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Nonreciprocal two-photon transmission and statistics in a chiral waveguide QED system

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We study the nonreciprocal properties of transmitted photons in a chiral waveguide quantum electrodynamics (QED) system, including single- and two-photon transmissions and second-order correlations. For the single-photon transmission, the nonreciprocity is induced by the effects of chiral coupling and atomic dissipation in the weak coupling region. It vanishes in the strong coupling regime when the effect of atomic dissipation becomes ignorable. In the case of two-photon transmission, there exist two ways of going through the emitter: independently as plane waves and formation of bound state. Besides the nonreciprocal behavior of plane waves, the bound state that differs in two directions also alters transmission probabilities. In addition, the second-order correlation of transmitted photons depends on the interference between plane wave and bound state. The destructive interference leads to the strong antibunching in the weak coupling region, while the effective formation of bound state leads to the strong bunching in the intermediate coupling region. However, the negligible interactions for left-propagating photons hardly change the statistics of the input coherent state.

Keywords: chiral waveguide QED, nonreciprocal transmissions, nonreciprocal correlations

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1. Introduction

Recently, intensive attention has been paid to exploration of chiral interfaces to tailor atom–light interactions for important applications in quantum devices and quantum information techniques.[1–4] The novel effect arises from the peculiar polarization exhibited by strongly and transversely confined optical fields in nano-photonic structures such as nanofibers and photonic crystal waveguides (PCWs).[5–8] The transverse confinement naturally leads to dependence of polarization properties of light on the propagation direction, which is a manifestation of optical spin–orbit coupling.[9] As a result, the interaction between light and a quantum emitter (QE) depends on propagation direction of light and polarization of the transition dipole moment of the QE, in contrast to the identical interactions between the QE and photons of the either two propagation directions in regular waveguides. In practice, chiral interface facilitates the realization of optical nonreciprocity.

Optical nonreciprocity means that the properties of transmitted fields become asymmetry when exchanging the positions of source and detector.[10] Various nonreciprocal phenomena have been intensively investigated, including nonreciprocal transmission and amplification,[11–16] nonreciprocal (unconventional) photon blockade,[17–20] nonreciprocal signal routing,[21–23] and nonreciprocal quantum entanglement,[24–26] etc. The strategies for realization of nonreciprocity cover magneto-optical materials in conjunction with a magnetic field,[27,28] time modulation of the optical properties,[29] optical nonlinearity,[13,30,31] and synthetic magnetism,[32] etc. In addition, nonreciprocity based on a new paradigm referred to as "chiral quantum optics" has been investigated, where the nonreciprocal behavior is controllable by the spin state of the QE.[33–36] Furthermore, the scalable quantum techniques involving miniaturization and fabrication problems in nanophotonic waveguide systems have been assessed, in which chiral coupling is considered to be well-characterized and robust, and has potential implementation in a range of applications.[37–39]

Nonreciprocal phenomena in chiral systems are mainly focused on the single-photon level, and the properties at two- or multi-photon levels are less studied. The two-photon transport has been addressed in regular waveguide systems using the several interrelated theoretical techniques such as multi-particle scattering theory,[40–44] input-output formalism,[45–48] approaches based on Lippmann–Schwinger equations[49] and Lehmann–Symanzik–Zimmermann reduction.[50] Scattering theory is a common framework to study scattering of waves and particles in real space within the Schröngdinger picture, and applied in different branches of physics. The input-output formalism was mainly developed for understanding light-matter interaction in momentum space based on the Heisenberg picture,[51] and its extension to scattering of multiple photons by multiple interacting and noninteracting emitters in a 1D continuum has been investigated.[48,52] In theory, the bidirectional case in no matter the regular or chiral waveguides can be treated through generalizing the case of propagation in a single direction.[52] Therefore, the approach taken in regular waveguide systems can be extended to study the two- or multi-photon chiral waveguide systems.

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Here we follow the approach of scattering wave function in a real-space formalism to construct scattering matrixes via imposing open boundary conditions and the incoming plane wave functions, and to study the nonreciprocal two-photon transmission and statistics in a chiral waveguide system. Via introducing the bright and dark optical modes, the bright mode effectively couples to the QE while the dark mode is decoupled, and the system reduces to the case of an effective single direction of propagation. To investigate the two-photon transmission, it is necessary to analyze single-photon transmission firstly, where the nonreciprocity is induced by the joint effects of chiral coupling and atomic dissipation in the weak coupling regime. In the strong coupling regime, the effect of atomic dissipation becomes ignorable, and the nonreciprocity disappears. For the two-photon case, there are two ways for photons going through the QE. One is to pass by the QE independently as plane waves and gain a phase factor, and the other is to bind together to form a bound state. Plane-wave term is similar to the single-photon transmission. Due to the different interactions in two directions, left-propagating photons hardly form the bound state in which transmission is mainly determined by the plane-wave term, while bound state formed by right-propagating photons further alters the transmission probability. In addition, the second-order correlations of transmitted photons in the two directions are discussed. For the right-propagating photons, the destructive interference between plane wave and bound state leads to the significant antibunching in the weak coupling regime, and the effective formation of bound state leads to the strongest bunching at the intermediate coupling. For the left-propagating photons, the negligible interaction hardly changes the statistics of the input coherent state.

The paper is organized as follows. In Section 2, the chiral waveguide QED system is introduced, and the nonreciprocal transmissions of single- and two-photon Fock states are analyzed in Sections 3 and 4. In Section 5, the nonreciprocal second-order correlations of transmitted photons are discussed, and lastly the conclusion is drawn.

2. Chiral coupling between the light and QE

We consider a typical chiral quantum system consisting of a two-level QE coupled to light field that is transversally confined at the subwavelength scale in an optical fiber as shown in Fig. 1, which has been experimentally realized in different setups.[35,53,54] For an ideal unidirectional case, the polarizations at the position of QE are circular and orthogonal for opposite propagation directions, and a circularly polarized QE is matched to the local polarization of one propagation direction. Here we consider a more practical case where the QE couples to the field in both propagation directions but with different strengths. Explicitly, the system is described by the Hamiltonian ($\hbar = 1$)

$$\begin{aligned}\hat{H} =& \left(\omega_{\rm e} - {\rm i}\frac{\gamma}{2}\right)|e\rangle\langle e| + \int {\rm d}x\left[\hat{a}_{\rm R}^{\dagger}(x)\left(\omega_{\rm a} - {\rm i}\upsilon_{\rm g}\partial_x\right)\hat{a}_{\rm R}(x)\right.\\ &\left.+\hat{a}_{\rm L}^{\dagger}(x)\left(\omega_{\rm a} + {\rm i}\upsilon_{\rm g}\partial_x\right)\hat{a}_{\rm L}(x)\right]\\ &+g_{\rm R}\int {\rm d}x\delta(x)\left[\hat{a}_{\rm R}^{\dagger}(x)\hat{\sigma}_- + \hat{\sigma}_+\hat{a}_{\rm R}(x)\right]\\ &+g_{\rm L}\int {\rm d}x\delta(x)\left[\hat{a}_{\rm L}^{\dagger}(x)\hat{\sigma}_- + \hat{\sigma}_+\hat{a}_{\rm L}(x)\right],\end{aligned} \tag{1}$$

where $\hat{a}_{\rm R}^{\dagger}(x)$ and $\hat{a}_{\rm L}^{\dagger}(x)$ are the creation operators of right- and left-propagating photons in the real space, $\omega_{\rm e}$ is the atomic transition frequency between the ground state $|g\rangle$ and excited state $|e\rangle$ of the QE, γ is the atomic decay rate into channels other than the 1D continuum, and $\hat{\sigma}_- = |g\rangle\langle e|$ is the atomic lowering operator. Here $\omega_{\rm a}$ can be an arbitrary frequency that is away from the cutoff frequency of the photonic waveguide, and $\upsilon_{\rm g}$ is the group velocity.[40] For the strongly transversely confined waveguide system, we consider the chiral light-QE interaction where the coupling strengths between the QE and right- and left-propagating photons are $g_{\rm R}$ and $g_{\rm L}$, respectively. Then, we introduce a free Hamiltonian

$$\hat{H}_0 = \omega_{\rm a}|e\rangle\langle e| + \omega_{\rm a}\int {\rm d}x\left[\hat{a}_{\rm R}^{\dagger}(x)\hat{a}_{\rm R}(x) + \hat{a}_{\rm L}^{\dagger}(x)\hat{a}_{\rm L}(x)\right] \tag{2}$$

and a unitary operator $\hat{U}(t) = \exp(-{\rm i}\hat{H}_0 t)$, and the Hamiltonian is transformed into the interaction picture following the relation

