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Citation: Chin. Phys. B, 2022, 31 (**5**): 054205. DOI: 10.1088/1674-1056/ac40f5
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Nonclassicality of photon-modulated spin coherent states in the Holstein–Primakoff realization

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(Received 17 October 2021; revised manuscript received 25 November 2021; accepted manuscript online 8 December 2021)

Two new photon-modulated spin coherent states (SCSs) are introduced by operating the spin ladder operators $J_{\pm}$ on the ordinary SCS in the Holstein–Primakoff realization and the nonclassicality is exhibited via their photon number distribution, second-order correlation function, photocount distribution and negativity of Wigner distribution. Analytical results show that the photocount distribution is a Bernoulli distribution and the Wigner functions are only associated with two-variable Hermite polynomials. Compared with the ordinary SCS, the photon-modulated SCSs exhibit more stronger nonclassicality in certain regions of the photon modulated number k and spin number j, which means that the nonclassicality can be enhanced by selecting suitable parameters.

Keywords: the photon-modulated spin coherent state, nonclassicality, photocount distribution, Wigner distribution function

PACS: 42.50.–p, 03.65.–w **DOI:** 10.1088/1674-1056/ac40f5

1. Introduction

As a new class of physical carriers, non-Gaussian states play a significant role for many quantum information theory applications. This is mainly because they can improve some important quantum information processes addressing the practical problems of state generation, entanglement and nonclassicality.[1–3]

At present, there has been considerable interest in the engineering of non-Gaussian states, especially, their theoretical and experimental aspects. Both photon-addition and photon-subtraction operations are considered as two kinds of effective methods for producing the non-Gaussian states[4–8] that may provide advantages in metrology and quantum computation.[9–11] The notion of photon addition to a quantum state was suggested by Agarwal and Tara[12,13] and subsequently a number of earlier studies of photon-added and photon-subtracted states was carried out.[14–18] For example, the experimental realization and tomographic analysis of quantum states by adding and subtracting single photon to a classical thermal field have been clarified in Ref. [19]. In addition, the potential usefulness of entangled states concerning precision measurements using entangled input states has been presented.[20]

Spin coherent states (SCSs) are analogous to ordinary coherent states in that they may be considered as near-classical states, non-Gaussian states of their corresponding systems. Experimentally, the SCSs can be easily generated via the cavity QED and trapped ions.[21] This also promotes the theoretical development of the SCSs. For example, the authors of Ref. [19] presented an improved phase estimation scheme and demonstrated that increasing the spin number leads to the smallest variance in comparison to NOON states ($|\mathrm{NOON}\rangle = (1/\sqrt{2})[|N,0\rangle + |0,N\rangle]$). The photon-added SCSs have been introduced by Berrada,[22] the statistical properties of these states are investigated in terms of hypergeometric and Meijer G-functions. As a further research of the SCSs, we now introduce two new photon-modulated SCSs in the Holstein–Primakoff realization and investigate their nonclassicality via the photon number distributions, second-order correlation function, and photocount distribution. Moreover, we discuss the Wigner functions and their marginal distributions of the two new photon-modulated SCSs and investigate their nonclassical behaviors via the appearance of their negative values. This paper is organized as follows. In Section 2, two new photon-modulated SCSs are theoretically constructed via the Holstein–Primakoff realization of angular momentum algebra. Sections 3–6 are devoted to investigating their nonclassical properties analytically and numerically. The main results of this article are summarized in Section 7.

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2. Photon-modulated spin coherent states

Noting that the Holstein–Primakoff realization of the spin ladder operators is given as $J_+ = a^\dagger\sqrt{(2j-n)}$, $J_- = \sqrt{(2j-n)}a$, $J_z = n-j$, which satisfy the spin Lie algebra $[J_+,J_-] = J_z$, $[J_\pm,J_z] = \mp J_\pm$,[23] where a and $a^\dagger$ are the annihilation and creation operators, $[a,a^\dagger] = 1$, and the number j presents the total angular momentum quantum number, thus acting the exponential operator $\mathrm{e}^{zJ_+ + z^*J_-}$ on single-mode vacuum and in the the Fock state space we can expand the SCSs as

$$|z,j\rangle = \frac{1}{(1+|z|^2)^j}\sum_{n=0}^{2j}\binom{2j}{n}^{1/2} z^n|n\rangle. \tag{1}$$

