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Relativistic motion on Gaussian quantum steering for two-mode localized Gaussian states

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Realistic quantum systems always exhibit gravitational and relativistic features. In this paper, we investigate the properties of Gaussian steering and its asymmetry by the localized two-mode Gaussian quantum states, instead of the traditional single-mode approximation method in the relativistic setting. We find that the one-side Gaussian quantum steering will monotonically decrease with increasing observers of acceleration. Meanwhile, our results also reveal the interesting behavior of the Gaussian steering asymmetry, which increases for a specific range of accelerated parameter and then gradually approaches to a finite value. Such finding is well consistent and explained by the well-known Unruh effect, which could significantly destroy the one-side Gaussian quantum steering. Finally, our results could also be applied to the dynamical studies of Gaussian steering between the Earth and satellites, since the effects of acceleration are equal to the effects of gravity according to the equivalence principle.

Keywords: relativistic quantum information, quantum nonlocality, Unruh effect

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1. Introduction

Quantum physics, one of the most important scientific achievements in the past years, has fantastic properties that distinguish it from classical physics. One of these fantastic features is the well-known EPR paradox firstly proposed by Einstein, Podolsky, and Rosen (EPR) in 1935,[1] which was subsequently followed by the illustration of the concept of quantum steering in the same year.[2,3] Quantum steering, an intermediate type of quantum correlation between quantum entanglement and Bell nonlocality, is also recognized as the nonlocal quantum correlation (i.e., Alice allows to steer the state of a distant party Bob through local measurements). Actually, many important results have been obtained in the past few years, focusing on the experimental applications of quantum steering. For instance, considering the inherent asymmetric features of EPR steering, its potential applications in the one-side device-independent quantum key distribution is highly emphasized.[4] The asymmetry of quantum steering in different quantum systems has also attracted worldwide attention, with the generation of various theoretical works[5–13] and experimental demonstrations.[14–26] Moreover, a significant amount of work has been done to develop quantum steering characterization and quantification in non-relativistic quantum mechanics, in particular for the Gaussian state. However, it should be pointed out that these works considered only the inertial coordinate system, which indicates the necessity of taking relativistic effects into account in realistic non-inertial coordinate systems. Such conclusion is strongly supported by the recent work,[27] which demonstrates that the Gaussian quantum steering between the Earth and satellites will be strongly affected by the curved space-time around the Earth.

On the other hand, when one part of a multi-body system is involved in relativistic effects, the Unruh effect[28] should be taken into consideration. The traditional method to solve the quantum field mode of scalar or spinor fields (which corresponds to Klein–Gordon or Dirac fields, respectively) is the so-called single mode approximation method, which has been widely used in quantum information tasks quantifying how uniform acceleration affects quantum resources. From theoretical point of view, many efforts have been made to understand quantum systems under different relativistic settings, e.g., the dynamics of quantum steering under the influence of dynamical Casimir effect,[29] the Hawking radiation,[30] and accelerated motions.[31] More related work on relativistic quantum information can be found in the literature.[32–41] In addition, some attempts have previously been made to perform experimental tests of quantum resources in relativistic coordinate systems, with the realization of photonic entanglement in an accelerated setting.[42] However, it is well known that one cannot get the full information from a global mode in the experiment, since the method of single mode approximation is delocalized in practice.[43] Such issue has been extensively discussed in several recent works, taking gravitational

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or accelerational field as lossy channel rather than global free models, and furthermore calculating the fidelity of the lossy channels.[27,44] Dragan *et al.*[45,46] proposed a new idea, localized mode, to successfully realize the localization of Gaussian states, by storing the squeezed Gaussian states in a simulated cavity capable of transport quantum states. In this case, more attention has been paid to the dynamics of quantum entanglement in localized Gaussian state, with the recent findings that the quantum entanglement increases with the observer's proper acceleration, but decreases with the width of cavity and central frequency.[47] However, quantum steering in relativistic coordinate system by using localized Gaussian state is still an uncharted territory. In this work, we will study the quantum steering with localized two-mode Gaussian quantum states in relativistic coordinate system. Since the quantification of Gaussian quantum steering has explicit definition, it can be straightly applied to arbitrary two-mode Gaussian states. More specifically, we consider a two-mode squeezed vacuum state with squeezing parameter r of two localized wave packets as input state, and transform it to an accelerated frame of reference corresponding to a pair of uniformly accelerated observers (named Alice and Bob). In the framework of two cases, i.e., Alice shares the equal or unequal acceleration with Bob, we will show how the observer's proper acceleration affects the Gaussian quantum steering.

This paper is organized as follows: In Section 2, we briefly describe the localization process of quantum states and select the input state. In Section 3, we introduce the definition and measurements of a bipartite Gaussian quantum steering. The dynamics of Gaussian quantum steering in accelerated coordinate systems are presented in Section 4. Finally, we summarize our conclusions in Section 5. The natural units $G = c = \hbar = k_{\rm B} = 1$ are employed throughout the paper.

2. The localization process of quantum states

The localization process of quantum states is equivalent to a noisy Gaussian channel, which can be characterized by explicit analytic expressions with no approximations. Moreover, it will not be constrained to the geometry of the Rindler chart, which enables observers to have arbitrary proper accelerations and the minimal distance between them. In this section, we will briefly introduce the noisy quantum Gaussian channel (a detailed description could be found in Section II of Ref. [47]).