$$\hat{H}_I = \hat{U}^{\dagger}(t)\hat{H}\hat{U}(t) + {\rm i}\left[\partial_t\hat{U}^{\dagger}(t)\right]\hat{U}(t), \tag{3}$$

which becomes

$$\begin{aligned}\hat{H}_{\rm I} =& \left(\varepsilon - {\rm i}\frac{\gamma}{2}\right)|e\rangle\langle e| - {\rm i}\upsilon_{\rm g}\\ &\times\int {\rm d}x\left[\hat{a}_{\rm R}^{\dagger}(x)\partial_x\hat{a}_{\rm R}(x) - \hat{a}_{\rm L}^{\dagger}(x)\partial_x\hat{a}_{\rm L}(x)\right]\\ &+g_{\rm R}\int {\rm d}x\delta(x)\left[\hat{a}_{\rm R}^{\dagger}(x)\hat{\sigma}_- + \hat{\sigma}_+\hat{a}_{\rm R}(x)\right]\\ &+g_{\rm L}\int {\rm d}x\delta(x)\left[\hat{a}_{\rm L}(x)\hat{\sigma}_- + \hat{\sigma}_+\hat{a}_{\rm L}(x)\right]\end{aligned} \tag{4}$$

with the detuning $\varepsilon = \omega_{\rm e} - \omega_{\rm a}$.

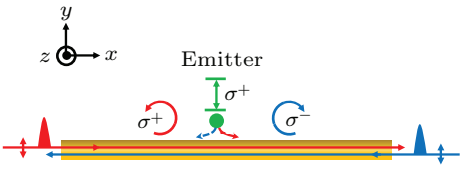


Fig. 1. Schematic of chiral photons in evanescent fields coupled to a σ^+-polarized emitter. Polarization properties of the evanescent light field that surrounds an optical nanofiber are: a y-polarized light field that propagates in the right ($+x$) direction is mainly σ^+ polarized in the ($z = 0$) plane. If it propagates in the left ($-x$) direction, it is mainly σ^- polarized.

Now it is necessary to introduce the bright and dark optical modes

$$\hat{a}_{\rm B}^{\dagger}(x) = \frac{1}{g_{\rm B}}\left[g_{\rm R}\hat{a}_{\rm R}^{\dagger}(x) + g_{\rm L}\hat{a}_{\rm L}^{\dagger}(-x)\right],$$

$$\hat{a}_{\rm D}^{\dagger}(x) = \frac{1}{g_{\rm B}}\left[g_{\rm L}\hat{a}_{\rm R}^{\dagger}(x) - g_{\rm R}\hat{a}_{\rm L}^{\dagger}(-x)\right], \tag{5}$$

with the effective light-QE coupling strength $g_{\rm B} = \sqrt{g_{\rm R}^2 + g_{\rm L}^2}$. The Hamiltonian is then split into two decoupled parts: $\hat{H}_{\rm I} = \hat{H}_{\rm B} + \hat{H}_{\rm D}$, with

$$\begin{aligned}\hat{H}_{\rm B} &= \left(\varepsilon - {\rm i}\frac{\gamma}{2}\right)|e\rangle\langle e| - {\rm i}\upsilon_{\rm g}\int {\rm d}x\hat{a}_{\rm B}^{\dagger}(x)\partial_x\hat{a}_{\rm B}(x) \\ &\quad + g_{\rm B}\int {\rm d}x\delta(x)\left[\hat{a}_{\rm B}^{\dagger}(x)\hat{\sigma}_- + \hat{\sigma}_+\hat{a}_{\rm B}(x)\right], \\ \hat{H}_{\rm D} &= -{\rm i}\upsilon_{\rm g}\int {\rm d}x\hat{a}_{\rm D}^{\dagger}(x)\partial_x\hat{a}_{\rm D}(x),\end{aligned} \tag{6}$$

where the bright mode effectively couples to QE with strength $g_{\rm B}$, while the dark mode is free from the interaction. Thus, the system becomes an effective light-QE interaction in one direction of propagation. For simplicity, we will take $\upsilon_{\rm g}$ as 1 hereafter.

3. Nonreciprocal transmissions of single-photon Fock state

Before the discussion of two-photon transport, we first consider the single-photon scattering process, which is necessary to construct the two-photon scattering matrix later. The stationary one-excitation state in the bright-state subspace is given by

$$|\psi_1\rangle = \int {\rm d}x f_1(x)\hat{a}_{\rm B}^{\dagger}(x)|0,g\rangle + e_1|0,e\rangle, \tag{7}$$

where $|0,g\rangle$ is the vacuum state with the QE in its ground state, and $|0,e\rangle$ is the vacuum state with the QE in its excited state. From the eigenvalue equation $\hat{H}_{\rm B}|\psi_1\rangle = E_1|\psi_1\rangle$, we can obtain

$$\begin{aligned}\left(\varepsilon - {\rm i}\frac{\gamma}{2}\right)e_1 + g_{\rm B}f_1(0) &= E_1 e_1, \\ -{\rm i}\partial_x f_1(x) + g_{\rm B}\delta(x)e_1 &= E_1 f_1(x),\end{aligned} \tag{8}$$

where $f_1(0) \equiv 1/2\,[f_1(0^+) + f_1(0^-)]$, and the eigenvalue $E_1 = k_1$. It should be noted that the function $f_1(x)$ is piecewise continuous because the derivative of $f(x)$ is the ordinary derivative plus contributions from the jump discontinuities.[43] Via eliminating e_1 and at $x = 0$ where the interaction occurs, we have the following boundary condition:

$$\begin{aligned}f_1(0^+) &= \bar{t}_{k_1} f_1(0^-), \\ \bar{t}_{k_1} &= \frac{k_1 - \varepsilon + {\rm i}\frac{\gamma}{2} - {\rm i}\frac{\Gamma_{\rm B}}{2}}{k_1 - \varepsilon + {\rm i}\frac{\gamma}{2} + {\rm i}\frac{\Gamma_{\rm B}}{2}},\end{aligned} \tag{9}$$

where $\Gamma_{\rm B} = g_{\rm B}^2$ indicates the spontaneous emission rate from the QE to the 1D continuum, and $\bar{t}_{k_1}$ is the transmission amplitude for the bright mode. For $x < 0$, the incident state is a plane wave which is

$$|k_1\rangle_{\rm B} = \int {\rm d}x h_{k_1}(x)\hat{a}_{\rm B}^{\dagger}(x)|0,g\rangle, \tag{10}$$

with $h_{k_1}(x) = \langle x|k_1\rangle = \frac{1}{\sqrt{2\pi}}{\rm e}^{{\rm i}k_1 x}$. Thus, $f_1(0^-) = h_{k_1}(0) = 1/\sqrt{2\pi}$, and according to the boundary condition, the wave function becomes $f_1(x) = \bar{t}_{k_1}h_{k_1}(x)$ for $x > 0$. Totally the wave function can be written as

$$f_1(x) = h_{k_1}(x)\left[\theta(-x) + \bar{t}_{k_1}\theta(x)\right], \tag{11}$$

and $\theta(x)$ is the Heaviside step function, which is consistent to the ansatz in Refs. [55,56]. According to the Lippmann–Schwinger formalism,[43,57] the exact scattering eigenstates can be used to construct the scattering matrix. Before and after the scattering, the photons are away from the QE, and the S matrix characterizes the evolution from the "input" (before scattering) to "output" (after scattering) states, which in the bright-state subspace is

$$\hat{S}_{\rm B}^{(1)} = \int {\rm d}k_1 |\phi_{\rm out}^{(1)}\rangle_{\rm BB}\langle\phi_{\rm in}^{(1)}|, \tag{12}$$

where the input and output states are given by $|\phi_{\rm in}^{(1)}\rangle_{\rm B} = |k_1\rangle_{\rm B}$ and $|\phi_{\rm out}^{(1)}\rangle_{\rm B} = \bar{t}_{k_1}|k_1\rangle_{\rm B}$.