Further, by repeatedly operating the spin ladder operators J_+ and J_- on the ordinary SCSs $|z,j\rangle$ for k times, we obtain the photon-modulated SCSs $|z,j\rangle_a$ and $|z,j\rangle_s$, which are respectively represented as

$$\begin{aligned}|z,j\rangle_a &= \mathcal{N}_a^{-1/2}\sum_{n=0}^{2j-k}\mathcal{D}_a^{1/2}z^n|n+k\rangle,\\ |z,j\rangle_s &= \mathcal{N}_s^{-1/2}\sum_{n=k}^{2j}\mathcal{D}_s^{1/2}z^n|n-k\rangle,\end{aligned} \tag{2}$$

thus the normalization factors $\mathcal{N}_a$ and $\mathcal{N}_s$ read

$$\mathcal{N}_a = \sum_{n=0}^{2j-k}\mathcal{D}_a|z|^{2n},\quad \mathcal{N}_s = \sum_{n=k}^{2j}\mathcal{D}_s|z|^{2n}, \tag{3}$$

where the parameters $\mathcal{D}_a$ and $\mathcal{D}_s$ are

$$\begin{aligned}\mathcal{D}_a &= \frac{(n+k)!(2j)!}{(n!)^2(2j-n-k)!},\\ \mathcal{D}_s &= \frac{(2j-n+k)!(2j)!}{[(2j-n)!]^2(n-k)!}.\end{aligned} \tag{4}$$

3. Photon number distribution

For the photon-modulated SCSs $|z,j\rangle_{a(s)}$, their photon number distributions are given as

$$\begin{aligned}P_a(m) &= \mathcal{N}_a^{-1}\frac{m!(2j)!\,|z|^{2(m-k)}}{(2j-m)!\,[(m-k)!]^2},\\ P_s(m) &= \mathcal{N}_s^{-1}\frac{(2j)!(2j-m)!\,|z|^{2(m+k)}}{m!\,[(2j-m-k)!]^2}.\end{aligned} \tag{5}$$

For a given value of $|z|$ (e.g., $|z|=1$), we plot the photon number distributions $P_{a(s)}(m)$ of the states $|z,j\rangle_{a(s)}$ for different values of the parameters k and j in Fig. 1. Figure 1 shows that the photon number distribution only appears in several photon numbers and almost approaches zero for others. With increasing the parameters k and j, the peak of the state $|z,j\rangle_a$ moves to a higher photon number m, but both a smaller k and a larger j can lead to a wider photon number distribution. However, for the state $|z,j\rangle_s$, with increasing j or decreasing k, the peak moves to a higher value of photon number and shows a wider photon number distribution.

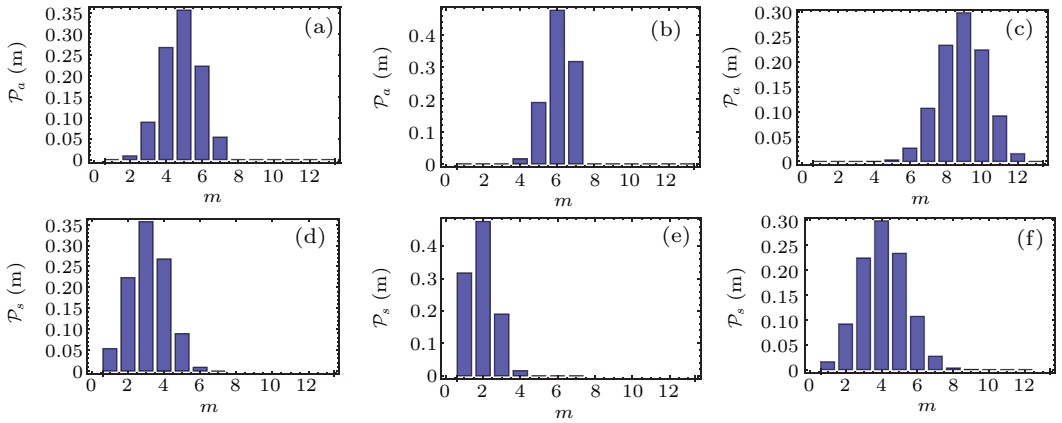


Fig. 1. Photon number distribution of the state $|z,j\rangle_a$ as a function of photon number m with $|z|=1$ for (a) $j=3$, $k=1$; (b) $j=3$, $k=3$; (c) $j=5.5$, $k=3$. Photon number distribution of the state $|z,j\rangle_s$ as a function of photon number m with $|z|=1$, for the same values of $|z,j\rangle_a$, respectively, refer to (d), (e) and (f).