2.1. The Gaussian channel

For a real scalar quantum field $\hat{\Phi}$ with a mass m in $(1+1)$-dimensional Minkowski spacetime, its evolution is governed by the Klein–Gordon equation $(\Box + m^2)\hat{\Phi} = 0$, where $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$. The implicit Klein–Gordan scalar product can be written as

$$(\phi_1, \phi_2) = \mathrm{i}\int_\Sigma \mathrm{d}x\,(\phi_1^\star \partial_t \phi_2 - \phi_2 \partial_t \phi_1^\star), \tag{1}$$

where Σ is a space-like Cauchy surface and the imaginary factor guarantees that $(\phi_1, \phi_2) = (\phi_2, \phi_1)^\star = -(\phi_2^\star, \phi_1^\star)$.[48]

In our scheme, we consider two uniformly accelerating observers, Alice and Bob, in an accelerated frame of reference which transforms a two-mode state prepared in an inertial frame from initial frame to the frame of the observers. At the same time, we are interested in analyzing the situation that two Rindler wedges meet at the origin. The inertial modes will be denoted as ϕ_k, with associated annihilation operators $\hat{f}_k$ which only contains positive frequencies with respect to the Minkowski time-like Killing vector field. In the accelerated frame of reference, the two operators will be denoted as ψ_k and $\hat{d}_k$ which consist only of positive frequencies with respect to the Rindler time-like Killing vector field, respectively. Then, the field operator Φ can be decomposed by these two sets of orthonormal modes with respect to the Klein–Gordon inner product

$$\hat{\Phi} = \sum_k [\phi_k \hat{f}_k + \text{H.c.}] = \sum_k [\psi_k \hat{d}_k + \text{H.c.}]. \tag{2}$$

In this paper, we will investigate the scenario in which all modes ϕ_k of the Minkowski frame decomposition are empty except for the two modes $\phi_{\rm I}$ and $\phi_{\rm II}$ with their corresponding annihilation operators $\hat{f}_{\rm I}$ and $\hat{f}_{\rm II}$ in an arbitrary quantum state. Out of the set of accelerated modes, we restrict our attention to two modes $\psi_{\rm I}$ and $\psi_{\rm II}$. Although $\psi_{k\neq\{\rm I,II\}}$ are not empty, we just consider two uniformly accelerating observers corresponding to $\psi_{\rm I}$ and $\psi_{\rm II}$, therefore we choose discard the other modes. Our goal is to perform such Gaussian channel to transform the state of $\phi_{\rm I}$ and $\phi_{\rm II}$ into the state of $\psi_{\rm I}$ and $\psi_{\rm II}$.

In order to do this, we introduce the quadrature operators associated with ϕ_Λ ($\Lambda \in \{\rm I, II\}$)

$$\hat{q}_\Lambda^{(f)} = \frac{\hat{f}_\Lambda + \hat{f}_\Lambda^\dagger}{\sqrt{2}}, \qquad \hat{p}_\Lambda^{(f)} = \mathrm{i}\frac{\hat{f}_\Lambda^\dagger - \hat{f}_\Lambda}{\sqrt{2}}. \tag{3}$$

It is well known that Gaussian states are completely described by first moments and second moments of their quadrature operators. Moreover, we can obtain the first moment of inertial modes associated with $\hat{q}_\Lambda^{(f)}$ and $\hat{p}_\Lambda^{(f)}$

$$\hat{X}^{(f)} = \left(\hat{q}_{\rm I}^{(f)}, \hat{p}_{\rm I}^{(f)}, \hat{q}_{\rm II}^{(f)}, \hat{p}_{\rm II}^{(f)}\right)^{\rm T}. \tag{4}$$

And the second moment $\sigma^{(f)}$ known as the covariance matrix is given by

$$X_k^{(f)} = \left\langle \hat{X}_k^{(f)} \right\rangle, \tag{5a}$$

$$\sigma_{kl}^{(f)} = \left\langle \left\{ \hat{X}_k^{(f)} - X_k^{(f)}, \hat{X}_l^{(f)} - X_l^{(f)} \right\} \right\rangle, \tag{5b}$$

where the anticommutator is defined as $\{\hat{A}, \hat{B}\} = \hat{A}\hat{B} + \hat{B}\hat{A}$. Similarly, for the accelerated modes, we can just replace the

letter f in Eqs. (4) and (5) with d to obtain its first and second moments

$$\hat{X}^{(d)} = \left(\hat{q}_{\rm I}^{(d)}, \hat{p}_{\rm I}^{(d)}, \hat{q}_{\rm II}^{(d)}, \hat{p}_{\rm II}^{(d)}\right)^{\rm T}, \tag{6a}$$

$$X_k^{(d)} = \left\langle \hat{X}_k^{(d)} \right\rangle, \tag{6b}$$

$$\sigma_{kl}^{(d)} = \left\langle \left\{ \hat{X}_k^{(d)} - X_k^{(d)}, \hat{X}_l^{(d)} - X_l^{(d)} \right\} \right\rangle. \tag{6c}$$

Then the Gaussian channel can be described fully in terms of the first and the second moments as[49]

$$X^{(d)} = MX^{(f)}, \tag{7a}$$

$$\sigma^{(d)} = M\sigma^{(f)}M^{\rm T} + N, \tag{7b}$$

where M and $N = N^{\rm T}$ are 4×4 real matrices. We note that the matrix N denotes the noise in the quantum channel, while M is symplectic,[50] whose specification uniquely characterizes the channel. It means that we only need to calculate two matrices M and N.

