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Quantum speed limit of the double quantum dot in pure dephasing environment under measurement

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The quantum speed limit (QSL) of the double quantum dot (DQD) system has been theoretically investigated by adopting the detection of the quantum point contact (QPC) in the pure dephasing environment. The Mandelstam–Tamm (MT) type of the QSL bound which is based on the trace distance has been extended to the DQD system for calculating the shortest evolving time. The increase of decoherence rate can weaken the capacity for potential speedup (CPS) and delay the evolving process due to the frequently measurement localizing the electron in the DQD system. The system needs longer time to evolve to the target state as the enhancement of dephasing rate, because the strong interaction between pure dephasing environment and the DQD system could vary the oscillation of the electron. Increasing the dephasing rate can sharp the QSL bound, but the decoherence rate would weaken the former effect and vice versa. Moreover, the CPS would be raised by increasing the energy displacement, while the enhancement of the coupling strength between two quantum dots can diminish it. It is interesting that there has an inflection point, when the coupling strength is less than the value of the point, the increasing effect of the CPS from the energy displacement is dominant, otherwise the decreasing tendency of the CPS is determined by the coupling strength and suppress the action of the energy displacement if the coupling strength is greater than the point. Our results provide theoretical reference for studying the QSL time in a semiconductor device affected by numerous factors.

Keywords: quantum speed limits, the DQD system, the pure dephasing environment, quantum measurement

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1. Introduction

The quantum speed limit (QSL) has been a hot research field with the development of quantum computation and quantum information technology.[1–10] The QSL is the minimum evolution time between the initial and distinguishable states, which is usually applied to prolong the life-time of coherence and shorten the evolving time.[11–16] Originally, Mandelstam–Tamm (MT) type and Margolus–Levitin (ML) type of the QSL bound have been proposed under the unitary dynamics. People have summarized the definiting expression of the QSL time which has described the speed of system evolving to orthogonal state by combining the two types of bound.[17–19] In recent years, people have paid much attention on the QSL in open systems. The QSL bounds of the ML and the MT type have been derived by using geometric approach, which is based on the bures angle between states of system, and those bounds are generalized to generic time-dependent dynamics of open quantum system.[20] Furthermore, a QSL bound which contains the energy fluctuation term has been proposed and it is applicable to general open system.[21]

The QSL bound determines the upper limit of evolving speed. It also represents the maximal degree which the system can be accelerated, and it is significant to improve the efficiency of quantum computing.[22–24] Recently, lots of works have been done for different way to accelerate the evolution and sharp the QSL bound.[25–30] It proves that the non-Markovian effects can speed up quantum evolution and therefore lead to a smaller QSL time in dissipative environment.[20] How to shorten the minimum evolving time under the pure dephasing environment is the important field of research. For a single quantum dot system which is coupled to purely-dephasing bosonic reservoir, the potential acceleration of the quantum evolution could be induced by the periodic dynamical decoupling pulses.[31] Moreover, the double quantum dot (DQD) system with its strong controllability has become an useful model for developing the unit devices of quantum computing.[32,33] So the variation of the QSL bound of DQD system in the pure dephasing environment is an attractive object of study.

In this paper, we explore the quantum evolution of the DQD system under the pure dephasing environment and analyze the effect of the dephasing rate (Γ_ϕ), the decoherence rate (Γ_d), the energy displacement (ε) and the coupling strength (Ω) between qubits to the QSL bound. The ratio of the QSL time (τ_qsl) to the driving time (τ) represent the CPS of the DQD system. Firstly, the CPS reduce to steady level along the driving time. The altitude of the stable value is risen by enhancing the Γ_d, and it reveals that increasing the frequency

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of measurement can weaken the CPS to some extent in the DQD system. By contrast, the increase of Γ_ϕ is more efficient on decreasing the CPS, which is according to that the strong coupling with the pure dephasing environment would accelerate the diminish of coherence. Secondly, the enlargement of the range of the QSL bound owing to the increase of Γ_d is narrowed by Γ_ϕ. However, the expansion of the scope of the QSL bound according to the increase of Γ_ϕ is also reduced by Γ_d. Thirdly, the CPS would be raised with the increasing of the energy displacement, while it decreases for enhancing the coupling strength between two quantum dots. It is interesting that there has an inflection point, and the reducing effect of the CPS from the Ω would be dominant if the Ω is greater than the point, otherwise the increasing influence of the CPS from the ε would suppress the impact of the coupling strength. Our results and analysis can not only reveal the accelerating mechanism of evolution in the DQD system, but also provide reference for the fabrication of quantum computing devices.

