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Evolution of donations on scale-free networks during a COVID-19 breakout

Xian-Jia Wang(王先甲)[1,2] and Lin-Lin Wang(王琳琳)[1,†]

[1] *Economics and Management School, Wuhan University, Wuhan* 430072, *China*
[2] *Institute of Systems Engineering, Wuhan University, Wuhan* 430072, *China*

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Having a large number of timely donations during the early stages of a COVID-19 breakout would normally be considered rare. Donation is a special public goods game with zero yield for donors, and it has the characteristics of the prisoners' dilemma. This paper discusses why timely donations in the early stages of COVID-19 occurred. Based on the idea that donation is a strategy adopted by players during interconnection on account of their understanding of the environment, donation-related populations are placed on social networks and the inter-correlation structures in the population are described by scale-free networks. Players in donation-related populations are of four types: donors, illegal beneficiaries, legal beneficiaries, and inactive people. We model the evolutionary game of donation on a scale-free network. Donors, illegal beneficiaries and inactive people learn and update strategies under the Fermi update rule, whereas the conversion between legal beneficiaries and the other three types is determined by the environment surrounding the players. We study the evolution of cooperative action when the agglomeration coefficient, the parameters of the utility function, the noise intensity, the utility coefficient, the donation coefficient and the initial states of the population on the scale-free network change. For population sizes of 50, 100, 150, and 200, we give the utility functions and the agglomeration coefficients for promoting cooperation and study the corresponding steady states and structural characteristics of the population. We identify the best ranges of the noise intensity K, the donation coefficient α and the utility coefficient β for promoting cooperation at different population sizes. Furthermore, with the increase of the population size, the donor traps are found. At the same time, it is discovered that the initial states of the population have a great impact on the steady states; thus the upper and lower triangle phenomena are proposed. We also find that the population size itself is also an important factor for promoting donation, pointing out the direction of efforts to further promote donation and achieve better social homeostasis under the donation model.

Keywords: donation, public goods game, evolutionary game, scale-free network

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1. Introduction

On January 23, 2020, Wuhan was closed down for 76 days for pneumonia caused by COVID-19. At the early stage of the pneumonia, the medical equipment used for COVID-19 treatment was in serious shortage, and there was also a temporary shortage of general goods. It was at this time that a large number of donations including medical devices, protective devices, daily necessities, basic living materials and money were provided. Many donations were spontaneous and even anonymous. The phenomenon of voluntary donation is rare in normal society. Donation seems to be an irrational action, so how did this massive donation happen during the closure of Wuhan?

Donation can be regarded as a special form of the multi-subject public goods games; its particularity is manifested in the fact that the yield of the public goods for suppliers is zero, and donation only produces income to individuals other than the suppliers.[1–3] From a rational point of view, the multi-subject public goods game is the prisoner dilemmas' game, and so naturally donation is also the prisoners' dilemma game. We refer to the action of providing supply in the multi-subject public goods game as the "cooperative action".[4] Under complete rationality, the irrational action of cooperation in a multi-subject public goods game is impossible and cooperative action is unlikely to arise. However, in reality, especially during the closure of Wuhan in 2020, a large number of donation (cooperative action) occurred naturally. We speculate that this irrational cooperative action arises from the continuous learning and evolution of the population on social networks under finite rationality.

Donation is affected by the strategies of neighbors, and this mutual relationship structure between players in the population constitutes a social network.[5–10] Due to limits imposed by geographical factors, players in society can not have the same opportunity to contact and communicate, and each player's ability to connect with other players is also different. The more contacts that a player has with others, the more easily they affect other players in society. The few players that are associated with a great number of others can be very influential. Bearing in mind these properties of social networks, this paper considers the donation model on complex networks. It is generally believed that the properties of social networks such

†Corresponding author. E-mail: 2015301580305@whu.edu.cn

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as these can be characterized by scale-free networks, which are a type of complex networks. As a part of the generation process, the new nodes added to the network are connected to the original nodes with some probabilities, which is directly proportional to the degrees. The scale-free network has sufficient heterogeneity because the numbers of players' neighbors are different, and the players joining later are randomly connected according to the degree distribution of the existing players.

This paper suggests that there are four possible strategies or types of players in the social network. Players have four strategies: donation, corruption, disaster and inaction. These strategies correspond, respectively, to four types, donors, illegal beneficiaries, legal beneficiaries, and inactive people. Donors donate free of charge. Illegal beneficiaries represent institutions or individuals who embezzle medical or essential materials during the course of epidemics. Legal beneficiaries represent victims of epidemics. Inactive people neither donate nor benefit.

The strategy or type of a player in a social network depends upon her strategy update rules under finite rationality, and the strategy update rule is based on her learning rule or is influenced by the overall environment. There are less barriers to changing their strategies among those not in the epidemic, i.e., donors, illegal beneficiaries, and inactive people. If a donor, for example, finds that her neighbor who chooses inaction gains more, then she may update her strategy with a certain probability and become an inactive person. However, updating strategies depends not only on the will of the player, but more on the types of her neighbors. If a legal beneficiary is in a very serious epidemic, then it is most likely that she will continue to be a victim. Correspondingly, even if a player does not want to be caught in the epidemic, she is highly likely to be infected if she has many infected neighbors. Thus donors, illegal beneficiaries and inactive people update strategies by the Fermi update rule. When the proportion of donors among the neighbors of a legal beneficiary reaches a certain threshold, the legal beneficiary's income can naturally be enough to bring her out of trouble. At this time, the legal beneficiary will choose the strategy that is most common among her neighbors. Meanwhile, when the proportions of legal beneficiaries among the neighbors of donors, illegal beneficiaries, or inactive people reach certain thresholds, these three types will convert into legal beneficiaries. Otherwise, they can only update strategies under the Fermi update rule among themselves. The renewal of players' strategies at each time step determines the state of the entire population at that time, and this is the evolution of population states under the players' strategy updating rules. In this paper, we study the results of the dynamic evolution of a special public goods game on the scale-free network.

What follows is a review of the previous classical literature from four aspects: games, donation, scale-free networks and the cooperative evolution on scale-free networks. The analysis of the four-strategy donation evolution game on the scale-free network is an innovation.

Game theory provides a widely applicable theoretical framework for studying interactions between players or groups.[11–14] Evolutionary game is developed from game theory, combines the game theory and the dynamic evolution process, and studies the dynamic balance of populations in evolution.[15,16] In many theoretical models, the public goods game (PGG) is a general model that can illuminate multiplayer decision-making action.[17] The dilemma of public land tragedy cannot be avoided without any interference mechanisms in the typical public goods game model.

Early researchers used experimental methods to confirm the impact of different family backgrounds on donation. Dawes *et al.* and Van de Kragt *et al.* successively used the experimental method to verify the authenticity of the above cases in a simulated experimental environment.[18] After studying numerous donation game experiments, Bagnoli and Lipman raised the question of donor incentives, and asked whether the donor had at least one constant incentive for her continuous donation, thus maximizing the group gain. In a later study, Bagnoli further confirmed that the number of donations gradually tended to a dynamic equilibrium.[19] Chaudhuri focused on donation game experiments after 1995 and focused on failed experiments. In a large number of failed experiments, the hitchhiking effect emerged as a very important factor.[20] Similarly, there are many studies on altruism. Experiments by Wilkinson showed that unrelated bats exchanged blood with each other in captivity.[21] Some studies have highlighted the importance of friendliness in solving social difficulties.[22] Wang *et al.* found that reducing anonymity could effectively promote cooperation, thereby bringing higher returns to collaborators, and thus inhibition ultimately favored betrayal.[23] At present, there is still very little research on donation motivation in China, and those that exist are mainly in the form of standardized research and questionnaire surveys.

Studies on complex networks are currently very popular.[24–26] Barabasi and Albert presented a scale-free network model (BA) satisfying the power-law distribution, and proposed two generative mechanisms (growth and preference).[27] Scale-free networks are typically heterogeneous networks, characterized by different degrees of nodes. These scale-free networks can be both robust and fragile.[28,29] Holme and Kim studied and proposed models capable of regulating the agglomeration coefficient of networks, and through which they were able to generate a scale-free network with tunable agglomeration coefficients.[30] Anna Brodio and Clauset devised a statistical method to test whether networks obey scale-free properties. They performed rigorous tests for nearly 1000 networks to conclude that scale-free networks are not universal in the real world.[31] Subsequently, Barabási also responded to the challenge on Barabási Lab. The degree dis-

tributions of many (or even the large majority of) large-scale networks have long tail features, where the degrees of a small number of nodes are relatively large. In other words, scale-free networks are indeed very common if they are defined as networks with long-tail degree distribution.