4. Photocount distribution

Nonclassicality of light fields can be measured using a photon counting apparatus to judge the nonclassical features in experiment, which greatly simplifies the theoretical research of photocount distribution. For a single-mode quantum state ρ of light field, the new formula of calculating the photocount distribution is defined as[24]

$$\mathcal{P}(m) = \left(\frac{\xi}{\xi-1}\right)^m\int\frac{\mathrm{d}^2z}{\pi}\mathrm{e}^{-\xi|z|^2}\times L_m(|z|^2)\left\langle\sqrt{1-\xi}z\right|\rho\left|\sqrt{1-\xi}z\right\rangle, \tag{6}$$

where $\langle\sqrt{1-\xi}z|\rho|\sqrt{1-\xi}z\rangle$ is Q function of the state ρ since $|\sqrt{1-\xi}z\rangle$ is a coherent state with the amplitude $\sqrt{1-\xi}z$, ξ is the quantum efficiency of the detector and $L_m(\cdot)$ is a Laguerre polynomial. Clearly, when the Q function of ρ in the coherent state $|\sqrt{1-\xi}z\rangle$ is known, the photocount distribution can directly be obtained via performing the integral of the variable z in Eq. (6). For this purpose, using the technique of method within an ordered product of operator, we now obtain the Q

function of the state $|z,j\rangle_a$ as

$$\left\langle \sqrt{1-\xi}z\middle|z,j\rangle_{aa}\langle z,j\middle|\sqrt{1-\xi}z\right\rangle = \mathcal{P}_{n+k}e^{-(1-\xi)|z|^2}\frac{(1-\xi)^{n+k}|z|^{2(n+k)}}{(n+k)!}, \tag{7}$$

where $\mathcal{P}_{n+k}=\mathcal{D}_a|z|^{2n}/\mathcal{N}_a$. By virtue of the relation between Hermite polynomials and Laguerre polynomials, that is, $L_m(|z|^2)=\frac{(-1)^m}{m!}H_{m,m}(z,z^*)$, and the differential form of two-variable Hermitian polynomials, i.e., $H_{m,n}(\xi,\xi^*)=\partial_s^m\partial_t^n\,\mathrm{e}^{-st+s\xi+t\xi^*}\big|_{s=t=0}$, the photocount distribution of the state $|z,j\rangle_a$ is calculated as

$$\mathcal{P}_a(m) = \frac{\mathcal{P}_{n+k}\xi^m}{(n+k)!m!(1-\xi)^{m-n-k}}\int\frac{\mathrm{d}^2z}{\pi}|z|^{2(n+k)} \times\partial_s^m\partial_t^m\exp(-|z|^2-st+sz+tz^*)\Big|_{s=t=0}. \tag{8}$$

Furthermore, using the mathematical integral formula

$$\int\frac{\mathrm{d}^2z}{\pi}z^nz^{*m}\exp(h|z|^2+sz+\eta z^*) = \mathrm{e}^{-s\eta/h}\sum_{l=0}^{\min(m,n)}\frac{m!n!s^{m-l}\eta^{n-l}}{l!(m-l)!(n-l)!(-h)^{m+n-l+1}}, \tag{9}$$

we can obtain the distribution $\mathcal{P}_a(m)$ as

$$\mathcal{P}_a(m)=\sum_{n=0}^{2j-k}\mathcal{P}_{n+k}\binom{n+k}{m}\xi^m(1-\xi)^{n+k-m}. \tag{10}$$

Similarly, we can obtain the photocount distribution of the state $|z,j\rangle_s$, i.e.,

$$\mathcal{P}_s(m)=\sum_{n=k}^{2j}\mathcal{P}_{n-k}\binom{n-k}{m}\xi^m(1-\xi)^{n-k-m}, \tag{11}$$

where $\mathcal{P}_{n-k}=\mathcal{D}_s|z|^{2n}/\mathcal{N}_s$. Clearly, Eqs. (10) and (11) show that $\mathcal{P}_{a(s)}(m)$ are the familiar Bernoulli distributions, which indicate that counting the success probability for m out of $n\pm k$ photons depends linearly on the probability of counting m photons ξ^m multiplied by the probability of counting $n\pm k-m$ photons $(1-\xi)^{n\pm k-m}$.