Accordingly, the S matrix in the dark-state subspace is just an identity operator since the dark mode is decoupled from the interaction, which is

$$\hat{S}_{\rm D}^{(1)} = \int {\rm d}k_1 |\phi_{\rm in}^{(1)}\rangle_{\rm DD}\langle\phi_{\rm in}^{(1)}|, \tag{13}$$

with the input state $|\phi_{\rm in}^{(1)}\rangle_{\rm D} = |k_1\rangle_{\rm D} = \int {\rm d}x h_{k_1}(x)\hat{a}_{\rm D}(x)|0\rangle$. Therefore, the S matrices in the bright- and dark-state subspaces become

$$\begin{aligned}\hat{S}_{\rm B}^{(1)} &= \int {\rm d}k_1 \bar{t}_{k_1}|k_1\rangle_{\rm BB}\langle k_1|, \\ \hat{S}_{\rm D}^{(1)} &= \int {\rm d}k_1 |k_1\rangle_{\rm DD}\langle k_1|,\end{aligned} \tag{14}$$

and the full S matrix for one-excitation state is defined as

$$\hat{S}^{(1)} = \hat{S}_{\rm B}^{(1)} \otimes \hat{S}_{\rm D}^{(0)} + \hat{S}_{\rm B}^{(0)} \otimes \hat{S}_{\rm D}^{(1)}, \tag{15}$$

with $\hat{S}_{\rm B}^{(0)} = \mathbf{1}$ and $\hat{S}_{\rm D}^{(0)} = \mathbf{1}$, from which the transmission properties of an arbitrary one-excitation incident state can be calculated.

To evaluate the nonreciprocal transmission, we consider the inputs of single-photon Fock states in the right- and left-propagating directions respectively, which are described by the wave packet[58]

$$\begin{aligned}|1_\alpha\rangle_j &= \int_{-\infty}^{\infty} {\rm d}k\alpha(k)\hat{a}_j^{\dagger}(k)|0\rangle \\ &= \int_{-\infty}^{\infty} {\rm d}k\alpha(k)|k\rangle_j, \quad (j = {\rm R},{\rm L})\end{aligned} \tag{16}$$

in the momentum space. We take into account of Gaussian-type wave packet with the spectral amplitude

$$\alpha(k) = \left(2\pi\Delta^2\right)^{-1/4}\exp\left[-\frac{(k-k_0)^2}{4\Delta^2}\right], \tag{17}$$

and take $k_0 = \varepsilon$ for the central frequency of wave packet on resonance with the QE, which means the strongest interaction between the photon and QE. Moreover, we take the central momentum k_0 much larger than the bandwidth of wave packet Δ, so that the narrow-band wave packet is satisfied. Via employing the S matrix, the right- and left-propagating output states are

$$|\mathrm{out}_\alpha^{(1)}\rangle_j = \int_{-\infty}^{\infty} \mathrm{d}k\alpha(k)\hat{S}^{(1)}|k\rangle_j. \tag{18}$$

To evaluate the transmission amplitudes for right- and left-propagating input states, we should use the relations $|k\rangle_\mathrm{R} = (g_\mathrm{R}|k\rangle_\mathrm{B} + g_\mathrm{L}|k\rangle_\mathrm{D})/g_\mathrm{B}$ and $|k\rangle_\mathrm{L} = (g_\mathrm{L}|k\rangle_\mathrm{B} - g_\mathrm{R}|k\rangle_\mathrm{D})/g_\mathrm{B}$ in momentum space. The transmission probabilities can be calculated, respectively, as

$$\begin{aligned} P_\mathrm{R}^{(1)} &= \int \mathrm{d}k|_\mathrm{R}\langle k|\mathrm{out}_\alpha^{(1)}\rangle_\mathrm{R}|^2 = \int \mathrm{d}k\alpha(k)^2|t_{k,\mathrm{R}}|^2, \\ P_\mathrm{L}^{(1)} &= \int \mathrm{d}k|_\mathrm{L}\langle k|\mathrm{out}_\alpha^{(1)}\rangle_\mathrm{L}|^2 = \int \mathrm{d}k\alpha(k)^2|t_{k,\mathrm{L}}|^2, \end{aligned} \tag{19}$$

where the amplitudes are

$$\begin{aligned} t_{k,\mathrm{R}} &= {}_\mathrm{R}\langle k|\hat{S}^{(1)}|k\rangle_\mathrm{R} = (\Gamma_\mathrm{R}\bar{t}_k + \Gamma_\mathrm{L})/\Gamma_\mathrm{B}, \\ t_{k,\mathrm{L}} &= {}_\mathrm{L}\langle k|\hat{S}^{(1)}|k\rangle_\mathrm{L} = (\Gamma_\mathrm{L}\bar{t}_k + \Gamma_\mathrm{R})/\Gamma_\mathrm{B}, \end{aligned} \tag{20}$$

with $\Gamma_j = g_j^2$. In Fig. 2 we present the nonreciprocal transmissions of right- and left-propagating single-photon Fock states as functions of Γ_R under the condition of chiral coupling $\Gamma_\mathrm{L} = 0.1\Gamma_\mathrm{R}$ and atomic decay rate $\gamma = 0.2$. The nonreciprocal behavior is most obvious at $\Gamma_\mathrm{R} = 0.2$, where the transmission probability of right-propagating input state is much smaller than that of left-propagating input state. The nonreciprocal single-photon transmission can be explained from the perspective of mode conversion, where the photonic mode is absorbed by the QE and then re-emitted into the waveguide or the environment.[59] The transmission amplitudes of right- and left-propagating input states are

$$t_{k,\mathrm{R}} = \left[k + \mathrm{i}\left(\frac{\gamma}{2} + \frac{\Gamma_\mathrm{L}}{2} - \frac{\Gamma_\mathrm{R}}{2}\right)\right] \Big/ \left[k + \mathrm{i}\left(\frac{\gamma}{2} + \frac{\Gamma_\mathrm{L}}{2} + \frac{\Gamma_\mathrm{R}}{2}\right)\right],$$

$$t_{k,\mathrm{L}} = \left[k + \mathrm{i}\left(\frac{\gamma}{2} + \frac{\Gamma_\mathrm{R}}{2} - \frac{\Gamma_\mathrm{L}}{2}\right)\right] \Big/ \left[k + \mathrm{i}\left(\frac{\gamma}{2} + \frac{\Gamma_\mathrm{L}}{2} + \frac{\Gamma_\mathrm{R}}{2}\right)\right],$$

respectively, which are equal by exchanging Γ_R and Γ_L. If the emitter's decay to the environment is neglected, i.e., $\gamma = 0$, the single-photon transmission probabilities $|t_{k,\mathrm{R}}|^2$ and $|t_{k,\mathrm{L}}|^2$ are equal, although $t_{k,\mathrm{R}} \neq t_{k,\mathrm{L}}$. The nonreciprocal single-photon transmission cannot be achieved. When the atomic decay rate is taken into account, $|t_{k,\mathrm{R}}|^2$ and $|t_{k,\mathrm{L}}|^2$ become different due to the chiral coupling. Especially when $\gamma = \Gamma_\mathrm{R} - \Gamma_\mathrm{L}$ and $k = 0$, we have $t_{k,\mathrm{R}} = 0$ while $t_{k,\mathrm{L}} = \frac{\Gamma_\mathrm{R} - \Gamma_\mathrm{L}}{\Gamma_\mathrm{R}}$. Under the condition, the resonant single photon injected from the left port of the waveguide cannot reach the right port due to the critical coupling, which completely decays into the environment. However, the single photon injected from the right port will reach the left port with a near unity probability with $\Gamma_\mathrm{L} \ll \Gamma_\mathrm{R}$. Therefore, both the chiral coupling and the emitter's decay to the environment are necessary to realize the nonreciprocal transmission. However, in the strong coupling regime, i.e., $\Gamma_\mathrm{R} \gg \gamma$, the transmission probabilities become almost equal, i.e., $|t_{k,\mathrm{R}}|^2 \approx |t_{k,\mathrm{L}}|^2$. The nonreciprocity vanishes due to the negligible effect of atomic dissipation, which is shown by the inset in Fig. 2.

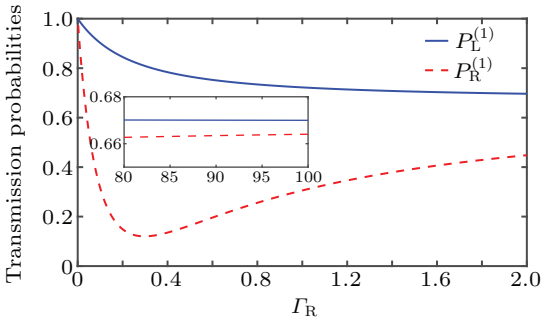


Fig. 2. Nonreciprocal single-photon transmission probabilities $P_\mathrm{R}^{(1)}$ and $P_\mathrm{L}^{(1)}$ for the right- and left-propagating input Fock states as functions of Γ_R. Incident field is on resonance with the QE ($k_0 = \varepsilon$), the atomic dissipation is $\gamma = 0.2$, chiral interactions fulfill $\Gamma_\mathrm{L} = 0.1\Gamma_\mathrm{R}$, and the spectral width is $\Delta = 0.1$.

