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Citation: Chin. Phys. B, 2022, 31 (**6**): 060401. DOI: 10.1088/1674-1056/ac48ff
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Nonlinear dynamical wave structures of Zoomeron equation for population models

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(Received 26 September 2021; revised manuscript received 16 December 2021; accepted manuscript online 11 January 2022)

The nonlinear dynamical exact wave solutions to the non-fractional order and the time-fractional order of the biological population models are achieved for the first time in the framwork of the Paul–Painlevé approach method (PPAM). When the variables appearing in the exact solutions take specific values, the solitary wave solutions will be easily obtained. The realized results prove the efficiency of this technique.

Keywords: (2+1)-dimensional non-fractional Zoomeron equation, time-fractional biological population model, Paul–Painlevé approach method, traveling wave solutions

PACS: 04.20.Jb, 05.45.Yv, 94.05.Fg **DOI:** 10.1088/1674-1056/ac48ff

1. Introduction

The biological population process is one of the most important principal pillars on which the population regulation process depends. The principal axes that determine this scheme and play a vital role in the population regulation process are position, time, and density. Two important mathematical models are established to study these population processes, namely (i) the (2+1)-dimensional non-fractional Zoomeron model (NFZM) mentioned in Refs. [1,2], which is a forward stretch for the well-known (1+1)-NFZM "extensively investigated in the literature" that describes special cases of solitons achieved in different branches of physics; (ii) the time-fractional biological population model (TFBPM) mentioned in Ref. [3] which is the other well-known model that represents the population processes. Particularly, Jabbari *et al.*[4] gave a good model to the TFBPM for animals which is a special case of the (2+1)-NFZM.

Many scientists used several methods to study the nonlinear partial differential equations. Most of these methods are included in Refs. [5–21]. In the literature, recent studies have been introduced to discuss these suggested equations. Especially Abazari[22] obtained the periodic and soliton solutions to Zoomeron equation by means of (G'/G)-expansion method, Khan *et al.*[23] used the (G'/G)-expansion method to find the traveling wave solutions to the (2+1)-dimensional Zoomeron equation. In the relevant subject, there exist other modern articles about solving nonlinear evolution equations that describe the blood flow in arterial vessel that will make revolution in developing the human health and animals. For example Bi *et al.*[24] derived a new model that describes the envelope solitary wave's propagation of the pulsating flow in the blood and discussed and analyzed the influencing factors of stroke volume which provide a strong theoretical basis for some blood diseases and Wenhui *et al.*[25] deduced the (2+1)-dimensional mZK by using multi-scale analysis and reduced perturbations method from the dimensionless equation of (2+1)-dimensional cold dust plasma to describe the propagation law of dust acoustic solitary wave and obtained new model that has high nonlinearity, which can describe the solitary wave on the surface by using the fractional variational method and semi-inverse method.

This work is mainly to realize new impressive regulations for the biological population models through obtaining the exact solution to these two models (which contain some variables) by using the PPAM.[26–28] When these variables take specific values, we will achieve the traveling wave solutions.

2. Technique description of PPAM

Now we first propose the general form of the nonlinear evolution equation We introduce R as a function of $\phi(x,t)$, and its partial derivative is expressed as

$$R(\phi, \phi_x, \phi_t, \phi_{xx}, \phi_{tt}, \ldots) = 0, \tag{1}$$

which involves the highest order derivatives and nonlinear terms. With the aid of the transformation $\phi(x,t) = \phi(\zeta)$, $\zeta = x - kt$, we can have the following ordinary differential equation (ODE):

$$S(\phi', \phi'', \phi''', \ldots) = 0, \tag{2}$$

where S is a function of $\phi(\zeta)$ and its total derivative.

According to PPAM[24] the exact solution to the nonlinear ordinary differential equation can be written in the following

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form:

$$\phi(\zeta) = A_0 + A_1 W(X)\, \mathrm{e}^{-N\zeta}, \quad X = R(\zeta)) \tag{3}$$

or

$$\begin{aligned} &\phi(\zeta) = A_0 + A_1 W(X)\, \mathrm{e}^{-N\zeta} + A_2 W^2(X)\, \mathrm{e}^{-2N\zeta}, \\ &X = R(\zeta), \end{aligned} \tag{4}$$

where $X = R(\zeta) = C_1 - (\mathrm{e}^{-N\zeta}/N)$ and $W(X)$ that appears in Eq. (4) satisfies the Riccati equation in the form of $W_X - AW^2 = 0$.