Network reciprocity is an important mechanism for promoting cooperation.[32] Nowak and May believed that collaborators on networks are able to form close clusters to defend against the betrayal of invaders. Since then, networks have become an important tool for researchers exploring cooperation.[7] Santos *et al.* investigated the effects of social diversity on cooperative evolution in the BA scale-free network, and found that the structures of heterogeneous networks could facilitate cooperation to a higher degree than those of regular networks.[33] Gardene *et al.* also investigated the evolution of cooperation under the prisoner's dilemma game model in scale-free and ER stochastic networks.[34] Perc and Szolnoki considered scaling factors with uniform, exponential, and scale-free distributions, thus distinguishing the effects of three different aspects of social diversity on cooperation in the spatial prisoner's dilemma game.[35] Rong *et al.* investigated the effects of correlations of degrees on the evolution of cooperation in scale-free networks, and found that cooperators with greater degrees could support each other to jointly defend against betrayal.[36] Xu *et al.* found that in addition to cooperative and betrayal strategies, blackmail strategies could form a stable alliance structure with cooperative strategies and together resist the invasion of betrayal strategies.[37] Assenza *et al.* investigated the promoting effect[38] of clustering on the evolution of cooperative action on scale-free networks. Du *et al.* studied the effects of asymmetric cost on the cooperative behavior in the snowdrift game on scale-free networks.[39] Li *et al.* introduced a return redistribution mechanism, interpreted as compassion that facilitated the evolution of cooperation.[40]

As mentioned earlier, researches on donation have usually been conducted in the form of experiments, and the topic of cooperation on networks is usually modeled with external mechanisms in a prisoners' dilemma or simply considers the impacts of network structures on cooperation. However, donation, as a special public goods game with zero income for cooperators, has its own characteristics, and is also worth us building models on scale-free networks for analysis. Therefore, this paper builds a four-strategy evolutionary game model on a scale-free network to study donation in the epidemic, which is a fusion of those researches fields mentioned above. Each player in this model has four strategies, thus we divide them into four types based on whether they pay costs for donation, whether they benefit from the allocation of donations, and whether they get utilities from victims. Then we calculate the benefits of each player in combination with networks. In addition, phase diagrams of two parameters is mostly studied at home and abroad. This paper innovatively introduces the expression of the steady-state equilibrium under three parameters.

The second part of this paper introduces our theoretical model, which is primarily the donation model and its evolution on the scale-free network; the third part discusses the evolutionary dynamics and its robustness under different parameters. At last, this paper concludes and gives the realistic significance of the model, clarifies our limits and raises future work.

2. Donation model on the scale-free network and its evolution

2.1. Scale-free network

Here N is the number of players in the population, namely, the number of nodes in the network; P_N is the set of players, and also the set of nodes in the network; E is the connection between these nodes. The scale-free network can be represented as $G = (P_N, \Omega_i | i \in P_N)$, and each node i represents each player i in the population, while Ω_i represents the neighbors of the player i, $\Omega_i = \{j | j \in P_N, (i,j) \in E\}$. Defining N different groups in the scale-free network, each including player i and her k_i neighbors, $k_i = |\Omega_i|, i \in P_N$, the groups can be represented as $\{i\} \cup \Omega_i$. The sizes of these groups are different and the number of neighbors of each player fluctuates around the average connectivity of scale-free networks. The probability that a player has k neighbors follows the power law $P(k) \approx k^{-\gamma}$, $2 \leq \gamma \leq 3$. Each player i participates in $k_i + 1$ group games, and the central players participate in much more group games.

The following is a simple overview of generating BA scale-free networks:[27]

(i) Growth: Start with a fully-connected network with m_0 nodes. Add one node to the network each time, and select m nodes existing for connection, where $m < m_0$.

(ii) Preference connection: When the new node selects nodes existing for connection, the probability that the node i is selected is $\Pi_i = \frac{k_i}{\sum_j k_j}$.

(iii) Repeat steps (i) and (ii), and after $N - m_0$ steps, get a network with N nodes and $m(N - m_0) + A_{m_0}^2$ edges, where $A_{m_0}^2$ is the permutation of 2 nodes from m_0 nodes.

In this paper, $m_0 = 3$ and the initial network is fully connected. $m = 2$ means when the new node joins she selects two existing nodes to connect. The new node is connected to the neighbor of the selected node with probability q, or it is connected according to the connection mechanism of the BA model with probability $1 - q$. The value of the agglomeration coefficient q represents the agglomeration level of the scale-free network, and the greater the value, the higher agglomeration level of the network.

2.2. The donation model on the scale-free network

After building the BA scale-free network, we want to combine the public goods game with the network. The donation model on scale-free networks focuses on how each player updates strategies, thus leading to the evolution of the population and facilitating donation.

At each round, every player in the population is a donor, an illegal beneficiary, a legal beneficiary, or an inactive person, and the population states $p^t = \{p_c^t, p_d^t, p_l^t\}$, $p_n^t = 1 - p_c^t - p_d^t - p_l^t$, where $p_c^t, p_d^t, p_l^t, p_n^t$ are the proportions of donors, illegal beneficiaries, legal beneficiaries and inactive people in round t.

In round t, the player i plays $k_i + 1$ games in the $k_i + 1$ groups centered on $\{i\} \cup \Omega_i$, and adopts the same strategy. After completing each round, the player i calculates her total gain, $f_i(t)$. If the player is a donor, she will pay c for each of the $k_i + 1$ groups. Her total cost is $(k_i + 1)c$ in round t.

As shown in Table 1, x_i denotes whether player i pays c in the donation game. $x_i = 1$ when i is a donor, and $x_i = 0$ when i is other types. y_i denotes whether player i benefits from the allocation of donations. $y_i = 1$ when i is a beneficiary, otherwise, $y_i = 0$. z_i denotes whether player i obtains utilities from the benefits of legal beneficiaries. $z_i = 1$ when i is a donor or an illegal beneficiary, and $z_i = 0$ when i is an other types.

Table 1. Income components.

Type	Cost (x)	PGG benefit (y)	Utility (z)
Donor	1	0	1
Illegal beneficiary	0	1	1
Legal beneficiary	0	1	0
Inactive person	0	0	0

According to the ideas proposed by Brandt and Sigmund,[41] and extended researches by Wang *et al.*[42] and Szolnoki and Perc,[43] this paper combines with scale-free networks and then obtains formulas (1)–(3) and (5), which are benefits of legal beneficiaries, donors, illegal beneficiaries, and inactive people, respectively. Then the benefit of a legal beneficiary i in round t is $f_i^l(t)$,

$$f_i^{\mathrm{l}}(t) = \sum_{j=1}^{N} A_{ij} \frac{r\left(\sum\limits_{h=1}^{N} A_{jh}x_h c + x_j c\right)}{k_j - \sum\limits_{h=1}^{N} A_{jh}x_h - \sum\limits_{h=1}^{N} A_{jh}(1-x_h)(1-y_h) + y_j - x_j}. \tag{1}$$

Here A_{ij} is the adjacency matrix of the network. $A_{ij} = A_{ji} = 1$ if i and j are connected; otherwise, $A_{ij} = 0$, and $A_{ii} = 1$. r is the synergistic coefficient and $r > 1$.

The benefit of a donor i in round t is $f_i^c(t)$,

$$f_i^{\mathrm{c}}(t) = U_c\left(\sum_{j=1}^{N} A_{ij} \sum_{h=1}^{N} A_{jh} P_h(t)\right) - g_i^t c,$$
$$P_i = \begin{cases} f_i^{\mathrm{l}}(t), & \text{if } i \text{ is a legal beneficiary,} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

The former term of formula (2) is utilities that the donor obtains from legal beneficiaries among her neighbors and her neighbors' neighbors. The latter term is the cost the donor pays in donation games centered on her and her neighbors. In each round, as long as there is a beneficiary in the donor's group, the donor will pay c in this group to donate. g_i^t represents the number of groups where donor i donates. Although the donor donates in groups having legal beneficiaries or illegal beneficiaries, her utilities depends only on the incomes of legal beneficiaries. When a legal beneficiary belongs to the group centered on donor i as well as the groups centered on i's neighbors, then donor i cannot repeat the utilities from the legal beneficiary.

The benefit of an illegal beneficiary i in round t is $f_i^d(t)$,

$$f_i^{\mathrm{d}}(t) = \beta U_{\mathrm{d}}\left(\sum_{j=1}^{N} A_{ij} \sum_{h=1}^{N} A_{jh} P_h(t)\right) + \alpha\left(\sum_{j=1}^{N} A_{ij} \frac{r\left(\sum\limits_{h=1}^{N} A_{jh}x_h c_h + x_j c_j\right)}{k_j - \sum\limits_{h=1}^{N} A_{jh}x_h - \sum\limits_{h=1}^{N} A_{jh}(1-x_h)(1-y_h) + y_j - x_j}\right). \tag{3}$$

The first item in formula (3) is the utilities from the benefits of legal beneficiaries among i's neighbors and her neighbors' neighbors, which can be explained by the subjective and objective benefits that institutions and individuals embezzling medical supplies derive from the control of the outbreak. The latter item in formula (3) is the incomes from donations which can be understood as institutions and individuals disguised as transit platforms or people affected by the epidemic can obtain donations and profit from them.

The α is the the donation coefficient of illegal beneficiaries and $\alpha > 0$. β is the utility coefficient, $\beta > 0$. The donation coefficient measures the public goods that illegal beneficiaries can get from donation games relative to legal beneficiaries. The utility coefficient measures the utilities that illegal beneficiaries can derive from the benefits of legal beneficiaries relative to donors.