4. Nonreciprocal transmissions of two-photon Fock state

Further, we consider the two-photon scattering processes, where the eigenstate within the two-excitation bright state subspace is given by

$$\begin{aligned} |\psi_2\rangle &= \int \mathrm{d}x_1 \int \mathrm{d}x_2 f_2(x_1,x_2)\hat{a}_\mathrm{B}^\dagger(x_1)\hat{a}_\mathrm{B}^\dagger(x_2)|0,g\rangle \\ &\quad + \int \mathrm{d}x_1 e_2(x_1)\hat{a}_\mathrm{B}^\dagger(x_1)|0,e\rangle. \end{aligned} \tag{21}$$

The function $f_2(x_1,x_2)$ is symmetric due to the bosonic statistics and satisfies $f_2(x_1,x_2) = f_2(x_2,x_1)$, and $e_2(x)$ is the probability amplitude distribution of one photon when the QE is in the excited state. From the eigenvalue equation $\hat{H}_\mathrm{B}|\psi_2\rangle = E_2|\psi_2\rangle$ with $E_2 = k_1 + k_2$, we can obtain the equations

$$\begin{aligned} &[-\mathrm{i}(\partial_{x_1} + \partial_{x_2}) - (k_1 + k_2)] f_2(x_1,x_2) \\ &\quad + \frac{g_\mathrm{B}}{2}[\delta(x_1)e_2(x_2) + \delta(x_2)e_2(x_1)] = 0, \\ &\left[\varepsilon - \mathrm{i}\frac{\gamma}{2} - \mathrm{i}\partial_x - (k_1 + k_2)\right] e_2(x) \\ &\quad + g_\mathrm{B}[f_2(0,x) + f_2(x,0)] = 0, \end{aligned} \tag{22}$$

where $f_2(0,x) \equiv 1/2[f_2(0^+,x) + f_2(0^-,x)]$, $f_2(x,0) \equiv 1/2[f_2(x,0^+) + f_2(x,0^-)]$, and $f_2(0,x) = f_2(x,0)$ for the symmetric statistics. The function $f_2(x_1,x_2)$ is piecewise continuous for the existence of $\delta(x)$, while $e_2(x)$ is continuous. The light–QE interaction occurs at the coordinate axes $x_1 = 0$ and $x_2 = 0$ in the real space, and for the region of $x_1 \neq 0$ and $x_2 \neq 0$ in the absence of interactions, the equation of motion becomes

$$[-\mathrm{i}(\partial_{x_1} + \partial_{x_2}) - (k_1 + k_2)] f_2(x_1,x_2) = 0. \tag{23}$$

The continuity and self-consistency conditions are necessary to determine $f_2(x_1,x_2)$. According to the bosonic symmetry, we can solve the function $f_2(x_1,x_2)$ by first considering the half-space $x_2 \geq x_1$, and $f_2(x_1,x_2)$ in $x_2 \leq x_1$ region is achievable from $f_2(x_1,x_2)$ in $x_2 \geq x_1$ region by the symmetrization. In this case, the $x_2 > x_1$ region is dissected by x_1 and x_2 axes into three subregions: (I) $0 > x_2 > x_1$; (II) $x_2 > 0 > x_1$; (III) $x_2 > x_1 > 0$, as shown in Fig. 3. In subregion I, it corresponds to an incident plane-wave function before interactions which is described by $f_2^{(\mathrm{I})}(x_1,x_2) = \frac{1}{2}\left[h_{k_1}(x_1)h_{k_2}(x_2)+h_{k_1}(x_2)h_{k_2}(x_1)\right]$. Without loss of generality, we suppose $k_1 \leq k_2$ in the following, and the integration fulfills the relation[43]

$$\int_{-\infty}^{\infty} \mathrm{d}k_2 \int_{-\infty}^{k_2} \mathrm{d}k_1 = \frac{1}{2}\int_{-\infty}^{\infty} \mathrm{d}k_2 \int_{-\infty}^{\infty} \mathrm{d}k_1. \tag{24}$$

Through the integration of Eq. (22) around the boundary between subregions I and II ($x_2 = 0$), it gives the following equations:

$$\begin{aligned} &-\mathrm{i}\left[f_2^{(\mathrm{II})}(x_1,0^+) - f_2^{(\mathrm{I})}(x_1,0^-)\right] + \frac{g_\mathrm{B}}{2}e_2(x_1) = 0, \\ &\left(\varepsilon - \mathrm{i}\frac{\gamma}{2} - \mathrm{i}\partial_{x_1} - k_1 - k_2\right)e_2(x_1) \\ &\quad + g_\mathrm{B}\left[f_2^{(\mathrm{II})}(x_1,0^+) + f_2^{(\mathrm{I})}(x_1,0^-)\right] = 0. \end{aligned} \tag{25}$$

Then via eliminating $e_2(x_1)$ we have the continuity condition at $x_2 = 0$,

$$\begin{aligned} &\left(\varepsilon - \mathrm{i}\frac{\gamma}{2} - \mathrm{i}\partial_{x_1} - k_1 - k_2 - \mathrm{i}\frac{\Gamma_\mathrm{B}}{2}\right) f_2^{(\mathrm{II})}(x_1,0^+) \\ &= \left(\varepsilon - \mathrm{i}\frac{\gamma}{2} - \mathrm{i}\partial_{x_1} - k_1 - k_2 + \mathrm{i}\frac{\Gamma_\mathrm{B}}{2}\right) f_2^{(\mathrm{I})}(x_1,0^-). \end{aligned} \tag{26}$$

Because of $f_2^{(\mathrm{I})}(x_1,0^-) = \frac{1}{4\pi}\left(\mathrm{e}^{\mathrm{i}k_1x_1} + \mathrm{e}^{\mathrm{i}k_2x_1}\right)$ on the boundary of subregion I, and then on the boundary of subregion II

$$\begin{aligned} f_2^{(\mathrm{II})}(x_1,0^+) &= \frac{1}{4\pi}\left(\bar{t}_{k_1}\mathrm{e}^{\mathrm{i}k_2x_1} + \bar{t}_{k_2}\mathrm{e}^{\mathrm{i}k_1x_1}\right) \\ &+ A\mathrm{e}^{\left[-\frac{\gamma}{2}-\frac{\Gamma_\mathrm{B}}{2}-\mathrm{i}\varepsilon+i(k_1+k_2)\right]x_1}, \end{aligned} \tag{27}$$

where

$$\bar{t}_{k_1} = \frac{k_1 - \varepsilon + \mathrm{i}\frac{\gamma}{2} - \mathrm{i}\frac{\Gamma_\mathrm{B}}{2}}{k_1 - \varepsilon + \mathrm{i}\frac{\gamma}{2} + \mathrm{i}\frac{\Gamma_\mathrm{B}}{2}}, \quad \bar{t}_{k_2} = \frac{k_2 - \varepsilon + \mathrm{i}\frac{\gamma}{2} - \mathrm{i}\frac{\Gamma_\mathrm{B}}{2}}{k_2 - \varepsilon + \mathrm{i}\frac{\gamma}{2} + \mathrm{i}\frac{\Gamma_\mathrm{B}}{2}},$$

and A is an undetermined coefficient. Using the constraint equation in subregion II, i.e.,

$$\left[-\mathrm{i}\left(\partial_{x_1} + \partial_{x_2}\right) - (k_1 + k_2)\right] f_2^{(\mathrm{II})}(x_1,x_2) = 0, \tag{28}$$

we can obtain

$$\begin{aligned} f_2^{(\mathrm{II})}(x_1,x_2) &= \frac{1}{4\pi}\left[\bar{t}_{k_1}\mathrm{e}^{\mathrm{i}(k_2x_1+k_1x_2)} + \bar{t}_{k_2}\mathrm{e}^{\mathrm{i}(k_1x_1+k_2x_2)}\right] \\ &+ A\mathrm{e}^{\left(\frac{\gamma}{2}+\frac{\Gamma_\mathrm{B}}{2}+\mathrm{i}\varepsilon\right)(x_2-x_1)}\mathrm{e}^{\mathrm{i}(k_1+k_2)x_1}. \end{aligned} \tag{29}$$

Here A should be equal to zero due to the divergence of the term $\mathrm{e}^{\left(\frac{\gamma}{2}+\frac{\Gamma_\mathrm{B}}{2}\right)(x_2-x_1)}$ when $x_2 - x_1 \to \infty$. Hence, $f_2(x_1,x_2)$ in subregion II is given by

$$f_2^{(\mathrm{II})}(x_1,x_2) = \frac{1}{4\pi}\left[\bar{t}_{k_1}\mathrm{e}^{\mathrm{i}(k_2x_1+k_1x_2)} + \bar{t}_{k_2}\mathrm{e}^{\mathrm{i}(k_1x_1+k_2x_2)}\right]. \tag{30}$$

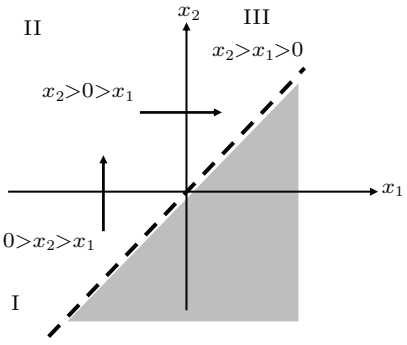


Fig. 3. The $x_2 > x_1$ region is dissected into three subregions: (I) $0 > x_2 > x_1$, (II) $x_2 > 0 > x_1$, (III) $x_2 > x_1 > 0$, due to interactions at the coordinate axes $x_1 = 0$ and $x_2 = 0$.