For convenience, we denote $\alpha_{\rm I} = (\psi_{\rm I}, \phi_{\rm I})$, $\beta_{\rm I} = -(\psi_{\rm I}, \phi_{\rm I}^{\star})$, $\alpha_{\rm II} = (\psi_{\rm II}, \phi_{\rm II})$ and $\beta_{\rm II} = -(\psi_{\rm II}, \phi_{\rm II}^{\star})$, then we can obtain the matrix M, which is

$$M = \begin{pmatrix} {\rm Re}(\alpha_{\rm I} - \beta_{\rm I}) & -{\rm Im}(\alpha_{\rm I} + \beta_{\rm I}) & 0 & 0 \\ {\rm Im}(\alpha_{\rm I} - \beta_{\rm I}) & {\rm Re}(\alpha_{\rm I} + \beta_{\rm I}) & 0 & 0 \\ 0 & 0 & {\rm Re}(\alpha_{\rm II} - \beta_{\rm II}) & -{\rm Im}(\alpha_{\rm II} + \beta_{\rm II}) \\ 0 & 0 & {\rm Im}(\alpha_{\rm II} - \beta_{\rm II}) & {\rm Re}(\alpha_{\rm II} + \beta_{\rm II}) \end{pmatrix}. \tag{8}$$

Moreover, previous research[47,51] has shown that the values of β_Λ are much less than α_Λ. Hence, we can ignore β_Λ, and the matrix M is then simplified to

$$M = \alpha_{\rm I}\mathbb{1} \oplus \alpha_{\rm II}\mathbb{1}, \tag{9}$$

where $\mathbb{1}$ is a 2×2 identity matrix. The matrix N is then given by

$$N = (1-\alpha_{\rm I}^2)\mathbb{1} \oplus (1-\alpha_{\rm II}^2)\mathbb{1}. \tag{10}$$

Then we will determine the mode functions in the next subsection, and one can see that α_Λ can be written as a function of the proper acceleration.

2.2. Gaussian wave packet in accelerated coordinates

In general, a photon can be modeled by a wave packet of excitations of a bosonic field.[52] In order to achieve the real scalar quantum field with the relativistic effect, we first focus on the photon wave packet in rest frames, and then extend it to in accelerated coordinates. Here we adopt the following form of the wave packets which are approximately localized and consist of only positive frequency in respective rest frames. This wave packet form has been widely discussed in Refs. [45–47,53]. The input mode consists of a Gaussian develop and sinusoidal modulation as

$$\begin{aligned} \phi_\Lambda(x,0) &= \pm C\exp\left[-2\left(\frac{x_0}{L}\log\frac{x}{x_0}\right)^2\right] f(x), \\ \partial_t\phi_\Lambda(x,0) &= -{\rm i}\Omega_0\phi_\Lambda(x,0), \end{aligned} \tag{11}$$

where $f(x) = \sin\left(\sqrt{\Omega_0^2 - m^2}(x-x_0)\right)$, x_0 is the central position of the mode function, L denotes its width, C is a normalization constant, and the upper (lower) sign $\pm$ refers to $\Lambda = {\rm I}$ (II). The frequency Ω_0, about which the spectrum of the mode function is centered, has to be sufficiently large to effectively damp the negative frequencies, i.e., satisfying $\Omega_0 \gg 1/L$. Additionally, we introduce a zero-frequency cutoff to safely remove the effects of negative-frequency part.

The input modes do not directly determine the output modes ψ_Λ, however, which can be obtained by replacing the trigonometric functions with modified Bessel functions and substituting the Rindler coordinates in place of inertial ones. The Rindler coordinates χ and η are given by

$$\begin{aligned} t &= \chi\sinh(a\eta), \\ x &= \chi\cosh(a\eta), \end{aligned} \tag{12}$$

where x and t are Minkowski coordinates, and a is an arbitrary positive parameter (interpreted as the proper acceleration of a trajectory). The output mode functions are given by

$$\begin{aligned} \psi_\Lambda(\chi,0) &= C'\exp\left[-2\left(\frac{x_0}{L}\log\frac{\chi}{x_0}\right)^2\right] g(\chi), \\ \partial_\tau\psi_\Lambda(\chi,0) &= \mp{\rm i}\Omega_0\psi_\Lambda(\chi,0), \end{aligned} \tag{13}$$

where C' is the normalization constant and the upper (lower) sign $\mp$ corresponds to $\Lambda = {\rm I}$ (II). The modulating function of the output mode is replaced by the combination of modified Bessel functions of the first kind $K_{{\rm i}\nu}$, with

$$g(\chi) = {\rm Im}\left[K_{-{\rm i}\frac{\Omega_0}{\mathcal{A}_\Lambda}}(m|x_0|)K_{{\rm i}\frac{\Omega_0}{\mathcal{A}_\Lambda}}(m|\chi|)\right],$$

and $|x_0| = 1/\mathcal{A}_\Lambda$. Furthermore, we need to make sure that the proper acceleration $\mathcal{A}_\Lambda$ satisfies the boundary condition $1/\mathcal{A}_\Lambda \gg L$ due to the fact that the accelerating mode functions ψ_Λ are far from the event horizon compared to their size L. In Fig. 1, we plot the shape of the localized wave packets, with fixed values $L = 2$, $\Omega_0 \approx 5$, $\mathcal{A}_{\rm I} = \mathcal{A}_{\rm II} = 0.1$ and the field mass $m = 0.1$ (just to show the properties of the wave packet).