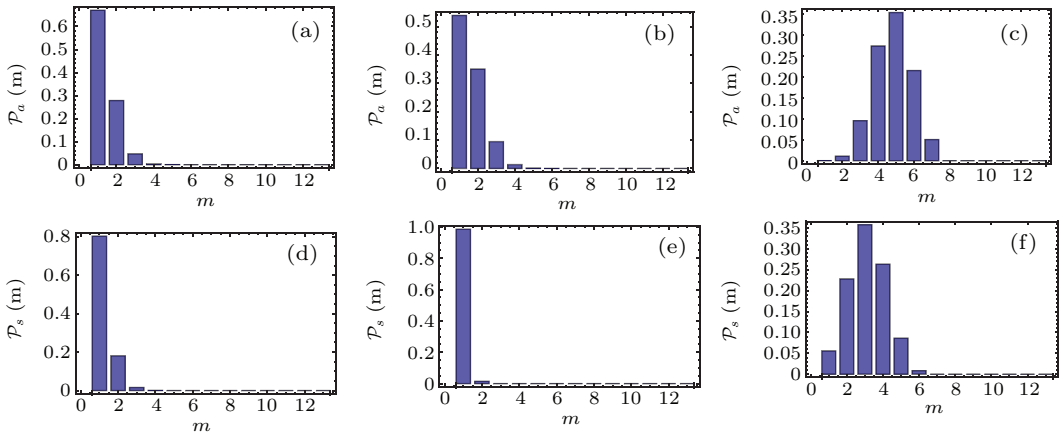


Fig. 2. Photocount distribution of the state $|z,j\rangle_a$ versus the number of detected photons m for some different values of k and ξ, with $|z|=1$, $j=3$ (a) $k=1$, $\xi=0.1$; (b) $k=5$, $\xi=0.1$; (c) $k=1$, $\xi=0.99$. Photocount distribution of the state $|z,j\rangle_s$ versus the number of detected photons m, with the same values as $|z,j\rangle_a$, respectively, refer to (d), (e) and (f).

Here we are interested in how the photon counting distributions $\mathcal{P}_{a(s)}(m)$ are affected by varying the parameters ξ and k, which reveal the variations of the distributions $\mathcal{P}_{a(s)}(m)$ with the quantum efficiency ξ of the detector and photon modulated number k. From Fig. 2, we can see that the distributions $\mathcal{P}_{a(s)}(m)$ are related to ξ and k, only emerge in several numbers of counting m, and are close to zero for others, which indicates that the probability of counting m has more complicated nonclassical features. For a fixed poor efficiency ξ (e.g., $\xi=0.1$), the maximum probability of counting photons gradually decreases and the range being larger as k increases for the state $|z,j\rangle_a$. However, for the state $|z,j\rangle_s$, the changes of photocount distributions with the increase of k are just contrary [see Figs. 2(a), 2(b), 2(e), and 2(f)]. For a fixed modulated number k (e.g., $k=1$), as quantum efficiency ξ increases, the maximum probability of counting photons slowly decreases and its position moves to the large counting m almost to 0.35 regardless of $|z,j\rangle_{a(s)}$. In sum, the states $|z,j\rangle_{a(s)}$ can become more robust and highly nonclassical by modulating the quantum efficiency ξ and photon modulated number k.

5. Second-order correlation function

Nonclassicality of the photon-modulated SCSs $|z,j\rangle_{a(s)}$ can be investigated via its second-order correlation function defined as $g^{(2)}(0)=\langle a^{\dagger2}a^2\rangle/\langle a^\dagger a\rangle^2$.[25] For the case of $g^{(2)}(0)<1$, the states $|z,j\rangle_{a(s)}$ can exhibit sub-Poissonian distribution representing nonclassicality. From Eq. (2), we find that the average values ${}_{a(s)}\langle a^\dagger a\rangle_{a(s)}$ and ${}_{a(s)}\langle a^{\dagger2}a^2\rangle_{a(s)}$ are obtained as

$${}_a\langle a^\dagger a\rangle_a=\mathcal{N}_a^{-1}\sum_{n=0}^{2j-k}\mathcal{D}_a(n+k)|z|^{2n}, \tag{12}$$

$$ {}_s\langle a^{\dagger}a\rangle_s = \mathcal{N}_s^{-1}\sum_{n=k+1}^{2j}\mathcal{D}_s(n-k)\,|z|^{2n}, \tag{13}$$

$$ {}_a\langle a^{\dagger 2}a^2\rangle_a = \mathcal{N}_a^{-1}\sum_{n=0}^{2j-k}\mathcal{D}_a(n+k)(n+k-1)\,|z|^{2n}, \tag{14}$$

$$ {}_s\langle a^{\dagger 2}a^2\rangle_s = \mathcal{N}_s^{-1}\sum_{n=k+2}^{2j}\mathcal{D}_s(n-k)(n-k-1)\,|z|^{2n}. \tag{15}$$