One can solves this equation to obtain

$$W(X) = \frac{1}{AX + X_0}. \tag{5}$$

3. Exact solution to (2+1)-NFZM equation

In this section, we will use the PPAM as a new technique to realize the exact solution to the (2+1)-NFZM[1] "in terms of some variables", we can easily obtain the traveling wave solutions when these variables take specific values. According to Refs. [1,2], the (2+1)-NFZM equation can be written as

$$\left(\frac{\phi_{xy}}{\phi}\right)_{tt} - \left(\frac{\phi_{xy}}{\phi}\right)_{xx} + 2\left(\phi^2\right)_{xt} = 0, \tag{6}$$

where $\phi(x,y,t)$ is the amplitude of the relative wave mode. By transforming $\phi(x,y,t) = \phi(\zeta)$ and $\zeta = x + y - kt$, equation (6) becomes

$$k^2\left(\frac{\phi''}{\phi}\right)'' - \left(\frac{\phi''}{\phi}\right)'' - 2\left(\phi^2\right)'' = 0. \tag{7}$$

Integrating Eq. (7) twice with respect to ζ we obtain

$$(k^2 - 1)\phi'' - 2k\phi^3 + k_1\phi = 0, \tag{8}$$

where k_1 is the integration constant, the homogeneous balance for the proposed model implies $m = 1$. Thus the solution form according to the proposed technique is

$$\phi(\zeta) = A_0 + A_1 W\, \mathrm{e}^{-N\zeta}. \tag{9}$$

Consequently,

$$\phi_\zeta = -NA_1\, \mathrm{e}^{-N\zeta} W - AA_1 W^2 \mathrm{e}^{-2N\zeta}, \tag{10}$$

$$\begin{aligned} \phi_{\zeta\zeta} = {} & N^2 A_1 W \mathrm{e}^{-N\zeta} + 3AA_1 N W^2 \mathrm{e}^{-2N\zeta} \\ & + 2A_1 A^2 W^3 \mathrm{e}^{-3N\zeta}. \end{aligned} \tag{11}$$

Inserting ϕ, ϕ_ζ, $\phi_{\zeta\zeta}$ into Eq. (8) and equating the coefficients of different powers of $W^i\, \mathrm{e}^{-iN\zeta}$ to zero, the following system of equations will be obtained:

$$\begin{aligned} &k_1 A_0 - 2kA_0^3 = 0, \\ &N^2 A_1 (k^2 - 1) - 6kA_1 A_0^2 + A_1 k_1 = 0, \\ &3ANA_1 (k^2 - 1) - 6kA_0 A_1^2 = 0, \\ &2A^2 A_1 (k^2 - 1) - 2kA_1^3 = 0. \end{aligned} \tag{12}$$

Solving this system of equations, the following results will be achieved:

$$A_0 = -\frac{\sqrt{k_1}}{\sqrt{2k}},\ A_1 = -\frac{A}{\sqrt{k}\sqrt{k^2-1}} + \frac{Ak^{3/2}}{\sqrt{k^2-1}},$$
$$N = -\frac{\sqrt{2k_1}}{\sqrt{k^2-1}}, \tag{13a}$$

$$A_0 = -\frac{\sqrt{k_1}}{\sqrt{2k}},\ A_1 = \frac{A}{\sqrt{k}\sqrt{k^2-1}} - \frac{Ak^{3/2}}{\sqrt{k^2-1}},$$
$$N = \frac{\sqrt{2k_1}}{\sqrt{k^2-1}}, \tag{13b}$$

$$A_0 = \frac{\sqrt{k_1}}{\sqrt{2k}},\ A_1 = \frac{A}{\sqrt{k}\sqrt{k^2-1}} - \frac{Ak^{3/2}}{\sqrt{k^2-1}},$$
$$N = -\frac{\sqrt{2k_1}}{\sqrt{k^2-1}}, \tag{13c}$$

$$A_0 = \frac{\sqrt{k_1}}{\sqrt{2k}},\ A_1 = -\frac{A}{\sqrt{k}\sqrt{k^2-1}} + \frac{Ak^{3/2}}{\sqrt{k^2-1}},$$
$$N = \frac{\sqrt{2k_1}}{\sqrt{k^2-1}}. \tag{13d}$$