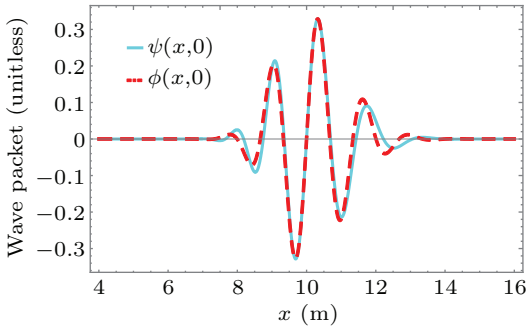


Fig. 1. The shape of input and output modes. We fixed $\mathcal{A}_{\rm I}=\mathcal{A}_{\rm II}=0.1$, $L=2$, $\Omega_0\approx 5$, and the field mass $m=0.1$.

In fact, all the units of these parameters (e.g., acceleration $\mathcal{A}$, length L and central frequency Ω_0) can not be chosen until the geometry of the cavity is completely determined. One of the reference is the imaginary cavity in Ref. [56], the length L and the travel time W were set in order of tens cm and 10^{-7} s, respectively, and the accelerations were accordingly calculated as in order of 10^{33} ms^{-2}. The mode transforms from Minkowski to Rindle spacetime through Bogoliubov coefficient (see Appendix A for more detailed calculations). From the Bogoliubov coefficient $\alpha_\Lambda=(\psi_\Lambda,\phi_\Lambda)$, one can clearly see that α_Λ is inversely proportional to $\mathcal{A}_\Lambda$ with the mode functions from Eqs. (11) and (13). As shown in Fig. 2, with the increase of $\mathcal{A}_\Lambda$, α_Λ decreases monotonically, which also means that the noise in the channel increases.

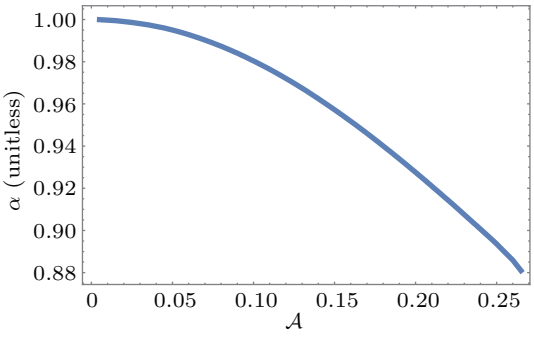


Fig. 2. The relationship between α_Λ and $\mathcal{A}_\Lambda$, with $L=2$, $\Omega_0\approx 5$ and the field mass $m=0.1$.

3. The measurement of Gaussian quantum steering

In this section we briefly review the measurement of quantum steering for a bipartite $(m+n)$ mode Gaussian state. The general Gaussian $(n+m)$-mode state $\rho^{(AB)}$ of a bipartite system comprises of a subsystem A of n modes and a subsystem B of m modes. For each mode i in subsystem A or B, one can define the phase-space operators $\hat{x}_i^{A(B)}$, $\hat{p}_i^{A(B)}$, and then labeled for convenience into the vector $\hat{R}=(\hat{x}_1^A,\hat{p}_1^A,\ldots,\hat{x}_n^A,\hat{p}_n^A,\hat{x}_1^B,\hat{p}_1^B,\ldots,\hat{x}_m^B,\hat{p}_m^B)^{\rm T}$. The vector $\hat{R}$ satisfies the canonical commutation relations $[\hat{R}_i,\hat{R}_j]=i\Omega_{ij}$, with $\Omega=\bigoplus_1^{n+m}\left(\begin{smallmatrix}0&1\\-1&0\end{smallmatrix}\right)$ being the symplectic form. Any Gaussian state ρ_{AB} is completely specified by its covariance matrix (CM)

$$\sigma_{AB}=\begin{pmatrix}A&C\\C^{\rm T}&B\end{pmatrix},\tag{14}$$

where the elements $\sigma_{ij}={\rm Tr}\left[\{\hat{R}_i,\hat{R}_j\}_+\rho_{AB}\right]$. Here the submatrices A and B are the CMs correspoding to the reduced states of A and B subsystems, respectively.

Then quantify how much a bipartite Gaussian state with CM σ_{AB} is steerable (by Gaussian measurements on Alice's side). Based on the invariance under local unitary and Gaussian measurement, Kogias and Adesso *et al.* have proposed a computable measurement of steering for arbitrary bipartite Gaussian states of continuous variable systems[55]

$$\mathcal{G}^{A\to B}(\sigma_{AB}):=\max\left\{0,-\sum_{j:\bar{\nu}_j^B<1}\ln(\bar{\nu}_j^B)\right\},\tag{15}$$

where $\bar{\nu}_j^B$ are the symplectic eigenvalues of the Schur complement of A in the CM σ_{AB}. By defining the Schur complement $\det\sigma_{AB}=\det A\det M_\sigma^B$, where $M_\sigma^B=B-C^{\rm T}A^{-1}C$ is the Schur complement of A in the CM σ_{AB}, and employing the definition of the Rényi-2 entropy, the Gaussian steerability of $A\to B$ can be written as

$$\begin{aligned}\mathcal{G}^{A\to B}(\sigma_{AB})&=\max\left\{0,\frac{1}{2}\ln\frac{\det A}{\det\sigma_{AB}}\right\}\\&=\max\left\{0,\mathcal{S}(A)-\mathcal{S}(\sigma_{AB})\right\},\end{aligned}\tag{16}$$

where the Rényi-2 entropy $\mathcal{S}$ reads $\mathcal{S}(\sigma)=\frac{1}{2}\ln(\det\sigma)$[54] for a Gaussian state with CM σ. However, different from the definition of quantum entanglement and quantum coherence, quantum steering has asymmetric property.[55] One can swap the roles of A and B to obtain the measurement of Gaussian steering $B\to A$.