The $U(\cdot)$ is the utility function in formula (4), indicating that as the benefits of legal beneficiaries increase, the utilities of donors or illegal beneficiaries also increase. Utility functions are widely used in many areas[44] and also have many forms.[45] There are three parameters in the power function and this paper adopts the power function as the utility function. Then, as the parameters change, we may include concave functions, convex functions, additive functions, subtraction functions and so on. According to the previous analysis, both donors and illegal beneficiaries use the power function

$$U(x) = a + (bx)^v. \tag{4}$$

Inactive people are neither involved in the donation nor distribution. In each round, their benefits are 0,

$$f_i^{\mathrm{n}}(t) = 0. \tag{5}$$

In each round, players update their strategies according to the Fermi update rule. Overall, this makes players tend to update to higher-yielding strategies in the current state, but also allows for the occurrence of mutations. The Fermi update rule is widely used in the study of evolutionary games.[46–49] In each round, players compare their own benefits with those of their neighbors, and formula (5) shows that the state transfer probability of players from strategy X to strategy Y, $W_{X\to Y}$, is negatively associated with the difference between the two benefits. That is to say, the greater the benefit of a player choosing strategy X and the grater the revenue gap between strategy X and strategy Y, the smaller the probability that the player will move from strategy X to strategy Y. When $(P_X - P_Y)$ is negative infinity, the probability of players moving from strategy X to Y is close to 1; when $(P_X - P_Y)$ is 0, the probability of players moving from strategy X to Y is 0.5; and when $(P_X - P_Y)$ is close to positive infinity, the probability is almost 0. As individuals update strategies, the proportions of the four types change and the state of the population changes accordingly in each round. Therefore, the state transfer can be determined to obtain the population state in the next round. The Fermi update rule can be represented as follows:

$$W_{X\to Y} = \frac{1}{1+\exp\left[(P_X - P_Y)/K\right]}. \tag{6}$$

Here $\mathrm{W}_{X\to Y}$ indicates the state transfer probability of players from strategy X to strategy Y. P_X indicates the benefits of the player who adopted the strategy X, and P_Y indicates the benefits of her neighbor, who adopted strategy Y. K is the noise intensity, reflecting to what extent players want to receive higher payments when updating strategies. If $K = 0$, players always update considering their neighbors' benefits, but when $K \to \infty$, payment is inconsequential.

There are no restrictions on conversion among donors, illegal beneficiaries and inactive people, but conversion between legal beneficiaries and other types cannot be simply determined by the Fermi update rule. This is because when the benefits of legal beneficiaries reach a certain threshold, victims get enough help. Then these victims may convert into other types. In other words, the proportion of legal beneficiaries in the population should be negatively related to their benefits. Here, when the proportion of donors among the neighbors of a legal beneficiary reaches a certain threshold, the benefit of the legal beneficiary naturally reaches the corresponding threshold. Then the legal beneficiary will choose the strategy most common among her neighbors. Meanwhile, when the proportions of legal beneficiaries among the neighbors of donors, illegal beneficiaries and inactive people reach another thresholds, these three types convert into legal beneficiaries. The idea that decisions may be influenced by network structures (neighbors) can be seen in some researches.[50,51]

In each round when the following formula is satisfied, legal beneficiaries transfer into donors, illegal beneficiaries, or inactive people:

$$\frac{\sum_{j=1}^{N} A_{ij}x_j}{k_i} \geq 0.5. \tag{7}$$

Here, $x_i = 1$ when i is a donor, and $x_i = 0$ when i is a legal beneficiary, an illegal beneficiary, or an inactive person.

When the following is satisfied, donors, illegal beneficiaries or inactive people transfer into legal beneficiaries:

$$\frac{\sum_{j=1}^{N} A_{ij}y_j(1-z_j)}{k_i} \geq 0.5. \tag{8}$$

Here, $y_i = 1$ and $z_i = 0$ when i is a legal beneficiary, and otherwise when i is a donor, an illegal beneficiary, or an inactive person.

Figure 1 shows the strategy transfer process of a player. If the strategy chosen by a player at round t is donation, then the strategies that the player can choose at round $t+1$ are donation, corruption, disaster and inaction. According to the above, the state transfer probability for a donor to become an illegal beneficiary, a legal beneficiary or an inactive person is $W_{\mathrm{c}i\to\mathrm{d}}$, $W'_{\mathrm{c}i\to\mathrm{l}}$, or $W_{\mathrm{c}i\to\mathrm{n}}$, so the probability that she remains a donor at the next round is $1 - W_{\mathrm{c}i\to\mathrm{d}} - W'_{\mathrm{c}i\to\mathrm{l}} - W_{\mathrm{c}i\to\mathrm{n}}$. Similarly, the probability that an illegal beneficiary transfers to a donor, a legal beneficiary, or an inactive person is $W_{\mathrm{d}j\to\mathrm{c}}$, $W'_{\mathrm{d}j\to\mathrm{l}}$, or $W_{\mathrm{d}j\to\mathrm{n}}$, so the probability that she remains an illegal beneficiary in the next round is $1 - W_{\mathrm{d}j\to\mathrm{c}} - W'_{\mathrm{d}j\to\mathrm{l}} - W_{\mathrm{d}j\to\mathrm{n}}$. The probability that a legal beneficiary transfers to a donor, an illegal beneficiary, or an inactive person is $W'_{\mathrm{l}k\to\mathrm{c}}$, $W'_{\mathrm{l}k\to\mathrm{d}}$, or $W'_{\mathrm{l}k\to\mathrm{n}}$, so the probability that she remains a legal beneficiary in the next round is $1 - W'_{\mathrm{l}k\to\mathrm{c}} - W'_{\mathrm{l}k\to\mathrm{d}} - W'_{\mathrm{l}k\to\mathrm{n}}$. The probability for an inactive person to become a donor, a legal beneficiary or an illegal beneficiary is $W_{\mathrm{n}m\to\mathrm{c}}$, $W_{\mathrm{n}m\to\mathrm{d}}$, or $W'_{\mathrm{n}m\to\mathrm{l}}$, so the probability that she remains an inactive person in the next round is $1 - W_{\mathrm{n}m\to\mathrm{c}} - W_{\mathrm{n}m\to\mathrm{d}} - W'_{\mathrm{n}m\to\mathrm{l}}$.

The discrete state-transfer equations of the population are as follows:

$$E(p_{\rm c}^{t+1}) = \frac{-\sum_{i=0}^{N\cdot p_{\rm c}^t}(W_{{\rm c}i\to{\rm d}}+W'_{{\rm c}_i\to{\rm l}}+W_{{\rm c}i\to{\rm n}})+\sum_{j=0}^{N\cdot p_{\rm d}^t}W_{{\rm d}_j\to{\rm c}}+\sum_{k=0}^{N\cdot p_{\rm l}^t}W'_{{\rm l}_k\to{\rm c}}+\sum_{m=0}^{N\cdot p_{\rm n}^t}W_{{\rm n}_m\to{\rm c}}}{N}, \tag{9}$$

$$E(p_{\rm d}^{t+1}) = \frac{\sum_{i=0}^{N\cdot p_{\rm c}^t}W_{{\rm c}_i\to{\rm d}}-\sum_{j=0}^{N\cdot p_{\rm d}^t}(W_{{\rm d}_j\to{\rm c}}+W'_{{\rm d}_j\to{\rm l}}+W_{{\rm d}_j\to{\rm n}})+\sum_{k=0}^{N\cdot p_{\rm l}^t}W'_{{\rm l}_k\to{\rm d}}+\sum_{m=0}^{N\cdot p_{\rm n}^t}W_{{\rm n}_m\to{\rm d}}}{N}, \tag{10}$$

$$E(p_{\rm l}^{t+1}) = \frac{\sum_{i=0}^{N\cdot p_{\rm c}^t}W'_{{\rm c}_i\to{\rm l}}+\sum_{j=0}^{N\cdot p_{\rm d}^t}W'_{{\rm d}_j\to{\rm l}}-\sum_{k=0}^{N\cdot p_{\rm l}^t}(W'_{{\rm l}_k\to{\rm c}}+W'_{{\rm l}_k\to{\rm d}}+W'_{{\rm l}_k\to{\rm n}})+\sum_{m=0}^{N\cdot p_{\rm n}^t}W'_{{\rm n}_m\to{\rm l}}}{N}, \tag{11}$$

$$E(p_{\rm n}^{t+1}) = \frac{\sum_{i=0}^{N\cdot p_{\rm c}^t}W_{{\rm c}_i\to{\rm n}}+\sum_{j=0}^{N\cdot p_{\rm d}^t}W_{{\rm d}_j\to{\rm n}}+\sum_{k=0}^{N\cdot p_{\rm l}^t}W'_{{\rm l}_k\to{\rm n}}-\sum_{m=0}^{N\cdot p_{\rm n}^t}(W_{{\rm n}_m\to{\rm c}}+W_{{\rm n}_m\to{\rm d}}+W'_{{\rm n}_m\to{\rm l}})}{N}, \tag{12}$$

where $p_{\rm c}^{t+1}$, $p_{\rm d}^{t+1}$, $p_{\rm l}^{t+1}$, and $p_{\rm n}^{t+1}$, respectively, indicate the proportions of donors, illegal beneficiaries, legal beneficiaries, and inactive people in time $t+1$, and the left sides of formula (12) express the expectations of proportions of the four types.