These four various results in which $k \neq 0$, $k \neq \pm 1$, $k_1 \neq 0$ will generate four different solutions. For simplicity and similarity, we design the solution corresponding to one of these results, say, the first one, which is

$$A_0 = -\frac{\sqrt{k_1}}{\sqrt{2k}},\ A_1 = -\frac{A}{\sqrt{k}\sqrt{k^2-1}} + \frac{Ak^{3/2}}{\sqrt{k^2-1}},$$
$$N = -\frac{\sqrt{2k_1}}{\sqrt{k^2-1}}.$$

This result is dependent on the sign of k and k_1 which will govern the behavior of the resulting solution definitely if they are both positive or negative at the same time, this result will have real values while if one of them is positive and the other is negative which will have imaginary value. We will discuss the two cases separately.

Case 1 When k and k_1 have any two positive or negative real values at the same time, this result can be simplified into

$$A_0 = -1,\ A_1 = 1,\ A = 0.8,\ N = -3.3. \tag{14}$$

Thus the solution is

$$\phi(\zeta) = A_0 + \frac{A_1\, \mathrm{e}^{-N\zeta}}{AX + X_0}, \tag{15}$$

$$\phi(\zeta) = -1 + \frac{\mathrm{e}^{3.3\zeta}}{0.8\left(1 + \left(\mathrm{e}^{3.3\zeta}/3.3\right)\right) + 1},$$

$$\phi(\zeta) = -1 + \frac{3.3\,\mathrm{e}^{3.3\zeta}}{5.9 + 0.8\,\mathrm{e}^{3.3\zeta}}. \tag{16}$$

Case 2 When either of k and k_1 has positive real value and the other has negative real value at the same time, this result can be simplified into

$$A_0 = -\mathrm{i},\ A_1 = 1,\ A = 0.8,\ N = -1.6\mathrm{i}. \tag{17}$$

Thus the solution is

$$\phi(\zeta) = -\mathrm{i} + \frac{\mathrm{e}^{1.6\mathrm{i}\zeta}}{0.8\left(1+(\mathrm{e}^{1.6\mathrm{i}\zeta}/1.6\mathrm{i})\right)+1},$$

$$\phi(\zeta) = -\mathrm{i} + \frac{1.6\mathrm{i}\,\mathrm{e}^{1.6\mathrm{i}\zeta}}{0.8\left(1.6\mathrm{i}+\mathrm{e}^{1.6\mathrm{i}\zeta}\right)+1.6\mathrm{i}}, \tag{18}$$

$$\mathrm{Re}\,\phi(\zeta) = \frac{7.2\cos 1.6\zeta}{13.96+7.2\sin 1.6\zeta}, \tag{19}$$

$$\mathrm{Im}\,\phi(\zeta) = -1 - \frac{2+7.2\sin 1.6\zeta}{13.96+7.2\sin 1.6\zeta}. \tag{20}$$

In the same manner we can design the other three solutions that are identical to the remaining three results.

4. Some knots about fractional calculus

Before we apply the PPAM mentioned above to the TFBPM,[3] we first give some notes about the fractional calculus.[27]

4.1. Caputo's fractional derivative[28–36]

Although the fractional calculus is a powerful tool to describe the physical phenomena system which has long-term memory and long-range spatial interaction, there exists the distortion between fractional mild solution and strong solution. This confusion is due to the fact that the fractional differential equation has a non-differentiable solution, and this makes solution process difficult, but an approximate continuous solution can be realized if we observe the discontinuous solution. In this section we build preliminary notes and remind some distinctive relations to Riemann–Liouville derivatives.