The paper is organized as follows. In Section 2, we make a brief presentation with the measurement of the DQD system, and extend the QSL bound of ML type to a general two-level system. In Section 3, we apply the equation of QSL time to the evolution of the DQD system under the dephasing environment and analyze the influence to the QSL time for different factors. In Section 4, we summarize the conclusion drawn from the present study.

2. Theoretical method

2.1. The QSL bound of ML type

The QSL bound is always represented by using the inequality of time, because it originates from the uncertain relationship between energy and time. To quantify the evolving velocity, the trace distance is defined for judging how close between two differentiable states of system. For two states ρ_1 and ρ_2, the trace distance between them is $\mathcal{D}(\rho_1,\rho_2) = \frac{1}{2}\parallel \rho_2-\rho_1 \parallel_{\mathrm{tr}} = \frac{1}{2}\mathrm{Tr}|\rho_2-\rho_1|$, the $\mathrm{Tr}|X| = \sqrt{X^\dagger X}$ is the modulus of an operator X and it refers to the trace norm on the set of trace class operators. The trace distance is a qualified metric to measure and it conforms to $0 \le \mathcal{D} \le 1$.[34] Furthermore, it has a contractivity property that a quantum process acting on two quantum states would not increase their distinguishability, and can also estimate probability of error in the quantum information process.[35–37] The QSL time has become the familiar standard for estimating the speed of evolution because of the uncertainty of the velocity in evolving path.

As is well known, the reduced density matrix of an open quantum system is depending on the Liouville equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = \mathcal{L}\rho(t). \tag{1}$$

In order to quantify the distinguishability between an initial state $\rho(0)$ and a final state $\rho(t)$, the trace distance has been employed to measure the distance between two states. Then combining with the von-Neumann trace inequality one can derive the QSL bound of ML type[38]

$$\tau_{\mathrm{qsl}} \ge \frac{M_\mathrm{D}(\rho(0),\rho(t))}{\frac{1}{2}\overline{\sum_{i=1}^{n} q_i}}, \tag{2}$$

where the time average in denominator of this inequality is represented as $\overline{A} = \tau^{-1}\int_0^\tau \mathrm{d}tA$, and q_i are the singular values of the matrix $\mathcal{L}\rho(t)$. The ML type of the QSL bound is tight and it can be attained when the decoherence dynamics is Markovian in the equilibrium environment.

For a general two-level system, the form of density matrix $\rho(t)$ is expressed as the two order matrix, then the elements of $\rho(t)$ are plugged in to the trace norm for deriving the geodesic distance M_D of Eq. (2) between the initial and final states, which is shown as

$$\begin{aligned} M_\mathrm{D}(\rho(0),\rho(t)) &= \frac{1}{2}\parallel \rho(t)-\rho(0) \parallel_{\mathrm{tr}} \\ &= \sqrt{[\rho_{11}(t)-\rho_{11}(0)]^2 + \mid \rho_{12}(t)-\rho_{12}(0) \mid^2}. \end{aligned} \tag{3}$$

In the derivation of Eq. (3), the conditions $\rho_{11}^*(t) = \rho_{11}(t)$, $\rho_{11}(t) = 1-\rho_{22}(t)$, $\rho_{12}^*(t) = \rho_{21}(t)$ has been used. Meanwhile, the sum of the singular values in Eq. (2) is equal to the trace norm of the matrix $\mathcal{L}\rho(t)$, so the denominator of Eq. (2) can be expressed as

$$\begin{aligned} m_\mathrm{p}(\rho(0),\rho(t)) &= \frac{1}{2}\int_0^\tau \mathrm{d}t \parallel \mathcal{L}\rho(t) \parallel_{\mathrm{tr}} \\ &= \int_0^\tau \mathrm{d}t \sqrt{[\dot{\rho}_{11}(t)]^2 + \mid \dot{\rho}_{12}(t) \mid^2}. \end{aligned} \tag{4}$$