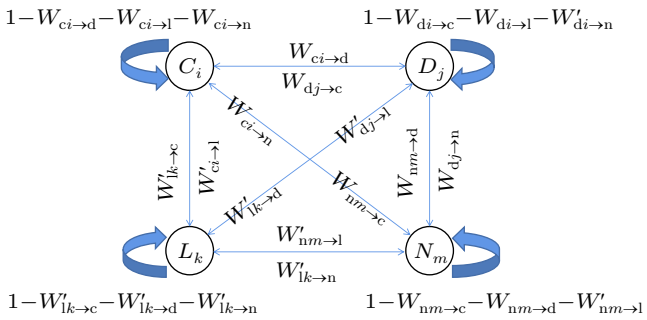


Fig. 1. The strategy transfer process of a player. If the strategy chosen by a player at time t is donation, then the strategies that the player can choose at time $t+1$ are donation, corruption, disaster and inaction. According to above, the state transfer probability for a donor to become illegal, legal or inactive is $W_{ci\to{\rm d}}$, $W'_{ci\to{\rm l}}$, or $W_{ci\to{\rm n}}$, so the probability that she remains a donor at the next stage is $1-W_{ci\to{\rm d}}-W'_{ci\to{\rm l}}-W_{ci\to{\rm n}}$.

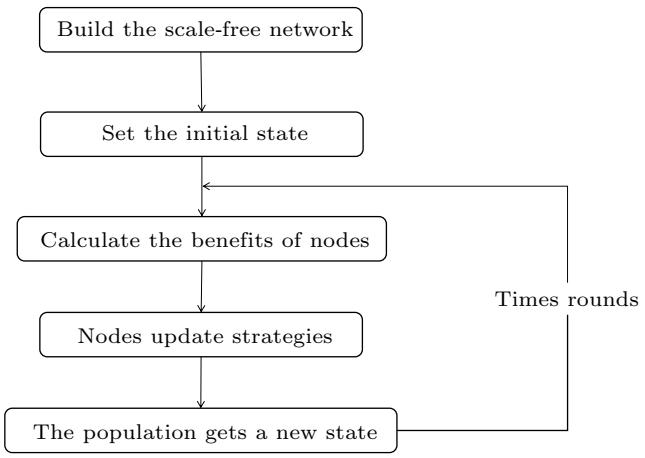


Fig. 2. The evolutionary process of the donation game on a scale-free network. First, build a scale-free network in a population of size N under some agglomeration coefficient. Second give a strategy for each node to set the initial state of the population. After that, enter an iteration in which play games for Times rounds, and the state of the population will achieve steady. In each round, we calculate the benefits of nodes. Then nodes update their strategies, and consequently, the state of the population updates.

Thus the static states of the population can be defined. States of the population can be considered stationary when the proportions of all four types are changeless, or at least when regular fluctuations of these proportions are in a minimal range.

Altogether, the algorithm tree in this paper is shown in Fig. 2. We first build a scale-free network in a population with size N under some agglomeration coefficient. Then we give a strategy for each node to set the initial state of the population. After that, people play games for Times rounds and the state of the population will achieve steady. In each round, we calculate the benefits of nodes on the scale-free network based on formulas (1)–(5). Then nodes update their strategies according to formulas (6)–(8).When the strategies of all players (nodes) in the population have been updated, the state of the population updates in accordance with formulas (9)–(12).

3. Evolution of the donation model

In the simulation conducted in this paper, when there are no beneficiaries among groups centered on a donor and her neighbors, the donor does not donation. In addition, donations are allocated between beneficiaries, but the benefit of a donor is affected only by legal beneficiaries. This is because the utility part of the donor is a function of the benefits of all the legal beneficiaries in groups centered on the donor and her neighbors (if a legal beneficiary is both in the group centered on the the donor and the group centered on the donor's neighbor, calculation should not be repeated. And if a legal beneficiary is simultaneously in the groups centered on different neighbors of the donor, calculation should not be repeated, too). The utility components of illegal beneficiaries are calculated in the same way.

A legal beneficiary can repeat getting profits in all groups she is involved in, and the same applies to the incomes from donations of illegal beneficiaries.

3.1. The utility function and the agglomeration coefficient

This section identifies the intervals where the utility function and the agglomeration coefficient q maximize the proportion of donors in steady states at different population sizes, and focuses on the evolutionary characteristics of the population within this interval. Parameters are set from the actual situation of the society with a epidemic. we set the initial proportion of donors as $p_1 = 0.3$, the initial proportion of illegal beneficiaries as $p_2 = 0.07$, the initial proportion of legal beneficiaries as $p_3 = 0.08$, the coefficients of illegal beneficiaries as $\alpha = \beta = 0.25$, the noise intensity as $K = 1000$, and noting that K makes benefits have a weak influence on strategy update; the synergistic coefficient as $r = 2$; and the cost of a single donation in a group as $c = 1$ and it does not affect the results.

Figure 3 shows changes of the proportion of donors for different parameters of the utility function and agglomeration coefficients at population sizes of 50, 100, 150, and 200. The axes represent three parameters of the utility function (formula (4)): the abscissa a, the ordinate b, and the high axis v. The size of circles represents the proportion of donors, and the larger these circles, the more donors there are in the steady state. Changes in circle size are less obvious when the population size increases. This indicates that the donation is less sensitive to parameters of the utility function as the population size increases. Only a few parameters enable the proportion of donors to reach 0.9 at the population size of 50 in the steady state, while the proportion of donors in the steady state can reach 0.9 for most parameters tested at the population size of 200. The phenomenon whereby donation is less sensitive to parameters with increasing population size also appears in later sections.

At the population size of 50, a higher proportion of donors appears with lower values of agglomeration coefficient q and higher values of v. In addition, utility functions are subtraction functions. This phenomenon weakens with increasing population size. A higher proportion of donors can also be reached with negative values of v at population sizes of 200, and the utility functions are additive functions. According to formulas (2) and (3), the benefits of legal beneficiaries have more significant effects on the utility part of illegal beneficiaries in smaller population sizes, while the benefits of legal beneficiaries have more significant effects on the utility of donors in larger population sizes, when the utility from legal beneficiaries will promote donation.

Next this paper selects combinations of parameters discussed in Subsection 3.1 that can promote donation at different population sizes, and we consider population structures in order to analyse evolutionary trends. We select parameters from Fig. 3 to reach higher proportions of donors at different population sizes, and analyze the evolution of populations according to these the parameters.

Figure 4 shows the evolution at different population sizes under parameters from Table 2. Obviously the proportion of donors increases with population sizes increasing.

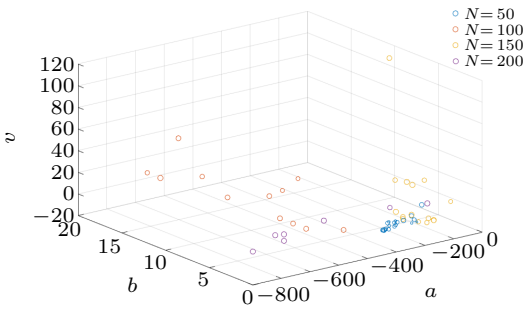


Fig. 3. Changes of the proportion of donors for different parameters of the utility function and agglomeration coefficients at population sizes of 50 (blue circles), 100 (red circles), 150 (yellow circles), and 200 (purple circles). The axes represent three parameters of the utility function: the horizontal axis a, the longitudinal axis b, and the high axis v. The size of the circles represents the proportion of donors in steady states, and the larger these circles, the more donors there are in the steady state. Other parameter settings are $p_1 = 0.3$, $p_2 = 0.07$, $p_3 = 0.08$; $c = 1$, $r = 2$, $K = 1000$; $\alpha = 0.25$, $\beta = 0.25$. Note that detailed data can be found in Appendix A.

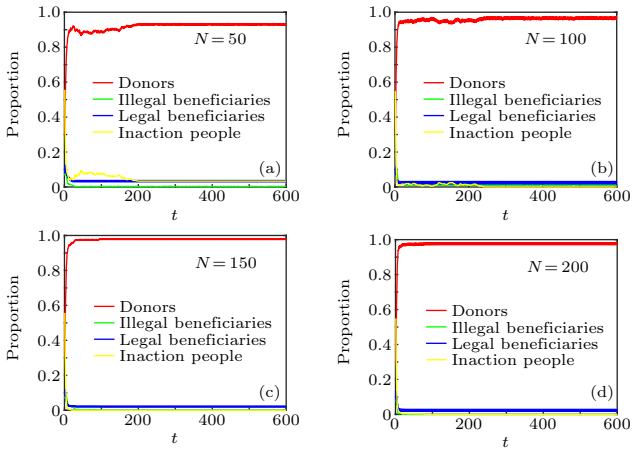


Fig. 4. (a)–(d) The evolution under parameters from Table 2 at population sizes of 50, 100, 150, and 200, respectively. The proportion of donors increases with population sizes increasing. Parameter settings are $p_1 = 0.3$, $p_2 = 0.07$, $p_3 = 0.08$; $c = 1$, $r = 2$, $K = 1000$; $\alpha = 0.25$, $\beta = 0.25$.

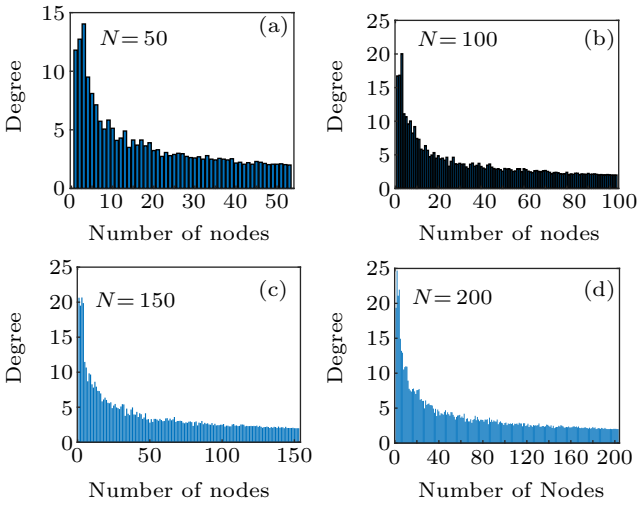


Fig. 5. Degree distributions under the parameters from Table 2. Panels (a)–(d) correspond to the population sizes of 50, 100, 150, and 200, respectively. Parameter settings are $p_1 = 0.3$, $p_2 = 0.07$, $p_3 = 0.08$; $c = 1$, $r = 2$, $K = 1000$; $\alpha = 0.25$, $\beta = 0.25$.