Definition 1 A real function $g(x)$, $x>0$ is said to be in space D_α, $\alpha\in\Re$ if there exists real number $q>\alpha$, such that $g(x)=x^q g_1(x)$, where $g_1(x)\in D[0,\infty)$.

Definition 2 A real function$g(x)$, $x>0$ is said to be in space D_α^m, $m\in N\cup\{0\}$ if $g^m\in D_\alpha$.

Definition 3 Let $g\in D_\alpha$, and $\alpha\geq -1$, then the left-hand side of Riemann–Liouville integral of order μ, $\mu>0$ is given by

$$I_t^\mu\, g(x,t) = \frac{1}{\Gamma(\mu)}\int (t-\tau)^{\mu-1} g(x,\tau)\,\mathrm{d}\tau,\ \tau>0. \tag{21}$$

Definition 4 The (left-sided) Caputo partial fractional derivative of g with respect to t, $g\in D_{-1}^m$, $m\in N\cup\{0\}$, is defined as

$$D_t^\mu\, g(x,t) = \frac{\partial^m}{\partial t^m} g(x,t),\ \mu=m, \tag{22}$$

$$D_t^\mu\, g(x,t) = I_t^{\mu-m}\frac{\partial^m}{\partial t^m} g(x,t),$$
$$m-1<\mu<m,\ m\in N, \tag{23}$$

$$I_t^\mu D_t^\mu\, g(x,t) = g(x,t) - \sum_{k=0}^{m-1}\frac{\partial^k g}{\partial t^k}(x,0)\frac{t^k}{k!},$$
$$m-1<\mu<cm,\ m\in N, \tag{24}$$

$$I_t^\mu t^v = \frac{\Gamma(v+1)}{\Gamma(\mu+v+1)} t^{\mu+v}. \tag{25}$$

Furthermore, according to the Grünwald–Letnikov, the Caputo & Riemann–Liouville[28,29] and the modified Riemann–Liouville derivative by Jumarie,[30,31] these definitions and some properties for the Jumarie's derivative of order α are listed as follows:

$$D_t^\alpha t^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)} t^{r-1}, \tag{26}$$

$$D_t^\alpha[f(t)g(t)] = f(t)D_t^\alpha[g(t)] + g(t)D_t^\alpha[f(t)], \tag{27}$$

$$D_t^\alpha f[g(t)] = f'[g(t)]D_t^\alpha[g(t)] = D_t^\alpha[g(t)](g'(t))^\alpha, \tag{28}$$

$$D_t^\alpha(c) = 0. \tag{29}$$

Moreover, the operator D_t^α satisfies the following basic properties:

$$D_t^\alpha D_t^\beta\, g(x) = D_t^{\alpha+\beta} g(x), \tag{30a}$$

$$D_t^\alpha x^\gamma = \left(\frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)}\right) x^{\gamma-\alpha}, \tag{30b}$$

$$D_t^\alpha I_t^\beta\, g(t) = g(t), \tag{30c}$$

$$I_t^\beta D_t^\alpha\, g(t) = g(t) - \sum_{k=0}^{m} g^k(0^+)\left(\frac{t^k}{k!}\right). \tag{30d}$$

Also, we introduce the conformable fractional derivative which is stated as follows.

If the function $\phi:[0,\infty)\to 0$, then the conformable fractional derivative of $\phi(x)$, $x>0$ is defined as

$$D_x^\gamma(\phi(x)) = \lim_{H\to 0}\frac{\phi(x+Hx^{1-\gamma})-\phi(x)}{H},\ \gamma\in(0,1], \tag{31}$$

for which all original differentiation rules that are applied to the ordinary functions have been realized.[40]

Now, we propose the general form of the NLFPDE in terms of the modified Riemann–Liouville derivatives D_t^α, D_x^α as follows:

$$R(\phi, D_t^\alpha\phi, D_x^\alpha\phi, D_{t^2}^{2\alpha}\phi, D_{x^2}^{2\alpha}\phi,\ldots) = 0,\ \ 0<\alpha\leq 1. \tag{32}$$