Therefore, the QSL time of the DQD system has received a concise expression form

$$\tau_{\mathrm{qsl}} = \frac{\sqrt{[\rho_{11}(t)-\rho_{11}(0)]^2 + \mid \rho_{12}(t)-\rho_{12}(0) \mid^2}}{\tau^{-1}\int_0^\tau \mathrm{d}t \sqrt{[\dot{\rho}_{11}(t)]^2 + \mid \dot{\rho}_{12}(t) \mid^2}}. \tag{5}$$

Actually, this equation not only relates the initial state ($\rho(0)$) and the target state ($\rho(t)$) closely, but also intuitively shows the ratio relationship of the QSL time (τ_{qsl}) to the driving time (τ). It is convenient for us to observe the changes of QSL in the evolution of a specific system.

2.2. The quantum measurement in DQD system

The DQD system have important research value for realizing the measurement of QSL in experiment. We show the DQD system in Fig. 1, and the Hamilton of the system is written as

$$\begin{aligned} H &= H_\mathrm{S} + H_\mathrm{PC} + H_\mathrm{I}, \\ H_\mathrm{S} &= E_\mathrm{L} c_\mathrm{l}^\dagger c_\mathrm{l} + E_\mathrm{R} c_\mathrm{r}^\dagger c_\mathrm{r} - \Omega(c_\mathrm{l}^\dagger c_\mathrm{r} + c_\mathrm{r}^\dagger c_\mathrm{l}), \end{aligned} \tag{6}$$

$$H_{\rm D} = \sum_{\rm a} E_{\rm a} c_{\rm a}^{\dagger} c_{\rm a} + \sum_{\rm b} E_{\rm b} c_{\rm b}^{\dagger} c_{\rm b} + \sum_{\rm a,b} \left(P c_{\rm b}^{\dagger} c_{\rm a} + P' c_{\rm b} c_{\rm a}^{\dagger} \right),$$
$$H_{\rm I} = \sum_{\rm a,b} \delta P c_{\rm r}^{\dagger} c_{\rm r} (c_{\rm b}^{\dagger} c_{\rm a} + c_{\rm b} c_{\rm a}^{\dagger}), \tag{7}$$

represent the Hamiltonians of the DQD, the quantum point-contact (QPC) detector and its interaction term, respectively.[39–41]

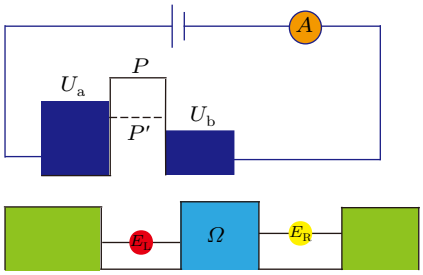


Fig. 1. A point-contact detector monitoring the electron position in the double quantum dots, $U_{\rm a}$ and $U_{\rm b}$ denote the chemical potentials in the left and right reservoirs. $E_{\rm L}$ and $E_{\rm R}$ represent the energy level of electrons in two quantum dots. If the electron stay in right (left) dot, the tunnelling probability of the electrons in QPC detector will be P (P').