For the states $|z,j\rangle_{a(s)}$, substituting Eqs. (12)–(15) into the original definition of $g^{(2)}(0)$, we plot the second-order correlation function $g^{(2)}_{a(s)}(0)$ versus the amplitude $|z|$ for different values of photon modulated number k and spin number j in Fig. 3. Clearly, there are always $g^{(2)}_{a(s)}(0)<1$ for any parameter values of k and j, which corresponds to a sub-Poissonian distribution. Notably, for a given value of j (e.g., $j=3$), $g^{(2)}_a(0)$ always increases and $g^{(2)}_s(0)$ gradually decreases with increasing k. For a fixed k (e.g., $k=2$), $g^{(2)}_{a(s)}(0)$ always increases with increasing j. Especially, for large $|z|$, $g^{(2)}_a(0)$ tends to a fixed value that only depends on the photon modulated spin number j, as shown in Fig. 3(a). This indicates that the states $|z,j\rangle_{a(s)}$ exhibit different nonclassical behaviors, and much stronger nonclassicality than the original SCSs $|z,j\rangle$ by choosing appropriate parameters.

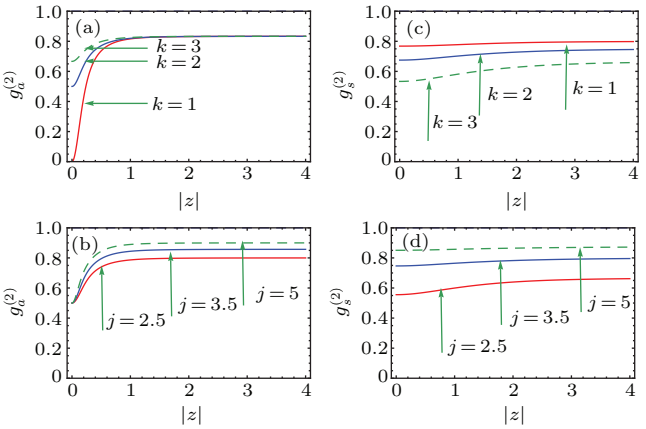


Fig. 3. The second-order correlation function of the state $|z,j\rangle_a$ as a function of amplitude $|z|$ (a) $j=3$, $k=1,2,3$; (b) $k=2$, $j=2.5,3.5,5$. The second-order correlation function of the state $|z,j\rangle_s$ as a function of amplitude $|z|$ with the same values as $|z,j\rangle_a$, respectively, refer to (c) and (d).

6. Wigner distribution

Usually, for a single-mode quantum state ρ, its Wigner distribution is defined as $W(\alpha)=\mathrm{tr}[\Delta(\alpha)\rho]$, where $\Delta(\alpha)=\pi^{-1}D(2\alpha)(-1)^N$ is the coherent state representation of single-mode Wigner operator,[26–29] $D(2\alpha)=\mathrm{e}^{2(\alpha a^{\dagger}-\alpha^* a)}$ is displacement operator and $N=a^{\dagger}a$ is number operator. To obtain the Wigner distributions of the states $|z,j\rangle_{a(s)}$, we first calculate the normal ordering product of the operator $a^m D(\alpha)a^{\dagger n}$. Inserting the completeness of coherent states into the operator $a^m D(\alpha)a^{\dagger n}$ leads to

$$ a^m D(\alpha)a^{\dagger n} = \int\frac{\mathrm{d}^2 z}{\pi}a^m D(\alpha)D(z)\,|0\rangle\langle z|\,z^{*n}. \tag{16}$$