According to Refs. [33,34], the fractal derivative can be generally defined as

$$\frac{\partial t}{\partial x^\alpha} = \Gamma(1+\alpha)\lim_{\substack{x\to x_0\\ \Delta x-x-x_0\to 0}}\frac{t-t_0}{(x-x_0)^\alpha}, \tag{33}$$

where x_0 is the smallest scale beyond which there is no physical understanding. Now, using the fractional complex nonlinear transformation[30,34]

$$\phi(x,t) = \phi(\zeta),\ \ \zeta = \frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ct^\alpha}{\Gamma(1+\alpha)} + \zeta_0, \tag{34}$$

where k, c, and ζ_0 are constants with $k,c\neq 0$, we transform Eq. (32) into the following ODE with integer order which is

$$S(\phi', \phi'', \phi''', \ldots) = 0, \tag{35}$$

where the prime $' = \mathrm{d}/\mathrm{d}\xi$.

According to Ref. [3], the TFBPM is given as

$$\frac{\partial^\alpha \phi}{\partial t^\alpha} = \frac{\partial^2}{\partial x^2}(\phi^2) + \frac{\partial^2}{\partial t^2}(\phi^2) + \lambda\left(\phi^2 - r\right) = 0, \tag{36}$$

where $t > 0$, $0 < \alpha \leq 1$, and $x, y \in \mathbb{R}$, while u and $\lambda(\varphi^2 - r)$ represent the density and the principle support of population respectively built on birth and death while λ and r are constants.

4.2. Exact solution to time-fraction order of biological population model

We apply the constructed approach to the time-fraction order of the biological population model (TFBPM) and consider this complex transformation

$$\phi(x,y,t) = \phi(\zeta),\ \zeta = x + \mathrm{i}y - \frac{ct^\alpha}{\Gamma(1+\alpha)}, \tag{37}$$

where c is constant and $\mathrm{i}^2 = -1$. Thus, according to this transformation, equation (36) becomes

$$c\phi' + \lambda\phi^2 - \lambda r = 0, \quad \phi' = \frac{\mathrm{d}\phi}{\mathrm{d}\zeta}. \tag{38}$$

Hence, according to the proposed method the solution is

$$\begin{aligned} \phi(\zeta) &= A_0 + A_1\,\mathrm{e}^{-N\zeta} W(X), \\ X(\zeta) &= R(\zeta) = C_1 - \frac{\mathrm{e}^{-N\zeta}}{N}. \end{aligned} \tag{39}$$

Inserting ϕ^2, ϕ' into Eq. (39) and equating the coefficients of different powers of $W^i\,\mathrm{e}^{-\mathrm{i}N\zeta}$ to zero we obtain the following system of equations:

$$\begin{aligned} &-\lambda A_0 + \lambda A_0^2 = 0, \\ &-CNA_1 - \lambda A_1 + 2\lambda A_0 A_1 = 0, \\ &-CAA_1 + \lambda A_1^2 = 0. \end{aligned} \tag{40}$$

When one solves this system manually, the following results are as follows:

$$A = \frac{\lambda A_1}{C}, \quad N = \frac{\lambda}{C}, \quad A_0 = 1. \tag{41}$$

This result can be simplified into

$$A = 0.5, \quad N = 0.5, \quad C_1 = A_0 = A_1 = 1, \quad \lambda = 0.5. \tag{42}$$

According to the proposed method the solution is

$$\phi(\zeta) = A_0 + \frac{A_1\,\mathrm{e}^{-N\zeta}}{AX + X_0}, \tag{43}$$

$$\phi(\zeta) = 1 + \frac{\mathrm{e}^{-0.5\zeta}}{0.5\left(1 - (\mathrm{e}^{-0.5\zeta}/0.5)\right) + 1}, \tag{44}$$

$$\phi(\zeta) = 1 + \frac{0.5\,\mathrm{e}^{-0.5\zeta}}{0.75 - 0.5\,\mathrm{e}^{-0.5\zeta}}, \tag{45}$$

where

$$\zeta = x + \mathrm{i}y - \frac{ct^\alpha}{\Gamma(1+\alpha)} = x + \mathrm{i}y - 1.1\sqrt{t}.$$

Remark 1 Many other solutions can be achieved according two different values of α, λ, and C.