Table 2. Parameter values ($N = 50$, 100, 150, and 200).

Population size	Parameter name	Parameter values	Utility function	The proportion of donors
53	a^*	−250	$U(x) = -250 - (0.8x)^{3.5}$	0.92893
	b^*	0.8		
	v^*	3.5		
	q^*	0.2		
103	a^*	−550	$U(x) = -550 - (5.1x)^{10}$	0.973
	b^*	5.1		
	v^*	10		
	q^*	0.1		
153	a^*	−210	$U(x) = -210 - (1.2x)^5$	0.976
	b^*	1.2		
	v^*	5		
	q^*	0.3		
203	a^*	−400	$U(x) = -400 - (5x)^{(-1)}$	0.977
	b^*	5		
	v^*	-1		
	q^*	0.1		

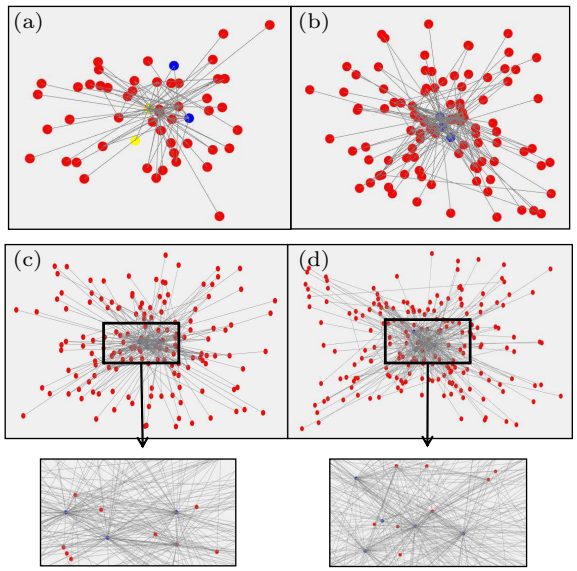


Fig. 6. Networks under the parameters from Table 2. As in Fig. 4, red represents donors, green represents illegal beneficiaries, blue represents legal beneficiaries, and yellow represents inactive people. Panels (a)–(d) correspond to the population sizes of 50, 100, 150, and 200, respectively. Parameter settings are $p_1 = 0.3$, $p_2 = 0.07$, $p_3 = 0.08$; $c = 1$, $r = 2$, $K = 1000$; $\alpha = 0.25$, $\beta = 0.25$.

Figures 5 and 6 explore the characteristics of network structures under q^* at different population sizes. It can be seen that the degree distribution of the network fulfills the characteristics of scale-free networks. As seen from Fig. 6, the legal beneficiaries or inactive people in the steady state are usually some nodes with core influence, which can be explained according to the model. First, donors that are not connected with beneficiaries do not play donation games, therefore, there is no motivation for these donors to convert into other types. When donors connected with legal beneficiaries play donation games, they may coexist with legal beneficiaries in homeostasis if the proportion of donors around legal beneficiaries is below the threshold, or the proportion of legal beneficiaries around donors is below the other threshold. Furthermore, legal beneficiaries are more likely to exist in nodes with high degrees in a steady state. Nodes with low degrees are more likely to be affected by neighbors because of the fact that there are fewer neighbors. However, nodes with high degrees are connected with more neighbors, thus the required transfer threshold is high. Generally speaking, it is more difficult for high-degree nodes to shift from legal beneficiaries to other types.

Proposition 1 The promoting effects of donors' utilities to the proportion of donors in homeostasis outweighs the inhibitory effects of illegal beneficiaries' utilities to the proportion of donors with population sizes increasing. Therefore, if it is a small outbreak, the concern of the population for the epidemic or the social praise for donation will not have a very significant incentive for the donation. However, if the epidemic breaks out in a large-scale society, the love to victims or the praise for donation will promote donation to a large extent.

Proposition 2 Nodes with high degrees are more likely to become legal beneficiaries. That is, people who are active in society are more likely to fall into the epidemic. This is consistent with reality, where in society, those who are more active can get more help but they also have a greater probability to get in touch with patients. Once they are sick, if they do not been isolated (change their original network structures), they find it difficult to recover and are very prone to reinfection.

3.2. The noise intensity and coefficients of illegal beneficiaries

We study the proportion of donors in steady states under different noise intensity K, the donation coefficient α and the utility coefficient β. We set the initial proportion of donors as $p_1 = 0.3$, the initial proportion of illegal beneficiaries as $p_2 = 0.07$, the initial proportion of legal beneficiaries as $p_3 = 0.08$, the utility function as $U(x) = -20 + (0.2x)^{0.35}$,

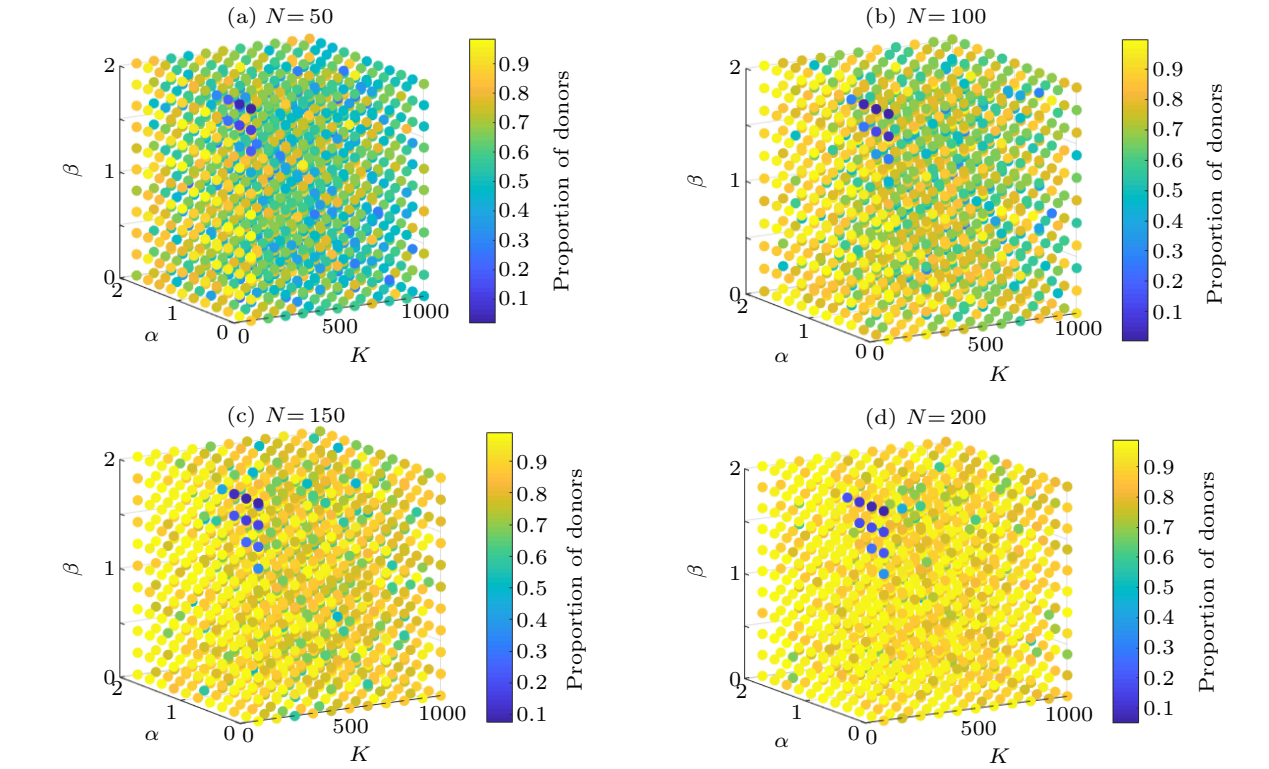


Fig. 7. The proportion of donors in a steady state when the donation coefficient α (horizontal axis), the noise intensity K (longitudinal axis), and the utility coefficient β (high axis) change. Panels (a)–(d) correspond to the population sizes of 50, 100, 150, and 200, respectively. Parameter settings are $q = 0.04$, $a = -20$, $b = 0.2$, $v = 0.35$; $p_1 = 0.3$, $p_2 = 0.07$, $p_3 = 0.08$; graphical rotation angles are $AZ = 3$, $EI = 3$. Note that, for analysis, the first value of noise intensity K is 0.5, and the subsequent values of K correspond with the coordinate.