4. The influence of accelerated effect on localized Gaussian states

We begin with a two-mode squeezed vacuum state characterized only by the squeezing parameter r as input state and the covariance matrix

$$\sigma^{(f)}=\begin{pmatrix}\cosh 2r&0&\sinh 2r&0\\0&\cosh 2r&0&-\sinh 2r\\\sinh 2r&0&\cosh 2r&0\\0&-\sinh 2r&0&\cosh 2r\end{pmatrix}.\tag{17}$$

Applying Eq. (17) to Eq. (7(b)), we can get the covariance matrix $\sigma^{(d)}$ as seen by two accelerating observers, which is

$$\sigma^{(d)}=\begin{pmatrix}X&0&Y&0\\0&X&0&-Y\\Y&0&Z&0\\0&-Y&0&Z\end{pmatrix},\tag{18}$$

where

$$\begin{aligned}X&=\alpha_{\rm I}^2\cosh(2r)-\alpha_{\rm I}^2+1,\\Y&=\alpha_{\rm I}\alpha_{\rm II}\sinh(2r),\end{aligned}$$

$$Z = \alpha_{\rm II}^2 \cosh(2r) - \alpha_{\rm II}^2 + 1. \tag{19}$$

One can easy see that the matrix has suffered from the noise channel, and the noise channel is characterized by $\alpha_{\rm I}$ and $\alpha_{\rm II}$ (relativistic effect). Combining Eqs. (16) and (18), we obtain an specific mathematic expression of Gaussian quantum steering from $A \to B$ in accelerated coordinate system which is given by

$$\mathcal{G}^{A\to B} = \max\left\{0, \ln \frac{1+2\alpha_{\rm I}^2 \sinh(r)^2}{1+2(\alpha_{\rm II}^2+\alpha_{\rm I}^2(1-2\alpha_{\rm II}^2))\sinh(r)^2}\right\}. \tag{20}$$

Unlike quantum entanglement, one of the most distinguishable properties of quantum steering is its asymmetry, which has been recently experimentally demonstrated in flat spacetime.[17,18] To understand this properties in the accelerated coordinate system, we also calculate the Gaussian quantum steering from $B \to A$, which is

$$\mathcal{G}^{B\to A} = \max\left\{0, \ln \frac{1+2\alpha_{\rm II}^2 \sinh(r)^2}{1+2(\alpha_{\rm II}^2+\alpha_{\rm I}^2(1-2\alpha_{\rm II}^2))\sinh(r)^2}\right\}. \tag{21}$$

When the relativistic effects vanish, the accelerated parameters $\alpha_{\rm I}$ and $\alpha_{\rm II}$ of Alice and Bob equal to one, and our results will return the non-relativistic results, i.e., the Gaussian steering of $\mathcal{G}_{\rm non}^{A\to B} = \mathcal{G}_{\rm non}^{B\to A} = \max\{0, \ln\cosh(2r)\}$, which improve the robustness of our results.

Here we consider two different cases.

Symmetric case In this case, the proper accelerations of two accelerating observers have the same magnitude, i.e., $\alpha_{\rm I} = \alpha_{\rm II}$. It is clear that the output state $\sigma^{(d)}$ is a symmetric state, also a state with balanced correlations.

Asymmetric case The two accelerating observers have unequal-magnitude proper acceleration, i.e., $\alpha_{\rm I} \neq \alpha_{\rm II}$. Although the output state $\sigma^{(d)}$ is no longer symmetric, it is still a balanced-correlated state.

For convenience, we use $\mathcal{A}_\Lambda$ to replace α_Λ. $\alpha_\Lambda = 1$ and $\mathcal{A}_\Lambda = 0$ mean that the proper acceleration equals to zero, where $\mathcal{A}_\Lambda$ is proportional to acceleration (please refer to Fig. 2 for more details).

We first focus on the symmetric case. In this case, the Gaussian quantum steering from $A \to B$ and $B \to A$ is equivalence, which means that $\mathcal{G}^{A\to B} = \mathcal{G}^{B\to A}$ with the symmetrical property. The numerically computed Gaussian steering with increased acceleration parameters $\mathcal{A}_{\rm I}$ and $\mathcal{A}_{\rm II}$ in this case are plotted in Fig. 3. We can see that $\mathcal{G}^{A\to B}$ and $\mathcal{G}^{B\to A}$ both monotonically decreases with the increase of accelerated parameters $\mathcal{A}_{\rm I} = \mathcal{A}_{\rm II}$ of observes Alice and Bob. This findings demonstrate that the Unruh effect will affect the Gaussian quantum steering and decrease it, which is consist with previous works.[27,28,44] Our results also indicate that, when performing more reliable quantum steering tasks, the relativistic effects cannot be ignored.