Following the same procedure through integrating around the boundary of subregions II and III ($x_1 = 0$), we can obtain

$$\begin{aligned} &-\mathrm{i}\left[f_2^{(\mathrm{III})}(0^+,x_2) - f_2^{(\mathrm{II})}(0^-,x_2)\right] + \frac{g_\mathrm{B}}{2}e_2(x_2) = 0, \\ &\left(\varepsilon - \mathrm{i}\frac{\gamma}{2} - \mathrm{i}\partial_{x_2} - k_1 - k_2\right)e_2(x_2) \\ &\quad + g_\mathrm{B}\left[f_2^{(\mathrm{III})}(0^+,x_2) + f_2^{(\mathrm{II})}(0^-,x_2)\right] = 0. \end{aligned} \tag{31}$$

Also, through eliminating $e_2(x_2)$ the continuity condition at $x_1 = 0$ becomes

$$\begin{aligned} &\left(\varepsilon - \mathrm{i}\frac{\gamma}{2} - \mathrm{i}\partial_{x_2} - k_1 - k_2 - \mathrm{i}\frac{\Gamma_\mathrm{B}}{2}\right) f_2^{(\mathrm{III})}(0^+,x_2) \\ &= \left(\varepsilon - \mathrm{i}\frac{\gamma}{2} - \mathrm{i}\partial_{x_2} - k_1 - k_2 + \mathrm{i}\frac{\Gamma_\mathrm{B}}{2}\right) f_2^{(\mathrm{II})}(0^-,x_2). \end{aligned} \tag{32}$$

Using the condition $f_2^{(\mathrm{II})}(0^-,x_2) = \frac{1}{4\pi}\left(\bar{t}_{k_1}\mathrm{e}^{\mathrm{i}k_1x_2} + \bar{t}_{k_2}\mathrm{e}^{\mathrm{i}k_2x_2}\right)$, we have

$$\begin{aligned} f_2^{(\mathrm{III})}(0^+,x_2) &= \frac{1}{4\pi}\bar{t}_{k_1}\bar{t}_{k_2}\left(\mathrm{e}^{\mathrm{i}k_2x_2} + \mathrm{e}^{\mathrm{i}k_1x_2}\right) \\ &+ B\mathrm{e}^{\left[-\frac{\gamma}{2}-\frac{\Gamma_\mathrm{B}}{2}-\mathrm{i}\varepsilon+\mathrm{i}(k_1+k_2)\right]x_2}, \end{aligned} \tag{33}$$

and B is an undetermined coefficient. Also, with the constraint equation $\left[-\mathrm{i}\left(\partial_{x_1} + \partial_{x_2}\right) - (k_1 + k_2)\right] f_2^{(\mathrm{III})}(x_1,x_2) = 0$ in the subregion III, we have

$$\begin{aligned} f_2^{(\mathrm{III})}(x_1,x_2) &= \frac{\bar{t}_{k_1}\bar{t}_{k_2}}{4\pi}\left(\mathrm{e}^{\mathrm{i}k_2x_2+\mathrm{i}k_1x_1} + \mathrm{e}^{\mathrm{i}k_1x_2+\mathrm{i}k_2x_1}\right) \\ &+ B\mathrm{e}^{\left(\frac{\gamma}{2}+\frac{\Gamma_\mathrm{B}}{2}+\mathrm{i}\varepsilon\right)(x_1-x_2)}\mathrm{e}^{\mathrm{i}(k_1+k_2)x_2}. \end{aligned} \tag{34}$$

To determine the coefficient B, we should follow the continuity condition of $e_2(x)$, i.e., $e_2(0^+) = e_2(0^-)$, and $f_2(x_1,x_2)$ at $x_1 = 0$ and $x_2 = 0$ yields

$$\begin{aligned} &f_2^{(\mathrm{III})}(0^+,0^+) - f_2^{(\mathrm{II})}(0^-,0^+) \\ &= f_2^{(\mathrm{II})}(0^-,0^+) - f_2^{(\mathrm{I})}(0^-,0^-), \end{aligned} \tag{35}$$

from which B can be determined as

$$B = -\frac{(\bar{t}_{k_1}-1)(\bar{t}_{k_2}-1)}{2\pi}. \tag{36}$$

Finally, through symmetrization in the scattering (output) region $x_1 > 0$ and $x_2 > 0$, $f_2(x_1,x_2) = \frac{1}{2}\left[f_2^{(\mathrm{III})}(x_1,x_2) + f_2^{(\mathrm{III})}(x_2,x_1)\right]$ becomes

$$\begin{aligned} f_2(x_1,x_2) &= \frac{1}{2}\bar{t}_{k_1}\bar{t}_{k_2}\left[h_{k_1}(x_1)h_{k_2}(x_2)+h_{k_1}(x_2)h_{k_2}(x_1)\right] \\ &\quad + \sum_Q B^{(2)}_{k_1,k_2}(x_{Q_1},x_{Q_2})\theta(x_{Q_1}). \end{aligned} \tag{37}$$

Here

$$\begin{aligned} & B^{(2)}_{k_1,k_2}(x_{Q_1},x_{Q_2}) \\ &= -\left(\bar{t}_{k_1}-1\right)\left(\bar{t}_{k_2}-1\right)\mathrm{e}^{-\left(\frac{\Gamma_\mathrm{B}}{2}+\frac{\gamma}{2}+\mathrm{i}\varepsilon\right)|x_{Q_2}-x_{Q_1}|} \\ &\quad \times h_{k_1}(x_{Q_2})h_{k_2}(x_{Q_2})\theta(x_{Q_2}-x_{Q_1}), \end{aligned} \tag{38}$$

and $Q = (Q_1,Q_2)$ is permutations of $(1,2)$ required to account for the bosonic symmetry of the wave function. Conceptually, two photons have two ways of going through the QE. One is to pass by the QE independently as plane waves and gain a phase factor, which is indicated by the first term of $f_2(x_1,x_2)$. The other way is to bind together and form a bound state, which is indicated by the second term. The formation of the bound state can be viewed as a result of stimulated emission: the first photon excites the QE and the passing of the second photon stimulates emission of the first photon into the same state, hence producing the bound state. The two-photon bound state must be included to guarantee the completeness of the basis, as previous discussions by Shen and Fan.[42,43]

The two-photon S matrix connects the freely incoming and outgoing photonic states (away from the QE), which is defined as

$$\hat{S}^{(2)}_\mathrm{B} = \int \mathrm{d}k_1 \int \mathrm{d}k_2 |\phi^{(2)}_\mathrm{out}(k_1,k_2)\rangle_{\mathrm{BB}}\langle\phi^{(2)}_\mathrm{in}(k_1,k_2)| \tag{39}$$

in the bright-state subspace. The incoming two-photon state in the real space is

$$\begin{aligned} & |\phi^{(2)}_\mathrm{in}(k_1,k_2)\rangle_B \\ &= \frac{1}{2}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \left[h_{k_1}(x_1)h_{k_2}(x_2)+h_{k_1}(x_2)h_{k_2}(x_1)\right] \\ &\quad \times \hat{a}^\dagger_\mathrm{B}(x_1)\hat{a}^\dagger_\mathrm{B}(x_2)|0\rangle, \end{aligned} \tag{40}$$

and the outgoing state is

$$|\phi^{(2)}_\mathrm{out}(k_1,k_2)\rangle_\mathrm{B} = \int \mathrm{d}x_1 \int \mathrm{d}x_2 f_2(x_1,x_2)\hat{a}^\dagger_\mathrm{B}(x_1)\hat{a}^\dagger_\mathrm{B}(x_2)|0\rangle. \tag{41}$$