Further, using the operator identity $D(\alpha)D(z)=D(\alpha+z)\,\mathrm{e}^{(z^*\alpha-z\alpha^*)/2}$, and the normal ordering of the density operator of vacuum, i.e., $|0\rangle\langle 0|=:\mathrm{e}^{-a^{\dagger}a}:$, as well as the integral formula (9), thus we can rewrite Eq. (16) as

$$\begin{aligned} a^m D(\alpha)a^{\dagger n} &= \int\frac{\mathrm{d}^2 z}{\pi}(a+z)^m\,|\alpha+z\rangle\langle z|\,z^{*n}\,\mathrm{e}^{(z^*\alpha-z\alpha^*)/2}\\ &= \mathrm{e}^{-|\alpha|^2/2}\sum_{l=0}^{m}\binom{m}{l}a^{m-l}:\mathrm{e}^{\alpha a^{\dagger}-a^{\dagger}a}\\ &\quad\times\int\frac{\mathrm{d}^2 z}{\pi}z^l z^{*n}\,\mathrm{e}^{-|z|^2+z(a^{\dagger}-\alpha^*)+z^* a}:\\ &= \mathrm{e}^{-|\alpha|^2/2}\sum_{l=0}^{m}\binom{m}{l}a^{m-l}\\ &\quad\times:\sum_{s=0}^{\min(l,n)}\frac{l!n!(a^{\dagger}-\alpha^*)^{n-s}a^{l-s}}{s!(l-s)!(n-s)!}\,\mathrm{e}^{\alpha a^{\dagger}-\alpha^* a}:, \end{aligned} \tag{17}$$

which leads to the matrix elements in the Fock basis, that is,

$$\begin{aligned} &\langle m\pm k|\,D(2\alpha)\,|n\pm k\rangle\\ &= \left[\frac{(n\pm k)!}{(m\pm k)!}\right]^{1/2}(2\alpha)^{m-n}\,\mathrm{e}^{-2|\alpha|^2}L^{m-n}_{n\pm k}(4|\alpha|^2). \end{aligned} \tag{18}$$

Substituting Eqs. (2) and (18) into the definition of Wigner distribution, we finally obtain the analytical Wigner distributions of the photon-modulated SCSs as follows:

$$\begin{aligned} W_a(\alpha) &= \sum_{m,n=0}^{2j-k}\frac{\mathcal{D}_a\mathcal{M}n!z^n z^{*m}}{\pi\mathcal{N}_a m!(n+k)!}\\ &\quad\times H_{m+k,n+k}(2\alpha,2\alpha^*)\,\mathrm{e}^{-2|\alpha|^2}, \end{aligned} \tag{19}$$

$$\begin{aligned} W_s(\alpha) &= \sum_{m,n=k}^{2j}\frac{\mathcal{D}_s(2j-n)!z^n z^{*m}}{\pi\mathcal{N}_s\mathcal{M}(2j-m)!(m-k)!}\\ &\quad\times H_{m-k,n-k}(2\alpha,2\alpha^*)\,\mathrm{e}^{-2|\alpha|^2}, \end{aligned} \tag{20}$$

where we have used the relation between the associated Laguerre polynomials $L_n^{m-n}(xy)$ and two-variable Hermite polynomials $H_{m,n}(x,y)$, that is, $(-1)^n x^{m-n}L_n^{m-n}(xy)=H_{m,n}(x,y)$, and $\mathcal{M}=[(2j-n-k)!/(2j-m-k)!]^{1/2}$.

In Fig. 4, we present the Wigner distributions of the photon-modulated SCSs $|z,j\rangle_{a(s)}$ as a function of $\mathrm{Re}\,\alpha$ and $\mathrm{Im}\,\alpha$ for different values of photon modulated number k, spin j and amplitude $|z|$. Intuitively, the distributions $W_{a(s)}(\alpha)$ always have negative regions and non-Gaussian features in phase space for certain parameter values, as a signal of the nonclassicality aspect of the quantum state of interest. Specially, the distribution $W_s(\alpha)$ of the state $|z,j\rangle_s$ exhibits the multi-peak structure with a main peak and a large negative region in phase space. For small $|z|$, the distribution $W_s(\alpha)$ loses completely its negative region and becomes a single upward Gaussian peak at the central position without the photon modulated number k and spin number j, which is similar to the

distribution for $2j = k$, whereas it exhibits gradually the negative region with increasing $|z|$. The negative region of the distribution $W_s(\alpha)$ increases with increasing integer j, however decreases with the increase of even (or odd) k. On the other hand, for the distribution $W_a(\alpha)$, its partial negative region becomes larger with increasing k for a fixed j and almost keeps unchanged with the spin j for a given k. Therefore, photon modulated operation can enhance the nonclassicality of the state $|z, j\rangle$ by modulating the parameters k, j and $|z|$.

