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Beating standard quantum limit via two-axis magnetic susceptibility measurement

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We report a metrology scheme which measures the magnetic susceptibility of an atomic spin ensemble along the x and z directions and produces parameter estimation with precision beating the standard quantum limit. The atomic ensemble is initialized via one-axis spin squeezing with optimized squeezing time and parameter ϕ (to be estimated) assumed as uniformly distributed between 0 and 2π while fixed in each estimation. One estimation of ϕ can be produced with every two magnetic susceptibility data measured along the two axes respectively, which has an imprecision scaling $(1.43\pm0.02)/N^{0.687\pm0.003}$ with respect to the number N of the atomic spins. The measurement scheme is easy to implement and is robust against the measurement fluctuation caused by environment noise and measurement defects.

Keywords: quantum metrology, spin-squeezing, standard quantum limit, fluctuation

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1. Introduction

Metrology is the cornerstone of scientific research and technology development. Quantum mechanics provides new opportunities to metrology and attracts attentions from different fields.[1–5] Given N probes, the lowest parameter estimation imprecision allowed by classical theory is in the order of $1/\sqrt{N}$, which is the so-called standard quantum limit (SQL). It comes from the well-known central limit theorem. Quantum metrology harnesses quantum features such as quantum entanglement and spin squeezing to beat the standard quantum limit. The lowest parameter estimation imprecision can be in the order of $1/N$ when quantum effects are fully utilized and thus results in a quadrature enhancement in estimation precision. It is the so-called Heisenberg limit (HL).[6,7] SQL can be beaten by using resources of quantum entanglement in different ways, such as weak measurement,[8] indefinite causal order,[9] spin-1 Dicke states,[10] and parametric amplifier-based photon correlation interferometers.[11]

Every metrology scheme involves three stages: 1) probe state initialization; 2) sensing parameter field; and 3) measurement of probes and information extraction. Among them, the second stage depends on the parameter field to be estimated. What we can do is to choose suitable probe systems and design initialization and measurement procedures carefully, which correspond to the first stage and the final stage design, respectively. Quantum entanglement and spin squeezing are generated in atomic ensembles, which can host 10^3–10^5 atoms. It is a platform of great potential in quantum metrology because of its sheer size as well as the strong quantum entanglement that can be generated between so many probes with technology available to many labs.[12–15] Atoms in ensembles are indistinguishable. Thus, neither manipulation nor measurement on individual probe is possible as required in many proposed metrology schemes.[16–20] Magnetic susceptibility $\hat{J}_n$ along a specific axis is the most common and convenient physical quantity to be measured for an atomic ensemble. When the average magnetic susceptibility $\langle\hat{J}_n\rangle$ is used to estimate the parameter field ϕ, the corresponding estimation imprecision would be $\Delta\phi = \Delta J_n \Big/ \left|\frac{\partial\langle\hat{J}_n\rangle}{\partial\phi}\right|$. Here $\Delta J_n = \sqrt{\langle(\hat{J}_n - \langle\hat{J}_n\rangle)^2\rangle}$ is the standard deviation of the magnetic susceptibility along the direction n. Thus, great fluctuation ΔJ_n would result in poor estimation. It is widely believed that "low-noise detection" is necessary to improve the parameter estimation precision.[12–15]

It has attracted plenty of attention in the endeavor to suppress the fluctuation of $\hat{J}_n$ and improve the single-atom resolution.[21,22] Others have worked to circumvent the single-atom resolution requirement. The main idea is to use nonlinear atomic interaction to suppress the fluctuation of $\hat{J}_n$, which is the so-called echo procedure. The echo procedure can utilize

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the same nonlinear interaction which is employed to generate multipartite entanglement at the first stage.[23–26] We find that measuring magnetic susceptibilities along two mutually orthogonal axes perpendicular to parameter field ϕ can produce parameter estimation precision beyond SQL. The importance of our work is two-fold. Firstly, it shows that neither single-atom resolution nor nonlinear interaction is necessary on final measurement stage of metrology to achieve precision improvement beyond SQL. Secondly, it employs the most convenient measurement and is robust to detection errors. Thus, it is one step further to achieve metrology gain beyond SQL promised by quantum mechanics in practice.

