.e., $\eta = 1$ and $1/6.64$. This is because the Fermi energy of lighter atoms is higher than that of heavier atoms for a given number density. And the polaron state is more favorable as the kinetic energy of background fermions surpasses the interaction energy binding the molecule state. Compared to the results in Ref. [43], our outcome shows that the transition point not only relates to the dimensionless parameter $\hbar\omega_z/E_F$, but also depends on the specific parameters of the Feshbach resonances used in experiments.

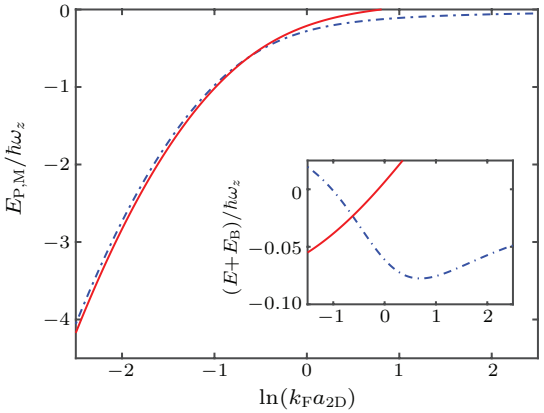


Fig. 1. Energies of the attractive polaron (blue dotted-dashed line) and the molecule (red solid line) states with mass ratios $\eta = 1$. Here we use the parameters for ^{40}K atoms and fix $\hbar\omega_z/E_F = 7.85$, which are adapted from the experiment configuration of Ref. [48]. The inset shows the result of $E + E_B$, and the transition point resides at $\ln(k_F a_{2D}) = -0.6224$.

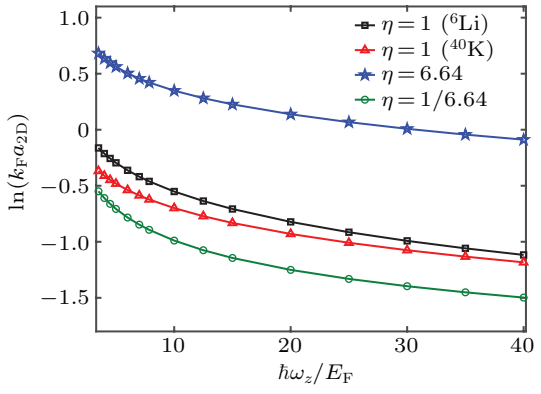


Fig. 2. The positions of transition point vary with $\hbar\omega_z/E_F$. The black square and red triangle lines represent the equal mass case, where the atoms are ^{6}Li and ^{40}K, respectively. From bottom to top the mass ratio is increasing.

The non-universal behavior is of potential importance when comparing theoretical results and experimental measurements. In many cold atom experiments such as Ref. [48], the quantum gases are trapped in a harmonic potential and the particle density varies from high to low by moving from the trap center to the edge. The response of the trapped gas under a global measurement is thus an average of all particles. Theoretical treatment can be performed within the local density approximation (LDA), under which the non-uniform system is considered as composed by segments with a local density fixed by the chemical potential with an on-site potential offset. The results shown in Fig. 2 suggest that the density dependence has to be taken into account when quantitative analysis is sought.

Another non-trivial finding of Fig. 2 is that the curves do not seem to saturate and remains non-universal for $\hbar\omega_z/E_F$ as large as 40. This ratio corresponds to a very strong confinement and the quasi-2D condition should be satisfied quite well at the first glance. However, one must notice that in this system there is another energy scale characterizing the interaction energy, which can be chosen as the binding energy of the two-body bound state formed by the impurity and one background fermion. If the binding energy is comparable or even exceeds $\hbar\omega_z$, the excited levels of the transversal trapping potential would be significantly populated and the system remains non-universal. To better reveal this property, we further show the fraction of the bare impurity, the dressed molecule, and the two-dimensional fermions in the polaron wave function ansatz in Fig. 3. The three components are defined as

$$
\begin{aligned}
|\gamma|^2 &= 1 - \sum_{\boldsymbol{q}} |\alpha_{\boldsymbol{q}}|^2 - \sum_{\boldsymbol{k},\boldsymbol{q}} |\beta_{\boldsymbol{k},\boldsymbol{q}}|^2, \\
\sum_{\boldsymbol{q}} |\alpha_{\boldsymbol{q}}|^2 &= \sum_{\boldsymbol{q}} |\gamma|^2 \left[\frac{\frac{\alpha_b}{E_{\boldsymbol{q}}} \cdot (V_b - \frac{\alpha_b^2}{E_{\boldsymbol{q}}})^{-1}}{(V_b - \frac{\alpha_b^2}{E_{\boldsymbol{q}}})^{-1} + \sum_{\boldsymbol{k}} E_{0,\boldsymbol{k},\boldsymbol{q}}^{-1}} \right]^2, \\
\sum_{\boldsymbol{k},\boldsymbol{q}} |\beta_{\boldsymbol{k},\boldsymbol{q}}|^2 &= \sum_{\boldsymbol{k},\boldsymbol{q}} |\gamma|^2 \left[\frac{E_{0,\boldsymbol{k},\boldsymbol{q}}^{-1}}{(V_b - \frac{\alpha_b^2}{E_{\boldsymbol{q}}})^{-1} + \sum_{\boldsymbol{k}_1} E_{0,\boldsymbol{k}_1,\boldsymbol{q}}^{-1}} \right]^2 .
\end{aligned} \tag{9}
$$