Due to the absence of light–QE interactions in the dark-state subspace, the corresponding two-photon S matrix is

$$\hat{S}^{(2)}_\mathrm{D} = \int \mathrm{d}k_1 \int \mathrm{d}k_2 |\phi^{(2)}_\mathrm{in}(k_1,k_2)\rangle_{\mathrm{DD}}\langle\phi^{(2)}_\mathrm{in}(k_1,k_2)| \tag{42}$$

with the incoming dark state $|\phi^{(2)}_\mathrm{in}(k_1,k_2)\rangle_\mathrm{D} = \frac{1}{2}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \left[h_{k_1}(x_1)h_{k_2}(x_2)+h_{k_1}(x_2)h_{k_2}(x_1)\right]\hat{a}^\dagger_\mathrm{D}(x_1)\hat{a}^\dagger_\mathrm{D}(x_2)|0\rangle$ in the real space. Totally, the two-photon scattering matrix $\hat{S}^{(2)}$ can be constructed in the form

$$\hat{S}^{(2)} = \hat{S}^{(2)}_\mathrm{B}\otimes\hat{S}^{(0)}_\mathrm{D} + \hat{S}^{(1)}_\mathrm{B}\otimes\hat{S}^{(1)}_\mathrm{D} + \hat{S}^{(0)}_\mathrm{B}\otimes\hat{S}^{(2)}_\mathrm{D}. \tag{43}$$

To investigate the nonreciprocal transmission properties of incident photons, we first consider the right-propagating two-photon input state

$$\begin{aligned} & |\phi^{(2)}_\mathrm{in}(k_1,k_2)\rangle_\mathrm{R} \\ &= \frac{1}{2}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \left[h_{k_1}(x_1)h_{k_2}(x_2)+h_{k_1}(x_2)h_{k_2}(x_1)\right] \\ &\quad \times \hat{a}^\dagger_\mathrm{R}(x_1)\hat{a}^\dagger_\mathrm{R}(x_2)|0\rangle, \end{aligned} \tag{44}$$

and the output state becomes

$$|\phi^{(2)}_\mathrm{out}(k_1,k_2)\rangle_\mathrm{R} = \hat{S}^{(2)}|\phi^{(2)}_\mathrm{in}(k_1,k_2)\rangle_\mathrm{R}. \tag{45}$$

With the use of relations between the right-propagating mode and the bright and dark modes, i.e., $|k\rangle_\mathrm{R} = \left(g_\mathrm{R}|k\rangle_\mathrm{B} + g_\mathrm{L}|k\rangle_\mathrm{D}\right)/g_\mathrm{B}$, the output state is expressed in the form

$$\begin{aligned} |\phi^{(2)}_\mathrm{out}(k_1,k_2)\rangle_\mathrm{R} &= \frac{1}{2}\int \mathrm{d}x_1 \int \mathrm{d}x_2 \Big[C_{\mathrm{RR}}(x_1,x_2)\hat{a}^\dagger_\mathrm{R}(x_1)\hat{a}^\dagger_\mathrm{R}(x_2) \\ &\quad + C_{\mathrm{LL}}(x_1,x_2)\hat{a}^\dagger_\mathrm{L}(x_1)\hat{a}^\dagger_\mathrm{L}(x_2) \\ &\quad + C_{\mathrm{RL}}(x_1,x_2)\hat{a}^\dagger_\mathrm{R}(x_1)\hat{a}^\dagger_\mathrm{L}(x_2) \\ &\quad + C_{\mathrm{LR}}(x_1,x_2)\hat{a}^\dagger_\mathrm{L}(x_1)\hat{a}^\dagger_\mathrm{R}(x_2)\Big]|0\rangle, \end{aligned} \tag{46}$$

with the coefficients

$$\begin{aligned} C_{\mathrm{RR}}(x_1,x_2) &= t_{k_1,R}t_{k_2,R}\left[h_{k_1}(x_1)h_{k_2}(x_2)+h_{k_2}(x_1)h_{k_1}(x_2)\right] \\ &\quad + 2\frac{\Gamma_\mathrm{R}^2}{\Gamma_\mathrm{B}^2}\left[B^{(2)}_{k_1,k_2}(x_1,x_2)+B^{(2)}_{k_1,k_2}(x_2,x_1)\right], \\ C_{\mathrm{LL}}(x_1,x_2) &= r_{k_1,R}r_{k_2,R}\left[h_{k_1}(x_1)h_{k_2}(x_2)+h_{k_2}(x_1)h_{k_1}(x_2)\right] \\ &\quad + 2\frac{\Gamma_\mathrm{R}\Gamma_\mathrm{L}}{\Gamma_\mathrm{B}^2}\left[B^{(2)}_{k_1,k_2}(x_1,x_2)+B^{(2)}_{k_1,k_2}(x_2,x_1)\right], \\ C_{\mathrm{RL}}(x_1,x_2) &= t_{k_1,R}r_{k_2,R}h_{k_1}(x_1)h_{k_2}(x_2) + r_{k_1,R}t_{k_2,R}h_{k_2}(x_1)h_{k_1}(x_2) \\ &\quad + 2\frac{\sqrt{\Gamma_\mathrm{R}^3\Gamma_\mathrm{L}}}{\Gamma_\mathrm{B}^2}\left[B^{(2)}_{k_1,k_2}(x_1,x_2)+B^{(2)}_{k_1,k_2}(x_2,x_1)\right], \\ C_{\mathrm{LR}}(x_1,x_2) &= r_{k_1,R}t_{k_2,R}h_{k_1}(x_1)h_{k_2}(x_2) + t_{k_1,R}r_{k_2,R}h_{k_2}(x_1)h_{k_1}(x_2) \\ &\quad + 2\frac{\sqrt{\Gamma_\mathrm{R}^3\Gamma_\mathrm{L}}}{\Gamma_\mathrm{B}^2}\left[B^{(2)}_{k_1,k_2}(x_1,x_2)+B^{(2)}_{k_1,k_2}(x_2,x_1)\right], \end{aligned} \tag{47}$$

where $t_{k_i,R} = \left(\Gamma_\mathrm{R}\bar{t}_{k_i}+\Gamma_\mathrm{L}\right)/\Gamma_\mathrm{B}$ and $r_{k_i,R} = \sqrt{\Gamma_\mathrm{R}\Gamma_\mathrm{L}}\left(\bar{t}_{k_i}-1\right)/\Gamma_\mathrm{B}$ $(i=1,2)$.

When the input state is a two-photon Fock state, it is the direct generalization of the single-photon wave packet in the momentum space

$$\begin{aligned} |2_\alpha\rangle &= \frac{1}{\sqrt{2}}\left[\int_{-\infty}^{\infty}\mathrm{d}k\alpha(k)\hat{a}^\dagger_\mathrm{R}(k)\right]^2|0\rangle \\ &= \int_{-\infty}^{\infty}\mathrm{d}k_1\int_{-\infty}^{k_1}\mathrm{d}k_2\alpha(k_1)\alpha(k_2)|\phi^{(2)}_\mathrm{in}(k_1,k_2)\rangle_\mathrm{R}, \end{aligned} \tag{48}$$

with the normalization condition $\int_{-\infty}^{\infty}|\alpha(k)|^2\mathrm{d}k=1$.[60] For convenience, we choose a Gaussian-type wave packet as the single-photon Fock state in Eq. (17). After scattering off the QE, the output state becomes

$$|\mathrm{out}_\alpha^{(2)}\rangle_\mathrm{R}=\frac{1}{2}\int_{-\infty}^{\infty}\mathrm{d}k_1\int_{-\infty}^{\infty}\mathrm{d}k_2\alpha(k_1)\alpha(k_2)|\phi_\mathrm{out}^{(2)}(k_1,k_2)\rangle_\mathrm{R}. \quad (49)$$