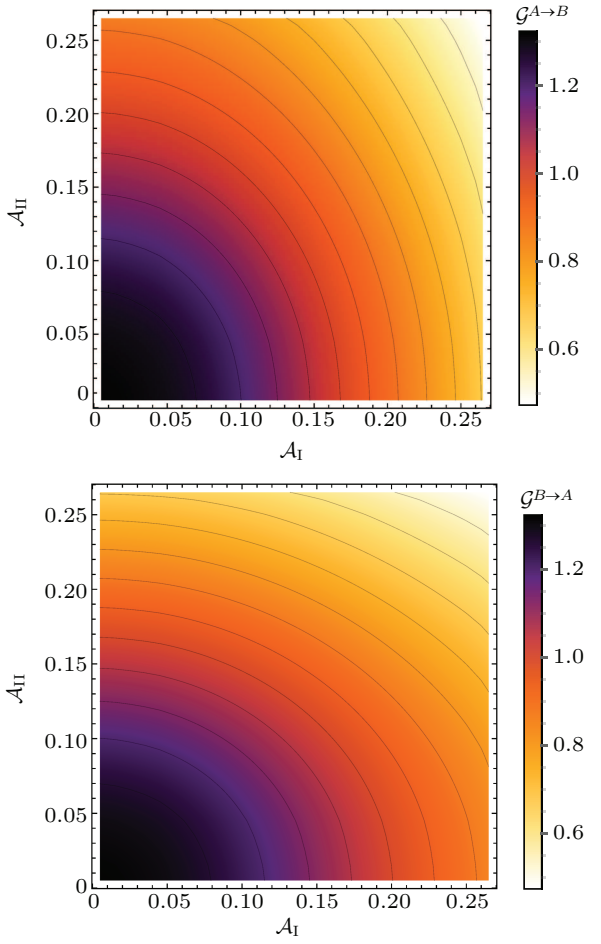


Fig. 3. The Gaussian quantum steering $\mathcal{G}^{A\to B}$ and $\mathcal{G}^{B\to A}$ between observers Alice and Bob (corresponding to wave packets $\psi_{\rm I}$ and $\psi_{\rm II}$) moving with equal-magnitude accelerations $\mathcal{A}_{\rm I} = \mathcal{A}_{\rm II} = \mathcal{A}$ are shown. The original state is a two-mode squeezed vacuum state characterized by a squeezing parameter $r = 1$ of the inertial wave packets $\phi_{\rm I}$ and $\phi_{\rm II}$.

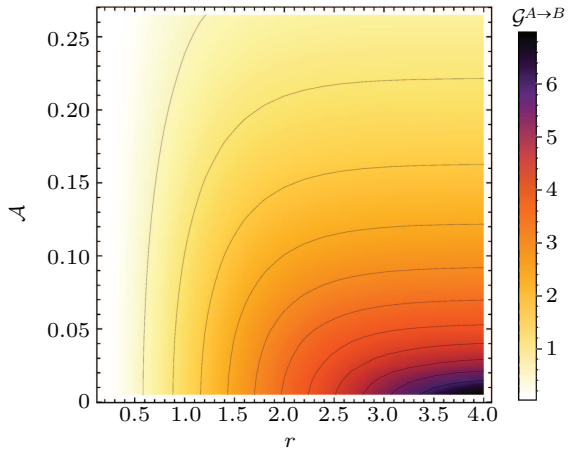


Fig. 4. The contour plot $\mathcal{G}^{A\to B}$ as a function of accelerated parameter $\mathcal{A}_{\rm I}$ ($\mathcal{A}_{\rm I} = \mathcal{A}_{\rm II}$) and squeezing parameter r.

In Fig. 4, we plot $\mathcal{G}^{A\to B}$ as a function of accelerated parameter $\mathcal{A}_{\rm I}$ and squeezing parameter r. We can observe a monotonic increase of $\mathcal{G}^{A\to B}$ with the increasing of the squeezing parameter r. It should be noted that, comparing with the accelerated parameter, the Gaussian steering $\mathcal{G}^{A\to B}$ changes more for different squeezing parameters, which indicates that

the initial quantum resource plays a more important role in the quantum steering. In other words, comparing with the squeezing parameter, the Gaussian steering is not easy to change with changing accelerated parameter. This gives us the guidance to choose appropriate physical parameters to perform more reliable quantum steering tasks with the inevitable relativistic effects.

Subsequently, we turn our attention to the asymmetric case. We plot the Gaussian quantum steering $\mathcal{G}^{A\to B}$ and $\mathcal{G}^{B\to A}$ as the functions of the Alice's acceleration and Bob's acceleration for fixed squeezing parameter $r=3$ in Fig 5. From this figure, we can see that, both the $\mathcal{G}^{A\to B}$ and $\mathcal{G}^{B\to A}$ will decrease with the increase of accelerated parameters. In additional, the Gaussian quantum steering degradation is inevitable since the mode mismatch between input and output modes cannot be removed by any amendments to the protocol.[46,47] Moreover, when $\mathcal{A}_{\rm I}=\mathcal{A}_{\rm II}$, it corresponds to the symmetric case. When $\mathcal{A}_{\rm I}\neq\mathcal{A}_{\rm II}$, it corresponds to the asymmetric case, which indicates that the asymmetric properties of Gaussian quantum steering will arise. To check the asymmetric degree of steerability by using local mode with relativistic effects, we calculate the Gaussian steering asymmetry and plot it as a function of accelerated parameter $\mathcal{A}_{\rm I}$ for different squeezing parameter r in Fig. 6. This allows us to have a better understanding of how the relativistic motions and the squeezing parameter of initial quantum state affect steering asymmetry. This figure clearly shows that the Gaussian steering asymmetry increase for a specific range of accelerated parameter, and then gradually approach to a finite value with increasing $\mathcal{A}_{\rm I}$. This is because that the Unruh effect almost completely destroys the one-side Gaussian quantum steering $\mathcal{G}^{A\to B}$ which means that one can no longer measure Alice to manipulate Bob. More importantly, we can see that the different squeezing parameters will change the inflection point, i.e., the smaller the squeezing parameter, the earlier the inflection point. This is reasonable, since the squeezing parameters of the initial state determine the quantization of the quantum resource.