5. Results and discussion

(i) From the point of view of the PPAM, we establish new periodic biological population distribution solutions of the (2+1)-dimensional Zoomeron model. The other achieved solutions for this model indicate that the biological population distribution is symmetric with respect to the axis. These new behaviors are documented via Figs. 1–3. The achieved solutions which were not realized before are new compared with those in Refs. [22,23] where is used the (G'/G)-expansion method that is obtained in Ref. [38] where the exp-function method is used. Furthermore, the simulation of the solitary solutions is achieved as shown in Ref. [39] for the bounded traveling wave solution of this model using the bifurcation method of dynamical system and numerical simulation method. Also some of the obtained solutions are in agreement with the solutions that are achieved in Ref. [40] in some cases but some are in disagreement in other cases. Moreover, the achieved solutions present the periodic behavior in some cases and the isomorphic distribution in other cases for the biological population distribution process.

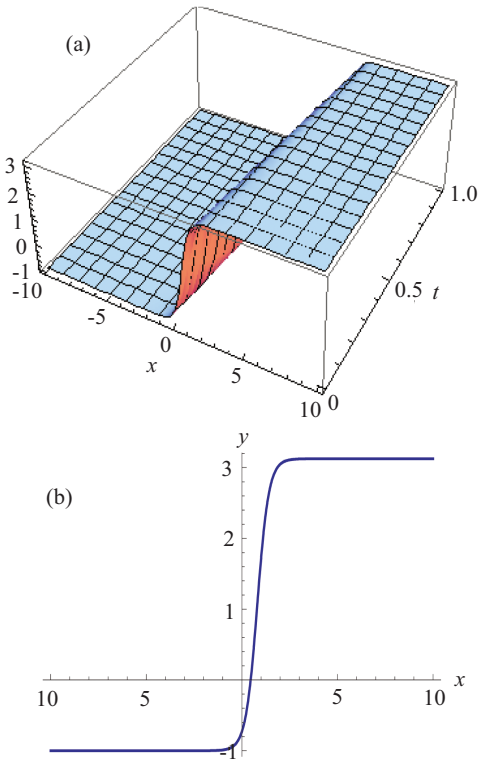


Fig. 1. Plot of 2D and 3D Eq. (16) with values: $A_0 = -1$, $A_1 = 1$, $A = 0.8$, $N = -3.3$, $C_1 = X_0 = 1$, $k = 2$, $k_1 = 4$, $t = 0.1$ for two dimensions.

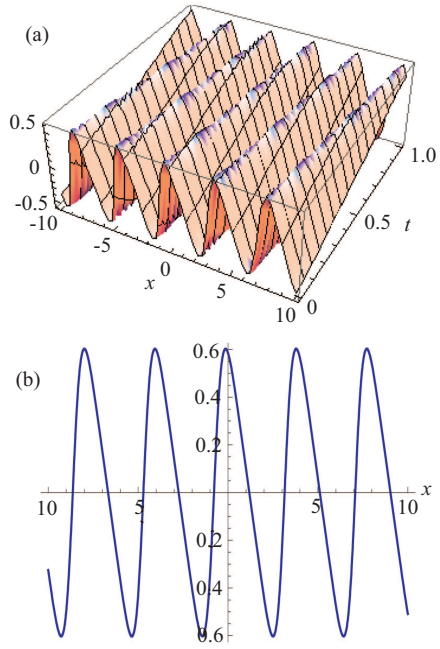


Fig. 2. Plot of real part of 2D and 3D Eq. (19) with values: $A_0 = -\mathrm{i}$, $A_1 = 1$, $A = 0.8$, $N = -1.6\mathrm{i}$, $C_1 = X_0 = 1$, $k = 2$, $k_1 = 4$, $t = 0.1$ for two dimensions.