In the weakly interacting limit, the dominant part of polaron wave function is the bare impurity component $|\gamma|^2$. In the intermediate and strong interaction regime, the dressed molecule fraction $\sum_{|q|} |\alpha_q|^2$ and fermionic contribution $\sum_{|k|,|q|} |\beta_q|^2$ are both significant. Remind that the dressed molecule is a phenomenological description of the excited levels of the transversal trap, and thus this observation suggests that the system is not strictly 2D yet for the range of interaction considered.

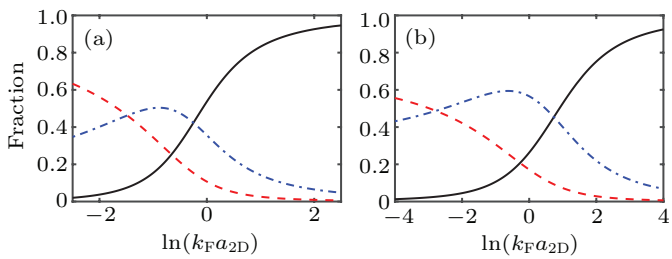


Fig. 3. The fraction of the three components in polaron wave function for (a) $\eta = 1$ and (b) $\eta = 6.64$. Here the three lines for $|\gamma|^2$ (black solid line), $\sum_{\boldsymbol{q}} |\alpha_{\boldsymbol{q}}|^2$ (red dashed line) and $\sum_{\boldsymbol{k},\boldsymbol{q}} |\beta_{\boldsymbol{q}}|^2$ (blue dotted-dashed line) represent the bare impurity, molecule and particle–hole excitation, respectively.

Next, we extend the discussion to finite center-of-mass momentum. The solution of Eqs. (7) and (8) at small but finite momentum $|\boldsymbol{Q}| \ll k_{\uparrow \mathrm{F}}$ can be expanded as

$$E(\boldsymbol{Q}) = E(0) + \frac{Q^2}{2m^*} \tag{10}$$

for both polaron and molecule states with m^* the effective mass. In Fig. 4, we show the effective mass for polaron and the inverse effective mass for molecule state. In the weakly interacting limit, the effective mass of the polaron state is the bare impurity mass $m^* \approx m_\downarrow$. With increasing interaction, the polaron state with zero center-of-mass momentum is always a metastable state with a positive effective mass. On the contrary, the molecule effective mass is negative for weak enough interaction, and the molecule solution with $Q = 0$ becomes unstable. In the strongly interacting regime, the effective mass for both the polaron and molecule states approach the same limiting value $m^* \approx m_\uparrow + m_\downarrow$.

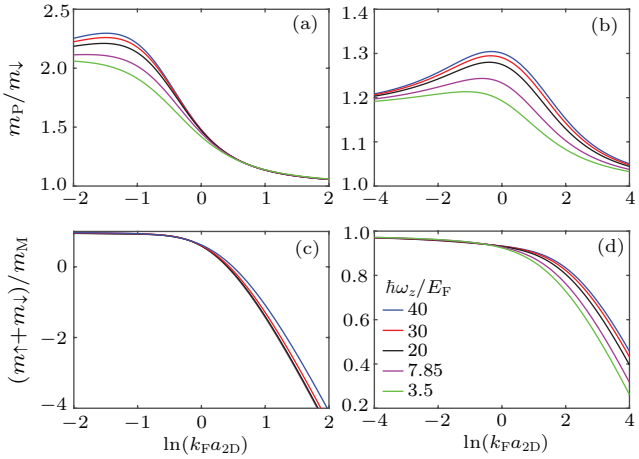


Fig. 4. Effective mass of polaron state for (a) $\eta = 1$, (b) $\eta = 6.64$ and inverse effective mass of molecule state for (c) $\eta = 1$, (d) $\eta = 6.64$. The five lines from top to bottom represent $\hbar\omega_z/E_{\mathrm{F}} = 40, 30, 20, 7.85$ and 3.5, respectively.