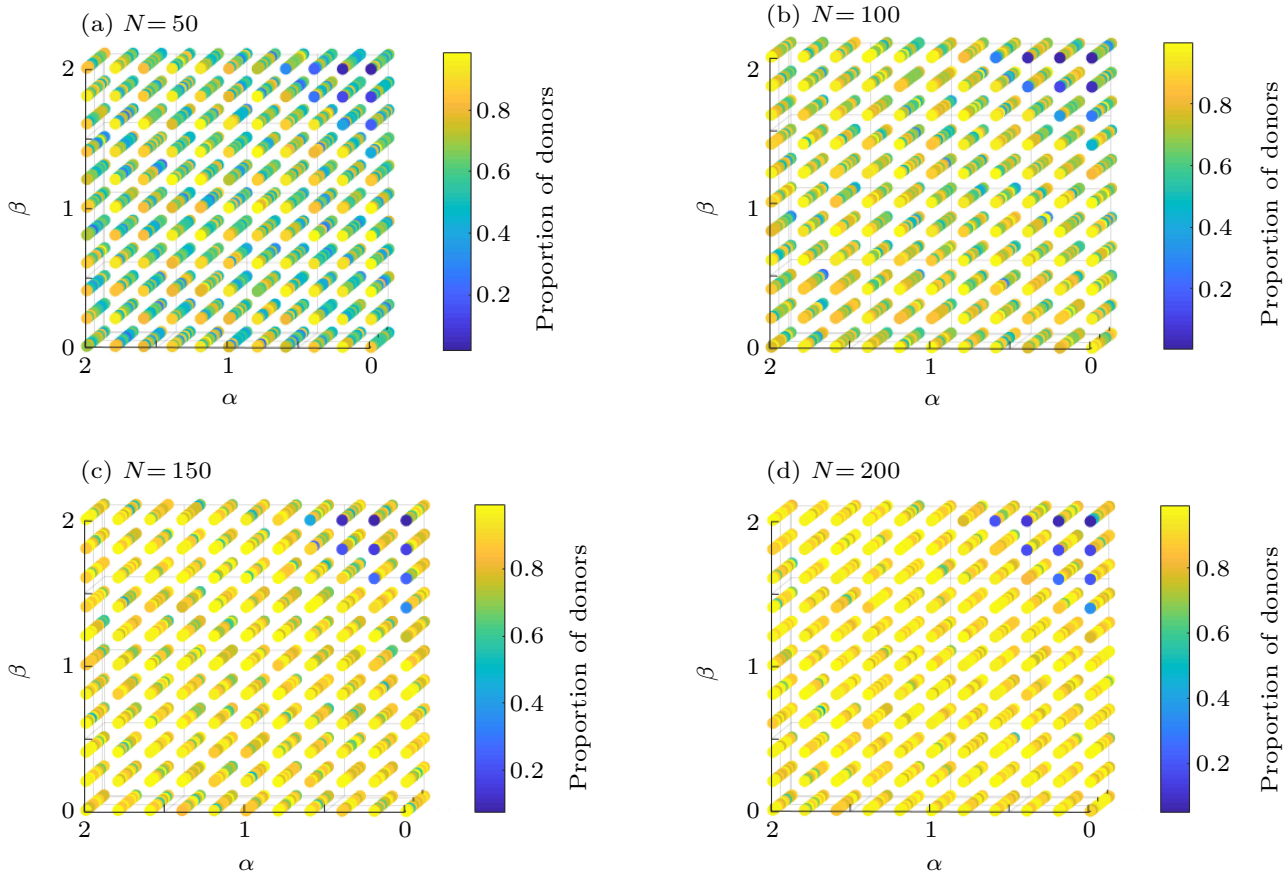


Fig. 8. The proportion of donors in the steady state when the donation coefficient α (horizontal axis), the noise intensity K (longitudinal axis), and the utility coefficient β (high axis) change. Panels (a)–(d) correspond to the population sizes of 50, 100, 150, 200, respectively. Parameter settings are $q = 0.04$, $a = -20$, $b = 0.2$, $v = 0.35$; $p_1 = 0.3$, $p_2 = 0.07$, $p_3 = 0.08$; $AZ = -86$, $EI = 3$. Note that, for analysis, the first value of noise intensity K is 0.5, and other values of K correspond with the coordinate.

the agglomeration coefficient as $q = 0.04$, the synergistic coefficient as $r = 2$, and the cost of making a single donation $c = 1$. Note that the utility function and the agglomeration coefficient are chosen arbitrarily, and we believe that their values have no effects on our conclusions of this section. It is seen from Figs. 7 and 8 that the proportion of donors generally increases in steady states as the population size increases.

As the population size increases, the proportion of donors basically increases. $K = 0.5$ is a relatively special value: at $K = 0.5$, $\alpha - \beta + 1.2 \leq 0$, the proportion of donors decreases significantly, which is the donor trap. We do further analysis in Fig. 8. It can be seen that when the parameters meet the conditions $\alpha - \beta + 1.2 \leq 0$, the proportion of donors is significantly smaller than that of others.

Proposition 3 A certain portion of the donation coefficient α and the utility coefficient β will form the donor trap and make donors reduce. Populations should avoid falling into the parameters of the donor trap regardless of their sizes. All of these mean that donors are more likely to die out when the utility coefficient of corrupt people is much larger than the donation coefficient. The donor trap may be contrary to our common sense — when corrupt people in society have much sympathy for the victims or benefit much from the control of the epidemic, and they do not profit much from corrupt donations, donors are relatively fewer when the society reaches a steady state. We further analyzes that when the value of α is large illegal beneficiaries will invade their donor-type and inaction-type neighbors very quickly, forming a stalemate between illegal beneficiaries and legal beneficiaries. This phenomenon is determined by heterogeneous networks together with the strategy update rule in the donation of this paper. Detailed descriptions will be presented in our subsequent papers.

3.3. The initial states

Some anomalies are observed during operation. Therefore, the effects of initial states on evolution will be discussed in this section. We consider changes of steady states when initial proportions of donors, illegal beneficiaries, and legal beneficiaries are within the interval $[0, 1/3]$. We set the utility function as $U(x) = -200 + (0.6x)^{0.5}$; the agglomeration coefficient as $q = 0.5$; the discount coefficient as $\alpha = \beta = 0.25$; the noise intensity as $K = 1000$; and the synergistic coefficient as $r = 2$; the cost of a single donation as $c = 1$. Note that, the utility function and the agglomeration coefficient can be chosen arbitrarily, and we believe that their values have no effects on the conclusions of this section.

First, we can intuitively observe that green blocks increase in the top and right of Fig. 9. This represents when initial proportions of donors (p_1) and legal beneficiaries (p_3) are both close to $1/3$, inactive people in the steady state die out and only donors and legal beneficiaries exist. Therefore, if there are a great number of victims and donors in society at the beginning, it is easier, in a larger population, to encourage people to participate in epidemic control.

Blue blocks are mainly on the left of Fig. 9 where initial proportion of donors (p_1) is close to 0. The blue blocks on the left decrease and the yellow and green blocks increase with population sizes increasing. This means that if donors are very few at the beginning, it is almost impossible for donation to exist in stable states. However, the larger the social scale, the more likely it is for the society to reach a better state. That is to say, even if there is almost no donation at the beginning, large social scales can promote donation and even eliminate the outbreak of an epidemic in steady states. We present the upper and lower triangle phenomena regarding donation in further analysis, below.

In addition, the turquoise blocks appear only at the bottom of Fig. 9. Turquoise blocks represent the fact that there are donors and inactive people in the stable state, which is the best steady state for society. Societies can not only eliminate the epidemic, but also have a good capacity to resist future epidemic in the best steady state. Further analysis will be performed below. The turquoise blocks represent the fact that there are no legal beneficiaries in a steady state, which equates to the end of the epidemic. Furthermore, there are no illegal beneficiaries, which means that society has formed a good atmosphere and is well prepared for resisting the next disaster. This is an ideal social state. We can see from Fig. 9 that this case exists almost only when $p_3 = 0$, and we provide further analysis in Fig. 10.

When donors' initial proportion (p_2) and legal beneficiaries' initial proportion (p_3) are both close to 0.333, there are more green blocks in Fig. 10. Meanwhile, yellow blocks transition to green blocks as population sizes increase. Green blocks are mainly concentrated on the upper right, which gradually increases with population size. We call this phenomenon as the upper triangle phenomenon, which means inactive people decrease in the steady state when $p_1 + p_3 - c_1 > 0$ (c_1 is non-negative and decreases when the population size increases).

When the initial proportion of donors (p_1) is close to 0, there are many blue blocks. The same applies when the initial proportion of legal beneficiaries (p_3) is close to 0. Blue blocks are distributed mostly on the lower left of Fig. 10, and gradually narrows with population sizes increasing. Here we define the lower triangle phenomenon: there is little donation in the steady state when $p_1 \approx 0$, and relatively less donation when $p_1 + p_3 - c_2 < 0$ (c_2 is non-negative and decreases when the population size increases).

In Fig. 11, the turquoise blocks increase when p_1 (initial proportion of donors) and p_2 (initial proportion of illegal beneficiaries) satisfy that $p_1 - p_2 + c_3 \geq 0$, $p_3 = 0$ (c_3 is non-negative and increases when the population size increases).

Proposition 4 It is easier for the population to achieve a better steady state when the population size increases.

Proposition 5 The upper and lower triangle phenomena.

The upper triangle phenomenon: Inaction decreases in the steady state when $p_1 + p_3 - c_1 > 0$ (c_1 is non-negative and

decreases when the population size increases). This shows that if the total number of victims and donors is above a certain threshold at the beginning of an outbreak (the larger the population size, the smaller the threshold will be), it is easier to call on onlookers to donate, and thus the society can further reduce the impact of epidemics.