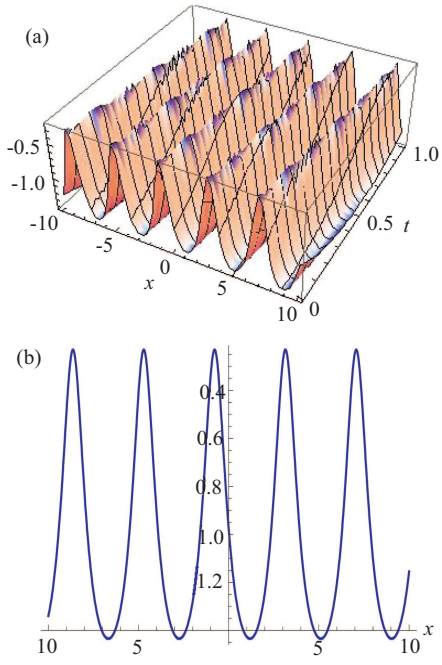


Fig. 3. Plot of imaginary part of 2D and 3D Eq. (20) with values: $A_0 = -\mathrm{i}$, $A_1 = 1$, $A = 0.8$, $N = -1.6\mathrm{i}$, $C_1 = X_0 = 1$, $k = 2$, $k_1 = 4$, $t = 0.1$ for two dimensions.

(ii) On the other hand, the achieved solutions for the time fraction population equation in the framework of the suggested technique indicate that the biological population distribution process is regular and symmetric one (see Fig. 4) which is new, very accurate compared with the obtained solutions in Refs. [36,37,41], where were used the Homotopy analysis Sumudu transformation method, the Homotopy perturbation method and Homotopy analysis respectivly.

It is clear that all achieved solutions for the fractional-order biological population models and non-fractional-order biological population models are stable, *i.e.* for consecutive intervals of time the behaviors of solutions will not change. now, we come to discuss this point. For example for the first solution for two consecutive different intervals of time we find that the behaviors of the solutions in [0–1] are the same as those in Refs. [2,3].

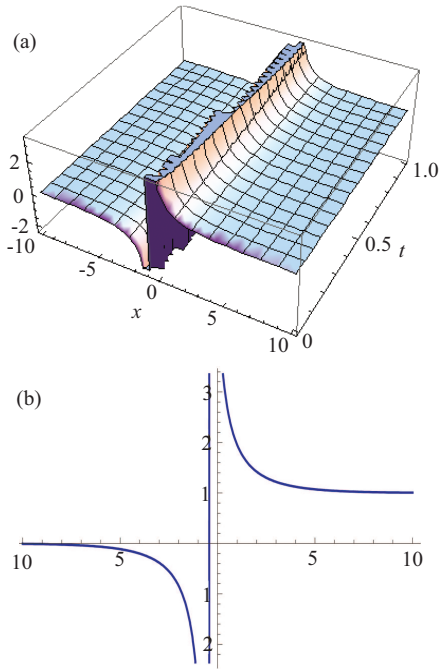


Fig. 4. Plot of 2D and 3D Eq. (45) with values: $A_0 = A_1 = c = C = C_1 = k = \lambda = X_0 = 1$, $A = 0.5$, $N = 0.5$, $\alpha = 0.5$, $t = 0.1$ for two dimensions.

6. Conclusions

The PPAM is effectively used for the first time to construct new impressive regulations for the biological population models with its different forms, which are the non-fractional-order biological population (2+1)-Zoomeron model (NFBPZM) (see Figs. 1–3) and the time-fractional-order biological population model (TFBPM) (see Fig. 4). Some of the obtained results agree with the results previously obtained in Refs. [24,25] for the NFBPZM and in Refs. [37–39,41] for the TFBPM in some cases, but some disagree in other cases. The achieved results and corresponding solutions that depend on the fundamental parameters which are the position, time, and density indicate that the population regulation process is symmetric in some cases and periodic in other cases. The satisfying results of the new accurate regulation population process prove this technique to be efficient.