2. One-axis spin squeezing

We consider an atomic ensemble consisting of N atoms. They are firstly initialized to the coherent spin state along the x direction. We then use the one-axis spin squeezing Hamiltonian $\hat{f}_z^2$ to generate multipartite entanglement among the atoms. Following the standard procedure of one-axis spin squeezing, a little adjustment along the x direction would also be implemented[27,28]

$$|\psi_0\rangle = \mathrm{e}^{\mathrm{i}\delta_{\mathrm{adj}}\hat{f}_x}\,\mathrm{e}^{-\mathrm{i}t_{\mathrm{s}}\hat{f}_z^2}\,|+\rangle^{\otimes N}. \tag{1}$$

The adjustment is $\delta_{\mathrm{adj}} = \frac{1}{2}\arctan\frac{B}{A}$ with $A = 1-(\cos 2t_{\mathrm{s}})^{N-2}$ and $B = 4\sin t_{\mathrm{s}}\,(\cos t_{\mathrm{s}})^{N-2}$. Here, $|+\rangle$ represents a single atomic spin directing along the positive x direction. We adopt the convention $\hbar = 1$. The squeezing time t_{s} (squeezing strength incorporated) can be adjusted in experiment.[12–15,23,24] That is the whole initialization stage. As shown in Figs. 1(a) and (b), the effect of the spin squeezing is suppressing the fluctuation on the z axis direction at the expense of increasing the fluctuation of spin along the two other directions.

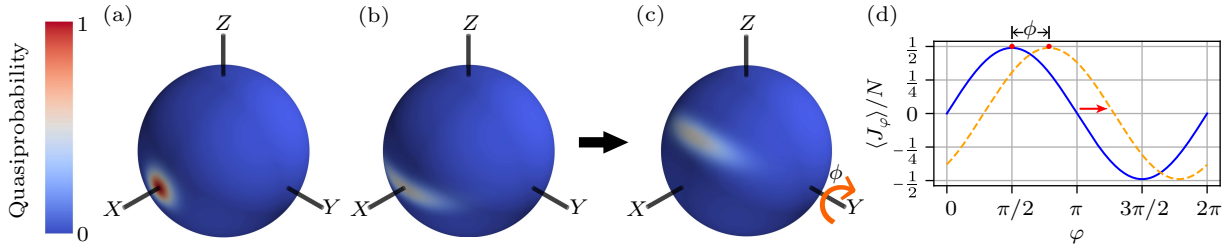


Fig. 1. Quasiprobability and magnetic susceptibility. The quasiprobability $P(\theta,\phi) = |\langle\theta,\phi|\psi\rangle|^2$ has been calculated for spin ensemble with $N = 100$ spins. $|\theta,\phi\rangle$ is the coherent spin state where all the spins pointing along the direction with polar angle θ and azimuth angle ϕ. The distribution of the quasiprobability is of similar shape for other values of N. The only difference is that the quasiprobability would be more concentrated as the number of spins grows larger. (a) The coherent spin state is initialized along the positive x direction. (b) The one-axis spin squeezed state. (c) The one-axis spin squeezed state rotates along the y axis clockwise through an angle ϕ. (d) Normalized magnetic susceptibility $\langle\hat{f}_\varphi\rangle/N$. The blue solid line describes the atomic ensemble initialized in an one-axis spin squeezed state. When the initial spin squeezed state rotates along the y axis through an angle ϕ, the curve would be shifted toward right by ϕ (orange dashed line). Note that $\langle J_\varphi\rangle$ is cyclic functions of φ and $N = 1000$ here.

3. Interference procedure

The atomic spins would be sent through a magnetic field pointing to the positive y direction. Let us assume that the overall quantum channel is $\hat{U}_\phi = \mathrm{e}^{-\mathrm{i}\phi\hat{f}_y}$. The output state of the atomic ensemble is

$$|\psi_\phi\rangle = \mathrm{e}^{-\mathrm{i}\phi\hat{f}_y}\,|\psi_0\rangle\,. \tag{2}$$

Measuring the atomic ensemble coming out of the magnetic field, we expect to estimate the value of ϕ and thus the value of the magnetic field.

4. Magnetic susceptibility measurements along two directions

It is widely believed that there is always an optimal measurement for a particular metrology scheme. In many cases, it can be proven that such kind of optimal measurement exists.[6] Others believe that adaptive feedback measurement can give the optimal information extraction.[16–20] Most of these measurement procedures are too complex to be realized on atomic ensembles. Magnetic susceptibility measurement is easy to be carried out on atomic ensembles. In most cases, it is not an optimal measurement scheme. Our goal here is to use the most simple measurement to achieve as high metrology precision as possible.