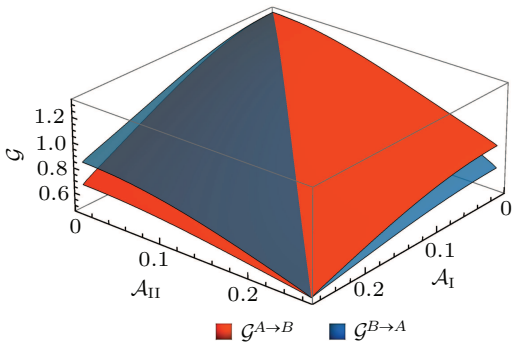


Fig. 5. The Gaussian quantum steering $\mathcal{G}^{A\to B}$ (red) and $\mathcal{G}^{B\to A}$ (gray) as the functions of the Alice's acceleration and Bob's acceleration for fixed squeezing parameter $r=3$.

Due to the Gaussian quantum steering asymmetry effect is arising, it is natural to wonder if this situation is caused by a mismatch of modules. In order to visualize Gaussian quantum steering asymmetry effect, we randomly generated 2000 sets of $\sigma^{(d)}$ with different values of $\mathcal{A}_{\rm I}$, $\mathcal{A}_{\rm II}\in[0.005,0.25]$ and $r\in[1,3]$. We now simply define the asymmetry as $P_{\rm asy}:=\log_2(\mathcal{A}_{\rm I}-\mathcal{A}_{\rm II})$. As shown in Fig. 7, we numerically computed the differential Gaussian quantum steering $\mathcal{G}^{B\to A}-\mathcal{G}^{A\to B}$ and marked it in cold/warm colors according to their values, and the specific color-value correlation can be found in colorbar. For convenience, we have plotted the dashed vertical line $P_{\rm asy}=0$, and the closer the dots are to the dashed vertical line, the lower the asymmetry. All the cool colored dots, i.e., the states with $\mathcal{G}^{B\to A}>\mathcal{G}^{A\to B}$, form an interesting chimney shape, while the other warm dots are concentrated in the upper left and upper right regions. It is also shown that no matter how much r increases, as long as the asymmetry stays within a certain range(about $[-0.2,0.2]$ here). As the final remark, we can conclude that the arising of Gaussian quantum steering asymmetry effect is not caused by a mismatch of modules.

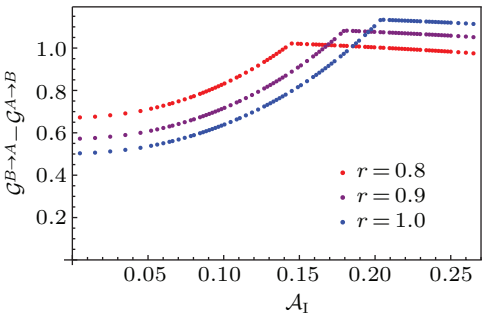


Fig. 6. The Gaussian quantum steering asymmetric $\mathcal{G}^{A\to B}-\mathcal{G}^{B\to A}$ as a function of the Alice's acceleration for different squeezing parameter, $r=0.8$ (red dashed line), $r=0.9$ (black dashed line) and $r=1$ (blue dashed line), respectively. The Bob's acceleration is fixed as $\mathcal{A}_{\rm II}=0$.

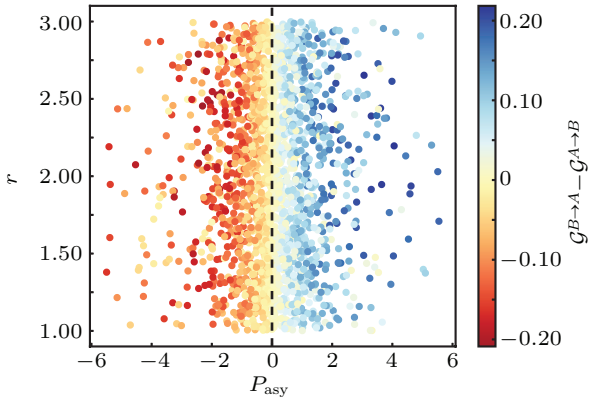


Fig. 7. The differential differential Gaussian quantum steering $\mathcal{G}^{B\to A}-\mathcal{G}^{A\to B}$ versus the corresponding value of squeezing parameter r and asymmetry $P_{\rm asy}$, which defined as $P_{\rm asy}:=\log_2(\mathcal{A}_{\rm I}-\mathcal{A}_{\rm II})$, for randomly generated states. The closer the dots are to the dashed vertical line, the higher the symmetry.

5. Conclusions

In this paper, we have studied Gaussian steering by using local mode instead of traditional single-mode approximation method in relativistic frame. We firstly considered two

uniformly accelerating observers, Alice and Bob, which transform a two-mode squeezed vacuum state prepared in an inertial frame from the initial frame to the frame of the observers, and this behavior can be represented as a Gaussian quantum channel. We obtained a simplified form of the channel and can use it to easily compute the output state.

Within the framework introduced there, we analyzed the Gaussian quantum steering and its asymmetry that can be extracted from the input two-mode squeezed vacuum state. We have shown that one-side Gaussian quantum steering monotonically decreases with increasing observers of acceleration. The findings demonstrate that the Unruh effect will affect the Gaussian quantum steering. In addition, the Gaussian quantum steering degradation is inevitable since the mode mismatch between the input and output modes cannot be removed by any amendments to the protocol. Our results also indicate that, when performing more reliable quantum steering tasks, the relativistic effects cannot be ignored. Comparing with the squeezing parameter, the Gaussian steering is not easy to change with changing accelerated parameter. This gives us the guidance to choose appropriate physical parameters to perform more reliable quantum steering tasks with the inevitable relativistic effects. Finally, we have shown that the Gaussian steering asymmetry increase for a specific range of accelerated parameter, and then gradually approach to a finite value with increasing observer's acceleration. This is because the Unruh effect almost completely destroys the one-side Gaussian quantum steering which means that one can no longer measure Alice to manipulate Bob.