The lower triangle phenomenon: When $p_1 \approx 0$, very little donation exists in the steady state. Furthermore, donation is relatively less when $p_1 + p_3 - c_2 < 0$ (c_2 is non-negative and decreases when the population size increases). The lower triangle phenomenon shows that if there is no donation at the beginning, it is difficult for the society to make donation arise in the natural course of evolution. Furthermore, when the total number of donors and legal beneficiaries is below a certain threshold (the larger the population size, the smaller the threshold will be), the results of natural evolution are detrimental to donation.

It can be seen that the upper and lower triangle phenomena have a lot to do with the sum of the initial proportions of donors and legal beneficiaries. Overall, the increase of $(p_1 + p_3)$ will promote donation even from scratch. Perhaps, in practice, this combination of p_1 and p_3 is not necessarily linear, and their weights in combination are not necessarily the same, but it is certain that the positive combination has significance in terms of promoting donation.

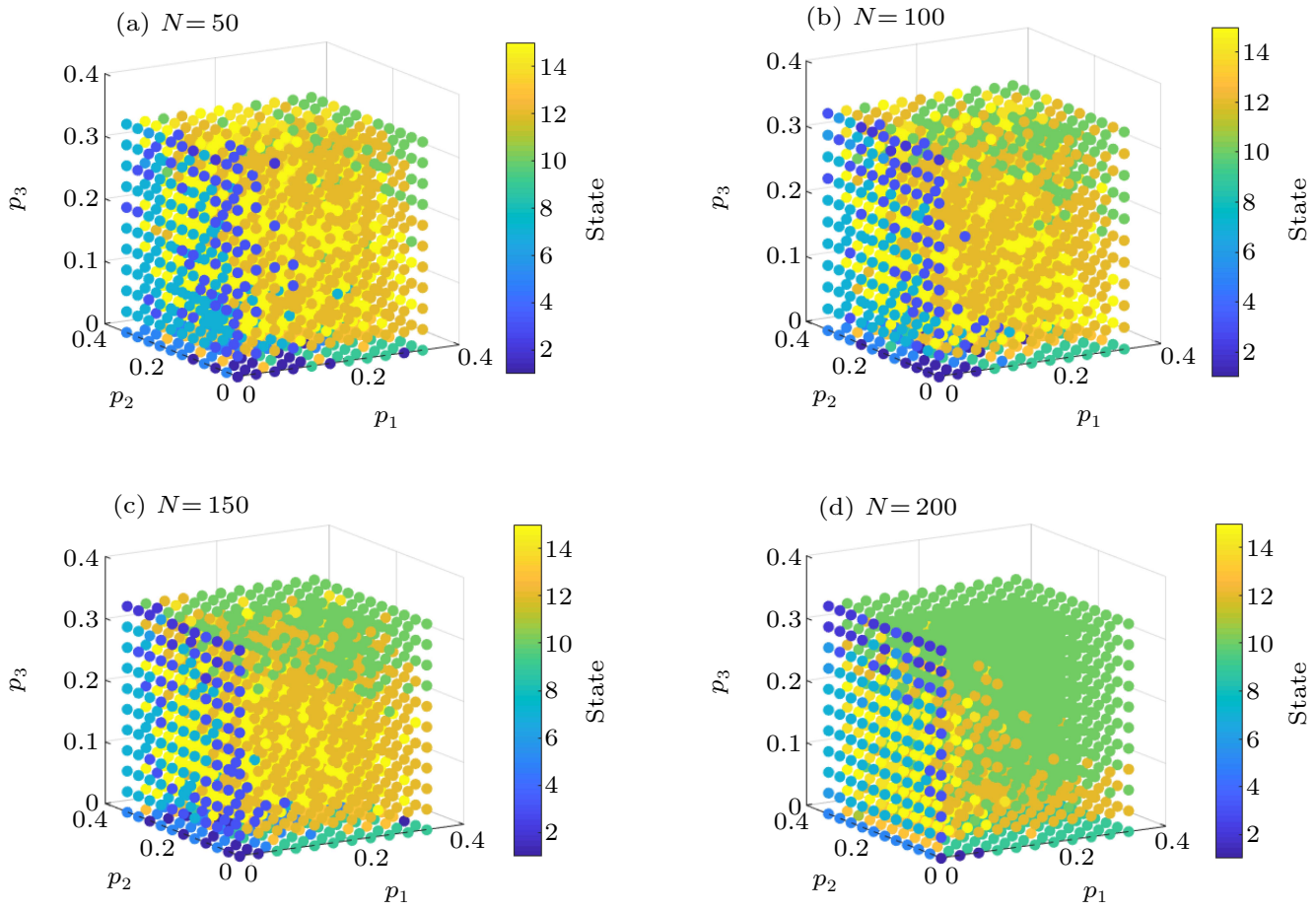


Fig. 9. Homeostasis when the initial proportion of donors p_1 (horizontal axis), the initial proportion of illegal beneficiaries p_2 (longitudinal axis), and the initial proportion of legal beneficiaries p_3 (high axis) change. Panels (a)–(d) correspond to the population sizes of 50, 100, 150, and 200, respectively. Parameter settings are $q = 0.5$; $a = -200$; $b = 0.6$; $v = 0.5$; $K = 1000$; $\alpha = 0.25$; $\beta = 0.25$. Note that different colors correspond to homologous values representing the existence of different types in the steady state: 1 represents that there only being inactive people in the steady state; 2 represents there only being legal beneficiaries; 3 represents there being legal beneficiaries and inactive people; 4 represents there only being illegal beneficiaries; 5 represents there being illegal beneficiaries and inactive people; 6 represents there being beneficiaries; 7 represents there being illegal beneficiaries, legal beneficiaries and inactive people. In the above 7 stable states, donors die out. In states 1 and 5, however, there are no donors, or legal beneficiaries, which means that the outbreak is under control. Legal beneficiaries still exist in states 2 and 3, but donors are extinct, which means that society has not reached a good states. In state 4 there are only illegal beneficiaries, which means that the epidemic has been controlled but the whole of society is vulnerable to the next outbreak. 8 represents there only being donors; 9 represents there being donors and inactive people; 10 represents there being donors and legal beneficiaries; 11 represents there being donors and illegal beneficiaries; 12 represents there being donors, legal beneficiaries and inactive people; 13 represents there being donors, illegal beneficiaries and inactive people; 14 represents there being donors, and beneficiaries; 15 represents all the four types coexisting in steady state. The figure contains only the following color blocks and corresponding steady states: deep blue on behalf of steady state 1, which contains only inactive people; deep medium blue on behalf of steady state 2, which contains only legal beneficiaries; light medium blue on behalf of steady state 3, which contains legal beneficiaries and inactive people; light blue on behalf of steady state 7, which contains illegal beneficiaries, legal beneficiaries and inactive people; turquoise on behalf of steady state 9, which contains donors and inactive people; green on behalf of steady state 10, which contains donors and legal beneficiaries; earth yellow on behalf of steady state 12, which contains donors, legal beneficiaries and inactive people; yellow on behalf of steady state 15, which contains all the four types in a steady state.

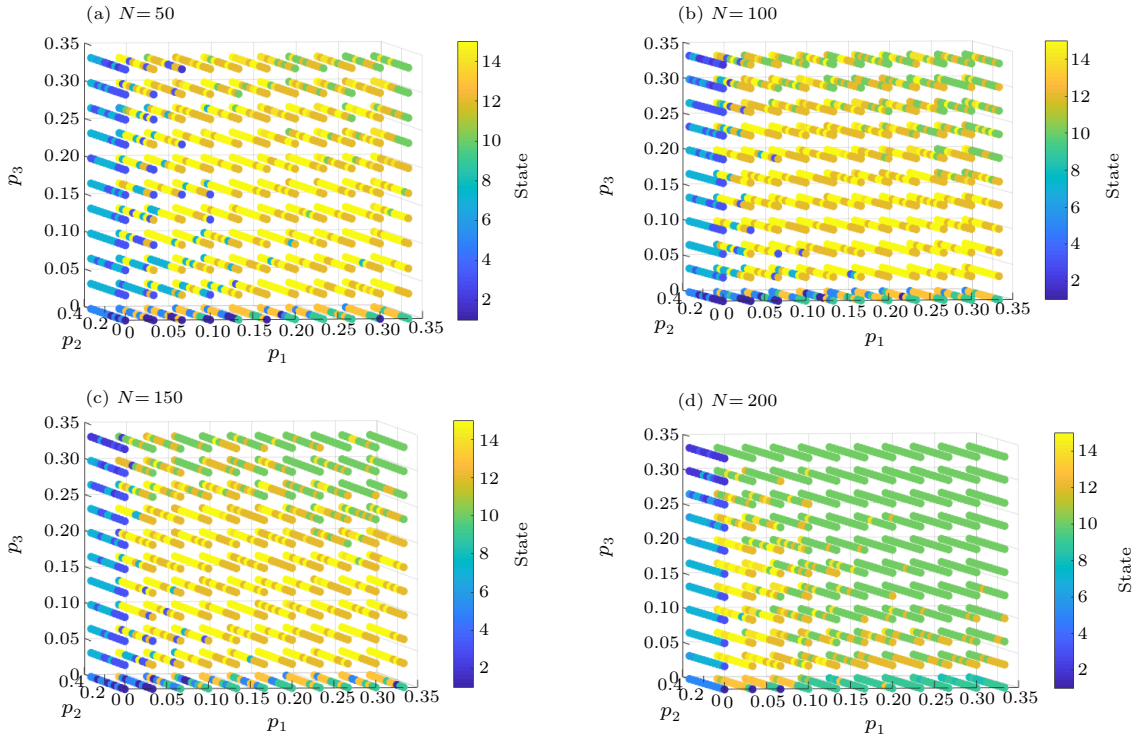


Fig. 10. Homeostasis when the initial proportion of donors p_1 (horizontal axis), the initial proportion of illegal beneficiaries p_2 (longitudinal axis), and the initial proportion of legal beneficiaries p_3 (high axis) change. Panels (a)–(d) correspond to the population sizes of 50, 100, 150, and 200, respectively. Parameter settings are $q=0.5$; $a=-200$; $b=0.6$; $v=0.5$; $K=1000$; $\alpha=0.25$; $\beta=0.25$; graphical rotation angles are $Az=-8$, $Ei=3$.