The reason that we choose two measurement directions of the magnetic susceptibility is of three folds. Firstly, the price is small. We usually need to collect a sufficient amount data to infer ϕ. Hence, the initialization-interference-measurement procedure has to be repeated R times with R sufficiently large. We divide the R experiments into two equal groups. Each group measures the magnetic susceptibility along a different direction from the other group. Let us assume the worst case where the magnetic susceptibility measurement on one of the directions provides no information of ϕ. Then only $R/2$ data samples are of use. The imprecision $\Delta\phi$ would increase by $\sqrt{2}$ which is relatively small when N is of the order of 10^3–10^5. Secondly, there is actually a simple correspondence between the parameter field ϕ and the magnetic susceptibility on the x–z plane of the Bloch sphere given that the atom ensemble is in a one-axis spin squeezed state (1), c.f., Fig. 1(d),

$$\langle\hat{f}_\varphi\rangle = \langle\hat{f}_x\rangle\sin\varphi + \langle\hat{f}_z\rangle\cos\varphi. \tag{3}$$

There is only one single peak of the $\langle\hat{f}_\varphi\rangle$ curve which is lo-

cated at $\varphi = \pi/2$. When the atomic ensemble initialized in a one-axis spin squeezed state comes out of the $\hat{U}_\phi$ channel, the peak of the $\langle \hat{J}_\varphi \rangle$ curve would shift by ϕ. $\langle \hat{J}_\varphi \rangle$ would be maximum if and only if $\varphi = \phi + \pi/2$,

$$\langle \hat{J}_\varphi \rangle = \sqrt{\langle \hat{J}_x \rangle^2 + \langle \hat{J}_z \rangle^2}\sin(\varphi - \phi). \tag{4}$$

Locating the peak $\langle \hat{J}_\varphi \rangle$ curve, we can obtain its position shift ϕ. Hence, ϕ can be calculated from $\langle \hat{J}_x \rangle$ and $\langle \hat{J}_z \rangle$, since

$$\sin\phi = -\frac{\langle \hat{J}_z \rangle}{\sqrt{\langle \hat{J}_x \rangle^2 + \langle \hat{J}_z \rangle^2}}, \quad \cos\phi = \frac{\langle \hat{J}_x \rangle}{\sqrt{\langle \hat{J}_x \rangle^2 + \langle \hat{J}_z \rangle^2}}. \tag{5}$$

In our scheme, we employ a slightly different evaluation formula from Eq. (5)

$$\phi_{\text{est}} = \text{atan2}(\tilde{s}, \tilde{c}), \tag{6}$$

with $\tilde{s} = -j_z/\sqrt{j_z^2 + j_x^2}$ and $\tilde{c} = j_x/\sqrt{j_z^2 + j_x^2}$. j_z and j_x are a pair of measurement results of the magnetic susceptibility collected in two separate experiments along the x and z directions, respectively. Note that the x axis magnetic susceptibility is usually carried out by firstly applying a $\pi/2$ pulse along the y axis to rotate the x axis to the z axis and followed by a magnetic susceptibility measurement along the z direction. Every $R = 2$ experiment measurement data produce one estimation ϕ_{est} of the real parameter ϕ. Thus, our estimation scheme (6) is "single-shot" in essence. Though the number of experiment data needed for one estimation is $R = 2$ instead of $R = 1$, we can obtain very good estimation with a small number of experiment data. This is fundamentally different from ensemble average estimation which demands sufficiently large number of data to produce good estimation.

4.1. Performance analysis

We have simulated our metrology scheme for up to 2480 atomic spins. $R = 2$ magnetic susceptibility measurements are carried out for estimating each ϕ, of which one is along the x axis and the other along the z axis. Thus, we have two experiments employing the same atomic ensemble, which goes through the same initialization and interference procedures plus a final magnetic susceptibility measurement at the end. The only difference is along which (x or z) direction the magnetic susceptibility measurement is carried out. The results of the two measurements are submitted to Eq. (6) to produce an estimation ϕ_{est} of ϕ. For each randomly chosen ϕ, we obtain its estimation ϕ_{est} by $R = 2$ experiments. Note that ϕ is fixed in these two experiments. We repeat the estimation $M = 1000$ times for different ϕ between 0 and 2π to evaluate $\Delta\phi$ in the entire range of values,

$$(\Delta\phi)^2 = \frac{R}{M}\sum_{i=1}^{M}\left[\Delta(\phi^{(i)}, \phi_{\text{est}}^{(i)})\right]^2. \tag{7}$$