Determining the value of variables is the aim of measurement, but the results of any kind of measurement are uncertain in quantum mechanics, and the best way for inferring the possible states of electron motion is to give the distribution of the results of probabilistic measurements. The probability of electron's transfer can be monitored by analyzing the output of the measurement device (the QPC) and it manifest the state's change of the DQD system.[42,43] In the DQD system, we set the energy state of the left and right dots as $E_{\rm L,R} = \mp(\varepsilon/2)$ and Ω is the coupling strength between two quantum dots. Ω represents the width of the tunneling channel connecting the qubits, and the greater the Ω is, the easier the transfer of electron could be.[44] Meanwhile, two sets of creation (annihilation) operators $c_{\rm l,r}$ ($c_{\rm l,r}^{\dagger}$) and $c_{\rm a,b}$ ($c_{\rm a,b}^{\dagger}$) correspond to the electron in the DQD system and the reservoirs in the QPC detector, respectively. Also, it will give rise to a noticeable change in tunnelling probability of the electrons in QPC detector as $\delta P = P - P'$, when the electron occupies the left or right dot.

Here we use the reduced density matrix $\rho(t)$ to describe the system. By employing the Liouville equation, the dynamics evolution of the system has been derived as

$$\frac{\rm d}{{\rm d}t}\rho(t) = -\frac{\rm i}{\hbar}[H, \rho(t)] + \mathcal{L}\rho(t). \tag{8}$$

The right-side of Eq. (8) consists of two parts. The first term represents the unitary evolution of the system, and the decoherence which produced by the coupling between the system and the environment is the second term.

For the influence of the detector in the whole system, it can be described in the following density-matrix elements:

$$\dot{\rho}_{\rm aa} = -{\rm i}\Omega(\rho_{\rm ab} - \rho_{\rm ba}),$$
$$\dot{\rho}_{\rm ab} = {\rm i}\varepsilon\rho_{\rm ab} - {\rm i}\Omega(\rho_{\rm aa} - \rho_{\rm bb}) - \frac{\Gamma_{\rm d}}{2}\rho_{\rm ab}, \tag{9}$$

where the diagonal entries $\rho_{\rm aa}$ ($\rho_{\rm bb}$) represent the probability of finding the electron in the quantum dots, and the off-diagonal matrix elements $\rho_{\rm ab}$ ($\rho_{\rm ba}$) are the coherence of the system and they meet the conditions $\rho_{\rm bb} = 1 - \rho_{\rm aa}$, $\rho_{\rm ab} = \rho_{\rm ba}^{*}$. The $\Gamma_{\rm d} = (\sqrt{I_1/e} - \sqrt{I_2/e})^2 (V/2\pi)$ generated in the off-diagonal matrix elements is called the decoherence rate, I_1 (I_2) represents the current of detector when the electron occupies the first (second) dots in DQD and $V = U_{\rm a} - U_{\rm b}$ is the bias voltage of detector. In fact, the change of $\Gamma_{\rm d}$ would affect the transfer in the DQD system for long time if the environment is ignored.[39] As shown in Eq. (9), the decoherence rate would result in the vanishment of coherence at long time limit, and it reproduce that the the density matrix collapse to a completely random mixture in the course of the measurement process. Especially, we pay close attention to the role $\Gamma_{\rm d}$ played in effecting the QSL bound.

3. The QSL time in pure dephasing environment

In this section, we systematically investigate the evolution of system in the pure dephasing environment. In order to facilitate the calculation, the DQD system has been transformed to a parallel two-level system by employing the following method. Bringing the first term of Eq. (7) into the Liouville equation is the primary step and the corresponding matrix L_0 of the derivation of density matrix with respect to time is shown as

$$L_0 = \begin{pmatrix} 0 & 0 & {\rm i}\Omega & -{\rm i}\Omega \\ 0 & 0 & -{\rm i}\Omega & {\rm i}\Omega \\ {\rm i}\Omega & -{\rm i}\Omega & -{\rm i}\varepsilon & 0 \\ -{\rm i}\Omega & {\rm i}\Omega & 0 & {\rm i}\varepsilon \end{pmatrix}. \tag{10}$$