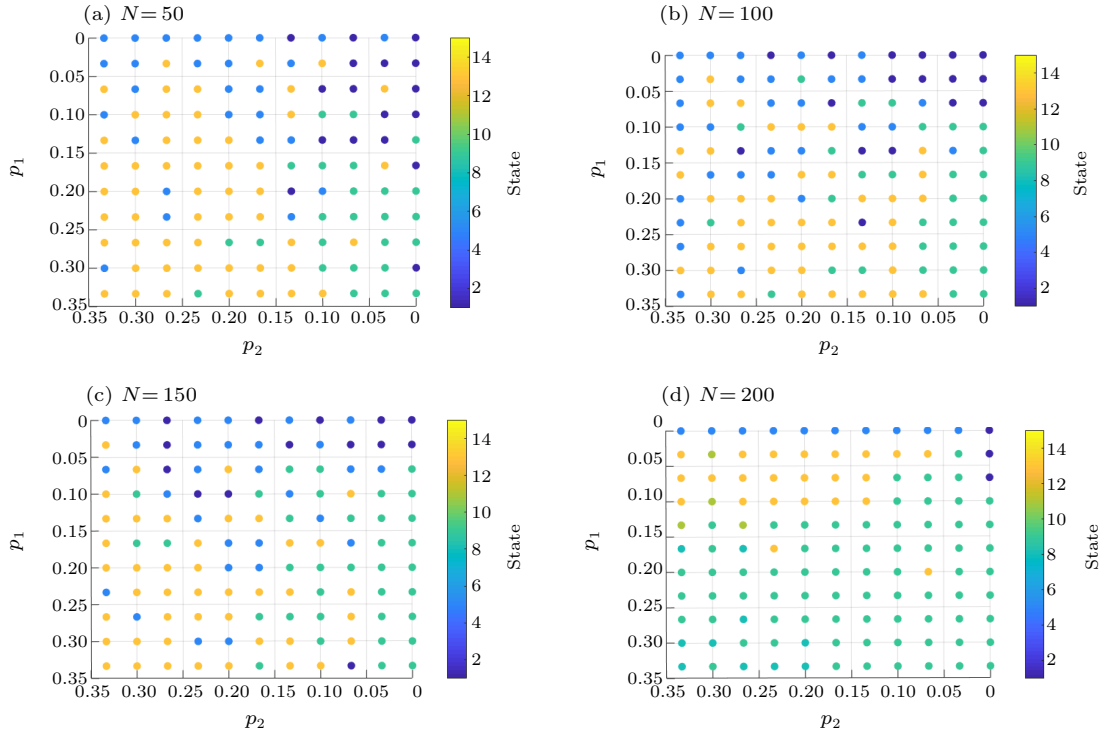


Fig. 11. Homeostasis when the initial proportion of donors p_1 (longitudinal axis) and the initial proportion of illegal beneficiaries p_2 (horizontal axis) change. Panels (a)–(d) correspond to the population sizes of 50, 100, 150, and 200, respectively. Parameter settings are $q=0.5$; $a=-200$; $b=0.6$; $v=0.5$; $K=1000$; $\alpha=0.25$; $\beta=0.25$; $p_3=0$; $Az=-90$, $Ei=-90$.

Proposition 6 Best social homeostasis effect.

When the initial proportions of legal beneficiaries are close to 0 ($p_3\approx 0$), and the initial proportions of donors (p_1) and the initial proportions of illegal beneficiaries (p_2) meet the conditions $p_1-p_2+c_3\geq 0$ (c_3 is non-negative and increases when the population size increases), society is more likely to achieve the best social homeostasis, where we can both promote donation and eliminate the epidemic.

These initial states for best social homeostasis show that if the initial number of victims is nearly zero, and corruption is not much more than donation at the beginning (the difference is less than a threshold), then the society is more likely to achieve the best social homeostasis after a period of time.

4. Conclusion and perspectives

The model constructed in this paper aims to truly reflect the generation and development of donation in social networks, and the six propositions indicate the direction of efforts to further promote donation and thus achieve better social homeostasis. We find that the promoting effects of utilities to the proportion of donors in homeostasis will enhance with population sizes increasing. Furthermore, nodes with high degrees are more likely to become legal beneficiaries. That is, people that are active in society fall more easily into the epidemic.

Based on this, we have discussed effects of the combination of the noise intensity K, the donation coefficient α and the utility coefficient β on promoting donation at different population sizes, and have proposed the donor trap. In other words, some combination of the donation coefficient α, the utility coefficient β and the noise intensity K will form the donor trap and make the proportion of donors reduce. Regardless of population sizes, we should avoid falling into the donor trap.

In addition, we find that the initial states of the population have a great impact on the steady states, and their effects vary with population sizes increasing. This paper has explored the critical initial states that make for the demise of donors, and proposed the upper and lower triangle phenomena. Moreover, we conclude that donation is less sensitive to the parameters proposed in all three parts of this paper as population sizes increase. Therefore, the population size is an important factor for promoting donation, and larger populations are more likely to achieve better homeostasis.

The work of this paper has two main limitations. First, due to the analytic complexity, evolutionary games on scale-free networks are mainly studied by computer technology for simulation analysis. Second, due to the limited computing power of our workstation, the utility function is only in the form of the power function. Furthermore, population sizes are 50, 100, 150, and 200, without studies on larger population sizes.

In future studies, we plan to further explore the analytical solutions on complex networks by pair approximation proposed by Zhang *et al.*[16] Second, we can take diverse forms of utility functions and appropriately expand the population size. Moreover, in this paper, we focus on the influence of some parameters on the homeostasis. In the subsequent study, the influence of the external mechanism can be tried, and results can be verified combined with management experiments.

Appendix A

Table A1. Data in Fig. 3.

N	a	b	v	q	Proportion of donors in homeostasis
50	−225	0.1	2.8	0.9	0.65975
	−240	0.2	2.4	0.8	0.68868
	−260	1.4	1.6	0.1	0.6993
	−280	1.5	1.8	0.4	0.7012
	−300	0.8	1.7	0.4	0.71981
	−330	0.25	1.6	0.4	0.7355
	−340	0.125	1.9	0.9	0.74717
	−330	0.2	2.7	0.1	0.76415
	−300	0.2	2.9	0.1	0.76824
	−290	0.4	3.1	0.1	0.77233
	−320	0.2	1.5	0.9	0.7739
	−270	1.2	1.4	0.1	0.78679
	−260	1.2	1.2	0.1	0.80346
	−225	0.4	3.9	0.6	0.81918
	−340	0.25	1.8	0.3	0.822
	−290	1.3	1.9	0.2	0.83176
	−210	1.2	4	0.1	0.86478
	−250	0.8	3.5	0.2	0.88145
	−150	2	8	0.6	0.8956
	−340	0.1	2.4	0.2	0.93082
100	−350	9.6	20	0.5	0.818445
	−450	8.1	20	0.7	0.828
	−650	17	20	0.1	0.87314
	−550	14	20	1	0.887
	−450	9.6	10	0.9	0.911
	−550	3.6	10	0.6	0.914
	−550	11.1	10	1	0.916
	−450	20	30	0.2	0.929
	−450	1.1	6	0.2	0.932
	−550	2.1	10	0.2	0.937
	−550	5.1	10	0.1	0.94
	−650	15.5	20	0.1	0.98
150	−110	0	15	0.6	0.8279
	−210	0.4	1.1	0.3	0.84
	−190	0.1	0.1	0.5	0.865
	−80	4	20	0.9	0.87538
	−60	8	6	0.3	0.89
	−240	1.2	5	1	0.8917
	−160	0.3	−0.3	0.5	0.911329
	−160	0.9	−0.3	0.1	0.913
	−80	8	120	0.4	0.923
	−240	1.2	3	0.3	0.96405
	−80	6	12	0.1	0.965
	−240	2	7	0.6	0.9677
	−210	1.2	5	0.3	0.969
	−160	0.3	−0.1	0.6	0.973
	−30	7	3	0.1	0.975
200	−200	4	2	0.9	0.876
	−600	3	−1	0.1	0.945
	−800	0	11	0.2	0.9578
	−400	5	−1	0.1	0.9703
	−600	4	1	0.2	0.9705
	−100	3	3	0.3	0.972
	−600	3	5	0.2	0.97225

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