Note that R in the above equation is to eliminate the metrology contribution of the size of data collected to estimate ϕ. ϕ is cyclic with period 2π and $\phi = 0$ and $\phi = 2\pi$ are considered to be of the same value. If the real parameter value is $\phi = 0$, the estimation $\phi_{\text{est}} = 2\pi - \varepsilon$ would be considered as a good estimation of $\phi = 0$ when $0 < \varepsilon \ll 1$. Hence we employ the unit circle distance[29] $\Delta(\phi^{(i)}, \phi_{\text{est}}^{(i)}) = \min\left\{|\phi^{(i)} - \phi_{\text{est}}^{(i)}|, 2\pi - |\phi^{(i)} - \phi_{\text{est}}^{(i)}|\right\}$ to reflect the cyclic nature of ϕ. We have optimized the squeezing time t_s to achieve the lowest $\Delta\phi$. Reference [28] tells us that the entanglement behavior of one-axis spin squeezed state has a time scale of $1/N^{2/3}$ and its quantum Fisher information reaches its maximum in the region of $t_s \gtrsim 1/N^{1/2}$. Hence, we choose the range of t_s to be $[0, 2/N^{1/2}]$) and optimize $t_s N^{2/3}$ instead of t_s.

As shown in Fig. 2(a), our two-axis magnetic susceptibility measurement scheme can produce parameter estimation

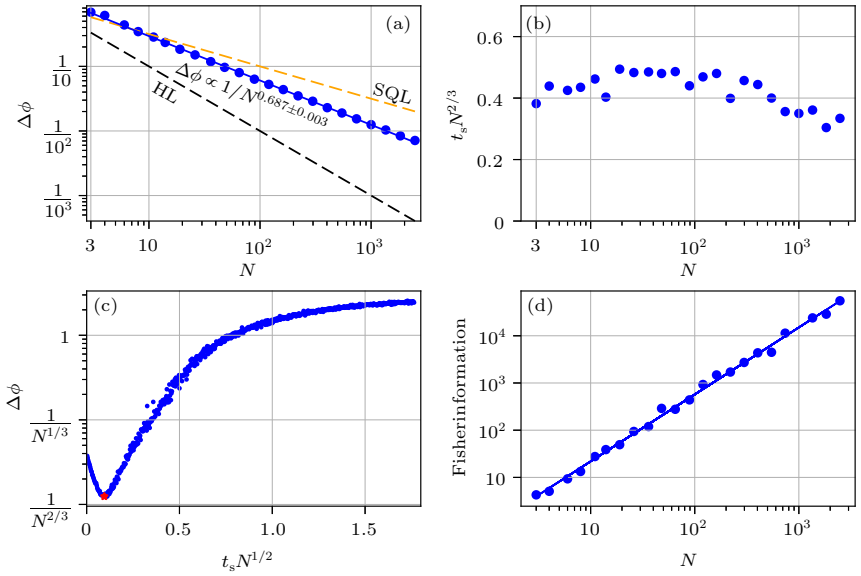


Fig. 2. Performance of the two-axis magnetic susceptibility measurement scheme for estimating ϕ. (a) Scaling of the imprecision $\Delta\phi$ versus atomic ensemble size N. (b) The optimal squeezing time t_s for different atomic ensemble size N. (c) The imprecision $\Delta\phi$ given by 1000 atomic spins with different squeezing time. There is a single valley of the $\Delta\phi$ versus t_s curve and the corresponding optimal squeezing time (marked red star) is employed in our scheme. (d) Fisher information of one-axis spin squeezed state given optimized squeezing time as shown in (b).

with imprecision below standard quantum limit for $N \gtrsim 16$. The scaling $\Delta\phi$ with respect to the atomic ensemble size is

$$\Delta\phi = \frac{1.43 \pm 0.02}{N^{0.687\pm0.003}}. \tag{8}$$

The optimal t_s is around the value of $0.4/N^{2/3}$ as shown in Fig. 2(b). Given the optimized squeezing time t_s, quantum Fisher information F of the corresponding one-axis spin squeezed state scales $F = 0.833N^{1.42}$ with respect to N. It matches the scaling of $(\Delta\phi)^2$ with respect to N, c.f., Eq. (8). The consistency of our numerical results obtained from different approaches confirms that they are valid and reliable. We must emphasis that t_s is optimized to obtain the highest estimation precision given by Eq. (6) instead of the highest quantum Fisher information. Note that the quantum Fisher information is[6]

$$F = 4\Delta J_y^2. \tag{9}$$

4.2. Robustness to detection noise

Further, we have investigated the robustness of our metrology scheme against the detection noise. The noise has two possible origins: a) defects of the detection apparatus and b) spin fluctuation agitated by surrounding environment. When measuring $\hat{J}_z$, we are actually counting how many atoms are occupying the spin up state $|\uparrow\rangle$. A defected detector would give us $n_\uparrow + \Delta n$ in one counting, when it should be $n_\uparrow$ if the detector is perfect. On the other hand, the environment would agitate a portion pN out of the total N atoms to flip.