Then diagonalize the result by utilizing the bonding state and antibonding state[44] $c_{\rm L} = \sin\frac{\theta}{2}c_{\rm l} - \cos\frac{\theta}{2}c_{\rm r}$, and $c_{\rm R} = \sin\frac{\theta}{2}c_{\rm r} + \cos\frac{\theta}{2}c_{\rm l}$, where $\tan\theta = -2\Omega/\varepsilon$. So the Hamiltonian of the DQD can be written as $H_{\rm s} = (\overline{\varepsilon}/2)(c_{\rm L}^{\dagger}c_{\rm L} - c_{\rm R}^{\dagger}c_{\rm R})$ with $\overline{\varepsilon} = \sqrt{\varepsilon^2 + 4\Omega^2}$. And we use the dephasing rate Γ_{ϕ} to represent the effect caused by environment. Under this condition, the evolution of density matrix can be summarized as

$$\dot{\rho}_{\rm aa} = 0, \tag{11}$$
$$\dot{\rho}_{\rm ab} = {\rm i}\overline{\varepsilon}\rho_{\rm ab} - \Gamma_{\phi}\rho_{\rm ab}. \tag{12}$$

It is worth noting that the dephasing rate will cause the vanishment of off-diagonal matrix elements in long-time limit. In the presence of the interaction between the QPC detector and the system, it is necessary to reverse the diagonalization by using the transfer matrix selected before. The corresponding evolution matrix reads as follows:

$$L=\begin{pmatrix} -2\Gamma_\phi\frac{\Omega^2}{\bar{\varepsilon}} & 2\Gamma_\phi\frac{\Omega^2}{\bar{\varepsilon}} & \mathrm{i}\Omega+\Gamma_\phi\frac{\Omega\varepsilon}{\bar{\varepsilon}^2} & -\mathrm{i}\Omega+\Gamma_\phi\frac{\Omega\varepsilon}{\bar{\varepsilon}^2} \\ 2\Gamma_\phi\frac{\Omega^2}{\bar{\varepsilon}} & -2\Gamma_\phi\frac{\Omega^2}{\bar{\varepsilon}} & -\mathrm{i}\Omega-\Gamma_\phi\frac{\Omega\varepsilon}{\bar{\varepsilon}^2} & \mathrm{i}\Omega-\Gamma_\phi\frac{\Omega\varepsilon}{\bar{\varepsilon}^2} \\ \mathrm{i}\Omega+\Gamma_\phi\frac{\Omega\varepsilon}{\bar{\varepsilon}^2} & -\mathrm{i}\Omega-\Gamma_\phi\frac{\Omega\varepsilon}{\bar{\varepsilon}^2} & -\mathrm{i}\varepsilon-\frac{\Gamma_\mathrm{d}}{2}-\frac{\Gamma_\phi}{2}\left(1+\frac{\varepsilon^2}{\bar{\varepsilon}^2}\right) & 2\Gamma_\phi\frac{\Omega^2}{\bar{\varepsilon}^2} \\ -\mathrm{i}\Omega+\Gamma_\phi\frac{\Omega\varepsilon}{\bar{\varepsilon}^2} & \mathrm{i}\Omega-\Gamma_\phi\frac{\Omega\varepsilon}{\bar{\varepsilon}^2} & 2\Gamma_\phi\frac{\Omega^2}{\bar{\varepsilon}^2} & \mathrm{i}\varepsilon-\frac{\Gamma_\mathrm{d}}{2}-\frac{\Gamma_\phi}{2}\left(1+\frac{\varepsilon^2}{\bar{\varepsilon}^2}\right) \end{pmatrix}. \tag{13}$$

Also we combine the matrix elements with Eq. (5) to derive the QSL time.

After taking an elementary operation to Eq. (5), the ratio of the QSL time to the driving time has derived. With the consideration of above details, the value of the ratio τ_{qsl}/τ reflects the potential capacity for the quantum dynamical speedup. If τ_{qsl}/τ equal to one, it represents that the system could not be speeded up, because the speed of evolution reaches the maximum in this condition. In addition, the greater the τ_{qsl}/τ is, the weaker the CPS could be.