Following the general procedure, the two-photon transmission probability of right-propagating input state is

$$P_\mathrm{RR}^{(2)}=\int_{-\infty}^{\infty}\mathrm{d}k_1\int_{-\infty}^{\infty}\mathrm{d}k_2|_\mathrm{R}\langle\phi_\mathrm{in}^{(2)}(k_1,k_2)|\mathrm{out}_\alpha^{(2)}\rangle_\mathrm{R}|^2, \quad (50)$$

which is explicitly expressed in the form

$$\begin{aligned}
P_\mathrm{RR}^{(2)}&=\int\mathrm{d}k_1\int\mathrm{d}k_2|t_\mathrm{R}(k_1,k_2)+\frac{\Gamma_\mathrm{R}}{\Gamma_\mathrm{L}}B_\mathrm{R}(k_1,k_2)|^2=\left(P_\mathrm{RR}^{(2)}\right)_\mathrm{PW}+\left(P_\mathrm{RR}^{(2)}\right)_\mathrm{BS},\\
\left(P_\mathrm{RR}^{(2)}\right)_\mathrm{PW}&=\int\mathrm{d}k_1\int\mathrm{d}k_2|t_\mathrm{R}(k_1,k_2)|^2,\\
\left(P_\mathrm{RR}^{(2)}\right)_\mathrm{BS}&=\int\mathrm{d}k_1\int\mathrm{d}k_2\left[\frac{\Gamma_\mathrm{R}}{\Gamma_\mathrm{L}}(t_\mathrm{R}(k_1,k_2)B_\mathrm{R}^*(k_1,k_2)+t_\mathrm{R}^*(k_1,k_2)B_\mathrm{R}(k_1,k_2))+\frac{\Gamma_\mathrm{R}^2}{\Gamma_\mathrm{L}^2}|B_\mathrm{R}(k_1,k_2)|^2\right],\\
t_\mathrm{R}(k_1,k_2)&=\alpha(k_1)\alpha(k_2)t_{k_1,R}t_{k_2,R},\\
B_\mathrm{R}(k_1,k_2)&=\left[\frac{-\mathrm{i}/2\pi}{k_1+\mathrm{i}\frac{\gamma}{2}+\mathrm{i}\frac{\Gamma_\mathrm{B}}{2}}+\frac{-\mathrm{i}/2\pi}{k_2+\mathrm{i}\frac{\gamma}{2}+\mathrm{i}\frac{\Gamma_\mathrm{B}}{2}}\right]\int\mathrm{d}k'\alpha(k')\alpha(k_1+k_2-k')r_{k',R}r_{k_1+k_2-k',R}.
\end{aligned} \quad (51)$$

The probability has two contributions: plane-wave term $\left(P_\mathrm{RR}^{(2)}\right)_\mathrm{PW}$, and bound-state term $\left(P_\mathrm{RR}^{(2)}\right)_\mathrm{BS}$. The plane-wave term is the direct transmission of individual incident photons, while the bound-state term indicates strong correlations between two photons.

If we change the direction of input state, the left-propagating two-photon input state is

$$|\phi_\mathrm{in}^{(2)}(k_1,k_2)\rangle_\mathrm{L}=\frac{1}{2}\int\mathrm{d}x_1\int\mathrm{d}x_2\left[h_{k_1}(x_1)h_{k_2}(x_2)+h_{k_1}(x_2)h_{k_2}(x_1)\right]\hat{a}_\mathrm{L}^\dagger(x_1)\hat{a}_\mathrm{L}^\dagger(x_2)|0\rangle.$$

Following the same procedure as above, the transmission probability is

$$\begin{aligned}
P_\mathrm{LL}^{(2)}&=\int\mathrm{d}k_1\int\mathrm{d}k_2|t_\mathrm{L}(k_1,k_2)+\frac{\Gamma_\mathrm{L}}{\Gamma_\mathrm{R}}B_\mathrm{L}(k_1,k_2)|^2=\left(P_\mathrm{LL}^{(2)}\right)_\mathrm{PW}+\left(P_\mathrm{LL}^{(2)}\right)_\mathrm{BS},\\
\left(P_\mathrm{LL}^{(2)}\right)_\mathrm{PW}&=\int\mathrm{d}k_1\int\mathrm{d}k_2|t_\mathrm{L}(k_1,k_2)|^2,\\
\left(P_\mathrm{LL}^{(2)}\right)_\mathrm{BS}&=\int\mathrm{d}k_1\int\mathrm{d}k_2\left[\frac{\Gamma_\mathrm{L}}{\Gamma_\mathrm{R}}(t_\mathrm{L}(k_1,k_2)B_\mathrm{L}^*(k_1,k_2)+t_\mathrm{L}^*(k_1,k_2)B_\mathrm{L}(k_1,k_2))+\frac{\Gamma_\mathrm{L}^2}{\Gamma_\mathrm{R}^2}|B_\mathrm{L}(k_1,k_2)|^2\right],
\end{aligned} \quad (52)$$

where

$$\begin{aligned}
t_\mathrm{L}(k_1,k_2)&=\alpha(k_1)\alpha(k_2)t_{k_1,L}t_{k_2,L},\\
B_\mathrm{L}(k_1,k_2)&=\left[\frac{-\mathrm{i}/2\pi}{k_1+\mathrm{i}\frac{\gamma}{2}+\mathrm{i}\frac{\Gamma_\mathrm{B}}{2}}+\frac{-\mathrm{i}/2\pi}{k_2+\mathrm{i}\frac{\gamma}{2}+\mathrm{i}\frac{\Gamma_\mathrm{B}}{2}}\right]\int\mathrm{d}k'\alpha(k')\alpha(k_1+k_2-k')r_{k',L}r_{k_1+k_2-k',L}
\end{aligned} \quad (53)$$

with $t_{k_i,L}=\left(\Gamma_\mathrm{L}\bar{t}_{k_i}+\Gamma_\mathrm{R}\right)/\Gamma_\mathrm{B}$ and $r_{k_i,L}=\sqrt{\Gamma_\mathrm{R}\Gamma_\mathrm{L}}\left(\bar{t}_{k_i}-1\right)/\Gamma_\mathrm{B}$ $(i=1,2)$.

In Fig. 4 we numerically present the nonreciprocal behavior of two-photon transmission as functions of Γ_R under the conditions of chiral coupling $\Gamma_\mathrm{L}=0.1\Gamma_\mathrm{R}$ and atomic decay rate $\gamma=0.2$. Distinct from the single-photon transmission, two-photon transmission is contributed by two parts: plane-wave term $\left(P_\mathrm{RR}^{(2)}\right)_\mathrm{PW}$ or $\left(P_\mathrm{LL}^{(2)}\right)_\mathrm{PW}$, and bound-state term $\left(P_\mathrm{RR}^{(2)}\right)_\mathrm{BS}$ or $\left(P_\mathrm{LL}^{(2)}\right)_\mathrm{BS}$. The plane-wave term is the direct transmission of individual incident photons, which behaves similarly to single-photon transmission with the significant nonreciprocity at $\Gamma_\mathrm{R}=\gamma$, and in the deep strong coupling limit, contributions to nonreciprocity become to vanish. The bound-state term indicating the strong correlations between two photons also has a substantial effect. For the chiral coupling $\Gamma_\mathrm{L}=0.1\Gamma_\mathrm{R}$, the interaction between left-propagating photons and QE is negligible compared to right-propagating photons. Therefore, $\left(P_\mathrm{LL}^{(2)}\right)_\mathrm{BS}$ is almost zero, and the two-photon transmission probability is determined by the plane-wave term, since the bound effect is hardly produced. For right-propagating transmission, $\left(P_\mathrm{RR}^{(2)}\right)_\mathrm{BS}$ turns to positive rapidly, which means that the presence of the bound state tends to increase two-photon transmission. The minimum value of transmission probability is at $\Gamma_\mathrm{R}=0.1$ as a joint result of plane-wave and bound states, where the nonreciprocal two-photon transmission is most obvious. It is different from the most obvious nonreciprocal be-

havior at $\Gamma_R = 0.2$ in single-photon transmission, which is also consistent to the minimum value of $\left(P_{RR}^{(2)}\right)_{PW}$. In Fig. 4 it can be seen that the bound-state term reaches the maximum at the intermediate coupling, instead at the deep strong coupling. This is because the formation of bound state requires the presence of both photons in the QE.[41] In the weak coupling regime, the interaction is too weak to produce a bound state for two-photon scattering. In the strong-coupling regime, the response of QE to the first photon is on the scale of $1/\Gamma_R$, so it is too fast for the second photon to form the bound state from the view of stimulated emission. Thus, the optimal coupling strength is at the intermediate coupling.

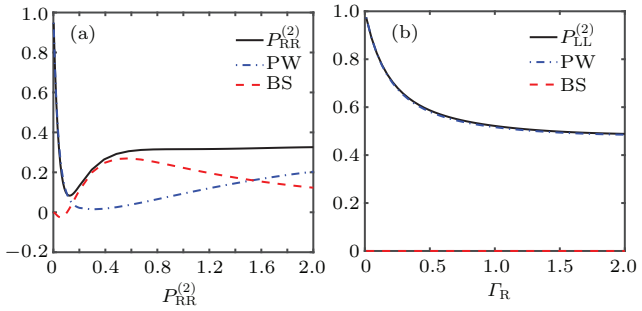


Fig. 4. Two-photon transmission probabilities $P_{RR}^{(2)}$ and $P_{LL}^{(2)}$ of right- and left-propagating input states as functions of Γ_R. (a) Probability for the right-propagating two-photon Fock state. (b) Probability for the left-propagating two-photon Fock state. The label PW refers to the contribution from the plane-wave term, while BS refers to all the other contributions involving bound-state terms. Incident photons are on resonance with the QE ($k_0 = \varepsilon$), the atomic dissipation rate is $\gamma = 0.2$, chiral interactions fulfill $\Gamma_L = 0.1\Gamma_R$, and $\Delta = 0.1$.