As the final remark, although the influence of Unruh effect on quantum steering cannot be detected by current technology, it on quantum steering or other quantum resources cannot be ignored, especially when performing relativistic quantum information tasks.

In addition, according to the equivalence principle, the effects of acceleration are equivalence with the effects of gravity, our results could be in principle applied to dynamics of Gaussian steering between the Earth and satellites. Since realistic quantum systems will always exhibit gravitational and relativistic features, our results should be significant both for giving pieces of advices to realize quantum information protocols such as quantum key distribution from the Earth to satellites and for a general understanding of quantum steering in relativistic quantum systems.

Appendix: Bogoliubov coefficients calculation

In the Minkowski coordinates, the Klein–Gordan equation could be solved in the form of the normalized plane waves

$$u_k = \frac{1}{\sqrt{4\pi\omega_k}} \mathrm{e}^{\mathrm{i}(kx-\omega_k t)}, \tag{A1}$$

where $\omega_k = \sqrt{k^2+m^2}$, and all the normalized solutions satisfy the orthonormal relations, i.e., $(u_k, u_l) = \delta(k-l)$,$(u_k^\star, u_l^\star) = -\delta(k-l)$ and $(u_k, u_l^\star) = 0$. In the Rindler coordinates, with the positive frequency limitation, the solutions with respect to the time-like Killing vector fields were given by[57]

$$\begin{aligned} w_{\mathrm{I}\Omega} &= \sqrt{\frac{\sinh\left(\frac{\pi\Omega}{a}\right)}{\pi^2 a}} K_{\mathrm{i}\frac{\Omega}{a}}(m\chi)\,\mathrm{e}^{-\mathrm{i}\Omega\eta} \\ &\quad \text{in Rindler I region}, \\ w_{\mathrm{II}\Omega} &= \sqrt{\frac{\sinh\left(\frac{\pi\Omega}{a}\right)}{\pi^2 a}} K_{\mathrm{i}\frac{\Omega}{a}}(-m\chi)\,\mathrm{e}^{\mathrm{i}\Omega\eta} \\ &\quad \text{in Rindler II region}, \end{aligned} \tag{A2}$$

where $K_{\mathrm{i}\nu}(x)$ is the modified Bessel function of the second kind of pure imaginary order. The orthonormal relations here were given by $(w_{\Lambda\Omega}, w_{\Lambda\Xi}) = \delta(\Omega-\Xi)$, $(w_{\Lambda\Omega}^\star, w_{\Lambda\Xi}^\star) = -\delta(\Omega-\Xi)$, and $(w_{\Lambda\Omega}, w_{\Lambda\Xi}^\star) = 0$.

The Bogoliubov coefficients α_Λ $(\Lambda \in \mathrm{I}, \mathrm{II})$ were defined as

$$\alpha_\Lambda = (\psi_\Lambda, \phi_\Lambda), \tag{A3}$$

where $\psi_\Lambda/\phi_\Lambda$ denoted the inertia/accelerated modes, respectively. We can rewrite Eq. (A3) as

$$\begin{aligned} \alpha_\Lambda &= (\psi_\Lambda, \phi_\Lambda) \\ &= \int_0^\infty \mathrm{d}\Omega \int_0^\infty \mathrm{d}k (\psi_\Lambda, w_{\Lambda\Omega})(w_{\Lambda\Omega}, u_k)(u_k, \phi_\Lambda). \end{aligned} \tag{A4}$$

By using Eqs. (A1), (A2) and some conversions ($|x_0| = \frac{1}{\mathcal{A}_\Lambda}$, $\omega_k = \sqrt{k^2+m^2}$, $(\phi_1, \phi_2) = \mathrm{i}\int_\Sigma \mathrm{d}x(\phi_1^\star \partial_t \phi_2 - \phi_2 \partial_t \phi_1^\star)$ and orthonormal relations) mentioned in the manuscript, each term of Eq. (A3) in the integration is obtained

$$\begin{aligned} (\psi_\Lambda, w_{\Lambda\Omega}) &= \frac{1}{a\chi}\left(\frac{a\Omega_0}{A} + \Omega\right) \\ &\quad \times \sqrt{\frac{\sinh\left(\frac{\pi\Omega}{a}\right)}{\pi^2 a}} K_{\frac{\mathrm{i}\Omega}{a}}(m\chi)\psi_\Lambda(\chi, 0), \end{aligned} \tag{A5}$$

$$(w_{\Lambda\Omega}, u_k) = \frac{\exp\left(\frac{\pi\Omega}{2a}\right)\exp\left(-\frac{\pi\Omega}{a}\right)\left(\frac{\sqrt{k^2+m^2}+k}{\sqrt{k^2+m^2}-k}\right)^{\frac{\mathrm{i}\Omega}{2a}}}{\sqrt{4\pi a\sqrt{k^2+m^2}\sinh\left(\frac{\pi\Omega}{a}\right)}}, \tag{A6}$$

$$(u_k, \phi_\Lambda) = \frac{\exp(-\mathrm{i}kx)\left(\sqrt{k^2+m^2}+\Omega_0\right)}{\sqrt{4\pi\sqrt{k^2+m^2}}}\phi_\Lambda(x, 0). \tag{A7}$$

Finally, we need to substitute the chosen modes (Eqs. (11) and (13) in the text) into Eq. (A5), and the relationship between $\mathcal{A}_\Lambda$ and α_Λ can be obtained by numerical integration.

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