Figure 2 shows the variation of τ_{qsl}/τ with respect to the driving time τ in the DQD system. The coupling strength between two qubits is $\Omega=1$ and the energy displacement is $\varepsilon=5$ (in units of Ω). There obviously displays that the value of τ_{qsl}/τ decreases to be steady with the increase of the driving time. The coherence of system diminishes continuously along the driving time which implies that the loss of coherence would promote the system evolving to a stable state. Just like the evolutionary trend of τ_{qsl}/τ which is shown in the dissipative environment,[28] the oscillation reduction behavior of τ_{qsl}/τ appears when the coupling between system and dephasing environment (Γ_ϕ) is chosen as zero in Figs. 2(a) and 2(b).

During the period of τ from 0 to 3, we notice that the decreasing process of τ_{qsl}/τ is delayed with the increase of Γ_d ($4\leq\Gamma_\mathrm{d}\leq 6$) in Fig. 2(c). It means that the evolution of system from the initial state to the stable state is hindered by enhancing the Γ_d in small time. Actually, it is because that the frequently measurement localizes the electron in the DQD system. In the region of $0\leq\tau\leq 3$, it is not difficult to find that the evolution from the initial state to the stable state is prolonged with the increase of Γ_ϕ ($4\leq\Gamma_\phi\leq 6$) in Fig. 2(d). It can be interpreted that the increasing rate of the CPS is impeded by the Γ_ϕ. In other words, the interaction with the pure dephasing environment has stimulative effect on the oscillation of the electron during the evolution.[45] The enhancement of the CPS represents that the oscillation of the electron is slowed down and the probability of the electron's tunnelling in the DQD system would be reduced.

The two figures with $\Gamma_\phi=0$ in Fig. 2(a) and $\Gamma_\phi=5$ in Fig. 2(c) show that τ_{qsl}/τ changes as the function of the decoherence rate (Γ_d). It is interesting that the decreasing range of τ_{qsl}/τ along τ turns to be narrow with the increase of Γ_ϕ by comparing the two graphs. It represents that Γ_ϕ determines the range of the CPS, and the range will be tighter when Γ_ϕ is larger. The CPS is weakened obviously with the increase of Γ_ϕ after the system arriving at the target state. In other words, the coupling between the system and the pure dephasing environment can accelerate the state's evolution of the DQD system.

It is obvious that the value of τ_{qsl}/τ tends to be zero along τ for $\Gamma_\phi=0$ with $\Gamma_\mathrm{d}=0$ in Fig. 2(b). On the other hand, the stable value of τ_{qsl}/τ for $\Gamma_\phi=0$ is near 0.4 with $\Gamma_\mathrm{d}=5$ in Fig. 2(d). Comparing two pictures, the stable value of τ_{qsl}/τ has been increased for different Γ_d. It is easy to find that Γ_d would shrink the reducing range of τ_{qsl}/τ. It means that the coupling between the system and the detector can improve the speed of the DQD system evolving to target state to a certain extent.

The competition between the two factors is discussed via Fig. 3(a), where the driving time is equal to one. Then the value of τ_{qsl}/τ corresponds to the degree of sharp of QSL bound under this condition. As is shown in Fig. 3(a), the scope of the change of τ_{qsl}/τ is from 0.2 to 0.8 under the condition of $\Gamma_\mathrm{d}=2$. While the rising range of τ_{qsl}/τ is from 0.25 to 0.65 when it meets $\Gamma_\phi=2$. The sharpening effect of the QSL bound is induced by increasing the decoherence rate, and it would be hindered by Γ_ϕ. Similarly, Γ_d could weaken the sharpening effect generated by enhancing the dephasing rate. The competition between Γ_ϕ and Γ_d determines the velocity of the evolution. It is worth noting that Γ_ϕ has more obvious tightening effect on the QSL bound compared with Γ_d. Based on this competitive effect, the value of τ_{qsl}/τ can reach the maximum via modulating the relative strength between Γ_ϕ and Γ_d. In other words, it would achieve the most efficient acceleration of evolution in the DQD system under the appropriate dephasing environment and conditions of measurement.