5. Nonreciprocal second-order correlations of transmitted photons

Now we turn to the statistical properties of transmitted photons to show the nonreciprocal photon-photon correlations induced by the chiral interactions. Explicitly, we calculate the second-order correlation function for scattering of continuous-mode coherent states, which is defined by[58]

$$|\{\alpha\}\rangle = e^{\hat{a}_\alpha^\dagger - \hat{a}_\alpha}|0\rangle \tag{54}$$

with $\hat{a}_\alpha^\dagger = \int dk\alpha(k)\hat{a}^\dagger(k)$ and the mean photon number $\bar{n} = \int dk|\alpha(k)|^2$. As above we consider a Gaussian-type wave packet

$$\alpha(k) = \frac{\sqrt{\bar{n}}}{(2\pi\Delta^2)^{1/4}} \exp\left[-\frac{(k-k_0)^2}{4\Delta^2}\right] \tag{55}$$

with the width $\Delta = 0.1$ as well as the resonant and narrow-band conditions $k_0 = \varepsilon$ and $k_0 \gg \Delta$. After scattering off the QE, the output state is

$$|\text{out}_\alpha^{(c)}\rangle = \sum_n \hat{S}^{(n)}|\{\alpha\}\rangle, \tag{56}$$

where $\hat{S}^{(n)}$ is the n-excitation scattering matrix.

Particularly, we consider the mean photon number $\bar{n} \leq 1$. In this case, the distribution of $n \geq 3$ number states is much lower than that of $n = 2$ number state. Therefore, it is appropriate to truncate to two-photon scattering of the continuous-mode coherent input state. We study the second-order correlation function of the transmitted field for the right-propagating input state that is defined as

$$g_R^{(2)} = \frac{\langle \text{out}_\alpha^{(c)}|\hat{a}_R^\dagger(x)\hat{a}_R^\dagger(x)\hat{a}_R(x)\hat{a}_R(x)|\text{out}_\alpha^{(c)}\rangle}{\langle \text{out}_\alpha^{(c)}|\hat{a}_R^\dagger(x)\hat{a}_R(x)|\text{out}_\alpha^{(c)}\rangle^2}. \tag{57}$$

With the use of single- and two-photon output states, the correlation is

$$g_R^{(2)} = \frac{\left|\int dk_1 \int dk_2 \alpha(k_1)\alpha(k_2)\left(t_{k_1,R}t_{k_2,R} - \frac{\Gamma_R}{\Gamma_L} r_{k_1,R}r_{k_2,R}\right)\right|^2}{\left|\int dk_1 \int dk_2 \alpha(k_1)\alpha(k_2) t_{k_1,R}t_{k_2,R}\right|^2}, \tag{58}$$

when neglecting the contributions from $n \geq 3$ number states. For the left-propagating input state, we follow the same procedure to calculate the second-order correlation function of transmitted field, which is

$$g_L^{(2)} = \frac{\left|\int dk_1 \int dk_2 \alpha(k_1)\alpha(k_2)\left(t_{k_1,L}t_{k_2,L} - \frac{\Gamma_L}{\Gamma_R} r_{k_1,L}r_{k_2,L}\right)\right|^2}{\left|\int dk_1 \int dk_2 \alpha(k_1)\alpha(k_2) t_{k_1,L}t_{k_2,L}\right|^2}. \tag{59}$$

The second-order correlations $g_R^{(2)}$ and $g_L^{(2)}$ of right- and left-propagating transmitted photons are also contributed by plane-wave and bound-state terms, and the interference between them will give rise to interesting phenomena.[41] In Fig. 5, the nonreciprocal behaviors for two directions with different chiral couplings are numerically presented. For the right-propagating photons, the interaction to QE induces the formation of photonic bound state. In the weak coupling regime, the destructive interference between plane-wave and bound states results in repulsive interaction between photons, and a significant antibunching is shown. With the increase of Γ_R, the constructive interference starts to induce attractive interactions, leading to $g_R^{(2)}$ much greater than unity and the change of statistics from antibunching to bunching. A remarkable bunching is obtained at the intermediate coupling, consistent to the effective formation of bound state at the intermediate coupling. For example, in the case of $\Gamma_L/\Gamma_R = 0.1$, the strongest antibunching is reached at $\Gamma_R = 0.12$ when $g_R^{(2)} \approx 0$, and then it starts to change from antibunching to bunching at $\Gamma_R = 0.14$ and reaches the strongest bunching at $\Gamma_R = 0.34$. For $\Gamma_L/\Gamma_R = 0.5$, the correlation of transmitted photons behaves similarly, with the strongest antibunching at $\Gamma_R = 0.13$ and bunching at $\Gamma_R = 0.54$. For the left-propagating photons in the case of chiral coupling $\Gamma_L/\Gamma_R = 0.1$, the interaction between left-propagating photons and QE is negligible, which fails to

form the bound state. The absence of bound state leads to the coherent transmitted field with $g_{\mathrm{L}}^{(2)}=1$. For $\Gamma_{\mathrm{L}}/\Gamma_{\mathrm{R}}=0.5$, the repulsive interaction between photons reaches strongest at $\Gamma_{\mathrm{R}}=0.54$, leading to the remarkable antibunching. For $\Gamma_{\mathrm{L}}/\Gamma_{\mathrm{R}}=1$, the nonreciprocity vanishes without the chiral couplings.

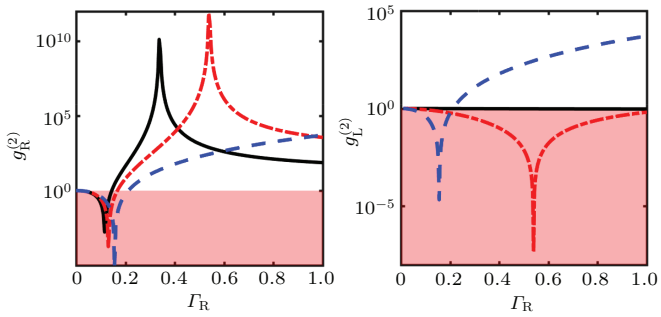


Fig. 5. Second-order correlation functions $g_{\mathrm{R}}^{(2)}$ and $g_{\mathrm{L}}^{(2)}$ of transmitted photons as functions of Γ_{R}. The curves of correlations vary with different $\Gamma_{\mathrm{L}}/\Gamma_{\mathrm{R}}$, where $\Gamma_{\mathrm{L}}/\Gamma_{\mathrm{R}}=0.1$ (black-solid line), $\Gamma_{\mathrm{L}}/\Gamma_{\mathrm{R}}=0.5$ (red dash-dotted line), $\Gamma_{\mathrm{L}}/\Gamma_{\mathrm{R}}=1$ (blue-dashed line). The other parameters are $k_0=\varepsilon$, $\gamma=0.2$, and $\Delta=0.1$.

6. Conclusions

In summary, we have studied the nonreciprocal single- and two-photon transmissions, and second-order correlations in the chiral waveguide QED system. For the single-photon transmission, the nonreciprocity is induced by the effects of chiral coupling and atomic dissipation, which is the most obvious at $\Gamma_{\mathrm{R}}=\gamma$. In the strong coupling regime, the effect of atomic dissipation becomes ignorable, and the nonreciprocity almost vanishes. For the two-photon case, the transmission is contributed by plane-wave and bound-state terms. The plane-wave term means that photons pass by the QE independently as plane waves and gain a phase factor, while bound-state term means that photons bind together and form a bound state. Plane wave behaves similarly to the single-photon transmission. However, due to the different interactions in two directions, left-propagating photons hardly form the bound state, while bound state of right-propagating photons alters the transmission probability further. Moreover, the interference between plane wave and bound state modifies the statistics of transmitted photons. The destructive interference leads to strong antibunching in the weak coupling regime, and constructive interference leads to strong bunching in the intermediate coupling regime which is consistent to the effective formation of bound state. In the left direction, the negligible interaction between photons and QE hardly changes the statistics of the input coherent state.

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