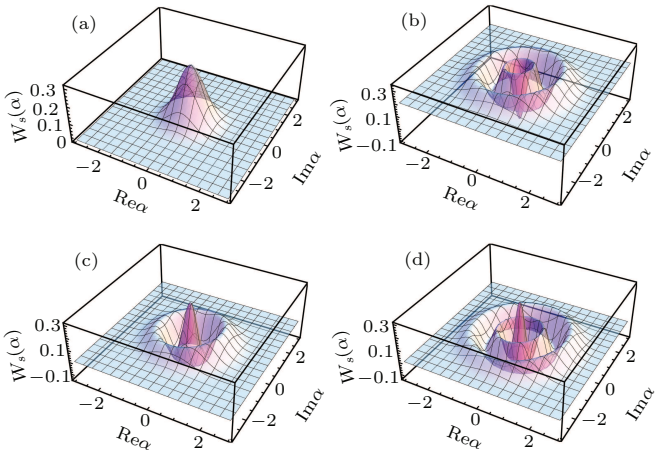


Fig. 4. The Wigner function of the state $|z, j\rangle_s$ for different parameters k, j and $|z|$, where the values of (k, j, $|z|$) are respectively (a) (2, 2.5, 0.01); (b) (2, 2.5, 20); (c) (3, 2.5, 20); (d) (2, 3, 20).

Next, we want to calculate two marginal distributions of $W_{a(s)}(\alpha)$. Using the definition of Hermite polynomials, i.e., $H_n(q) = \mathrm{e}^{q^2/2}(q - d/dq)^n \mathrm{e}^{-q^2/2}$,[28] we obtain the marginal distribution[30,31] of $W_a(\alpha)$ in the q- and p-direction as

$$P_{wa}(q) = \frac{\mathrm{e}^{-q^2}}{\sqrt{\pi}\mathcal{N}_a}\left|\sum_{n=0}^{2j-k}\mathcal{F}_a^{1/2} z^n H_{n+k}(q)\right|^2, \tag{21}$$

$$P_{wa}(p) = \frac{\mathrm{e}^{-q^2}}{\sqrt{\pi}\mathcal{N}_a}\left|\sum_{n=0}^{2j-k}\mathcal{F}_a^{1/2} z^n (-\mathrm{i})^{n+k} H_{n+k}(p)\right|^2, \tag{22}$$

which are related to the square of the modulus of single-mode Hermite polynomials $H_{n+k}(\cdot)$, $\mathcal{F}_a = \mathcal{D}_a/\left[(n+k)!2^{n+k}\right]$. Similarly, we can calculate the marginal distributions of $W_s(\alpha)$ in the q- and p-direction as

$$P_{ws}(q) = \frac{\mathrm{e}^{-q^2}}{\sqrt{\pi}\mathcal{N}_s}\left|\sum_{n=k}^{2j}\mathcal{F}_s^{1/2} z^n H_{n-k}(q)\right|^2, \tag{23}$$

$$P_{ws}(p) = \frac{\mathrm{e}^{-q^2}}{\sqrt{\pi}\mathcal{N}_s}\left|\sum_{n=k}^{2j}\mathcal{F}_s^{1/2} z^n (-\mathrm{i})^{n-k} H_{n-k}(p)\right|^2, \tag{24}$$

with $\mathcal{F}_s = \mathcal{D}_s/\left[(n-k)!2^{n-k}\right]$. Obviously, Eqs. (21)–(24) represent the probabilities of finding the particles with the given position q (momentum p) in the states $|z, j\rangle_{a(s)}$, which are just related to single-variable Hermite polynomials of orders $n \pm k$.