The variation of τ_{qsl}/τ under the condition of $\tau=1$ is plotted in Fig. 3(b). The value of τ_{qsl}/τ is decrease from 0.98 to 0.70 with the increase of ε for $\Omega=8$. The figure shows that the CPS is enhanced monotonously with increasing the energy

displacement for a fixed coupling strength. It can be ascribed to the property of ε that the greater the energy displacement is, the more difficult the transfer of electron could be. In addition, there has an inflection point which meets $\Omega = 0.74\varepsilon$ in the change of $\tau_{\rm qsl}/\tau$. For $\varepsilon = 5$, the value of $\tau_{\rm qsl}/\tau$ begins to rise with the increase of Ω for the condition $\Omega > 0.74\varepsilon$. The seducement of the CPS is interpreted that broadening the width of the tunneling channel connecting the qubits could increase the tunnelling probability of electron. Furthermore, the decreasing effect of the CPS from Ω is stronger than the increasing influence from ε under the case of Ω>0.74ε, but the increasing tendency of the CPS is suppressed by the action of the ε when $\Omega \leq 0.74\varepsilon$. It is the reason why the value of $\tau_{\rm qsl}/\tau$ decreases rapidly from 0.83 to 0.68 in the region $0 \leq \Omega \leq 0.74\varepsilon$.