Both of the effects are adding uncertainty to the measurement of $\hat{J}_z$ besides the inherent quantum fluctuation ΔJ_z. The same issues would emerge in measuring $\hat{J}_x$ which is carried out by counting the occupation number of spin up $|\uparrow\rangle$ after rotating the x axis to z axis. We assume that all the noise resources affect the atoms independently and the resulted fluctuation $\sigma = \sqrt{p(1-p)N}$ added to the measurement result $j_{x,z}$ is proportional to $\sqrt{N}$. Note that the effects of detection defects can be mimicked by random flipping of atomic spins caused by the environment noise before an ideal measurement. Hence, two kinds of noise can be depicted by the same model. The average of $\Delta n_\uparrow$ vanishes and its standard deviation is σ. Here we assume that every atomic spin is flipped independently. As shown in Fig. 3(a), our metrology scheme has strong robustness for noise fluctuation $\sigma \lesssim 0.2\sqrt{N}$. In the case of noise-induced fluctuation $\sigma = 0.1\sqrt{N}$, the estimation imprecision $\Delta\phi$ almost coincides with that of the noiseless case, c.f., Fig. 3(b). The scaling of the estimation imprecision $\Delta\phi$ with respect to the number of atoms indicates that SQL can be breached when $\sigma \lesssim 0.2\sqrt{N}$, c.f., Fig. 3(b) and Table 1.

Table 1. Scaling α of the imprecision $\Delta\phi$ with respect to N. α is calculated by fitting $\Delta\phi \propto 1/N^\alpha$. σ is measurement fluctuation induced by environment noise and measurement defects.

σ	0	$0.1\sqrt{N}$	$0.2\sqrt{N}$	$0.3\sqrt{N}$
α	0.687 ± 0.003	0.663 ± 0.004	0.615 ± 0.004	0.585 ± 0.003

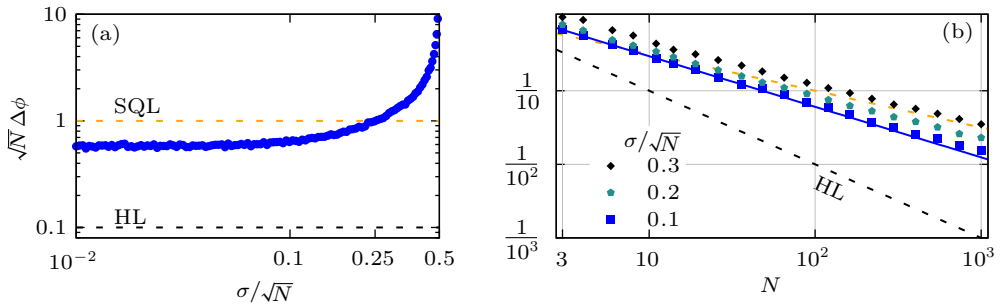


Fig. 3. Robustness against detection errors. The dashed orange line corresponds to SQL while the dashed black line to HL. (a) Imprecision $\Delta\phi$ versus fluctuation σ induced by environment noise and measurement defects, given $N = 100$. (b) Scaling of $\Delta\phi$ against different level of the detection defects and environment noise induced measurement fluctuation $\sigma = 0.1\sqrt{N}, 0.2\sqrt{N}, 0.3\sqrt{N}$. The solid blue line is the imprecision scale in the noiseless case as shown in Fig. 2(a).

5. Summary and outlooks

The magnetic susceptibility fluctuations along two orthogonal axes cannot be both very small. This is prohibited by the Heisenberg uncertainty relation. Our two-axis magnetic susceptibility measurement scheme can circumvent the worst effect of these fluctuations. By employing our scheme, neither sophisticated control scheme[16–20] nor echo procedure induced by nonlinear interaction[23–26] is needed to extract information from a quantum spin ensemble and SQL can be beaten. Besides, we need only two measurements to provide one estimation and thus a small number of data are enough to ensure good estimation. More importantly, our scheme promises estimation imprecision below SQL in the presence of the measurement defects and environment noise. When the defects and noise induced fluctuation is $\sigma \lesssim 0.2\sqrt{N}$, the performance of our scheme would deteriorate very slowly and SQL can be still beaten. The one-axis scheme would only produce estimation well in a small range of ϕ, while our scheme could work between 0 to 2π. It is interesting to ask whether HL can be reached or approached with such a simple measurement scheme.

Acknowledgments

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