References

[1] Calogero F and Degasperis A 1976 *Nuovo Cimento B* **32** 201
[2] Shakeri E and Dehghan M 2007 *Comput. Math. Appl.* **54** 1197
[3] Bekir A and Guner O 2013 *Chin. Phys. B* **22** 110202
[4] Jabbari A, Kheiri H and Bekir A 2011 *Comput. Math. Appl.* **62** 2177
[5] Bekir A and Zahran E H M 2020 *Optik* **223** 165233
[6] Li N and Liu X Q 2013 *Acta Phys. Sin.* **62** 160203 (in Chinese)
[7] Bekir A and Zahran E H M 2021 *Optik* **240** 166939

[8] Song C Q, Xiao D M and Zhu Z N 2017 *Chin. Phys. B* **26** 100204
[9] Bekir A and Zahran E H M 2021 *Phys. Scr.* **96** 055212
[10] Song M, Beidan W and Jun C 2020 *Chin. Phys. B* **29** 100206
[11] Shehata M S M, Rezazadeh H, Zahran E H M, Tala-Tebue E and Bekir A 2019 *Commun. Theor. Phys.* **71** 1275
[12] Bekir A and Zahran E H M 2021 *Pramana J. Phys.* **95** 158
[13] Shehata M S M, Rezazadeh H, Jawad A J M, Zahran E H M and Bekir A 2021 *Revista Mexicana de Fisica* **67** 050704
[14] Huang L L, Qiao Z J and Chen Y 2018 *Chin. Phys. B* **27** 020201
[15] Zahran E H M and Khater M M 2016 *Appl. Math. Model.* **40** 1769
[16] Khater M M, Lu D and Zahran E H M 2017 *Appl. Math. Inf. Sci.* **11** 1347
[17] Zahran E H M and Bekir A 2021 *Opt. Quantum Electron.* **53** 680
[18] Wazwaz A M 2004 *Appl. Math. Comput.* **154** 713
[19] Bekir A, Zahran E H M and Güner O 2020 *Int. J. Mod. Phys. B* **35** 2150025
[20] Zhang S, Jing-Lin T and Wei W 2008 *Phys. Lett. A* **372** 2254
[21] Fan E 2000 *Phys. Lett. A* **277** 212
[22] Abazari R 2011 *Appl. Math. Sci.* **5** 2943
[23] Khan K, Akbar M A, Salam A and Islam H 2014 *Ain Shams Engineering Journal* **5** 877
[24] Bi Y, Zhang Z, Liu Q and Liu T 2021 *Commun. Nonlinear Sci. Numer. Simul.* **102** 105918
[25] Wenhui Z, Hongwei Y and Dezhi G 2020 *Mathematical Modeling and Its Applications* **466** 73
[26] Bekir A, Ayhan B and Ozer M N 2013 *Chin. Phys. B* **22** 010202
[27] Bekir A and Zahran E H M 2020 *Int. J. Mod. Phys. B* **34** 2050270
[28] Bekir A and Zahran E H M 2021 *Opt. Quantum Electron.* **53** 118
[29] Caputo M and Mainardi F 1971 *Revista del Nuovo Cimento* **161** 198
[30] Caputo M 1967 *J. Royal Astronom. Soc.* **13529** 539
[31] Samko S G, Kilbas A A and Marichev O I 1993 *Fractional Integrals and Derivatives: Theory and Applications* (Gordon and Breach Science Publishers, Switzerland)
[32] Jumarie G 2006 *Comput. Math. Appl.* **51** 1367
[33] Jumarie G 2006 *Comput. Math. Appl.* **51** 1367
[34] Jumarie G 2009 *Appl. Math. Lett.* **22** 378
[35] Kilbas A A, Srivastava H M and Trujillo J J 2006 *Theory and Applications of Fractional Differential Equations-North-Holland Mathematical Studies* **204** (Amsterdam: Elsevier)
[36] Bekir A and Guner O 2014 *Ain Shams Eng. Journal* **5** 959
[37] Gupta V G and Kumar P 2013 *International Journal of Science and Research* **6** 14
[38] Gurtin M Eand Maccamy R C 1977 *Math. Biol. Sci.* **54** 35
[39] Lu Y G 2000 *Appl. Math. Lett.* **13** 123
[40] Liu Y, Li Z and Zhang Y 2011 *Fract. Diff. Calc.* **1** 117
[41] Khalil R, Al Horani M, Yousef A and Sababheh M 2014 *J. Comput. Appl. Math.* **265** 65
[42] Ain Q T and He J H 2019 *Thermal Science* **23** 1707