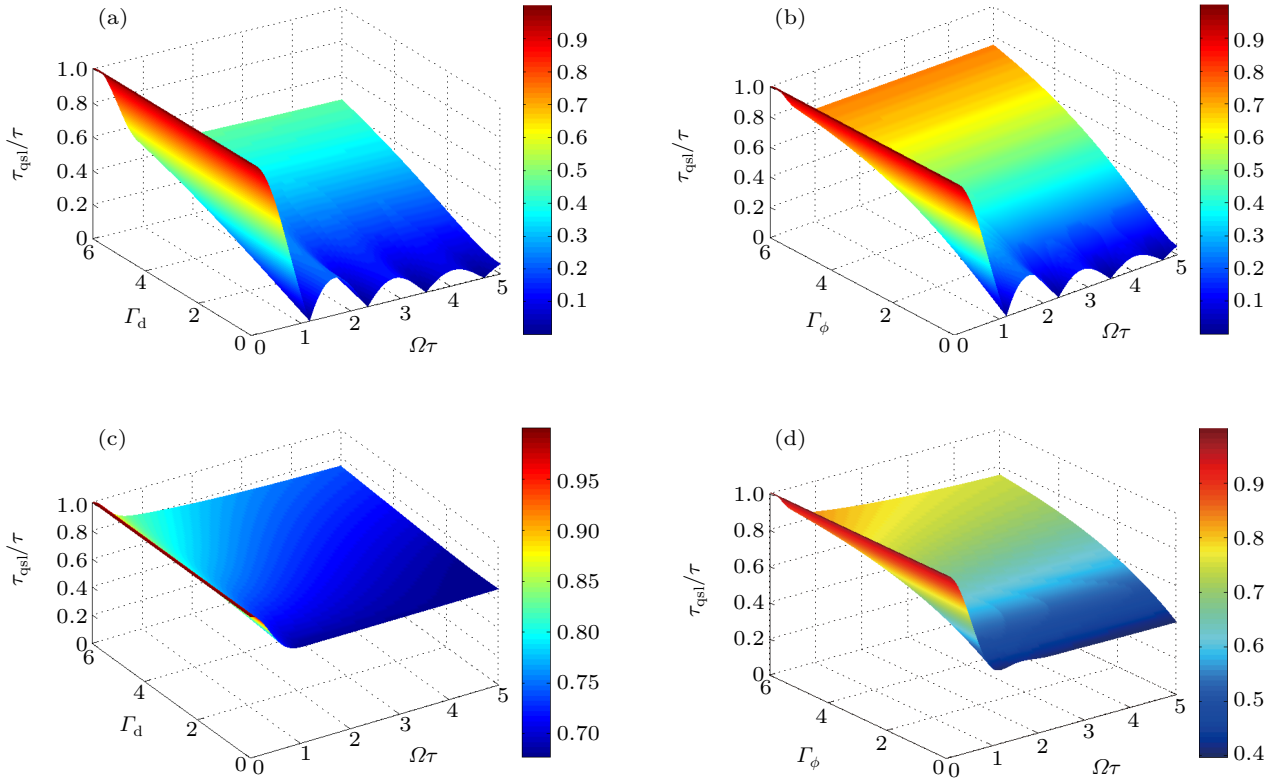


Fig. 2. The ratio of the QSL time to the driving time τ for the DQD system in the dephasing environment. The dephasing rate is chosen as $\Gamma_\phi = 0$ in (a) and $\Gamma_\phi = 5\Omega$ in (c). The decoherence rate is setting as $\Gamma_{\rm d} = 0$ in (b) and $\Gamma_{\rm d} = 5\Omega$ in (d). The energy displacement $\varepsilon = 5$ and the coupling strength parameter is $\Omega = 1$.

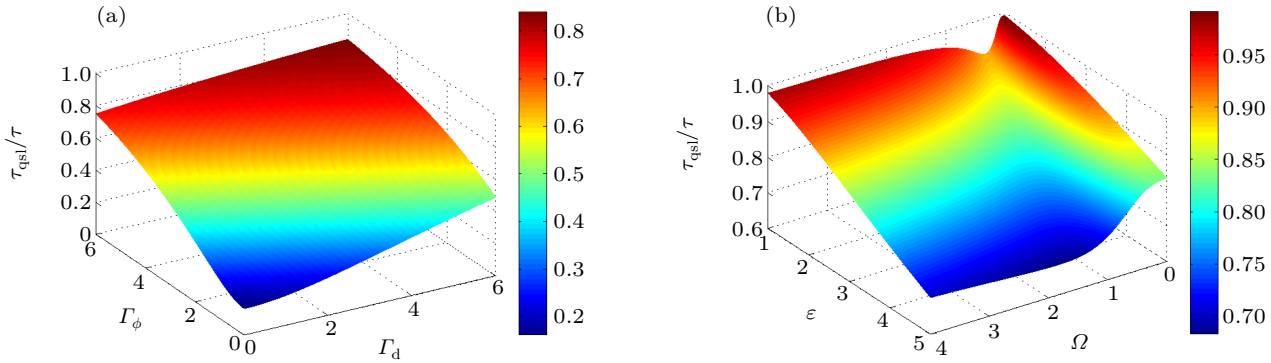


Fig. 3. (a) The $\tau_{\rm qsl}/\tau$ vs. the dephasing rate Γ_ϕ and the decoherence rate $\Gamma_{\rm d}$. The coupling strength between two quantum dots is $\Omega = 1$ and the energy displacement is $\varepsilon = 5$ (in units of Ω). The driving time is $\tau = 1$. (b) $\tau_{\rm qsl}/\tau$ vs. the energy displacement ε and the coupling strength Ω. The dephasing rate $\Gamma_\phi = 5$ and the decoherence rate $\Gamma_{\rm d} = 5$. The driving time is $\tau = 1$.

4. Conclusion

The QSL of the DQD system has been investigated by adopting the detecting of the quantum point contact (QPC) in the pure dephasing environment. The CPS would be decreased by enhancing $\Gamma_{\rm d}$ to some extent in the DQD system. And the process in which the system evolves to the stable state would be delayed for frequently measurement. Furthermore, the improvement of the CPS has been prolonged because of the enhancement of Γ_ϕ, namely, the pure dephasing environment would vary the oscillation of electron. The degree of

sharpen the QSL bound is enhanced with increasing Γ_ϕ, and Γ_d would weaken the sharpening effect and vice versa. The competition between the dephasing rate and the decoherence rate determines the degree of the evolution in the DQD system. The increase of energy displacement can raise the CPS because the strong ε could hinder the transfer of electron. However, the tunneling probability of electron would be increased by strengthening the coupling between two qubits, that is the reason for augmenting the CPS along Ω. It is interesting that there has an inflection point, and the increasing effect of the CPS from ε would be dominant if Ω is less than the point, otherwise the effect from ε would restrain the impact of Ω. More physical quantities can be connected with the QSL bound in the DQD system, such as the current of detector, the geometric phase of the system, etc.

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