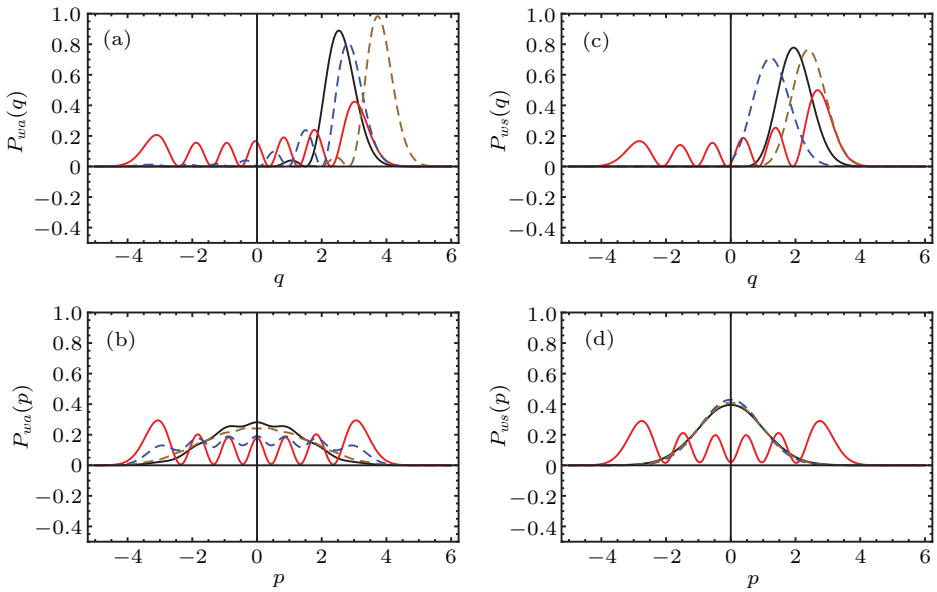


Fig. 5. Marginal distributions $P_{wa(ws)}$ for the functions $W_{a(s)}(\alpha)$ in the p–q phase space for different values of (j, k, $|z|$), where (3, 1, 1, black curves), (3, 1, 10, red curves), (3, 3, 1, blue curves), (5.5, 3, 1, brown curves), respectively.

Figure 5 shows the marginal distributions $P_{wa(s)}(q)$ and $P_{wa(s)}(p)$ for different values of j, k and $|z|$. As seen in Fig. 5, the probability distributions only occur in a certain range of q and p and tend to zero for other ranges. With the increase of k, the peak of the distributions $P_{wa}(q)$ and $P_{ws}(q)$ moves to the opposite direction along the q axis, and its peak value decreases. With increasing j, both the peak of the distributions $P_{wa}(q)$ and $P_{ws}(q)$ moves toward the larger q, and its peak value changes are quite on the contrary. In other words, the maximum marginal distribution probability moves to different positions q with the changing k and j. On the other hand, we have found that both the states $|z, j\rangle_{a(s)}$ show probability distributions $P_{wa(s)}(p)$ with a series of small peaks that have symmetrical feature. As k and j get larger the peak value of the

distribution $P_{wa}(p)$ decreases, however the distribution $P_{ws}(p)$ is just opposite. Moreover, with increasing $|z|$, the nonclassical oscillations are enhanced, and the probabilities to detect the particles decrease.

7. Conclusions

In summary, we have repeatedly acted the spin operators J_+ and J_- on the SCS that leads to theoretically producing two new photon-modulated SCSs, and studied their nonclassical properties via photon number distribution, photocount distribution, second-order correlation function and Wigner function. For the state $|z,j\rangle_a$, the maximum probability of counting photons gradually decreases as k increases, while it is completely contrary for the state $|z,j\rangle_s$. Regardless of the states $|z,j\rangle_{a(s)}$, the maximum probability tends to the large counting m and the value is almost to a fixed value as quantum efficiency ξ increases. Furthermore, using the coherent state representation of Wigner operator, we have obtained the Wigner functions and their marginal distributions for the states $|z,j\rangle_{a(s)}$ that are always related to Hermite polynomials. The Wigner distributions obviously exhibit the multi-peak structures and some negative regions, which shows that the photon modulated operations can enhance the nonclassicality to a certain degree. Moreover, we have particularly analyzed the peaks of the maximum marginal distribution probability along the q axis and interference fringes along the p axis with the different modulated numbers k and spins j. Noting that various types of nonclassicality can be obtained by selecting the appropriate control parameters, and the nonclassicality of the states can remarkably be improved, these results may make the features of the photon-modulated SCSs as a good reference implementation of different schemes of quantum optics and quantum information.

Acknowledgements

This work was supported by the National Natural Science Foundation of China (Grant No. 11347026) and the Natural Science Foundation of Shandong Province, China (Grant Nos. ZR2020MA085 and ZR2020MF113).

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