Finally, we investigate the properties of the repulsive polaron branch with a positive energy, which was recently realized in experiments in both three dimensions[57] and two dimensions.[58,59] We firstly write down the self-energy of a polaron state

$$\Sigma(\boldsymbol{Q}, E_{\mathrm{P}}) = \sum_{\boldsymbol{q}} \left[\frac{1}{U_{\mathrm{p}} - \frac{g_{\mathrm{p}}^2}{(\frac{1}{2}\varepsilon_{\boldsymbol{Q}+\boldsymbol{q}} + \nu_{\mathrm{b}}) + \varepsilon_{\boldsymbol{q}\downarrow} - E_{\mathrm{P}}}} + \sum_{\boldsymbol{k}} \frac{1}{E_{\boldsymbol{Q}\boldsymbol{k}\boldsymbol{q}} - \mathrm{i}0^+} - \sum_{\boldsymbol{k}} \frac{1}{2\varepsilon_{\boldsymbol{k}} + \hbar\omega_z} \right]^{-1}, \tag{11}$$

where the function $E_{\boldsymbol{Q}\boldsymbol{k}\boldsymbol{q}}$ is defined in Eq. (7). The spectral function thus takes the form of

$$A(\boldsymbol{Q}, E_{\mathrm{P}}) = -2\mathrm{Im}\frac{1}{E_{\mathrm{P}} + \mathrm{i}0^+ - \Sigma(\boldsymbol{Q}, E_{\mathrm{P}})}. \tag{12}$$

In Fig. 5(a), we plot the spectral function as a function of energy and interaction strength $\ln(k_{\mathrm{F}}a_{2\mathrm{D}})$ for $\boldsymbol{Q} = 0$ and $\eta = 1$. There exist two branches where the spectral function is strongly peaked and labeled by bright yellow color. The lower branch corresponds to the attractive polaron state, which intersects with the molecule state (red dotted line) at the polaron–molecule transition. The thick upper branch labels the repulsive polaron, which is a fairly well defined excited state in the strongly interacting regime with a positive energy. As moving towards the weakly interacting regime, the repulsive polaron branch merges into the molecule–hole continuum. At small momenta, the energy of the two branches of polaron can also be calculated from

$$E_{\mathrm{P}} = \mathrm{Re}[\Sigma(\boldsymbol{Q} = 0, E_{\mathrm{P}})], \tag{13}$$

which is also displayed as blue dotted-dashed line in Fig. 5(a). We also observe a non-universal density dependence of the repulsive polaron energy, as demonstrated in Figs. 5(b) and 5(c) for mass ratio $\eta = 1$ and 6.64, respectively.

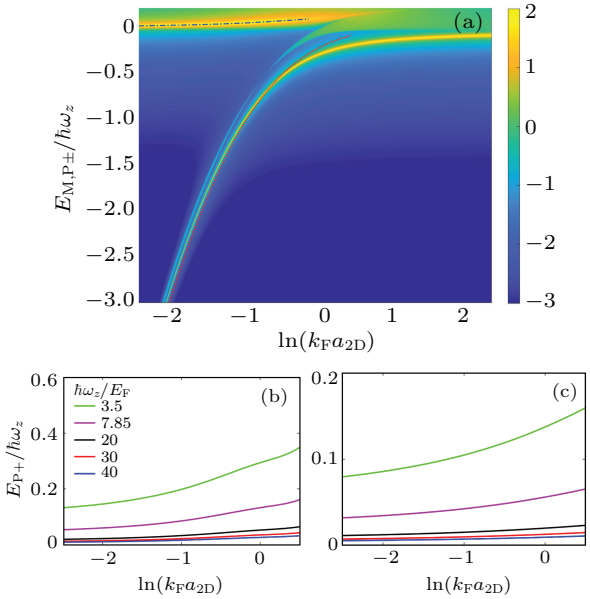


Fig. 5. (a) Contour plot of the spectral function $\mathrm{Log}_{10}A(\boldsymbol{Q} = 0, E_{\mathrm{P}})$ of the polaron state for a gas of ^{40}K with $\hbar\omega_z/E_{\mathrm{F}} = 7.85$. The red dotted line is the molecule energy obtained from Eq. (8). On the BCS side, the repulsive polaron branch (blue dotted-dashed line) merges into the molecule–hole continuum which is denoted by the broad light area. (b), (c) Repulsive polaron energy E_{P_+} with (b) $\eta = 1$ and (c) $\eta = 6.64$. The five lines from bottom to top represent $\hbar\omega_z/E_{\mathrm{F}} = 40, 30, 20, 7.85$ and 3.5, respectively. The color code is similar to Fig. 4.

4. Conclusion

We study the impurity problem in a background of fermionic particles trapped in a quasi-two-dimensional confinement. Using an effective two-channel model which incorporates the effect of excited states along the strongly confined direction, we investigate various properties of the polaron and molecule states, including the eigenenergy, wave function, effective mass, and the spectral function. Since the transversal trapping potential for a quasi-two-dimensional configuration is finite, the system acquires another energy scale of $\hbar\omega_z$, and hence breaks the universality which is present in a strictly

two-dimensional Fermi gas with contact interaction. Specifically, we find that the polaron–molecule transition point shows a strong density dependence on $\hbar\omega_z/E_F$ for all mass ratios. This observation is of particular interest when analyzing experimental results of cold atomic gases trapped in harmonic potentials, where the density is non-uniform.

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